Results in Physics 28 (2021) 104597

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Explicit wave phenomena to the couple type fractional order nonlinear evolution equations

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ARTICLE INFO

Keywords:
Solitary wave solution
Riemann-Liouville fractional derivative
Space–time fractional coupled Boussinesq equation
Space–time fractional-coupled Boussinesq Burger equation
The double-expansion method

MSC:
35C25
35C07
35C08
35Q20
76B25

ABSTRACT

We utilize the fractional modified Riemann-Liouville derivative in the sense to develop careful arrangements of space–time fractional coupled Boussinesq equation which emerges in genuine applications, for instance, nonlinear framework waves iron sound waves in plasma and in vibrations in nonlinear string and space–time fractional-coupled Boussinesq Burger equation that emerges in the investigation of liquids stream in a dynamic framework and depicts engendering of shallow-water waves. A decent comprehension of its solutions is exceptionally useful for beachfront and engineers to apply the nonlinear water wave model to the harbor and seaside plans. A summed-up partial complex transformation is correctly used to change this equation to a standard differential equation thus, many precise logical arrangements are acquired with all the free parameters. At this point, the traveling wave arrangements are articulated by hyperbolic functions, trigonometric functions, and rational functions, if these free parameters are considered as specific values. We obtain kink wave solution, periodic solutions, singular kink type solution, and anti-kink type solutions which are shown in 3D and contour plots. The presentation of the method is dependable and important and gives even more new broad accurate arrangements.

Introduction

Fractional calculus and from now into the foreseeable future the fractional order nonlinear differential equations have choked out the uncommon thought concerning various examiners for their importance to describe the inside segment of the possibility of the certifiable world. As of late, the partial differential equations (PDEs) assume a significant function in applied science, material science, science, building, science, signal handling, control hypothesis, frameworks identification, and fractional elements [1–4]. Furthermore, they are used in social sciences, for instance, food enhancements, environment, finance, and monetary issues. The fractional order models perform different scales, in particular, nanoscale, microscale, and macro scale. The poly-layer section of the human body is a particularly capable model system for applying fractional calculus. Along these lines, the investigation of accurate solutions for nonlinear fractional differential equations (NLFDEs) is turning into the key interest of the specialists and assumes a significant function in nonlinear science. Along these lines, the investigation of accurate solutions for NLFDEs is turning into the key interest of the specialists and assumes a significant function in nonlinear science. Subsequently, a ton of literature has been given to build up the particular arrangements of fractional PDEs and fractional ODEs of physical intrigue. Numerous ground-breaking, efficient methodologies have been mentioned to procure exact solutions and approximate solutions of NLFDEs, for example, the first integral method [5], the homotopy perturbation method [6,7], the variational iteration method [8,9], the homotopy analysis method [10–13], the fractional sub- equation method [14,15], the (G'/G)-expansion method [16–19], the improved

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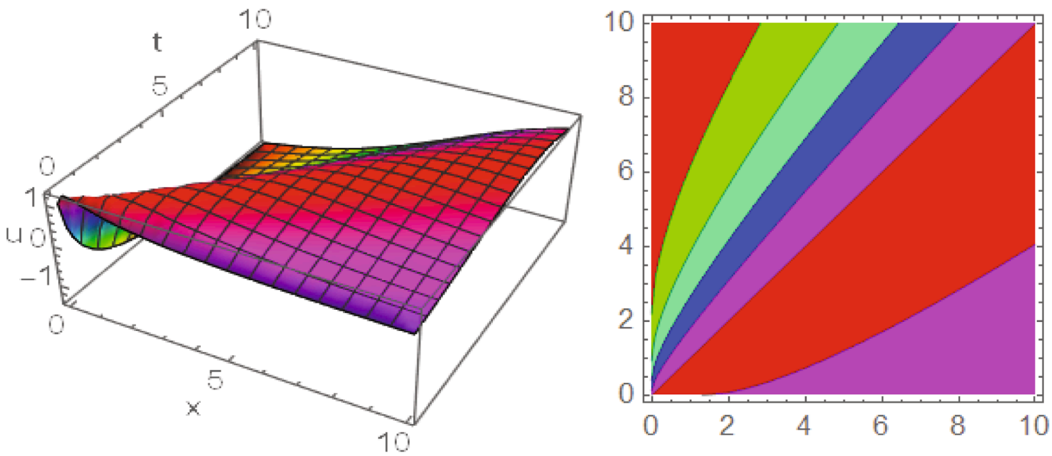
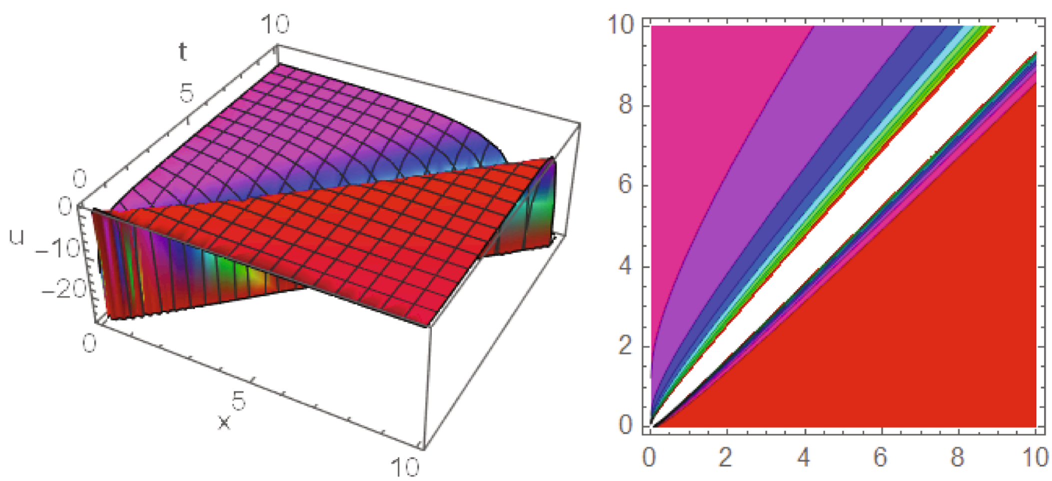
https://doi.org/10.1016/j.rinp.2021.104597
Received 28 June 2021; Received in revised form 19 July 2021; Accepted 22 July 2021
Available online 27 July 2021



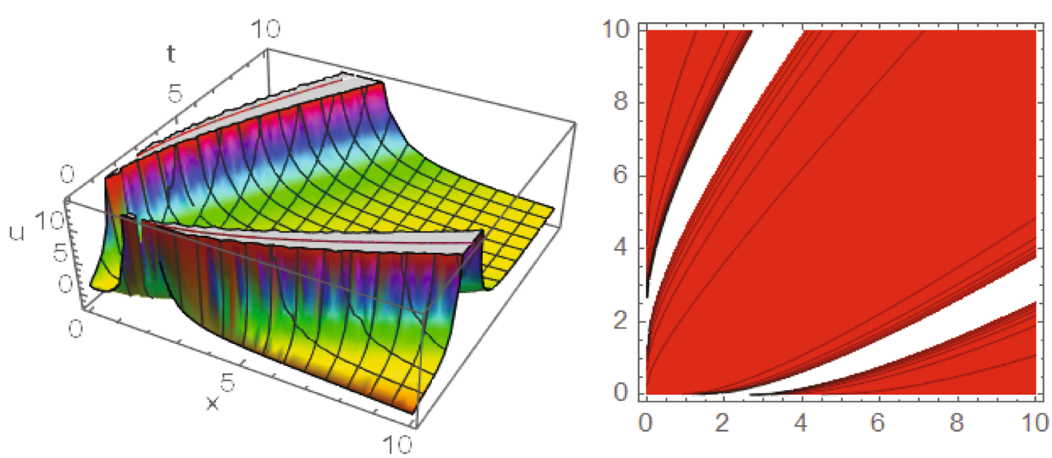
Three dimensional plotline. Plot of contour.

Fig. 1. Diagram of the kink type wave solution 3D (left section) and contour (right section) of $u_{1_2}(x,t)$ when $C_1 = a_0 = \mu = 0, C_2 = b_1 = a_1 = c = 1, \eta = \sigma = -1$, $\alpha = \frac{1}{2}, \beta = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.



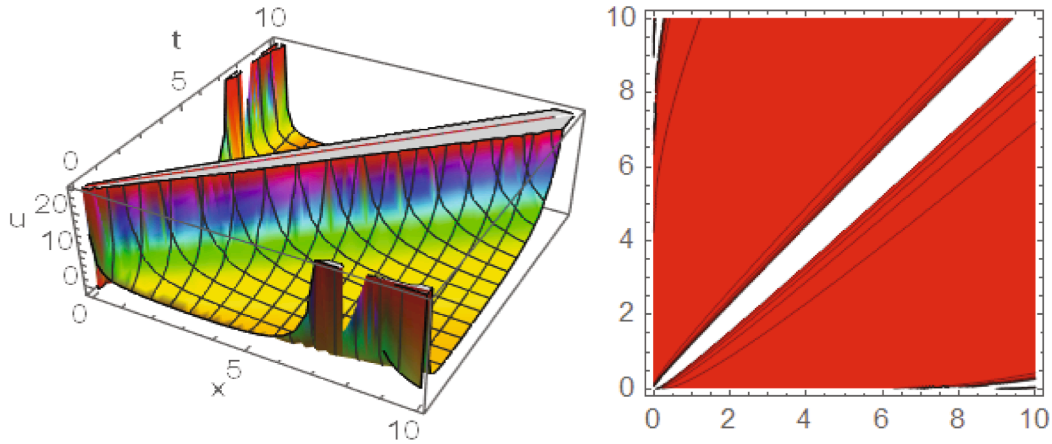
Three dimensional plotline. Plot of contour.

Fig. 2. Diagram of the singular kink type wave solution 3D (left section) and contour (right section) of $u_{1_3}(x,t)$ when $C_2 = a_0 = \mu = 0, C_1 = b_1 = a_1 = c = 1, \eta = \sigma = -1, \alpha = \frac{1}{2}, \beta = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.



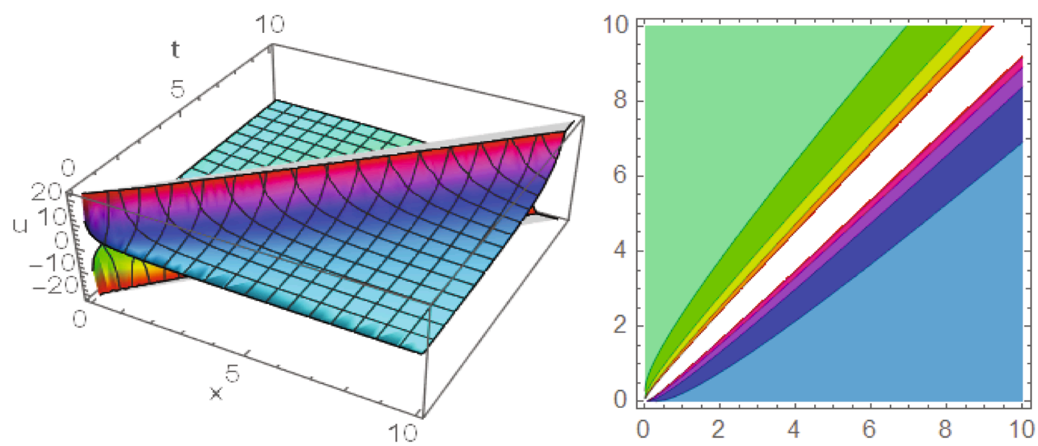
Three dimensional plotline. Plot of contour.

Fig. 3. Diagram of the periodic wave solution 3D (left section) and contour (right section) of $u_{1_5}(x,t)$ when $C_1 = \eta = b_1 = \sigma = c = a_1 = 1, C_2 = \mu = a_0 = 0, \alpha = \frac{1}{2}$, $\beta = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.



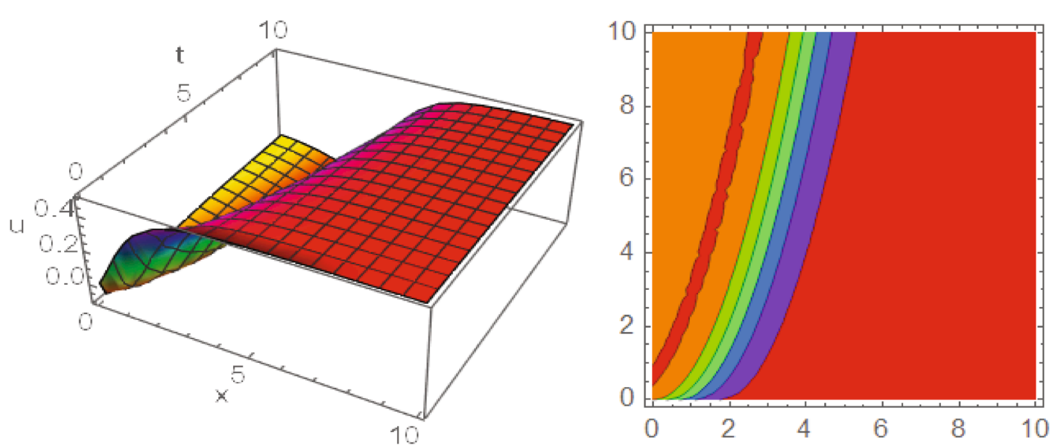
Three dimensional plotline. Plot of contour.

Fig. 4. Diagram of the double soliton wave solution 3D (left section) and contour (right section) of $u_{1_6}(x,t)$ when $C_2 = \eta = b_1 = \sigma = c = a_1 = 1, C_1 = \mu = a_0 = 0$, $\alpha = \frac{1}{2}, \beta = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.



Three dimensional plotline. Plot of contour.

Fig. 5. Diagram of the anti-kink type wave solution 3D (left section) and contour (right section) of $u_{1_7}(x,t)$ when $C_1 = \mu = 0, C_2 = b_1 = C_2 = a_1 = 1, c = 4, \alpha = \frac{1}{2}$, $\beta = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.



Three dimensional plotline. Plot of contour.

Fig. 6. Diagram of the kink type wave solution 3D (left section) and contour (right section) of $u_{2_2}(x,t)$ when $C_1 = \mu = 0, \eta = -1, \sigma = C_2 = c = 1, \alpha = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.

fractional sub-equation method [20,21], the exp-function method [22,23], the modified simple equation technique [24], the double $(G'/G, 1/G)$- expansion method [25,26], the discrete tanh method [27], etc. In light of the mentioned methods, an assortment of FDEs has been examined. The proposed fractional coupled Boussinesq equations given in [28] were developed by Hosseini and Ansari found its result using the modified Kudryashov method [29], Hoseini et al. explained it by applying exp $(-\phi(\varepsilon))$-expansion method [30], Yaslan and Girgin found the solution by using the first integral method [31]. Additionally, the proposed fractional-coupled Boussinesq Burgers equations [28] was developed by Kumar et al. explained it using the residual power series method [32] and Javeed et al. elucidated it using the first integral method [33], etc. To our appreciation, the recommended equations has not been concentrated through the double $(G'/G,1/G)$ -expansion method [34]. So the point of this investigation is to build up some fresh and additional broad precise results for previously mentioned conditions utilizing the double $(G'/G,1/G)$ -expansion method (see Figs. 1–3).

The article's layout is ordered as follows: In segment 2, we have offered the meaning and foreword, in segment 3, the double $(G'/G,1/G)$ -expansion method has been illustrated. In segment 4, we have built up the specific answer for the proposed condition by the previously mentioned technique. In segment 5, the findings and debates are evaluated using graphical representations and physical clarifications and in segment 6, investigations of results have been compared with others related works and lastly, conclusions are formed in Section 7** (see Figs. 4–6).

Meaning and foreword

Modified Riemann–Liouville fractional derivative established by Jumarie in 2006 [35]. We can transform NLFDEs into integer order differential equations employing a fractional derivative, variable transformation, and certain accommodating methods. Let $f: R\to R, x\to f(x)$ be a continuous function. The derivative of power α is ascertained as

$$D_x^\alpha f(x) = \begin{cases} \dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_0^x (x-\xi)^{-\alpha-1}[f(\xi)-f(0)]d\xi, \alpha<0 \\ \dfrac{1}{\Gamma(1-\alpha)}\dfrac{d}{dx}\displaystyle\int_0^x (x-\xi)^{-\alpha}[f(\xi)-f(0)]d\xi, 0<\alpha<1 \\ (f^{(n)}(x))^{(\alpha-n)}, n\le\alpha\le n+1, n>1 \end{cases} \tag{2.1}$$

Modified Riemann–Liouville derivative has two or three axioms, and the remaining four celebrated requirements are

$$D_t^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)}t^{(\gamma-\alpha)}, \gamma>0 \tag{2.2}$$

$$D_t^\alpha(af(t)+bg(t)) = aD_t^\alpha f(t)+bD_t^\alpha g(t) \tag{2.3}$$

wherever a and b stands for constants, and

$$D_x^\alpha f[u(x)] = f_u^\alpha(u)D_x^\alpha u(x) \tag{2.4}$$

$$D_x^\alpha f[u(x)] = D_u^\alpha f(u)(u'(x))^\alpha \tag{2.5}$$

which are the immediate results of

$$d^\alpha x(t) = \Gamma(1+\alpha)dx(t) \tag{2.6}$$

This holds for non-differentiable function. The Eqs. (2.3) and (2.4), u(x)is differentiable but non differentiable in Eq. (2.5) among the over Eqs. (2.3)–(2.5). The function u(x) is non-differentiable, and f(u)is differentiable in Eq. (2.4) and non-differentiable in Eq. (2.5). So the explanation Eqs. (2.3)–(2.5) should be applied closely.

The double (G'/G,1/G)- expansion method

The core part of the double $(G'/G,1/G)$-expansion method proposed by [36] to evaluate the particular traveling wave arrangement of NLFDEs has been addressed. Consider the second order ordinary differential equations.

$$G''(\xi)+\eta G(\xi) = \mu, \tag{3.1}$$

also, the accompanying relations

$$\psi = 1/G,\ \phi = G'/G. \tag{3.2}$$

Subsequently, it gives

$$\phi' = -\phi^2+\mu\psi-\eta, \psi' = -\phi\psi \tag{3.3}$$

The solution for Eq. (3.1) rely upon η as $\eta<0, \eta>0$, and $\eta=0$. For $\eta<0$, the complete solution of Eq. (3.1) will be as like as

$$G(\xi) = C_1\sinh(\sqrt{-\eta}\,\xi)+C_2\cosh(\sqrt{-\eta}\,\xi)+\frac{\mu}{\eta} \tag{3.4}$$

Taking into account that we get

$$\psi^2 = \frac{-\eta}{\eta^2\sigma+\mu^2}(\phi^2-2\mu\psi+\eta) \tag{3.5}$$

where $\sigma = {C_1}^2-{C_2}^2$.

On the off chance that $\eta>0$, the solution for Eq. (3.1) as follows;

$$G(\xi) = C_1\sin\left(\sqrt{\eta}\,\xi\right)+C_2\cos\left(\sqrt{\eta}\,\xi\right)+\frac{\mu}{\eta}. \tag{3.6}$$

Considering that we acquire

$$\psi^2 = \frac{\eta}{\eta^2\sigma-\mu^2}(\phi^2-2\mu\psi+\eta), \tag{3.7}$$

where $\sigma = C_1^2+C_2^2$.

When, $\eta=0$, the overall solution for condition (3.1) is as follows

$$G(\xi) = \frac{\mu}{2}\xi^2+C_1\xi+C_2. \tag{3.8}$$

Taking into account that we acquire

$$\psi^2 = \frac{1}{C_1^2-2\mu C_2}(\phi^2-2\mu\psi), \tag{3.9}$$

here C_1 and C_2 stand for constants and those are arbitrary.

In this part, we talk about the guideline a piece of proposed techniques to take exact traveling wave solutions for the NLFDE is as structure

$$p(u, D_t^\alpha u, D_x^\beta u, D_t^\alpha D_t^\alpha u, D_t^\alpha D_x^\beta u, D_x^\beta D_x^\beta, \ldots\ldots) = 0, 0<\alpha\le 1, 0<\beta\le 1, \tag{3.10}$$

here u speaks to an unidentified function of spatial subordinate x and transient subsidiary t and speaks to a polynomial of u(x, t)and its derivatives where the most maximum order of derivatives and nonlinear terms of the maximum order is associated.

At this point, u denotes an undiscovered spatial secondary function y and transient subsidiary t and speaks to a polynomial of u(y, t)and nonlinear terms of maximum order, and the most maximum order of derivatives are connected in its derivatives.

Step 1: Firstly, take travelling wave transformation

$$\xi = \frac{lx^\alpha}{\Gamma(1+\alpha)}-\frac{mt^\alpha}{\Gamma(1+\alpha)} \tag{3.11}$$

Where l and m are non-zero abstract constant.

By applying (3.11) into (3.10), it is reworked as:

$Q(u, u^{'}, u'', u''', \cdots\cdots\cdots) = 0$, (3.12) ordinary derivative of u with respect to ξ is represents prime here.

Step 2: Take the solution of Eq. (3.3) have been uncovered as polynomial in ϕ and ψ of the endorse category:

$$u(\xi) = \sum_{i=0}^{N} a_i\phi^i + \sum_{i=1}^{N} b_i\phi^{i-1}\psi, \tag{3.13}$$

here, a_i, b_i are stand for constants to be calculated afterwards.

Step 3: The maximum number of derivatives in linear and nonlinear terms seeming by homogeneous equilibrium in equation (3.12) stable the positive integer number N which decides the Eq. (3.13).

Step 4: Subbing (3.13) into (3.12) alongside (3.3) and (3.5) it diminishes to a polynomial in ϕ and ψ, having the degree one. Differentiating the polynomial of comparable terms with zero, gives a blueprint of algebraic equations that are analyzed by utilizing computational programming produces the assessments of $C_1, C_{2,}a_i, b_i, \mu,$ and η where $\eta < 0$ which give exaggerated function courses of action.

Step 5: Likewise, explore the assessments of a_i, b_i, μ, C_1, C_2 and η where$\eta > 0$, and $\eta = 0$ which are giving trigonometric and rational function results harmoniously.

$$-cv^{'} + A\left(u^2\right)^{'} - Eu^{''} = 0 \tag{4.4b}$$

From Eq. (4.4a), we obtain

$$v = cu \tag{4.5}$$

Surrogating Eq. (4.5) in Eq. (4.4b)

$$-c^2u + Au^2 - Eu^{''} = 0 \tag{4.6}$$

We acquire 2 + m = 2 m⇒m = 2, By means of the homogeneous equilibrium principle to Eq. (4.6), considering m to be set 2 in Eq. (3.13), we acquire type of the suggested arrangement of Eq. (4. 6) of the following form.

$$u(\xi) = a_0 + a_1\phi + a_2\phi^2 + b_1\psi + b_2\phi\psi \tag{4.7}$$

wherever, a_0, a_1, a_2, b_1 and b_2 are constant to be resolved.

Case 1: Substituting Eq. (4.7) into (4.6) using (3.3) and (3.5) for $\eta < 0$, plucking all the terms with equivalent power of (*G'/G,1/G*) together and a set of algebraic equations is obtained by reducing each coefficient to zero. The succeeding are the outcomes of solving these equations.

$a_0 = a_0, a_1 = a_1, a_2 = -\frac{b_1\sqrt{-\sigma}}{2\eta\sigma}, b_1 = b_1, b_2 = 0$ and $c = c$.

We get the solution for the fractional-coupled Boussinesq Eq. (4.1) as the structure by substituting these value

$$u_{1_1}(x,t) = a_0 + a_1 \times \frac{C_1\sqrt{-\eta}\cosh(\sqrt{-\eta}\xi) + C_2\sqrt{-\eta}\sinh(\sqrt{-\eta}\xi)}{C_1\sinh(\sqrt{-\eta}\xi) + C_2\cosh(\sqrt{-\eta}\,\xi) + \frac{\mu}{\eta}} - \frac{b_1\sqrt{-\sigma}}{2\eta\sigma} \times \left(\frac{C_1\sqrt{-\eta}\cosh(\sqrt{-\eta}\,\xi) + C_2\sqrt{-\eta}\sinh(\sqrt{-\eta}\,\xi)}{C_1\sinh(\sqrt{-\eta}\,\xi) + C_2\cosh(\sqrt{-\eta}\,\xi) + \frac{\mu}{\eta}}\right)^2 + \frac{b_1}{C_1\sinh(\sqrt{-\eta}\xi) + C_2\cosh(\sqrt{-\eta}\,\xi) + \frac{\mu}{\eta}} \tag{4.8}$$

Formulation of exact solution

The space–time fractional-coupled Boussinesq equations

We need to address can reenact the expansion of surface water waves through a profundity far not actually even scale, which is the halfway coupled Boussinesq equations in presence

$$D_t^{\alpha}u(x,t) - D_x^{\beta}v(x,t) = 0,$$

$$D_t^{\alpha}v(x,t) + AD_x^{\beta}\left(u^2(x,t)\right) - ED_{xxx}^{3\beta}u(x,t) = 0; 0 < \alpha, \beta \leq 1, \tag{4.1}$$

where $A$and E are developing parameters and D_t^{α} is the fractional derivative consisting the order α, where $0 < \alpha < 1$.

Introduce the following fractional transformation

$$\xi = \frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)} \tag{4.2}$$

Using Eq. (4.2) in Eq. (4.1), we get

$$-cu^{'} + v^{'} = 0 \tag{4.3a}$$

$$-cv^{'} + A\left(u^2\right)^{'} - Eu^{'''} = 0 \tag{4.3b}$$

where $u^{'} = \frac{du}{d\xi}$, By integrating one time regarding travelling wave variable factor ξ and believing the necessary consistent to be zero, we will acquire

$$-cu + v = 0 \tag{4.4a}$$

wherever $\xi = \frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}$ and $\sigma = C_1^2 - C_2^2$.

Since C_1 and C_2 are arbitrary constants, it may be self-assertively picked. In the event that we pick $C_1 = \mu = 0$ and $C_2 \neq 0$in Eq. (4.8), solitary wave solution is obtained as following manner.

$$u_{1_2}(x,t) = a_0 + a_1\sqrt{-\eta}\tanh\left(\sqrt{-\eta}\left(\frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right)\right) + \frac{b_1\sqrt{-\sigma}}{2\eta\sigma}tanh^2\left(\sqrt{-\eta}\left(\frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right)\right) + b_1 sech\left(\sqrt{-\eta}\left(\frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right)\right)$$

By choosing $C_1 \neq 0$ and $C_2 = \mu = 0$ in Eq. (4.8), it is possible to acquire the solitary wave solution.

$$u_{1_3}(x,t) = a_0 + a_1\sqrt{-\eta}\coth\left(\sqrt{-\eta}\left(\frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right)\right) + \frac{b_1\sqrt{-\sigma}}{2\eta\sigma}coth^2\left(\sqrt{-\eta}\left(\frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right)\right) + b_1 cosech\left(\sqrt{-\eta}\left(\frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right)\right)$$

Case 2: For$\eta > 0$, substituting Eq. (4.7) into (4.6) using (3.3) and (3.7), we achieve the resultant conclusion by solving mathematical problems with computer based math like Maple.

$$c = c, a_0 = a_{0,}a_1 = a_{1,}a_2 = \frac{b_1\sqrt{\sigma}}{2\eta\sigma}, b_1 = b_1, b_2 = 0$$

We get the solution for the fractional-coupled Boussinesq Eq. (4.1) as the structure by substituting these values into (4.7),

$$u_{1_4}(x,t)=a_0+a_1\times\frac{C_1\sqrt{\eta}\cos(\sqrt{\eta}\xi)-C_2\sqrt{\eta}\sin(\sqrt{\eta}\xi)}{C_1\sin(\sqrt{\eta}\xi)+C_2\cos\left(\sqrt{\eta}\,\xi\right)+\frac{\mu}{\eta}}+\frac{b_1\sqrt{\sigma}}{2\eta\sigma}\times\left(\frac{C_1\sqrt{\eta}\cos(\sqrt{\eta}\,\xi)-C_2\sqrt{\eta}\sin(\sqrt{\eta}\,\xi)}{C_1\sin(\sqrt{\eta}\xi)+C_2\cos\left(\sqrt{\eta}\,\xi\right)+\frac{\mu}{\eta}}\right)^2+\frac{b_1}{C_1\sin(\sqrt{\eta}\xi)+C_2\cos\left(\sqrt{\eta}\,\xi\right)+\frac{\mu}{\eta}} \tag{4.9}$$

wherever $\xi=\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}$ and $\sigma=C_1^2+C_2^2$.

It can be self-assertively chosen since C_1 and C_2 are arbitrary constants. In the event that we pick $C_1=\mu=0$ and $C_2\neq 0$in Eq. (4.9), then following solitary wave solution are obtained

$$u_{1_5}(x,t)=a_0-a_1\sqrt{\eta}\tan\left(\sqrt{\eta}\left(\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}\right)\right)+\frac{b_1\sqrt{\sigma}}{2\eta\sigma}tan^2\left(\sqrt{\eta}\left(\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}\right)\right)+b_1sec\left(\sqrt{\eta}\left(\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}\right)\right)$$

Over again, acquire the solitary wave solution, by selecting $C_1\neq 0$ and $C_2=\mu=0$ in Eq. (4. 9)

$$u_{1_6}(x,t)=a_0+a_1\sqrt{\eta}\cot\left(\sqrt{\eta}\left(\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}\right)\right)+\frac{b_1\sqrt{\sigma}}{2\eta\sigma}cot^2\left(\sqrt{\eta}\left(\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}\right)\right)+b_1cosec\left(\sqrt{\eta}\left(\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}\right)\right)$$

Case 3: Likewise, for the same organization, when $\eta=0$, setting equations (4.7) into (4.6) using (3.3) and (3.9), achieve the resultant conclusion by solving mathematical problems with computer based math like Maple.

$$c=c,a_0=a_{0,}a_1=a_{1,}a_2=\frac{b_1C_2}{{C_1}^2},b_1=b_1,b_2=0$$

We achieve to the rational function solution for the fractional-coupled Boussinesq Eq. (4.1) as the structure by substituting these values into (4.7),

$$u_{1_7}(x,t)=a_0+a_1\times\frac{\mu\xi+C_1}{\frac{\mu}{2}\xi^2+C_1\xi+C_2}+\frac{b_1C_2}{{C_1}^2}\times\left(\frac{\mu\xi+C_1}{\frac{\mu}{2}\xi^2+C_1\xi+C_2}\right)^2+\frac{b_1}{\frac{\mu}{2}\xi^2+C_1\xi+C_2} \tag{4.10}$$

wherever $\xi=\frac{x^\beta}{\Gamma(1+\beta)}-\frac{ct^\alpha}{\Gamma(1+\alpha)}$.

It is observable to see that the traveling waves arrangements of our proposed fractional-coupled Boussinesq equations are broadly new and general. These picked-up arrangements have not been checked in the previous investigation. These arrangements are advantageous to assign the above expressed phenomena.

The space–time fractional-coupled Boussinesq Burger equation

Take the following nonlinear time fractional coupled Boussinesq Burger Eq. [28]

$$D_t^\alpha u-\frac{1}{2}v_x+2uu_x=0$$

$$D_t^\alpha v-\frac{1}{2}u_{xxx}+2(uv)_x=0 \tag{4.11}$$

where D_t^α is the fractional derivative, consisting of order α, for $0<\alpha<1$.

Introduce the following fractional transformation

$$\xi=x-\frac{ct^\alpha}{\Gamma(1+\alpha)} \tag{4.12}$$

Applying Eq. (4.12) in Eq. (4.11), we have

$$-cu^{'}-\frac{1}{2}v'+2uu^{'}=0 \tag{4.13a}$$

$$-cv^{'}-\frac{1}{2}u'''+2(uv)^{'}=0 \tag{4.13b}$$

where $u^{'}=\frac{du}{d\xi}$, By integrating one time regarding travelling wave variable factor ξ and believing the necessary consistent to be zero, will acquire

$$-cu-\frac{1}{2}v+u^2=0 \tag{4.14a}$$

$$-cv-\frac{1}{2}u''+2uv=0 \tag{4.14b}$$

From Eq. (4.14a), we get

$$v=2\left(u^2-cu\right) \tag{4.15}$$

Surrogating Eq. (4.15) in Eq. (4.14b)

$$-\frac{1}{2}u^{''}+4u^3+cc^2u-6cu^2=0 \tag{4.16}$$

Using the homogeneous equilibrium principle to Eq. (4.6), acquire 2 + m = 3 m⇒m = 1. By compelling m to be set 2 in Eq. (3.13), we obtain type of the suggested arrangement of Eq. (4.16) by means of the following manner.

$$u(\xi)=a_0+a_1\phi+b_1\psi \tag{4.17}$$

wherever, a_0,a_1 and b_1 are constant to be resolved.

Case 1: Substituting Eq. (4.17) into (4.16) using (3.3) and (3.5) for $\eta<0$, plucking all the expressions with equivalent power of $(G'/G,1/G)$ together and each coefficient is equalized to zero, resulting in a collection of algebraic equations.. We have get the following results by solving these equations

Set 01: $c=\frac{\sqrt{-\eta}}{2},a_0=\frac{\sqrt{-\eta}}{4},a_1=\frac{1}{4},b_1=\frac{\sqrt{-\eta(\mu^2+\eta\sigma)}}{\eta}$.

Set 02: $c=\frac{\sqrt{-\eta}}{2},a_0=\frac{\sqrt{-\eta}}{4},a_1=-\frac{1}{4},b_1=\frac{\sqrt{-\eta(\mu^2+\eta^2\sigma)}}{\eta}$.

We get the solution for the fractional-coupled Boussinesq Burger Eq. (4.17) by substituting the values of set 01 into (4.11), as the structure

$$u_{2_1}(x,t)=\frac{\sqrt{-\lambda}}{4}+\frac{1}{4}\frac{C_1\sqrt{-\eta}\cosh(\sqrt{-\eta}\xi)+C_2\sqrt{-\lambda}\sinh(\sqrt{-\eta}\xi)}{C_1\sinh(\sqrt{-\eta}\xi)+C_2\cosh(\sqrt{-\eta}\xi)+\frac{\mu}{\eta}}+\frac{\sqrt{-\eta(\mu^2+\eta^2\sigma)}}{\eta}\times\frac{1}{C_1\sinh(\sqrt{-\eta}\xi)+C_2\cosh(\sqrt{-\eta}\xi)+\frac{\mu}{\eta}} \quad (4.18a)$$

$$v_{2_1}(x,t)=2\left((u_{2_1}(x,t))^2-cu_{2_1}(x,t)\right) \quad (4.18b)$$

wherever $\xi=x-\frac{\sqrt{-\eta}}{2}\frac{t^\alpha}{\Gamma(1+\alpha)}$ and $\sigma=C_1^2-C_2^2$.

Since $C_1$and C_2 are arbitrary constants, it may be self-assertively picked. In the event that we pick $C_1=\mu=0$ and $C_2\neq 0$in Eq. (4.18), solitary wave solution is obtained as the following fashion

$$u_{2_2}(x,t)=\frac{\sqrt{\eta}}{4}+\frac{1}{4}\sqrt{-\eta}\tanh\left(\sqrt{-\eta}\left(x-\frac{\sqrt{-\eta}}{2}\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\right)+\frac{\sqrt{-\eta(\eta^2\sigma)}}{\eta}sech\left(\sqrt{-\eta}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-\frac{cx-\frac{\sqrt{-\eta}}{2}\frac{t^\alpha}{\Gamma(1+\alpha)}t^\alpha}{\Gamma(1+\alpha)}\right)\right)$$

$$v_{2_2}(x,t)=2\left((u_{2_2}(x,t))^2-cu_{2_2}(x,t)\right)$$

Over again, for the selection of $C_1\neq 0$ and $C_2=\mu=0$ in Eq. (4. 18a) and (4.18b), we attain the solitary wave solution as following manner

$$u_{2_3}(x,t)=\frac{\sqrt{-\eta}}{4}+\frac{1}{4}\sqrt{-\eta}\coth\left(\sqrt{-\eta}\left(x-\frac{\sqrt{-\eta}}{2}\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\right)+\frac{\sqrt{-\eta(\eta^2\sigma)}}{\eta}cosech\left(\sqrt{-\eta}\left(x-\frac{\sqrt{-\eta}}{2}\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\right)$$

$$v_{2_3}(x,t)=2\left((u_{2_3}(x,t))^2-cu_{2_3}(x,t)\right)$$

Case 2: For$\eta>0$, substituting Eq. (4.17)) into (4.16) using (3.3) and (3.7), we achieve the resultant conclusion by solving mathematical problems with computer-based math like Maple.

Set 01: $c=\sqrt{\frac{\eta}{2}},a_0=\sqrt{\frac{\eta}{2}},a_1=0,b_1=\frac{1}{2}\sqrt{\eta\sigma}.$
Set 02: $c=\frac{1}{2}\sqrt{-\eta},a_0=\frac{1}{4}\sqrt{-\eta},a_1=\frac{1}{4},b_1=\frac{1}{2}\sqrt{\eta\sigma}.$

Substituting these values into (4.17), For set 01 we get the solution for the fractional-coupled Boussinesq Burger Eq. (4.11) as the structure

$$u_{2_4}(x,t)=\sqrt{\frac{\eta}{2}}+\frac{1}{2}\sqrt{\eta\sigma}\times\frac{1}{C_1\sin(\sqrt{\eta}\xi)+C_2\cos(\sqrt{\eta}\,\xi)+\frac{\mu}{\eta}} \quad (4.19a)$$

$$v_{2_4}(x,t)=2\left((u_{2_1}(x,t))^2-cu_{2_4}(x,t)\right) \quad (4.19b)$$

wherever $\xi=x-\sqrt{\frac{\eta}{2}}\frac{t^\alpha}{\Gamma(1+\alpha)}$ and $\sigma=C_1^2+C_2^2$.

Since C_1 and C_2 are arbitrary constants, it may be self-assertively picked. In the event that we pick $C_1=\mu=0$ and $C_2\neq 0$in Eqs. (4.19a) and (4.19b), we get solitary wave solution

$$u_{2_5}(x,t)=\sqrt{\frac{\eta}{2}}+\frac{1}{2}\sqrt{\eta\sigma}sec\left(\sqrt{\eta}\left(x-\sqrt{\frac{\lambda}{2}}\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\right)$$

$$v_{2_5}(x,t)=2\left((u_{2_5}(x,t))^2-cu_{2_5}(x,t)\right)$$

Over again, we obtain the solitary wave solution, if we pick $C_1\neq 0$ and $C_2=\mu=0$ in Eq. (4. 19a) and (4.19b)

$$u_{2_6}(x,t)=\sqrt{\frac{\eta}{2}}+\frac{1}{2}\sqrt{\eta\sigma}cosec\left(\sqrt{\eta}\left(x-\sqrt{\frac{\eta}{2}}\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\right)$$

$$v_{2_6}(x,t)=2\left((u_{2_6}(x,t))^2-cu_{2_6}(x,t)\right)$$

Case 3: Likewise, for the same arrangement, when $\eta=0$, setting Eq. (4.17) into (4.16) using (3.3) and (3.9), by applying computer based arithmetic, produces a solution to mathematical equations

Set 01: $c=\sqrt{\frac{-\eta}{2}},a_0=\frac{1}{2}\sqrt{\frac{-\eta}{2}},a_1=0,b_1=-\frac{1}{2}C_1.$

Set02: $c=\frac{1}{2}\frac{\left(\sqrt{C_1{}^2+2\eta C_2{}^2+2\sqrt{C_1{}^2\eta C_2{}^2+\eta^2C_2{}^4}}\right)-C_1{}^2}{\left(\sqrt{C_1{}^2+2\eta C_2{}^2+2\sqrt{C_1{}^2\eta C_2{}^2+\eta^2C_2{}^4}}\right)C_2},$

$$a_0=\frac{1}{2}\frac{\left(\sqrt{C_1{}^2+2\lambda\eta^2+2\sqrt{C_1{}^2\eta C_2{}^2+\eta^2C_2{}^4}}\right)-C_1{}^2}{\left(\sqrt{C_1{}^2+2\eta C_2{}^2+2\sqrt{C_1{}^2\eta C_2{}^2+\eta^2C_2{}^4}}\right)C_2},a_1=0,$$

$$b_1=\sqrt{C_1{}^2+2\eta C_2{}^2+2\sqrt{C_1{}^2\eta C_2{}^2+\eta^2C_2{}^4}}$$

.

Substituting the values of set 01 into (4.17), we get the solution for the fractional-coupled Boussinesq Burger Eq. (4.11) as the structure

$$u_{2_7}(x,t)=\frac{1}{2}\sqrt{\frac{-\eta}{2}}-\frac{1}{2}C_1\frac{1}{\frac{\mu}{2}\xi^2+C_1\xi+C_2}$$

$$v_{2_7}(x,t)=2\left((u_{2_7}(x,t))^2-cu_{2_7}(x,t)\right)$$

wherever $\xi=x-\sqrt{-\frac{\eta}{2}}\frac{t^\alpha}{\Gamma(1+\alpha)}$.

We observed that for aftereffect of constants specified in set 2 for both in (case 1, case 2 and case 3), we obtain new and simpler solitary wave solutions, which are similarly valuable in studying above mentioned subject.

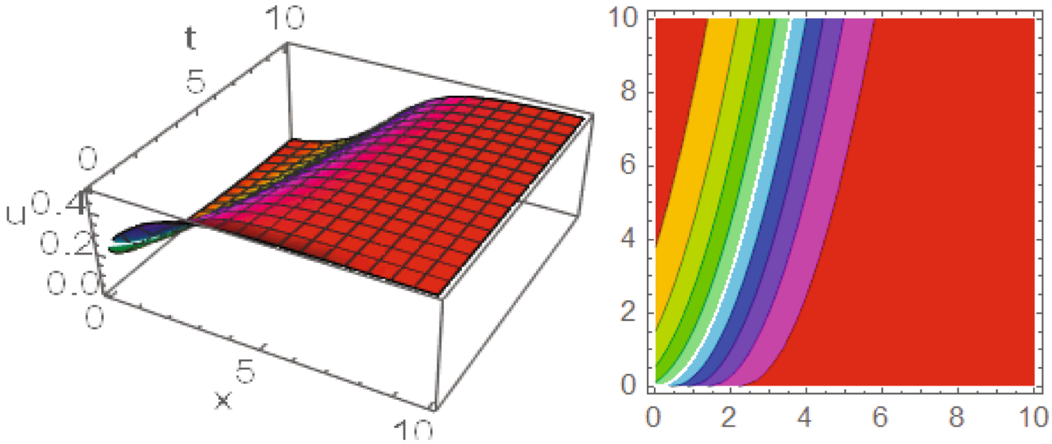
Brief discussion and graphical representation

In this segment, we give some graphical delineation of the obtained solutions of our equations. All of the figures in this article have been represented in three arrangements as 3D plot, and Contour plot contained by the specified domain $0\leq t\leq 10$, and $0\leq x\leq 10$. Mathematica, a computation package application, was used to construct all of the figures (see Figs. 7–9).

Results comparison

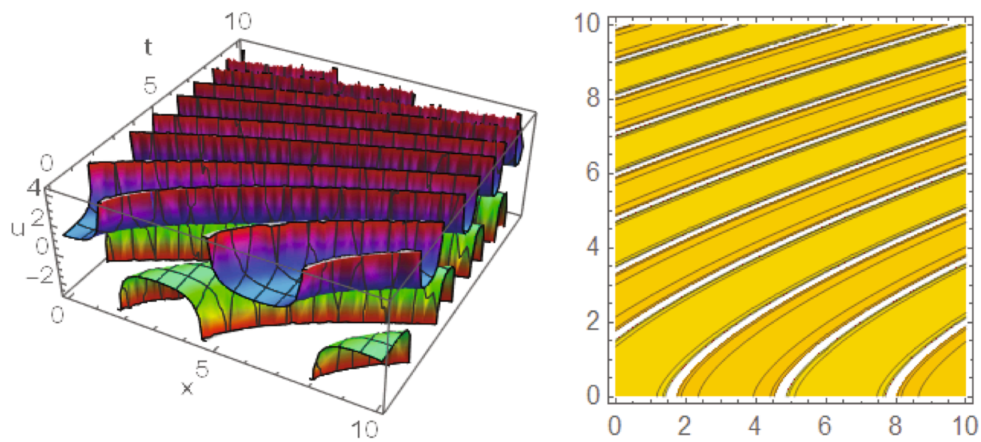
It is exceptional to think about the got results set up by a specific methodology with those found by different techniques accessible in the writing to approve the method a well as the outcomes. In this segment, we will contrast the accomplished solutions and those acquired by Kai Fan and Cunlong Zhou [28] in the accompanying Table 1.

The hyperbolic, trigonometric, and rational function solutions mentioned in preceding table are comparable, and they turn out to be indistinguishable when specific values of arbitrary constants are specified. In a nutshell, it is important to realize that the traveling waves solutions and solitary wave solutions of fractional-coupled Boussinesq equation and fractional-coupled Boussinesq Burger equation are all new except $u_{2_2}(x,t),u_{2_3}(x,t),u_{2_7}(x,t),v_{2_7}(x,t)$ and all are recent and very much significant, which were not originally in the previous works. It has



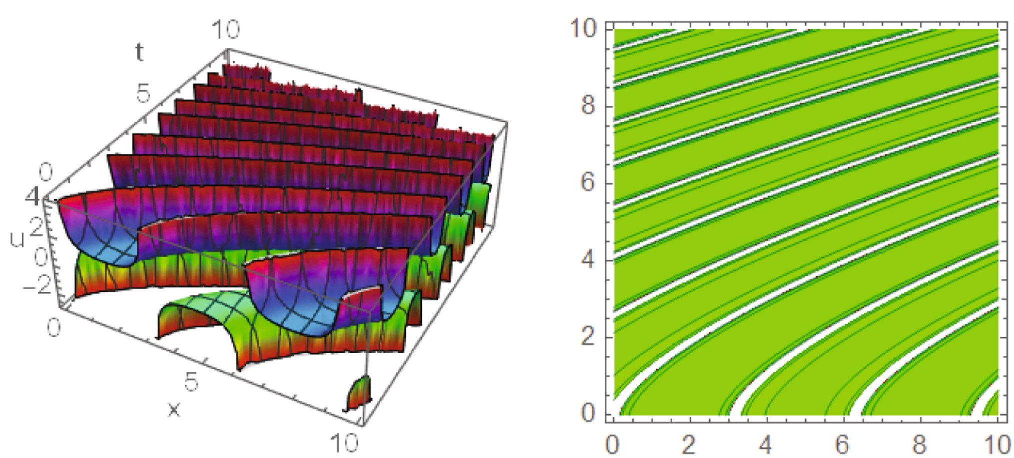
Three dimensional plotline. Plot of contour.

Fig. 7. Diagram of the kink type wave solution 3D (left section) and contour (right section) of $u_{2_3}(x,t)$ when $C_2 = \mu = 0, \eta = -1, \sigma = C_1 = c = 1, \alpha = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.



Three dimensional plotline. Plot of contour.

Fig. 8. Diagram of the periodic wave solution 3D (left section) and contour (right section) of $u_{2_5}(x,t)$ when $C_1 = \mu = 0, C_2 = \eta = \sigma = c = 1, \alpha = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.



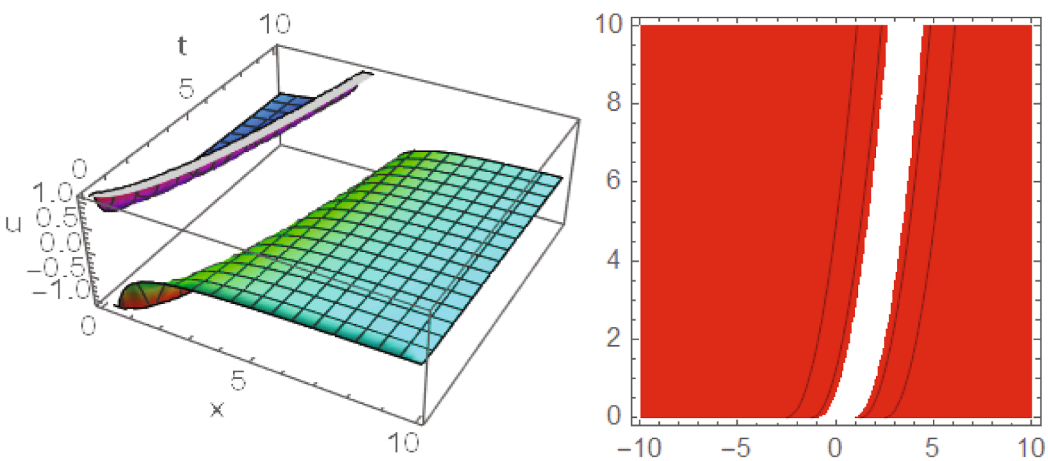
Three dimensional plotline. Plot of contour.

Fig. 9. Diagram of the periodic wave solution 3D (left section) and contour (right section) of $u_{2_6}(x,t)$ when $C_2 = \mu = 0, C_1 = \eta = \sigma = c = 1, \alpha = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.

been observed that the proposed equations are very much important to solve the above stated phenomena (see Fig. 10).

Table 1
Comparison between our obtained results and Kai Fan and Cunlong Zhou [28].

Kai Fan and Cunlong Zhou [28]	Obtained solutions
If $\lambda = 1$ and $\mu = 0$ then equation (61) becomes	If $\sigma = 0$ then $u_{2_2}(x,t)$ becomes
$\dot{u}_7^1(x,t) = \frac{-11}{24} - \frac{1}{12}\eta_1 tanh(\eta_1\xi + \xi_0)$	$u_{2_2}(x,t) = \frac{\sqrt{-\eta}}{4} + \frac{1}{4} \times \sqrt{-\eta}\tanh(\sqrt{-\eta}\xi)$.
If $\lambda = 1$ and $\mu = 0$ then equation (62) turn into	If $\sigma = 0$ then $u_{2_3}(x,t)$ turn into
$\ddot{u}_7^1(x,t) = \frac{-11}{24} - \frac{1}{12}\eta_1 coth(\eta_1\xi + \xi_0)$.	$u_{2_3}(x,t) = \frac{\sqrt{-\eta}}{4} + \frac{1}{4} \times \sqrt{-\eta}\coth(\sqrt{-\eta}\xi)$.
If $\lambda = 1$ and $\mu = 0$ then equation (65) turn out to be	If$\mu = 0$ then $u_{2_7}(x,t)$ turn out to be
$u_9^1(x,t) = \frac{-11}{24} - \frac{C_2}{2(C_1 + C_2\xi)}$.	$u_{2_7}(x,t) = \frac{1}{2}\sqrt{\frac{-\eta}{2}} - \frac{C_1}{2(C_1\xi + C_2)}$.
In equation (60b)	In equation (4.20b)
$\dot{v}_7^1(x,t) = 2\left(\dot{u}_7^1(x,t))^2 - c\dot{u}_7^1(x,t)\right)$.	$v_{2_7}(x,t) = 2((u_{2_7}(x,t))^2 - cu_{2_7}(x,t))$.



Three dimensional plotline. Plot of contour.

Fig. 10. Diagram of the kink type wave solution 3D (left section) and contour (right section) of $u_{2_7}(x,t)$ when $C_1 = 0, C_2 = 1, \mu = 0, \eta = 0, c = 1, \alpha = \frac{1}{2}, 0 \le t \le 10$, and $0 \le x \le 10$.

Conclusion

We have studied space–time fractional-coupled Boussinesq equations and space–time fractional-coupled Boussinesq Burger equation along with trigonometric, hyperbolic, in addition, rational function result having parameters through Riemann-Liouville fractional derivative on the double $(G'/G, 1/G)$-expansion method is considered. We realize the kink wave solution, periodic solution, singular kink type solution and anti-kink type solutions by using the proposed method which is shown in 3D and contour plots. The results achieved in this paper were verified by putting them back into NLFDEs and accurately found with computational software Maple and exceptionally useful for beachfront and engineers to spread over the nonlinear water wave model to coastal plans and harbor. To the best of our consciousness, it is mentionable that first-hand type of the acquired results in this learning has not been accomplished previously. Furthermore, the proficient solutions sanction that anticipated process affords an effective mathematical implement also seems to be sturdier, relaxed, and quicker employing the emblematic program calculation system. The proposed approach is a reliable, powerful, momentous, and especially persuading technique. It will also be applied to other NLFDEs structures that appear in nonlinear sciences. In addition, it might be used to screen a variety of NLFDEs in mathematical physics and coastal engineering.

CRediT authorship contribution statement

M. Ayesha Khatun: Methodology. **Mohammad Asif Arefin:** Validation, Formal analysis. **M. Hafiz Uddin:** Validation. **Dumitru Baleanu:** Funding acquisition, Validation. **M. Ali Akbar:** Formal analysis. **Mustafa Inc:** Conceptualization, Methodology.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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