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Research Article

Fractional Order Models of Industrial Pneumatic Controllers

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This paper addresses a new approach for modeling of versatile controllers in industrial automation and process control systems such as pneumatic controllers. Some fractional order dynamical models are developed for pressure and pneumatic systems with bellows-nozzle-flapper configuration. In the light of fractional calculus, a fractional order derivative-derivative (FrDD) controller and integral-derivative (FrID) are remodeled. Numerical simulations illustrate the application of the obtained theoretical results in simple examples.

1. Introduction

Fractional calculus is a powerful mathematical tool with a long history, but its application to engineering and modeling of physical systems has attracted much attention only in recent years [1]. This theory generalizes the classical differentiation and integration into noninteger order ones. It has been found that in interdisciplinary fields, many systems can be described more accurately and more conveniently by fractional differential equations (FDEs). For instance, fractional derivatives have been widely used in the mathematical modeling of viscoelastic materials [2]. The anomalous diffusion phenomena in nonhomogeneous media can be explained by noninteger derivative-based equations of diffusion [3]. Another example for an element with fractional order model is fractance, which is an electrical circuit with noninteger order impedance and has a property that lies between resistance and capacitance [4]. Moreover, it has been shown that the dynamical process of heat conduction can be modeled more adequately via fractional order calculus [5]. In biology, the membranes of the biological cells are proven to have fractional order electrical conductance and are classified among noninteger order systems [6, 7]. In economics, it is known that some finance systems can display fractional order dynamics [8]. For more examples of fractional order models see, for example, [9, 10] and the references therein. As an important industrial controller, fractional order versions of PID controllers have been considered in the literature [11]. More theoretical problems and recent applications can be found in [12].

Pneumatic controllers are essential parts of the industrial automation systems containing several diaphragms and bellows [13]. There have been a great development in low-pressure pneumatic controllers for industrial control systems in the past decades, and today they are used extensively in industrial processes. The advantages include safety and explosion-proof characteristic, simplicity, low cost, high compliance, and ease of maintenance [2]. For instance, recently, pneumatic servo systems have been applied in several systems, such as industrial robots, rehabilitation tools, and medical and caregiver robots, aspherical glass molding machines, and precise positioning [14, 15].

Recently in [16] a model of a pneumatic vibration isolation system (PVIS) has been presented. However, a thorough study on the modeling of the pneumatic elements via fractional order differential equations has not been considered yet. Because of compressibility of the air, the control action in practical actuating valves may not be positive; that is, an error may exist in the valve-stem position. To overcome this imperfection, a more accurate model of pneumatic controllers is needed. Using fractional order dynamics for diaphragms, bellows, and other devices that may have sprig-property yields a more accurate model for such industrial systems than the classical integer order models. In particular, relaxation processes deviating from the classical exponential behavior are often encountered in the dynamics of complex materials. In many cases experimentally observed relaxation functions exhibit a stretched exponential decay [17, 18]. Such behavior may be seen in the stress relaxation of viscoelastic materials, such as polymers or critical gels, in the change carrier transport in amorphous semiconductors and in the attenuation of seismic waves [19]. By this new approach, the available hysteresis in the diaphragms, which has been usually disregarded for simplicity in conventional approaches, can be easily modeled in the new approach. Fractional calculus allows for a rigorous and reliable modeling for such systems.

In this paper, in order to consider some real and nonideal features of a pneumatic structure, using a four-parameter fractional derivative Zener model for viscoelastic materials [20–22], we present a fractional order model for a nozzle-flapper-relay configuration. Then, under some simplified conditions, we provide a multiorder fractional derivative and integral controller, which considers the memory of the system in a compact form. The numerical simulations support the obtained results.

This paper is organized as follows. Section 2 provides the preliminary background from fractional calculus. In Section 3, the pneumatic and pressure systems are presented. Some illustrative examples are provided in Section 4. Finally, the conclusion remarks are given in Section 5.

2. Backgrounds

In this section, a brief background of fractional calculus is presented. The definition of the fractional integral is the extension of Cauchy formula for evaluating the integration. The qth order fractional integral of function $x(t)$ with respect to t is defined by

$$J^q x(t) = \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} x(s)\, ds, \tag{1}$$

in which $\Gamma(q) = \int_0^\infty e^{-z} z^{q-1} dz$, $q > 0$ is the Euler gamma function. In addition, there are some definitions for fractional derivatives such as Riemann-Liouville, Grunwald-Letnikov, and Caputo definitions [23]. For example, the Riemann-Liouville (RL) fractional derivative of order q for function $x(t)$ is defined by

$${}_0^{\mathrm{RL}}D_t^q x(t) = D^m J^{(m-q)} x(t), \tag{2}$$

where $m-1 < q < m, m \in \mathbf{Z}^+$.

The Laplace transform of RL fractional derivative of order q of function $x(t)$ is

$$L\left\{{}_0^{\mathrm{RL}}D_t^q x(t)\right\} = s^q X(s) - \sum_{k=0}^{m-1} s^k \cdot {}_0^{\mathrm{RL}}D_t^{q-k-1} x(0), \tag{3}$$

where $m-1 < q < m \in \mathbf{Z}^+$. As can be seen from (3) for evaluating the Laplace transform of RL fractional derivative operator, the fractional order derivative of the function $x(t)$ is needed as initial conditions. This is somewhat meaningless from the physical viewpoint. Therefore, the RL operator is useless in modeling of the physical systems.

Another definition for fractional derivative has been introduced by Caputo:

$$\begin{aligned} {}_0^{C}D_t^q x(t) &:= {}_0^{\mathrm{RL}}D_t^{-(m-q)} D^m x(t) \\ &= \frac{1}{\Gamma(m-q)} \int_0^t (t-s)^{m-q-1} x^{(m)}(s)\, ds, \end{aligned} \tag{4}$$

where $m-1 < q < m \in \mathbf{Z}^+$.

In contrast to RL operator, the Laplace transform of the Caputo operator needs the integer order derivative of the function as the initial conditions, as shown by the following relation:

$$L\left\{{}_0^{C}D_t^q x(t)\right\} = s^q X(s) - \sum_{k=0}^{m-1} s^{q-k-1} x^{(k)}(0), \tag{5}$$

where $m-1 < q < m \in \mathbf{Z}^+$.

For more information about fractional calculus see, for example, [24, 25].

3. Fractional Order Model of Pneumatic and Pressure Systems

Many industrial processes and pneumatic controllers involve the flow of a gas or air through connected pipelines and pressure vessels. Thus, it is logical to introduce a parameter for characterizing their specifications. Some quantities in pipelines and pressure vessels can be viewed as resistance (R) and capacitance (C) used in electrical circuits. Traditional definitions for these characteristics are as follows [2]:

$$R = \frac{d(\Delta P)}{dq}, \tag{6}$$

where $d(\Delta P)$ is a small change in the gas pressure difference between input and vessel gas and dq is a small change in the gas flow rate. Traditionally, the capacitance of the pressure vessel may be defined by

$$C = \frac{dm}{dp}, \tag{7}$$

where m and p are mass of gas stored in the vessel and gas pressure, respectively.

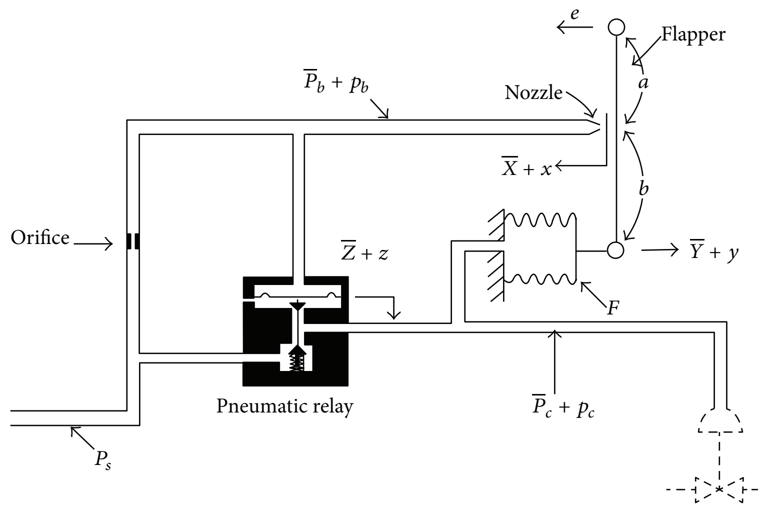

Figure 1: Schematic diagram of a force-distance type of pneumatic proportional controller.

It can be shown that during the change of state of a polytropic process, from isothermal to adiabatic state, the capacitance is constant and can be obtained as [2]

$$C = \frac{V}{nR_{\text{gas}}T}, \tag{8}$$

where n is the polytropic exponent, V, R_{gas}, and T are the volume of vessel, gas constant, and absolute temperature, respectively. It is worth noting that in some practical cases, the polytropic exponent n is approximately 1.0–1.2 for gases in uninsulated metal vessels.

3.1. A Fractional Order Model for the Nozzle-Flapper-Relay Controller. A conventional apparatus in the industrial pneumatic control systems is the nozzle-flapper configuration. The nozzle-flapper amplifier converts displacement into a pressure signal. Since typical industrial process control systems require large output power to operate large pneumatic actuating valves, the power amplification of the nozzle-flapper amplifier is usually insufficient. To overcome this problem, a pneumatic relay can be connected to the nozzle-flapper. Therefore, a complete pneumatic amplifier is composed of two stages: nozzle-flapper as the first and pneumatic relay as the second amplifier stages. A schematic diagram of such a configuration is depicted in Figure 1.

Assuming that the relationship between the variation in the nozzle back pressure p_b and the variation in the nozzle-flapper distance x is linear, one can write

$$p_b = k_1 x, \tag{9}$$

where k_1 is a positive constant. We now develop a new fractional order model for the pneumatic relay containing a diaphragm as mentioned in Section 1 which considers hereditary property of the configuration. The elasticity and damping properties of the diaphragm cannot be measured directly because the shape of the diaphragm in the deflated condition is quite different from that in the inflated condition, and the deflated diaphragm is very difficult to handle due to its flexibility. Therefore, the stiffness of the diaphragm is obtained usually from the experimental data. A simple bleed type of pneumatic relay is depicted in Figure 2.

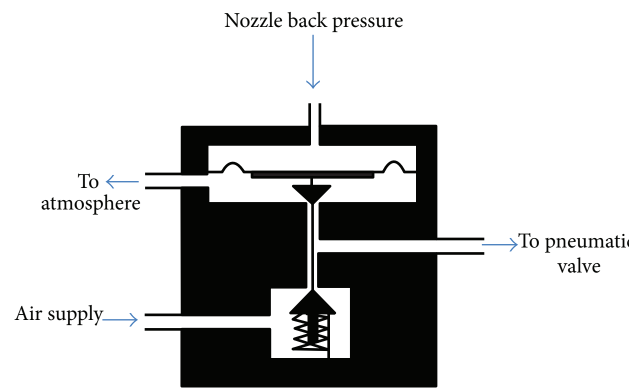

Figure 2: A simple model of bleed-type relay.

Following [26], a typical form of the four-parameter fractional derivative Zener model for viscoelastic materials can be written as

$$\sigma(t) + \tau^{\beta}\frac{d^{\beta}}{dt^{\beta}}\sigma(t) = -E_{\min}\varepsilon(t) - E_{\max}\tau^{\beta}\frac{d^{\beta}}{dt^{\beta}}\varepsilon(t), \tag{10}$$

where $\sigma(t)$ is the stress, $\varepsilon(t)$ is the strain, $E_{\max}$ and $E_{\min}$ are the maximum and minimum of elastic modulus, τ is the time constant, and β is the exponent of the fractional derivative. Based on the fact that for the pneumatic relay, the back pressure on the diaphragm is proportional to the stress and the displacement is proportional to the strain, (10) reduces to

$$p_b(t) + \tau_{\text{new}}^{\beta}\frac{d^{\beta}}{dt^{\beta}}p_b(t) = \Psi_{\min}z(t) + \Psi_{\max}\tau_{\text{new}}^{\beta}\frac{d^{\beta}}{dt^{\beta}}z(t), \tag{11}$$

where $p_b(t)$ is the back pressure acting on the top diaphragm of the pneumatic relay, z is the resulting displacement of

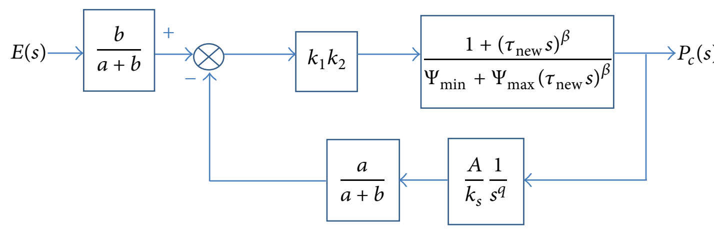

Figure 3: Block diagram for the fractional order nozzle-flapper-relay.

the diaphragm from the equilibrium, Ψ_{max} and Ψ_{min} are proportional to the maximum and minimum stiffness of the diaphragm, respectively, and τ_{new} is the new time constant which is a characteristic of the structure. Applying the Laplace transform on (11) yields

$$\frac{P_b(s)}{Z(s)} = \frac{\Psi_{min} + \Psi_{max}\tau_{new}^{\beta}s^{\beta}}{1 + \tau_{new}^{\beta}s^{\beta}}, \tag{12}$$

where $P_b(s)$ and $Z(s)$ are the Laplace transform of the back pressure signal $p_b(t)$ and the diaphragm displacement $z(t)$, respectively. The structure of the diaphragm is such that the relation between control pressure $p_c(t)$ and $z(t)$ is linear; that is,

$$P_c(s) = k_2 Z(s), \tag{13}$$

where k_2 is a positive constant and $P_c(s)$ is the Laplace transform of $p_c(t)$. Using (9), (12), and (13) one can conclude that

$$P_c(s) = k_1 k_2 \frac{1 + (\tau_{new}s)^{\beta}}{\Psi_{min} + \Psi_{max}(\tau_{new}s)^{\beta}} X(s). \tag{14}$$

For the flapper, since there are two small movements (e and y) in opposite directions, one may add up the results of two movements into one displacement x. It can be easily shown that for the flapper movement the following relationship is held:

$$x(t) = \frac{b}{a+b}e(t) - \frac{a}{a+b}y(t), \tag{15}$$

where y is the displacement of the end of the flapper due to the bellows expansion or contraction. A usual approach is to consider the bellows action as a linear spring. However, as was developed for the diaphragm, for every practical bellows a simple fractional order model can be considered as follows which we call it a pseudo-spring:

$$Ap_c(t) = k_s\, {}_0^C D_t^q y(t), \tag{16}$$

in which A is the effective area of the bellows and k_s is the equivalent pseudo-spring constant, that is, the stiffness due to the action of the corrugated side of the bellows. Using (9), (14), (15), and (16), one can develop a block diagram for the nozzle-flapper-relay, as shown in Figure 3.

The transfer function between the control pressure $P_c(s)$ and the error signal $E(s)$ can be obtained using Mason's formula as follows:

$$\begin{aligned}\frac{P_c(s)}{E(s)} &= \left(bk_s k_1 k_2 s^q \left(1 + (\tau_{new}s)^{\beta}\right)\right) \\ &\quad \times \left((a+b)k_s s^q \left(\Psi_{min} + \Psi_{max}(\tau_{new}s)^{\beta}\right)\right. \\ &\quad \left. + aAk_1k_2\left(1 + (\tau_{new}s)^{\beta}\right)\right)^{-1}.\end{aligned} \tag{17}$$

As can be seen, this controller configuration yields a fractional order proper transfer function. In the following sections, by inserting a restrictor in the feedback path, a fractional order derivative and integral performance are achieved.

Remark 1. It should be emphasized that in using transfer function (by the definition) all initial conditions must be set to zero. Since we have utilized the Caputo's fractional derivative, all initial conditions are those which have been considered in the integer order cases. In other words, from (5), in computing the Laplace transform of the Caputo's derivative, we need integer order derivatives of the signal; that is, $L\{{}_0^C D_t^q x(t)\} = s^q X(s) - \sum_{k=0}^{m-1} s^{q-k-1}x^{(k)}(0)$ in which necessity of $x^{(k)}(0)$ is apparent. So, setting these initial conditions to zero in the development of transfer functions is naïve. Indeed, if the Riemann-Liouville derivative had been used, setting the initial conditions to zero does not have any meaning. Moreover, as there is not any superfluous element in all proposed pneumatic structures, one can conclude that the system is completely characterized by its transfer function which implies that the contributions of the consistent initial conditions can be seen in the response given by the transfer function. Any initial conditions can be regarded in the simulation by setting the desired values in the integrator parts which is an option in Simulink package.

3.2. Pneumatic Controller with Fractional Order Derivative Performance. Consider the pneumatic controller configuration shown in Figure 4, in which a restriction in the negative feedback path modifies the previous controller introduced in (17).

Assuming a small positive step change in the error signal e, the control pressure p_c changes almost instantaneously. The restriction R momentarily prevents the feedback bellows from sensing the pressure change p_c. Thus, the feedback

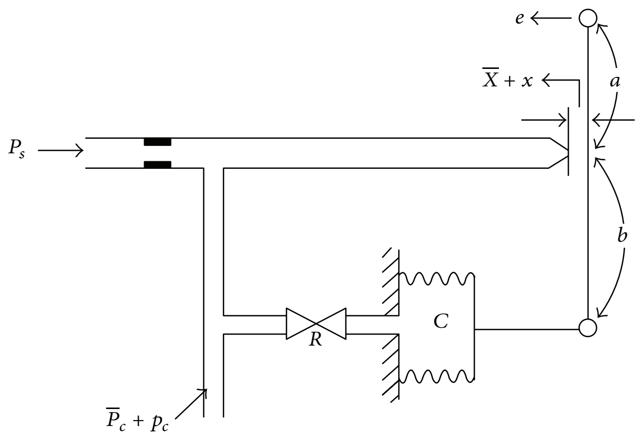

Figure 4: Pneumatic controller with a restriction in the feedback path.

bellows does not respond momentarily, and the pneumatic actuating valve feels the full effect of the flapper movement. As the time goes on, the feedback bellows expands. The expansion of the bellows critically depends on the type of the material used in the construction of the bellows. The usual materials used in the practical bellows are not purely spring and exhibit something between elastic and semiplastic behavior. The differential equation that could describe this behavior under pressure changes is very complicated. Using a fractional order operator to describe such a pseudo-spring bock is logical, because the order of the operator gives an additional freedom to match the behavior. As can be observed in a practical implementation, the signal p_c decays in time [27], and thus, the controller is of the derivative type or more accurately a proportional plus fractional order derivative. Indeed similar to the methodology discussed in Section 3.1, one can consider a fractional order model for this capacitance-resistance connection as $1/(RCs^{\nu}+1)$.

Thus, a simplified block diagram for the controller which is called fractional order derivative controller can be developed as depicted in Figure 5.

The transfer function of the control pressure with respect to the error signal is

$$\begin{aligned}
&\frac{P_c(s)}{E(s)} \\
&= \left(\frac{b}{a+b}k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right) \\
&\quad\times\left(1+k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right. \\
&\qquad\left.\times\left(\frac{a}{a+b}\right)\left(\frac{A}{k_ss^q}\right)\left(\frac{1}{RCs^{\nu}+1}\right)\right)^{-1} \\
&= \left(k_1k_2b\left(k_ss^q\right)\left(RCs^{\nu}+1\right)\left(1+(\tau_{\text{new}}s)^{\beta}\right)\right) \\
&\quad\times\left(\left(\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}\right)(a+b)\right. \\
&\qquad\left.\times\left(k_ss^q\right)\left(RCs^{\nu}+1\right)+k_1k_2aA\left(1+(\tau_{\text{new}}s)^{\beta}\right)\right)^{-1} \\
&= \left(K\left(RC\tau_{\text{new}}^{\beta}s^{q+\nu+\beta}+RCs^{q+\nu}+s^q+\tau_{\text{new}}^{\beta}s^{q+\beta}\right)\right) \\
&\quad\times\left(\kappa\left(\Psi_{\max}RC\tau_{\text{new}}^{\beta}s^{q+\nu+\beta}+\Psi_{\max}\tau_{\text{new}}^{\beta}s^{q+\beta}\right.\right. \\
&\qquad\left.\left.+\Psi_{\min}RCs^{q+\nu}+\Psi_{\min}s^q\right)+\eta+\eta\tau_{\text{new}}^{\beta}s^{\beta}\right)^{-1}
\end{aligned} \tag{18}$$

in which $K=k_1k_2k_sb$, $\kappa=k_s(a+b)$, and $\eta=k_1k_2aA$.

Remark 2. Note that if $|k_1k_2((1+(\tau_{\text{new}}s)^{\beta})/(\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}))(a/(a+b))(A/k_ss^q)(1/(RCs^{\nu}+1))|\gg 1$, then

$$\begin{aligned}
&\frac{P_c(s)}{E(s)} \\
&\cong\left(\frac{b}{a+b}k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right) \\
&\quad\times\left(k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right. \\
&\qquad\left.\times\left(\frac{a}{a+b}\right)\left(\frac{A}{k_ss^q}\right)\left(\frac{1}{RCs^{\nu}+1}\right)\right)^{-1} \\
&= M\left(RCs^{q+\nu}+s^q\right),
\end{aligned} \tag{19}$$

where $M=bk_s(aA)^{-1}$. It can be easily seen that this transfer function is a pure fractional order derivative controller with two tunable orders. For this reason, we call it a fractional order $D+D$ controller.

3.3. Pneumatic Controller with Fractional Order Integral Performance

Similar to the previous discussions in Section 3.2, we can develop a pneumatic controller with fractional order integral performance. Consider the controller configuration in Figure 6.

The bellows denoted by I is connected to the control pressure source without any restriction, though a restriction can be implemented for it. The bellows denoted by II is connected to the control pressure source through a restriction denoted by R. A small positive step change in the actuating error, e, will cause the back pressure in the nozzle to change instantaneously, and thus a change in the control pressure, p_c, also occurs instantaneously. Due to the restriction of the valve in the path to bellows II, there will be a pressure drop across the valve. As the time goes, the air flows across the valve in such a way that the pressure change in bellows II reaches p_c. Thus, bellows II will expand or contract as the time elapses causing to move the flapper an additional amount in the direction of the original displacement e. This will cause

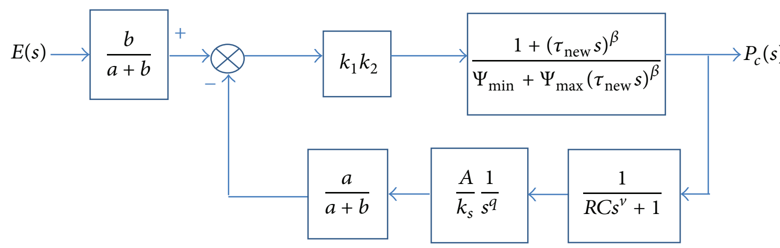

FIGURE 5: A simplified block diagram for fractional order PD controller.

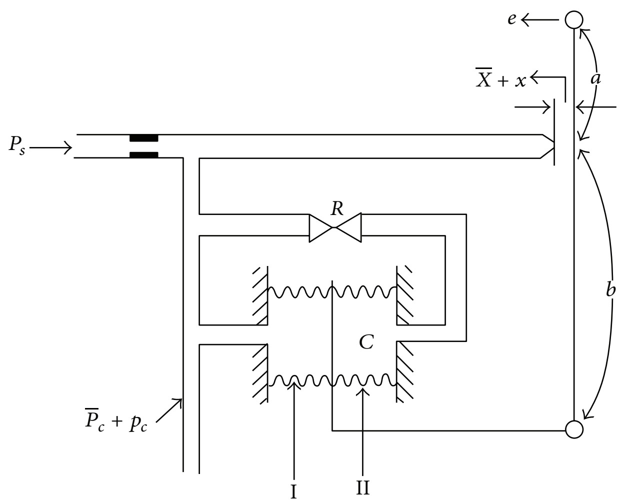

FIGURE 6: Pneumatic controller with a restriction in the feedback path which depicts an integral performance.

the back pressure, p_b, in the nozzle to change continuously. In other words in addition to the direct action of the bellows I, a positive feedback effect emerges as the time goes on. Using the methodology employed in the previous section, it can be deduced that the obtained performance is inherently an integral action. However, because of the resulted fractional order behavior of the bellows and the diaphragm of the pneumatic relay, one can draw a block diagram as shown in Figure 7. The effect of positive feedback can be observed easily from the figure.

In this block diagram we denote the fractional order model for the restrictor-bellows II with orders w. Note that the fractional order integral control action in the controller takes the form of slowly canceling the feedback that the proportional control originally provided.

Using Mason's formula, one can develop the transfer function for the block diagram sketched in Figure 7:

$$\begin{aligned}\frac{P_c(s)}{E(s)} &= \left(\frac{b}{a+b}k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right)\\ &\quad\times\left(1+k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right.\\ &\quad\left.\times\left(\frac{a}{a+b}\right)\left(\frac{A}{k_ss^q}\right)\left(1-\frac{1}{RCs^w+1}\right)\right)^{-1}\\ &= \left(bk_1k_2k_ss^q\left(RCs^w+1\right)\left(1+(\tau_{\text{new}}s)^{\beta}\right)\right)\\ &\quad\times\left((a+b)\,k_ss^q\left(RCs^w+1\right)\left(\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}\right)\right.\\ &\quad\left.+aAk_1k_2RCs^w\left(1+(\tau_{\text{new}}s)^{\beta}\right)\right)^{-1}.\end{aligned}\tag{20}$$

Notice that if $|k_1k_2((1+(\tau_{\text{new}}s)^{\beta})/(\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}))(a/(a+b))(A/k_ss^q)(RCs^w/(RCs^w+1))| \gg 1$, (20) can be simplified as follows:

$$\begin{aligned}\frac{P_c(s)}{E(s)} &\cong \left(\frac{b}{a+b}k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right)\\ &\quad\times\left(k_1k_2\left(\frac{1+(\tau_{\text{new}}s)^{\beta}}{\Psi_{\min}+\Psi_{\max}(\tau_{\text{new}}s)^{\beta}}\right)\right.\\ &\quad\left.\times\left(\frac{a}{a+b}\right)\left(\frac{A}{k_ss^q}\right)\left(1-\frac{1}{RCs^w+1}\right)\right)^{-1}\\ &= \frac{bk_ss^q\left(RCs^w+1\right)}{aARCs^w} = Ms^q\left(1+\frac{1}{RCs^w}\right)\end{aligned}\tag{21}$$

in which $M = bk_s(aA)^{-1}$. Note that if $w > q$, the transfer function indicates an integral plus derivative action, which can be called a fractional $I + D$ controller.

Remark 3. Using a similar method, one can develop a fractional PID controller for a nozzle-flapper-relay configuration.

4. Simulation Results

In this section some simulations are given for the developed theory in the previous sections.

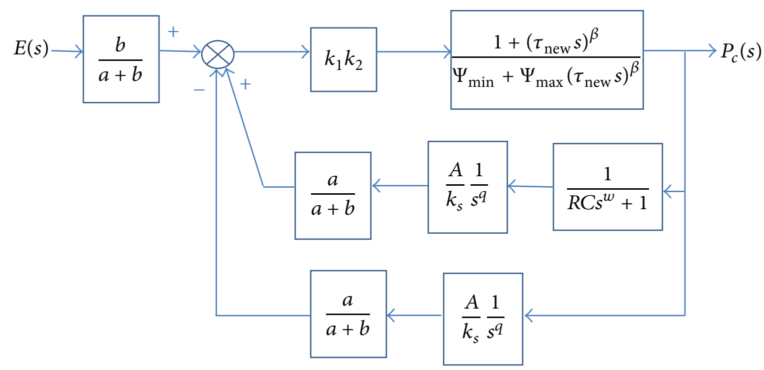

FIGURE 7: A simplified block diagram for fractional order PI controller.

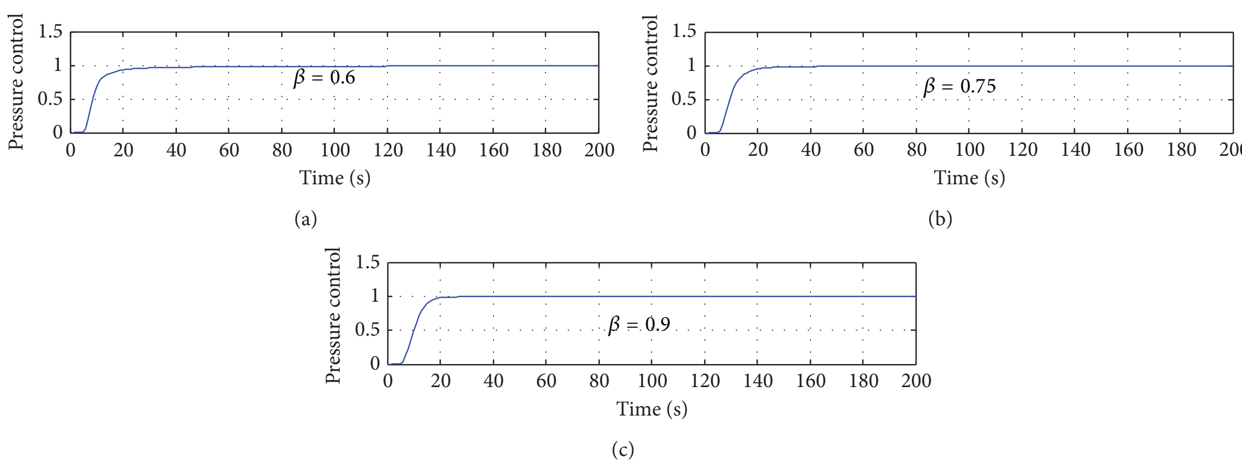

FIGURE 8: Step response of the nozzle-amplifier-relay (17) for various betas.

Example 4. Consider the following values for a nozzle-flapper amplifier discussed in Section 3.1: $b = 9$, $a = 1$, $k_1 = 0.1$, $k_2 = 0.1$, $A = 2$, $k_s = 20$, $\tau_{new} = 0.001$, $\Psi_{min} = 1$, $\Psi_{max} = 10$, $\beta = 0.8$, $q = 0.9,\ 0.99,\ 1$.

The step responses are depicted in Figure 8. As can be seen for larger β's the settling time will be better.

Remark 5. The values of k_1 and k_2 have been chosen from the procedure and graphs given in [28]. The tuning technique presented there is a rough approach to us for choosing the values of the pneumatic structures presented in this paper. We have used the algorithm and graphs proportionally to other parameters in the nozzle-flapper presented in our configurations. As the optimality of such values in the integer-order counterparts is a difficult problem (and in general is an open problem), its status in the fractional order cases is consequently more difficult and up to now there is no trend in choosing the parameters optimally.

Example 6. Now consider the nozzle-flapper-relay studied in Section 3.2. The parameters are similar to those of Example 4, which additionally $v = 0.94$ and $RC = 0.1$. Note that because of inherent hysteresis in the practical diaphragm and bellows, a delay is observed in the step responses which present a more accurate response than classical models using linear spring. Thus, considering fractional order model for the diaphragms and bellows is justified and is more consistent with practice.

Moreover the step changes in the error signal yield a differentiation performance for the apparatus designed in Figure 9.

Example 7. As the final example consider the nozzle-flapper-relay studied in Section 3.3. The parameters are similar to those of Example 4, which additionally $w = 0.94$ and $RC = 0.1$ and $k_1 = 2$, $k_2 = 5$, and $q = 0$. As can be seen from the figure if we set $q = 0$, the configuration shown in Figure 6 is a proportional plus fractional integral controller. The step responses are proposed in Figure 10.

Remark 8. We have employed the Simulink package to simulate these numerical examples. All standard blocks have been taken from its library. However, the fractional order derivative blocks have been generated via Oustaloup approximation frequency technique which is a routine and reliable method in simulating the fractional order control systems [29].

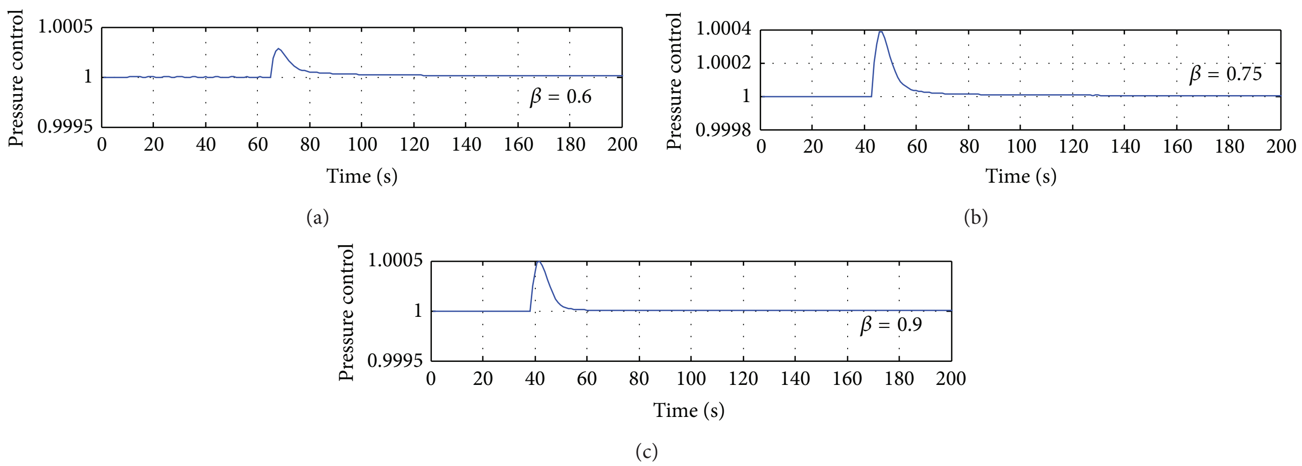

Figure 9: Step response of the fractional order derivative nozzle-flapper-relay for (20).

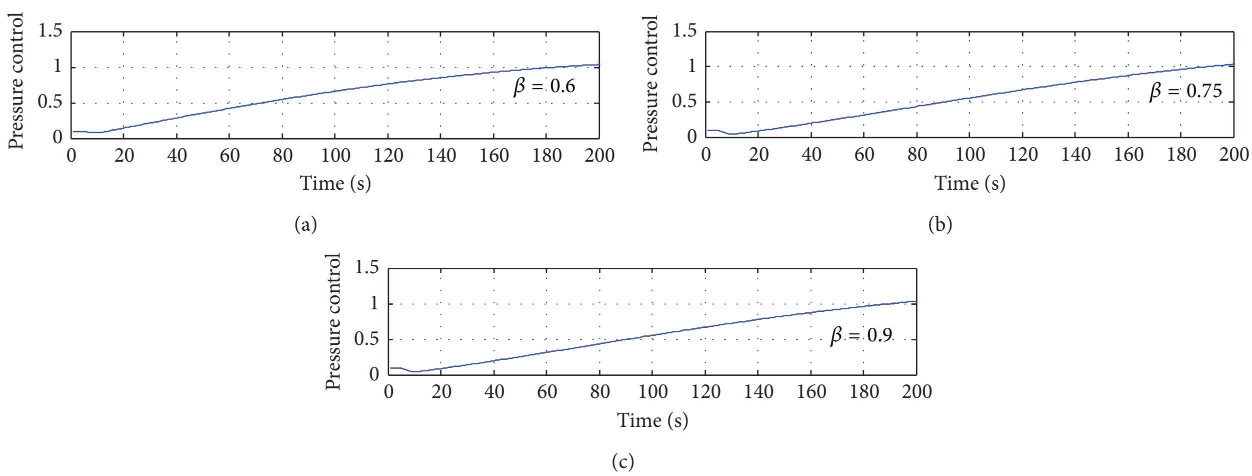

Figure 10: Step response of the fractional order derivative nozzle-flapper-relay for (21).

5. Conclusions

In this paper in the light of fractional calculus, some new models have been developed for three configurations of nozzle-flapper-relay amplifier. Indeed using a typical form of the four-parameter fractional derivative Zener model for viscoelastic materials, a fractional order model has been provided for the diaphragm of pneumatic relays and the spring property of the bellows. After developing block diagrams for each configuration, a fractional order derivative and integral controller with two freedoms in orders have been obtained. The fractional orders that are imposed in the models can be used as other adjustable parameters of the controllers which inherently depend upon the material of the bellows and diaphragms. Some numerical simulations are proposed to clarify the obtained results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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