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# Existence theory and approximate solution to prey–predator coupled system involving nonsingular kernel type derivative

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## Abstract

This manuscript considers a nonlinear coupled system under nonsingular kernel type derivative. The considered problem is investigated from two aspects including existence theory and approximate analytical solution. For the concerned qualitative theory, some fixed point results are used. While for approximate solution, the Laplace transform coupled with Adomian method is applied. Finally, by a pertinent example of prey–predator system, we support our results. Some graphical presentations are given using Matlab.

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**Keywords:** Prey–predator system; Existence theory; Laplace transform coupled with Adomian method

## 1 Introduction

In the last few decades significant interest has been shown in fractional calculus by researchers of different disciplines of science and engineering. The concerned area has many applications in modeling various real world problems, since fractional derivative is usually a definite integral including classical derivative as a special case. Also it geometrically produces the whole spectrum or accumulation of a function. The aforesaid derivative has been defined by a number of ways. In other words, various mathematicians have given different definitions of fractional order derivative [1]. The most notable definitions were given by Riemann–Liouville and Caputo, those definitions have been increasingly used in applications in the last decades, for details, see [2–4]. In fact fractional order differential equations (FODEs) have many applications in mathematical modeling of chemical, physical, and biological phenomena. The mentioned differential operator has the ability to describe many features of hereditary and memory materials more explicitly than that of classical order. Therefore significantly FODEs have been used in the last few decades in modeling various processes and phenomena (see for applications [5–16]).

Mathematical models are strong material to investigate many phenomena. For instance, occasionally in the previous time in many localities, unforeseen large potion in fish and

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animal takeover had been reported. This phenomenon gave birth to the well-known prey–predator relationship. Therefore, in 1920, Lotka and Volterra introduced their famous equations known as prey–predator model. Here we state that the concerned model given in (1) deals with the relationship of prey and predator in an ecological system as follows:

$$\begin{cases} \dot{u}(t) = a_1(t)u(t) - b_1(t)v(t)u(t) = \varphi_1(t, u(t), v(t)), \\ \dot{v}(t) = a_2(t)u(t)v(t) - b_2(t)v(t) = \varphi_2(t, u(t), v(t)), \\ u(0) = \alpha, \quad v(0) = \beta, \end{cases} \tag{1}$$

where  $\alpha, \beta \geq 0$ . Further the nonlinear functions  $\varphi_i (i = 1, 2) : \mathcal{J} \times \mathcal{R}^2 \rightarrow \mathcal{R}$  are continuous. Here we remark that  $u(t), v(t)$  represent the prey and the predator populations at time  $t$  respectively. Further  $a_1$  is the growth rate of species  $u$ , while  $b_1$  denotes the impact of predation on  $\dot{u}/u$ . Also  $b_2$  is the death rate of  $v$  and  $a_2$  is the growth rate (or immigration) of the predator population in response to the size of the prey population. The coefficients are linear continuous and bounded functions. So far the concerned model has been studied for various purposes and from various directions; for details, we refer to [17–19]. Also, model (1) has been investigated by using the homotopy perturbation method for ordinary Caputo derivative in [20].

Here, we remark that the definitions of fractional derivative were further extended from singular kernel to nonsingular kernel by Caputo and Fabrizio [21] in 2016. This definition has got much attention in the last few years. Some valuable results were investigated in [22, 23]. Further the aforesaid definition was generalized by replacing exponential kernel with Mittag-Leffler kernel. In this regard lots of research articles have been published, we refer to [24–26].

Motivated by the aforesaid work, in this work we undertake model (1) under CFFD as follows:

$$\begin{cases} {}^{CF}D_t^\omega u(t) = a_1(t)u(t) - b_1(t)u(t)v(t), \\ {}^{CF}D_t^\omega v(t) = a_2(t)u(t)v(t) - b_2(t)v(t), \\ u(0) = \alpha, \quad v(0) = \beta, \end{cases} \tag{2}$$

where  $\alpha, \beta \geq 0$  and  $\omega \in (0, 1]$ .

In the last two decades, to handle FODEs for their exact or numerical solutions, various techniques, methods, and theories were established. Because finding exact analytical solutions for every differential equation of fractional order is quite a difficult job, the well-known techniques including homotopy perturbation method, Adomian decomposition method, and many other numerical methods were utilized for the required results (for details, see [27]). As we know, the mentioned techniques were increasingly adopted for ordinary FODEs, but there are very few articles which study decompositions techniques coupled with integral transform for FODEs under CFFD, see [28–32].

Therefore, here, we construct existence theory of solution to the following semi-analytical results to the coupled system with  $t \in \mathcal{J} = [0, \tau]$  for the given prey–predator system in (2).

First we establish some qualitative results as the existence and uniqueness of the solution corresponding to the model we have considered. Fixed point theory is used to get these

results about solution due to Krasnoselskii and Banach. Also some approximate analytical results are established via the Laplace transform and Adomian decomposition tools. The concerned approximate results are illustrated by graphs via Matlab.

We arrange our paper as follows: In Sect. 1, we give introduction to the problem. In Sect. 2, we recall some preliminaries. In Sect. 3, we establish the existence results of the main work. In Sect. 4, we present general procedure for approximate solution. Finally, in Sect. 5, we give examples and brief conclusion.

### 2 Preliminaries

**Definition 1** ([22, 23]) Let  $\varphi \in \mathcal{H}^1(0, \tau)$ ,  $\tau > 0$ ,  $\omega \in (0, 1)$ , then the CFFD is defined as follows:

$${}^{CF}\mathcal{D}_t^\omega(\varphi(t)) = \frac{\mathcal{M}(\omega)}{1-\omega} \int_0^t \varphi'(t) \exp\left[-\omega \frac{t-\xi}{1-\omega}\right] d\xi,$$

$\mathcal{M}(\omega) = \frac{2\omega}{2-\omega}$  is known as a normalization function and statistics  $\mathcal{M}(1) = \mathcal{M}(0) = 1$ . Moreover, if  $\varphi$  does not fall in  $\mathcal{H}^1(0, \tau)$ , then the derivatives are given by

$${}^{CF}\mathcal{D}_t^\omega(\varphi(t)) = \frac{\mathcal{M}(\omega)}{1-\omega} \int_0^t (\varphi(t) - \varphi(\xi)) \exp\left[-\omega \frac{t-\xi}{1-\omega}\right] d\xi.$$

**Definition 2** ([22]) For  $\varphi \in \mathcal{H}^1(0, \tau)$ ,  $\tau > 0$ , the integral in Caputo–Fabrizio form is given as follows:

$${}^{CF}\mathcal{I}_t^\omega[\varphi(t)] = \frac{(1-\omega)}{\mathcal{M}(\omega)}\varphi(t) + \frac{\omega}{\mathcal{M}(\omega)} \int_0^t \varphi(\xi) d\xi, \quad \omega \in (0, 1].$$

**Definition 3** ([22]) The Laplace transform of  ${}^{CF}\mathcal{D}_t^\omega u(t)$  with  $\mathcal{M}(\omega) = 1$  is given as follows:

$$\mathcal{L}[{}^{CF}\mathcal{D}_t^\omega u(t)] = \frac{s\mathcal{L}[u(t)] - u(0)}{s + \omega(1-s)}, \quad s \geq 0, \omega \in (0, 1].$$

**Note** Corresponding to existence theory, let  $\mathcal{J} = [0, \tau]$  and  $0 \leq t \leq \tau < \infty$ , we define the space as  $\mathbf{Z} = C([0, \tau] \times \mathcal{R}^2, \mathcal{R})$  equipped with the norm  $\|(u, v)\| = \sup_{t \in \mathcal{J}} [|u(t)| + |v(t)|]$ .

**Theorem 1** ([33]) *Let  $\mathbb{B}$  be a convex subset of  $\mathbf{Z}$ , and we have two operators  $\mathbb{F}, \mathbb{G}$  with*

1.  $\mathbb{F}w + \mathbb{G}w \in \mathbb{B}$  for every  $w \in \mathbb{B}$ ;
2.  $\mathbb{F}$  is a contraction;
3.  $\mathbb{G}$  is continuous and compact.

*Then the operator equation  $\mathbb{F}w + \mathbb{G}w = w$  has at least one solution.*

### 3 Existence and uniqueness results of fractional order predator–prey equations

In this part some results about existence and uniqueness are given about the solution of the proposed model (2). Upon using integral operator  ${}^{CF}\mathcal{I}_t^\omega$  on both sides of (2) and putting the initial conditions, one has

$$\begin{cases} u(t) = \alpha + \frac{(1-\omega)}{\mathcal{M}(\omega)}\varphi_1(t, u, v) + \frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)} \int_0^t (t-\xi)^{\omega-1} \varphi_1(\xi, u(\xi), v(\xi)) d\xi, \\ \beta(t) = \beta + \frac{(1-\omega)}{\mathcal{M}(\omega)}\varphi_2(t, u, v) + \frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)} \int_0^t (t-\xi)^{\omega-1} \varphi_2(\xi, u(\xi), v(\xi)) d\xi, \end{cases} \tag{3}$$

which further may be written as

$$\mathbf{W}(t) = \mathbf{W}_0 + \Psi(t, \mathbf{W}(t)) \frac{(1-\omega)}{\mathcal{M}(\omega)} + \frac{\omega}{\mathcal{M}(\omega)} \int_0^t \Psi(\xi, \mathbf{W}(\xi)) d\xi, \tag{4}$$

where

$$\mathbf{W}(t) = \begin{cases} u(t), \\ v(t), \end{cases} \quad \mathbf{W}_0 = \begin{cases} \alpha, \\ \beta, \end{cases} \quad \Psi(t, \mathbf{W}(t)) = \begin{cases} \varphi_1(t, u, v), \\ \varphi_2(t, u, v). \end{cases} \tag{5}$$

Now, to derive our results, we define the following assumptions:

(A1) There exist constants  $L_\Psi > 0$  such that, for each  $\mathbf{W}, \bar{\mathbf{W}} \in \mathbf{Z}$ ,

$$|\Psi(t, \mathbf{W}(t)) - \Psi(t, \bar{\mathbf{W}}(t))| \leq L_\Psi [|\mathbf{W} - \bar{\mathbf{W}}|];$$

(A2) There exist constants  $C_\Psi, M_\Psi > 0$  and  $\mathbf{M}_\Psi > 0$  such that

$$|\Psi(t, \mathbf{W}(t))| \leq C_\Psi |\mathbf{W}| + M_\Psi.$$

Using (4) and (5), the operators are defined as follows:

$$\begin{aligned} \mathbb{F}(\mathbf{W}) &= \mathbf{W}_0(t) + \Psi(t, \mathbf{W}(t)) \frac{(1-\omega)}{\mathcal{M}(\omega)}, \\ \mathbb{G}(\mathbf{W}) &= \frac{\omega}{\mathcal{M}(\omega)} \int_0^t \Psi(\xi, \mathbf{W}(\xi)) d\xi. \end{aligned} \tag{6}$$

**Theorem 2** *With the help of (A1) and (A2), the integral system (4) has at least one solution provided that  $\frac{L_\Psi}{\mathcal{M}(\omega)} < 1$ .*

*Proof* Let  $\mathbb{B} = \{\mathbf{W} \in \mathbf{Z} : \|\mathbf{W}\| \leq \rho, \rho > 0\}$  be a closed and convex subset of  $\mathbf{Z}$ , we need to prove that  $\mathbb{F} : \mathbb{B} \rightarrow \mathbb{B}$  is a contraction. Let  $\mathbf{W} - \bar{\mathbf{W}} \in \mathbb{B}$ , we have

$$\begin{aligned} \|\mathbb{F}\mathbf{W} - \mathbb{F}\bar{\mathbf{W}}\| &= \sup_{t \in \mathcal{J}} \left| (\Psi(t, \mathbf{W}(t)) - (\Psi(t, \bar{\mathbf{W}}(t)))) \frac{(1-\omega)}{\mathcal{M}(\omega)} \right| \\ &\leq \frac{(1-\omega)}{\mathcal{M}(\omega)} L_\Psi \sup_{t \in \mathcal{J}} |\mathbf{W}(t) - \bar{\mathbf{W}}(t)| \\ &\leq \frac{L_\Psi}{\mathcal{M}(\omega)} \|\mathbf{W} - \bar{\mathbf{W}}\|. \end{aligned}$$

Hence  $\mathbb{F}$  is a contraction.

For  $\mathbb{G}$  to be compact and continuous, let any  $\mathbf{W} \in \mathbb{B}$ , we have

$$\begin{aligned} \|\mathbb{G}(\mathbf{W})\| &= \sup_{t \in \mathcal{J}} \left| \frac{\omega}{\mathcal{M}(\omega)} \int_0^t \Psi(\xi, \mathbf{W}(\xi)) d\xi \right| \\ &\leq \frac{\tau}{\mathcal{M}(\omega)} [C_\Psi \rho + \mathbf{M}_\Psi] := \Delta. \end{aligned} \tag{7}$$

From (7) we conclude that  $\mathbb{G}$  is bounded. Also  $\Psi$  is continuous, so is  $\mathbb{G}$ . Along the same lines, we can prove that  $\mathbb{G}$  is equicontinuous by taking  $t_1 < t_2 \in \mathcal{J}$ . By using Arzelà–Ascoli

theorem, the operator  $\mathbb{G}$  is completely continuous operator and also uniformly bounded as proved already. Hence  $\mathbb{G}$  is relatively compact. By Krasnoselskii’s fixed point theorem, problem (4) has at least one solution. Consequently, the considered system 2 has at least one solution.  $\square$

**Theorem 3** *Under assumption (A1), integral system (4) has unique solution if  $\frac{(1+\tau)L}{\mathcal{M}(\omega)} < 1$ . Consequently, the system under consideration has unique solution.*

*Proof* Let us define  $\mathbf{T} : \mathbf{Z} \rightarrow \mathbf{Z}$  by

$$\mathbf{T}(\mathbf{W}) = \mathbf{W}_0 + \Psi(t, \mathbf{W}(t)) \frac{(1-\omega)}{\mathcal{M}(\omega)} + \frac{\omega}{\mathcal{M}(\omega)} \int_0^t \Psi(\xi, \mathbf{W}(\xi)) d\xi.$$

Let  $\mathbf{W}, \bar{\mathbf{W}} \in \mathbf{Z}$ , we have

$$\begin{aligned} \|\mathbf{T}(\mathbf{W}) - \mathbf{T}(\bar{\mathbf{W}})\| &\leq \sup_{t \in \mathcal{J}} \frac{(1-\omega)}{\mathcal{M}(\omega)} |\Psi(t, \mathbf{W}(t)) - \Psi(t, \bar{\mathbf{W}}(t))| \\ &\quad + \frac{\omega}{\mathcal{M}(\omega)} \sup_{t \in \mathcal{J}} \int_0^t |\Psi(\xi, \mathbf{W}(\xi)) - \Psi(\xi, \bar{\mathbf{W}}(\xi))| d\xi \\ &\leq \frac{(1+\tau)L}{\mathcal{M}(\omega)} \|\mathbf{W} - \bar{\mathbf{W}}\|. \end{aligned} \tag{8}$$

Hence  $\mathbf{T}$  is a contraction and the concerned problem (4) has unique solution, and so the considered model (2) has unique solution.  $\square$

**4 Approximate solutions to predator–prey equations (2)**

To compute the required approximate solution, for easiness, take  $\mathcal{M}(\omega) = 1$ . Using the Laplace transform on both sides of system (2), we have

$$\begin{cases} \mathcal{L}[u(t)] = \frac{u(0)}{s} + \frac{s+\omega(1-s)}{s} \mathcal{L}[a_1 u(t) - b_1 u(t)v(t)], \\ \mathcal{L}[v(t)] = \frac{v(0)}{s} + \frac{s+\omega(1-s)}{s} \mathcal{L}[a_2 u(t)v(t) - b_2 v(t)]. \end{cases} \tag{9}$$

Now assume the solution in the series form as follows:

$$u(t) = \sum_{q=0}^{\infty} u_q(t), \quad v(t) = \sum_{q=0}^{\infty} v_q(t). \tag{10}$$

Further expressing the nonlinear terms  $u(t)v(t)$  by using the decomposition method

$$u(t)v(t) = \sum_{q=0}^{\infty} A_q(u, v), \tag{11}$$

where the “Adomian polynomial”  $A_q(u, v)$  can be defined as

$$A_q(u, v) = \frac{1}{q!} \frac{d^q}{d\lambda^q} \left[ \sum_{j=0}^p \lambda^j u_j(t) \sum_{j=0}^p \lambda^j v_j(t) \right] \Bigg|_{\lambda=0}.$$

Hence in view of (10) and (11), system (9) becomes

$$\begin{cases} \mathcal{L}[\sum_{q=0}^{\infty} u_q(t)] = \frac{u(0)}{s} + \frac{s+\omega(1-s)}{s} \mathcal{L}[a_1(t) \sum_{q=0}^{\infty} u_q(t) - b_1(t) \sum_{q=0}^{\infty} A_q(u, v)], \\ \mathcal{L}[\sum_{q=0}^{\infty} v_q(t)] = \frac{v(0)}{s} + \frac{s+\eta(1-s)}{s} \mathcal{L}[a_2(t) \sum_{q=0}^{\infty} A_q(u, v) - b_2(t) \sum_{q=0}^{\infty} v_q]. \end{cases} \tag{12}$$

From (12), we equate terms as follows:

$$\begin{cases} \mathcal{L}[u_0(t)] = \frac{u_0}{s}, & \mathcal{L}[v_0(t)] = \frac{v_0}{s}, \\ \mathcal{L}[u_1(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_1(t)u_0 - b_1(t)A_0(u, v)], \\ \mathcal{L}[v_1(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_2(t)A_0(u, v) - b_2(t)v_0], \\ \mathcal{L}[u_2(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_1(t)u_1 - b_1(t)A_1(u, v)], \\ \mathcal{L}[v_2(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_2(t)A_1(u, v) - b_2(t)v_1], \\ \mathcal{L}[u_3(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_1(t)u_2 - b_1(t)A_2(u, v)], \\ \mathcal{L}[v_3(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_2(t)A_2(u, v) - b_2(t)v_2], \\ \vdots \\ \mathcal{L}[u_{q+1}(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_1(t)u_q - b_1(t)A_q(u, v)], \\ \mathcal{L}[v_{q+1}(t)] = \frac{s+\omega(1-s)}{s} \mathcal{L}[a_2(t)A_q(u, v) - b_2(t)v_q], \quad q \geq 0. \end{cases} \tag{13}$$

*Case I:*

In the first case we take coefficients as constant functions  $a_1(t) = a_1, b_1(t) = b_1, c_1(t) = c_1, d_1(t) = d_1$  in (13). After performing simplification, we get

$$\begin{cases} u_0(t) = \alpha, & v_0(t) = \beta, \\ u_1(t) = [a_1\alpha - b_1\alpha\beta](1 + \omega(t - 1)), \\ v_1(t) = (1 + \omega(t - 1))[a_2\alpha\beta - b_2\beta], \\ u_2(t) = [(a_1 - b_1\beta)(a_1\alpha - b_1\alpha\beta) - b_1\alpha(a_2\alpha\beta - b_2\beta)](1 + \omega^2(t - 1)), \\ v_2(t) = [(\alpha a_2 - b_2)(a_2\alpha\beta - b_2\beta) + a_2\beta(a_2\alpha\beta - b_2\beta)](1 + \omega^2(t - 1)), \\ u_3(t) = [(a_1 - b_1\beta)((a_1 - b_1\beta)(a_1\alpha - b_1\alpha\beta) - b_1\alpha(a_2\alpha\beta - b_2\beta)) \\ \quad - b_1\alpha((\alpha a_2 - b_2)) - b_1(a_1\alpha - b_1\alpha\beta)(a_2\alpha\beta - b_2\beta)](1 + \omega^3(t - 1)), \\ v_3(t) = [(a_2\alpha - b_2)((\alpha a_2 - b_2)(a_2\alpha\beta - b_2\beta) + a_2\beta(a_2\alpha\beta - b_2\beta)) \\ \quad - b_1\alpha(a_2\alpha\beta - b_2\beta) + a_2(a_1\alpha - b_1\alpha\beta)(a_2\alpha\beta - b_2\beta) \\ \quad + a_2\beta((a_1 - b_1\beta)(a_1\alpha - b_1\alpha\beta)](1 + \omega^3(t - 1)) \end{cases} \tag{14}$$

and so on. In this way the other terms are computed.

*Case II:*

Here, we take some coefficients as linear functions  $a_1(t) = t, b_2(t) = t$  and  $a_2$  and  $b_1$  are constants. We obtain the resultant solution as follows:

$$\begin{cases} u_0(t) = \alpha, & v_0(t) = \beta, \\ u_1(t) = \alpha(t + \omega(\frac{t^2}{2!} - t)) - b_1\alpha\beta(1 + \omega(t - 1)), \\ v_1(t) = (1 + \omega(t - 1))a_2\alpha\beta - \beta(t + \omega(\frac{t^2}{2!} - t)), \end{cases} \tag{15}$$

and so on. The remaining terms may similarly be computed. The required solutions in both cases will be written as

$$\begin{cases} u(t) = u_0 + u_1(t) + u_2(t) + u_3(t) + \dots, \\ v(t) = v_0 + v_1(t) + v_2(t) + v_3(t) + \dots. \end{cases} \tag{16}$$

### 5 Results and discussion

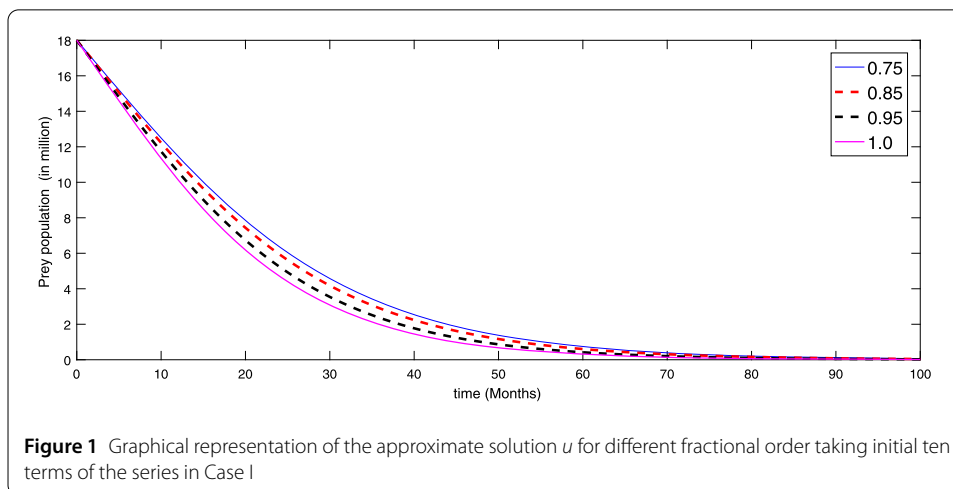
Here, by using Matlab, we present solutions (16) up to initial ten terms by graphs using the numerical values for parameters as given in Table 1. The solutions are displayed against various fractional orders in Figs. 1 and 2, respectively.

Further on using Matlab, we present solutions (16) up to initial ten terms by graphs using the numerical values for parameters as given in Table 2 in Case II. The solutions are displayed against various fractional orders in Figs. 3 and 4, respectively.

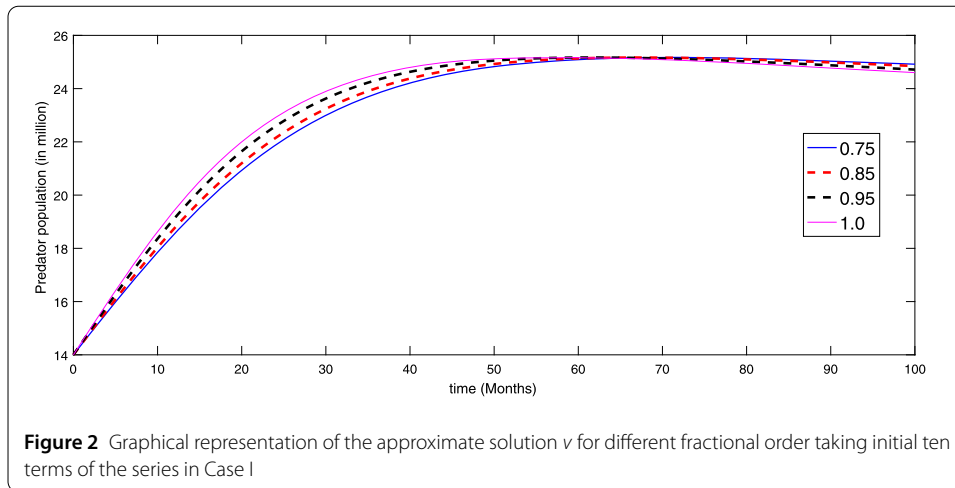
From Figs. 1, 3 we see that the population  $u$  is decreasing at different rate due to fractional order. The smaller the order, the faster the decay process, and hence stability occurs at smaller order first and then at greater one, as compared. In the same fashion the population  $v$  grows in Figs. 2 and 4, respectively, at different rate due to fractional order. The solution tends to the classical (integer) order solution when  $\omega \rightarrow 1$ . The solution obtained here for CFFD is close to the solution obtained by using the homotopy method in [20] by using Caputo ordinary derivatives. We have presented the solutions for both cases, i.e., Case I and Case II, in the aforesaid figures. From the figures, we conclude that CFFD can also be used as a powerful tool to investigate such systems.

**Table 1** Values of parameters taken for Case I

Parameters	Description of parameters	Numerical value
$a_1$	The growth rate of prey	0.009978
$b_1$	The rate at which predators destroy prey	0.00342
$a_2$	The death rate of predators	0.00342
$b_2$	The growth rate of predators	0.000765
$\alpha$	The initial population of prey	18
$\beta$	The initial population of predator	14

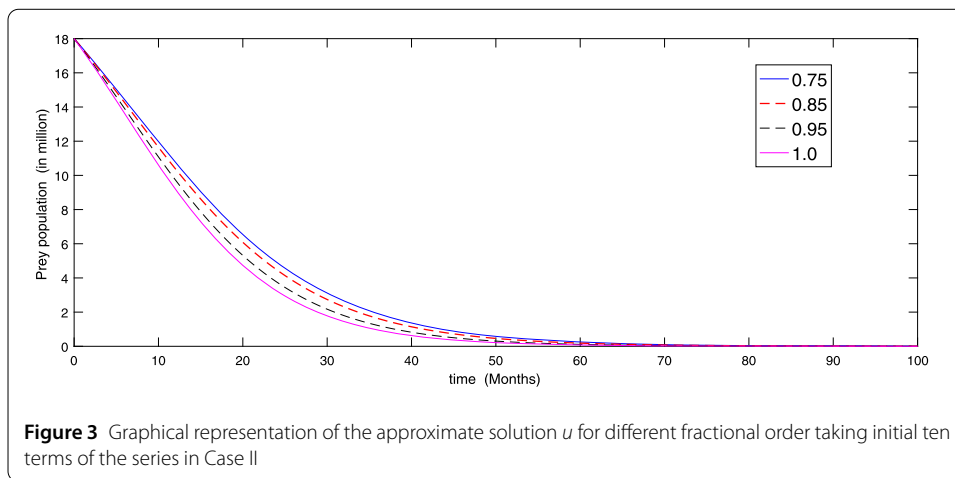


**Figure 1** Graphical representation of the approximate solution  $u$  for different fractional order taking initial ten terms of the series in Case I



**Table 2** Values of parameters taken for Case II

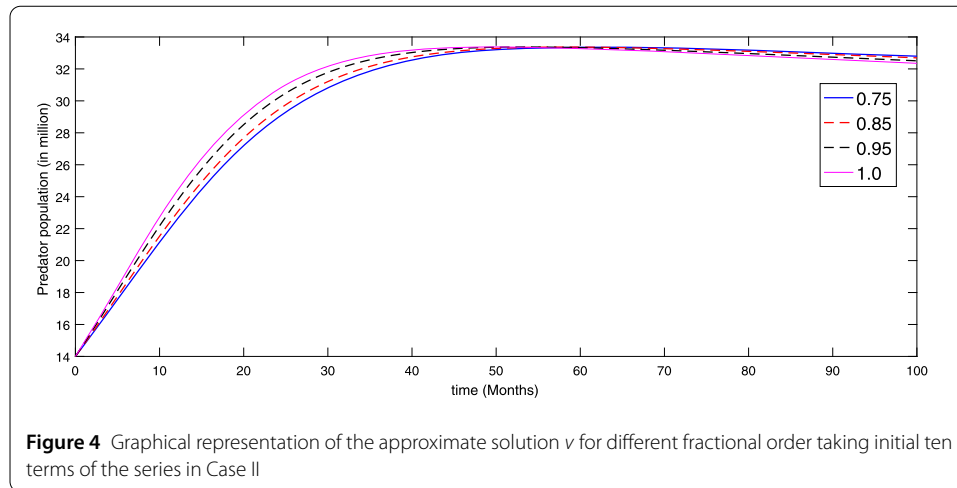
Parameters	Description of parameters	Numerical value
$a_1(t)$	The growth rate of prey	$t$
$b_1$	The rate at which predators destroy prey	0.00342
$a_2$	The death rate of predators	0.00342
$b_2(t)$	The growth rate of predators	$t$
$\alpha$	The initial population of prey	18
$\beta$	The initial population of predator	14



### 6 Conclusion

Since predator–prey models are debatably the building blocks of the bio- and ecosystems in which both the species depend on each other, we have taken two sets of parameter values in Tables 1 and 2. We have graphed the approximate solutions for different fractional order in Figs. 1–4, respectively. We see that the population of predators increases as shown in Figs. 2, 4, respectively. The growth rate is faster at smaller fractional order, and as the order increases, the solution behavior coincides with the solution at integer order. Consequently, the population of prey goes on decreasing as in Figs. 1 and 3. The decay rate is faster on smaller fractional order, while slower on greater order. Hence, the dynamical system addressing the relationship between prey and predator has been investigated under CFFD





from qualitative and analytical aspects. By using fixed point approach, the existence of the model has been verified. Also, by combining the Laplace transform with the decomposition method, some approximate analytical results have been established under two cases. In the first case for constant coefficients and in Case II the concerned results have been obtained for variable coefficients. Hence we concluded that taking few terms of the series solutions we can efficiently describe the model under investigation.

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#### Competing interests

The authors declare that they have no competing interests.

#### Authors' contributions

All authors equally contributed to this manuscript and approved the final version.

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