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Research article

Hilfer fractional neutral stochastic differential equations with non-instantaneous impulses

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Abstract: The aim of this manuscript is to investigate the existence of mild solution of Hilfer fractional neutral stochastic differential equations (HFNSDEs) with non-instantaneous impluses. We establish a new criteria to guarantee the sufficient conditions for a class of HFNSDEs with non-instantaneous impluses of order $0 < \beta < 1$ and type $0 \leq \alpha \leq 1$ is derived with the help of semigroup theory and fixed point approach, namely M$\ddot{o}$nch fixed point theorem. Finally, a numerical example is provided to validate the theoretical results.

Keywords: existence; non-instantaneous impulsive equation; Hilfer fractional stochastic system; semigroup theory; M$\ddot{o}$nch fixed point theorem
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1. Introduction

In recent years, the fractional calculus (FC) has enjoyed considerable importance in the field of science and engineering, physics, fluids mechanics, biological, chemical, finance markets and viscoelasticity. Moreover, FC is the more generalization of differentiation and integration. On the otherhand, the theory and practical application of the fractional differential equations (FDEs) in the field of science, finance and many other areas. The wide application of FDEs could be seen in the monographs [16, 17, 21, 25, 28, 30] and the references therein [11, 15, 19].

Hilfer [16] popularized a special kind of fractional derivative, which are includes both Riemann-Liouville (R-L) derivative and Caputo fractional derivative as a special kind such as the implication and application of Hilfer fractional derivative (HFD) implement in the theoretical simulation of rouse model, relaxation and diffusion models for biophysical phenomena, dielectric relaxation in glass forming materials, etc. Firstly, many researchers have been done in the field of existence of Hilfer fraction evolution equation and non-local condition (see [1–4, 19]).

On the other hand, deterministic models often fluctuate due to environmental noise. Therefore to have better performance in the models are widespread use. Therefore, it is necessary to move from deterministic case to stochastic ones. Stochastic differential equations (SDEs) are crucial application in many developement field of engineering and science . For other details on SDEs the authors can refer to the books [8, 20, 23, 26] and the articles therein [6, 7, 11]. Impulsive fractional differential equations (IFDEs) is an effective mathematical tool to model in both the physical and social sciences. There has a significant development in impulsive theory especially in the area of IFDEs with fixed moments and the references therein [5, 17, 18, 25, 30]. Although, all physical system which evolve with respect to time are suffered by small abrupt changes in the form of impulses. These impulse can be specified into two cases:
(i) Instantaneous impulsive differential equations (IIDEs).
(ii) Non-instantaneous impulsive differential equations (NIIDEs).

IIDEs: i.e., in the system, impulse occurs for a short time period which is negligible on comparing with overall time period is instantaneous impulse. The second type NIIDEs i.e., impulsive disturbance which starts at time and remains active on a finite time period is non-instantaneous impulsive. Inspite of, the action of instantaneous impulsive phenomena seen as do not describe some certain dynamics of evolution processes in pharmacotherapy. For example, high or low levels of glucose, one can prescribe some intravenous drugs (insulin). The introduction of the drugs in the blood stream and the consequent absorption for the body are gradual and continuous process. To this end, Hernandez and O'Regan [14] introduce the NIIDEs. It also can be broadly used in medical science, mechanical engineer and any other fields. For instance, bursting rhythm models in medicine, biological phenomena involving thresholds, learning control model and biology. For more details on NIIDEs see [12, 15, 24, 29]. To the best of our knowledge, there are finite works by considering the existence of HFSDEs with impulsive effect. Motivated by the above works HFNSDEs with non-instantaneous impluses, very recently, many researchers have done in the excellent field of the existence of mild solutions for a class of HFSDEs in Hilbert space see [1–4, 13, 19, 27].

Although, to the best of our knowledge the existence of HFNSDEs with non-instantaneous impluses has not been examined yet. Many researchers express the existence results by the familiar definitions of fractional derivatives defined by Caputo and R-L sense. HFD, it is universality of R-L fractional derivative and Caputo fractional derivative. The proposed work on the existence of HFNSDEs with non-instantaneous impluses is original to the literature and more general result than the existing literature. Therefore, in this work we consider the following HFNSDEs with non-instantaneous impluses to study the existence of mild solution:

$$\begin{aligned}
\mathfrak{D}_{0^+}^{\alpha,\beta}[\mathrm{u}(\mathrm{t})-\mathfrak{h}(\mathrm{t},\mathrm{u}(\mathrm{t}))] &= \mathfrak{A}[\mathrm{u}(\mathrm{t})-\mathfrak{h}(\mathrm{t},\mathrm{u}(\mathrm{t}))]+\mathfrak{f}(\mathrm{t},\mathrm{u}(\mathrm{t})) \\
&\quad + \int_0^s \mathfrak{g}(\tau,\mathrm{u}(\tau))dw(\tau),\ \mathrm{t}\in(\mathrm{s}_i,\mathrm{t}_{i+1}]\subset \mathrm{J}^{'} := (0,\mathrm{b}],\ i=0,1,2,\cdots,\mathbb{N} \\
\mathrm{u}(\mathrm{t}) &= \mathfrak{I}_i(\mathrm{t},\mathrm{u}(\mathrm{t})),\ \mathrm{t}\in(\mathrm{t}_i,\mathrm{s}_i],\ i=1,2,\cdots,\mathbb{N}
\end{aligned}$$

$$\mathfrak{I}_{0^+}^{(1-\gamma)}[\mathrm{u}(0) - \mathfrak{h}(0, \mathrm{u}(0))] \quad = \quad \mathrm{u}_0, \ \ \gamma = \alpha + \beta - \alpha\beta. \tag{1.1}$$

where $\mathrm{u}(\cdot) \in \mathfrak{X}$ a real separable Hilbert space; its inner product and norm are defined as follows: $< \cdot, \cdot >_{\mathfrak{X}}$, $\|\cdot\|_{\mathfrak{X}}$. Here $\mathrm{J} := [0, \mathrm{b}]$ and $\mathrm{J}' := (0, \mathrm{b}]$ denote the time intervals. The operators $\mathfrak{A} : \mathscr{D}(\mathfrak{A}) \subset \mathfrak{X} \to \mathfrak{X}$ is the infinitesimal generator of a strongly continuous semigroup of a bounded linear operator $\mathfrak{T}(t)$, $\mathrm{t} \geq 0$ on $\mathfrak{X}$, for more details on semigroup operators refer [26]. Let $\mathfrak{Y}$ be another separable Hilbert space, with norm $\|\cdot\|_{\mathfrak{Y}}$ and inner product $< \cdot, \cdot >_{\mathfrak{Y}}$. The functions $\mathfrak{h}, \mathfrak{f}$ and $\mathfrak{g}$ defined later.

The primary contribution and advantage of this article can be foreground as follows:

(1) For the first time in literature, existence of solution of HFNSDEs with non-instantaneous impluses is investigated.
(2) New set of sufficient conditions are established for the existence of mild solution of HFNSDEs with non-instantaneous impluses in system (1.1). This work generalizes many results obtained for fractional SDEs involving Caputo and R-L fractional derivatives.
(3) The property of Hausdorff measure of non compactness is adopted to prove the relatively compact conditions.
(4) The aimed of our technique relies on M*ö*nch fixed point theorem is effectively used to establish the new results.
(5) The proposed theoretical results through a numerical example .

The manuscript is formulated listed as follows: we will present some basic definitions for fractional operators and also the solution representation of HFNSDEs with non-instantaneous impluses will be discussed in Section 2. In Section 3, by applying M*ö*nch fixed point theorem and hypotheses, existence of mild solution of system (1.1) is proved. We illustrate the effectiveness of the theoretical results through a numerical example in Section 4. At last, conclusion is drawn in Section 5.

2. Preliminaries

This section contains basic preliminaries, and notations:

Let $(\Omega, \mathfrak{I}, \mathbb{P})$ be a complete probability space furnished with complete family of right continuous increasing sub σ-algebras $\{\mathfrak{I}_\mathrm{t}, \mathrm{t} \in \mathrm{J}\}$ satisfying $\mathfrak{I}_\mathrm{t} \in \mathfrak{I}$. The collection of all strongly measurable, p^{th} mean square integrable $\mathfrak{X}$-valued random variable, denoted by $\mathcal{L}^\mathrm{p}(\Omega, \mathfrak{I}, \mathbb{P}, \mathfrak{X}) \equiv \mathcal{L}_\mathrm{p}(\Omega, \mathfrak{X})$ with a Banach space equipped with norm

$$\|\mathrm{u}(\cdot)\|_{\mathcal{L}_\mathrm{p}(\Omega,\mathfrak{X})} = \left(\mathbb{E}\,\|\mathrm{u}(\cdot, w)\|_{\mathfrak{X}}^{\mathrm{p}}\right)^{1/\mathrm{p}}.$$

Let $\mathcal{L}(\mathfrak{Y}, \mathfrak{X})$ defined the space of all bounded linear operators from $\mathfrak{Y}$ into $\mathfrak{X}$, whenever $\mathfrak{X} = \mathfrak{Y}$, and denote by $\mathcal{L}(\mathfrak{Y})$. $Q \in \mathcal{L}(\mathfrak{Y})$ represents a non-negative self-adjoint operator. Let $\mathcal{L}_2^0 = \mathcal{L}_2(Q^{\frac{1}{2}}\mathfrak{Y}, \mathfrak{X})$ be the space of all Hilbert-Schmidt operators from $Q^{\frac{1}{2}}\mathfrak{Y}$ into $\mathfrak{X}$, $\psi \in \mathcal{L}_2^0$ is called a Q-Hilbert-Schmidt operator. For $\mathrm{a} \in [0, \mathrm{b})$ and $\gamma \in [0, 1]$, consider the weighted spaces of continuous functions

$$\mathfrak{C}_\gamma([\mathrm{a}, \mathrm{b}], \mathcal{L}_\mathrm{p}(\Omega, \mathfrak{X})) = \left\{\mathrm{u} \in \mathfrak{C}([\mathrm{a}, \mathrm{b}], \mathcal{L}_\mathrm{p}(\Omega, \mathfrak{X})) : (\mathrm{t} - \mathrm{a})^\gamma \mathrm{u}(\mathrm{t}) \in \mathfrak{C}([\mathrm{a}, \mathrm{b}], \mathcal{L}_\mathrm{p}(\Omega, \mathfrak{X}))\right\}.$$

Now, define $\mathfrak{C}([\mathrm{a}, \mathrm{b}], \mathcal{L}_\mathrm{p}(\Omega, \mathfrak{X}))$ is a Banach space with norm

$$\mathbb{E}\,\|\mathrm{u}\|_{\mathfrak{C}([\mathrm{a},\mathrm{b}],\mathcal{L}_\mathrm{p}(\Omega,\mathfrak{X}))} = (\sup_{\mathrm{t}\in(\mathrm{a},\mathrm{b}]} (\mathrm{t} - \mathrm{a})^\gamma \,\|\mathrm{u}(\mathrm{t})\|^\mathrm{p}).$$

Let $J_k = (s_k, t_{k+1}]$, $\bar{J}_k = [s_k, t_{k+1}]$ ($k = 0, 1, 2, \cdots, \mathbb{N}$), $\mathfrak{T}_i = (t_i, s_i]$, $\bar{\mathfrak{T}}_i = [t_i, s_i]$ ($k = 1, 2, \cdots, \mathbb{N}$). Let $\mathbb{H} = \mathscr{PC}_{1-\gamma}(J, \mathcal{L}_p(\Omega, \mathfrak{X})) = \{u : (t - s_k)^{1-\gamma} u \in J_k, \mathcal{L}_p(\Omega, \mathfrak{X})), \lim_{t \to s_k^+} (t - s_k)^{1-\gamma} u(t), u \in \mathfrak{C}(\mathfrak{T}_i, \mathcal{L}_p(\Omega, \mathfrak{X}))\}$ and $\lim_{t \to t_i^+} u(t)$ exist, $k = 0, 1, 2, \cdots, \mathbb{N}$, $i = 1, 2, \cdots, \mathbb{N}$, with

$$\begin{aligned} \|\cdot\|_{\mathbb{H}} &= \left\{\mathbb{E}\, \|u(t)\|^p_{\mathscr{PC}_{1-\gamma}(J, \mathcal{L}_p(\Omega, \mathfrak{X}))}\right\}^{\frac{1}{p}} \\ &= \max\left\{(\max_{k=0,1,2,\cdots,\mathbb{N}} \sup_{t \in J_k} \mathbb{E}\left\|(t - s_k)^{1-\gamma} u(t)\right\|^p)^{\frac{1}{p}}, (\max_{i=1,2,\cdots,\mathbb{N}} \sup_{t \in \mathfrak{T}_i} \mathbb{E}\, \|u(t)\|^p)^{\frac{1}{p}}\right\}. \end{aligned}$$

Definition 2.1. *[21] The Riemann-Liouville fractional integral operator of a function* $\mathfrak{f} : [0, +\infty) \to \mathbb{R}$ *with order* $\beta > 0$ *is*

$$\mathfrak{I}^{\beta}_{0^+} \mathfrak{f}(t) = \frac{1}{\Gamma(\beta)} \int_0^t \frac{\mathfrak{f}(s)}{(t - s)^{1-\beta}} ds, \ t > a.$$

Remark:

(i) For $\alpha = 0$ and $0 < \beta < 1$, the Hilfer fractional derivative leads as Riemann-Liouville fractional derivative:

$$\mathfrak{D}^{\alpha,\beta}_{0^+} \mathfrak{f}(t) = \mathfrak{I}^{1-\beta}_{0^+} \frac{d}{dt} \mathfrak{I}^{(1-\beta)}_{0^+} \mathfrak{f}(t) =^L \mathfrak{D}^{\beta}_{0^+} \mathfrak{f}(t).$$

(ii) For $\alpha = 1$ and $0 < \beta < 1$, the Hilfer fractional derivative becomes as Caputo derivative:

$$\mathfrak{D}^{1,\beta}_{0^+} \mathfrak{f}(t) = \mathfrak{I}^{1-\beta}_{0^+} \frac{d}{dt} \mathfrak{f}(t) =^C \mathfrak{D}^{\beta}_{0^+} \mathfrak{f}(t).$$

Lemma 2.2. *[13] The operators* $S_{\alpha,\beta}$ *and* P_β *satisfies,*

(i) $\left\{P_\beta(t),\ t > 0\right\}$ *is continuous.*
(ii) For any $t > 0$, $S_{\alpha,\beta}(t)$ *and* $P_\beta(t)$ *are bounded and linear operators,*

$$\begin{aligned} \left\|P_\beta(t) u\right\| &\leq \frac{M_T t^{\beta-1}}{\Gamma(\beta)} \|u\|. \\ \left\|S_{\alpha,\beta}(t) u\right\| &\leq \frac{M_T t^{\gamma-1}}{\Gamma(\gamma)} \|u\|,\ \gamma = (1 - \alpha)(1 - \beta). \end{aligned}$$

(iii) $\left\{P_\beta(t) : t > 0\right\}$ *and* $\left\{S_{\alpha,\beta}(t) : t > 0\right\}$ *are strongly continuous.*

Lemma 2.3. *[10] The Hausdorff measure of non compactness* $\mu(\cdot)$ *defined on each bounded subset* Λ *of the Banach space* $\mathfrak{X}$ *is given by* $\mu(\Lambda) = \inf\{\epsilon > 0; \Lambda \text{ has a finite } \epsilon - \text{ net in } \mathfrak{X}\}$. *The following are some important properties of* $\mu(\cdot)$. *If* $\mathfrak{X}$ *is a real Banach space and* $\Lambda, \Omega \subset \mathfrak{X}$ *are bounded, then the following properties hold:*

(i) Λ *is precompact iff* $\mu(\Lambda) = 0$.
(ii) $\mu(\Lambda + \Omega) \leq \mu(\Lambda) + \mu(\Omega)$, *where* $\Lambda + \Omega = \{u + v; u \in \Lambda, v \in \Omega\}$
(iii) If $W \subset \mathfrak{C}(J; \mathfrak{X})$ *is bounded and equicontinuous, then* $t \to \mu(W(t))$ *is continuous on* J, *and*

$$\mu(W) \leq \max_{t \in J} \mu(W(t)),\ \mu(\int_0^t W(s) ds) \leq \int_0^t \mu(W(s)) ds, \textit{ for all } t \in J,$$

where

$$\int_0^t W(s) ds = \left\{\int_0^t u(s) ds : \textit{ for all } u \in W, t \in J\right\}.$$

(iv) If $\{u_n\}_{n=1}^{\infty}$ *is a sequence of Bochner integrable functions from* J *into* $\mathfrak{X}$ *with* $\|u_n(\mathrm{t})\| \leq \tilde{m}(\mathrm{t})$ *for a.e.* $\mathrm{t} \in \mathrm{J}$ *and* $\forall n \geq 1$*, where* $\tilde{m}(\mathrm{t}) \in \mathcal{L}(\mathrm{J};\mathbb{R}^+)$*, then the function* $\psi(\mathrm{t}) = \mu(\{u_n\}_{n=1}^{\infty}) \in \mathcal{L}(\mathrm{J};\mathbb{R}^+)$ *and satisfies*

$$\mu(\left\{\int_0^{\mathrm{t}} u_n(\mathrm{s})d\mathrm{s} : n \geq 1\right\}) \leq 2\int_0^{\mathrm{t}} \psi(\mathrm{s})d\mathrm{s}.$$

Lemma 2.4. *[9] Let* $\mathrm{F} \subset \mathfrak{X}$ *be bounded and equicontinuous. Then* $\mu(\Lambda(\mathrm{t}))$ *is continuous on* $[0,\mathrm{b}]$*, and* $\mu(\Lambda) = \sup_{\mathrm{t}\in\mathrm{J}} \mu(\Lambda(\mathrm{t}))$*, where* $\mu(\Lambda(\mathrm{t})) = \{\mathrm{u}(\mathrm{t}) : \mathrm{u} \in \Lambda\}$*.*

Lemma 2.5. *[22] Suppose* D *is a closed convex subset of Banach space* $\mathbb{H}$*,* $0 \in \mathrm{D}$*. If* $\Phi : \mathrm{D} \to \mathbb{H}$ *is continuous and of Mönch type, (i.e.)* Φ *satisfies the property:* $\mathcal{M} \subseteq \mathrm{D}$*,* $\mathcal{M}$ *is countable,* $\mathcal{M} \subset \overline{co}(\{0\} \bigcup \Phi(\mathcal{M})) \Rightarrow \overline{\mathcal{M}}$ *is compact, then* Φ *has a fixed point in* D*.*

Lemma 2.6. *[11] For any* $\mathrm{p} \geq 1$ *and for arbitrary* $\mathcal{L}_2^0$*-valued predictable process* $\phi(\cdot)$ *such that*

$$\sup_{\mathrm{s}\in[0,\mathrm{t}]} \mathbb{E}\left\|\int_0^{\mathrm{s}} \phi(\mathrm{s})dw(\mathrm{s})\right\|_{\mathfrak{X}}^{2\mathrm{p}} \leq (\mathrm{p}(2\mathrm{p}-1))^{\mathrm{p}}\left(\int_0^{\mathrm{t}}\left[\mathbb{E}\,\|\phi(\mathrm{s})\|_{\mathcal{L}_2^0}^{2\mathrm{p}}\right]d\mathrm{s}\right)^{\mathrm{p}}, \quad \mathrm{t} \in [0,+\infty)$$

where $c_{\mathrm{p}} = (\mathrm{p}(2\mathrm{p}-1))^{\mathrm{p}}$*.*

Definition 2.7. *An* $\mathfrak{X}$*-valued* $\mathfrak{I}_{\mathrm{t}}$*-adopted stochastic process* $\mathrm{u}(\mathrm{t})$ *is called as mild solution of NIHFNSDEs (1.1) if the following integral equation is verified*

$$\mathrm{u}(\mathrm{t}) = \begin{cases} \mathrm{S}_{\alpha,\beta}(\mathrm{t})\mathrm{u}_0 + \mathfrak{h}(\mathrm{t},\mathrm{u}(\mathrm{t})) + \int_0^{\mathrm{t}} \mathrm{P}_\beta(\mathrm{t}-\mathrm{s})\mathfrak{f}(\mathrm{s},\mathrm{u}(\mathrm{s}))d\mathrm{s} \\ + \int_0^{\mathrm{t}} \mathrm{P}_\beta(\mathrm{t}-\mathrm{s})\left[\int_0^{\mathrm{s}} \mathfrak{g}(\tau,\mathrm{u}(\tau))dw(\tau)\right]d\mathrm{s} & \textit{for } \mathrm{t} \in [0,\mathrm{t}_1], \\ \mathfrak{I}_i(\mathrm{t},\mathrm{u}(\mathrm{t})), & \textit{for } \mathrm{t} \in (\mathrm{t}_i,\mathrm{s}_i], \\ \mathrm{S}_{\alpha,\beta}(\mathrm{t}-\mathrm{s}_i)\left[\mathfrak{I}_i(\mathrm{t},\mathrm{u}(\mathrm{s}_i))\right] + \mathfrak{h}(\mathrm{t},\mathrm{u}(\mathrm{s}_i)) + \int_0^{\mathrm{s}_i} \mathrm{P}_\beta(\mathrm{s}_i-\mathrm{s})\mathfrak{f}(\mathrm{s},\mathrm{u}(\mathrm{s}))d\mathrm{s} \\ + \int_0^{\mathrm{s}_i} \mathrm{P}_\beta(\mathrm{s}_i-\mathrm{s})\left[\int_0^{\mathrm{s}} \mathfrak{g}(\tau,\mathrm{u}(\tau))dw(\tau)\right]d\mathrm{s} \\ + \int_0^{\mathrm{t}} \mathrm{P}_\beta(\mathrm{t}-\mathrm{s})\mathfrak{f}(\mathrm{s},\mathrm{u}(\mathrm{s}))d\mathrm{s} + \int_0^{\mathrm{t}} \mathrm{P}_\beta(\mathrm{t}-\mathrm{s})\left[\int_0^{\mathrm{s}} \mathfrak{g}(\tau,\mathrm{u}(\tau))dw(\tau)\right]d\mathrm{s}, \\ & \textit{for } \mathrm{t} \in (\mathrm{s}_i,\mathrm{t}_{i+1}], \end{cases}$$

where

$$\begin{aligned} \mathrm{S}_{\alpha,\beta}(\mathrm{t}) &= \mathfrak{I}_{0^+}^{\alpha(1-\beta)}\mathrm{P}_\beta(\mathrm{t}), \\ \mathrm{P}(\mathrm{t}) &= \mathrm{t}^{\beta-1}(\mathrm{T})_\beta(\mathrm{t}), \\ T_\beta(\mathrm{t}) &= \int_0^\infty \beta\theta\psi_\beta(\theta)(\mathrm{T})(\mathrm{t}^\beta\theta)d\theta, \end{aligned}$$

$$\text{here } \psi_\beta(\theta) = \sum_{n=1}^{\infty} \frac{(-\theta^{n-1})}{(n-1)!\Gamma(1-n\beta)}, \ 0 < \beta < 1, \ \theta \in (0,\infty),$$

is Wright-type function which satisfies the following,

$$\int_0^\infty \theta^\xi \psi_\beta(\theta)d\theta = \frac{\Gamma(1+\xi)}{\Gamma(1+\beta\xi)} \text{ for } \theta \geq 0.$$

3. Main results

In order to prove the existence result, we impose the following hypotheses hold:

(H1) The function $\mathfrak{f} : J \times \mathfrak{X} \to \mathfrak{X}$ satisfies
(i) $u \to \mathfrak{f}(t, u)$ is continuous for a.e $t \in J$ and $t \to \mathfrak{f}(t, u)$ is strongly measurable for each $u \in \mathfrak{X}$.
(ii) $\exists$ a function $m_{\mathfrak{f}}(t) \in \mathcal{L}(J, \mathbb{R}^+)$ and non-decreasing continuous function $\Theta_1 : [0, \infty) \to (0, \infty)$ s.t for any $u \in \mathfrak{X}$ and each $t \in J$,

$$\mathbb{E}\,\|\mathfrak{f}(t, u(t))\|^p \le m_{\mathfrak{f}}(t)\Theta_1(\|u(t)\|^p_{\mathbb{H}}).$$

(iii) $\exists$ a function $\Theta_2 \in \mathcal{L}(J, \mathbb{R}^+)$ and a constant $\mathfrak{f}^* > 0$ with $\sup_{t\in J} \Theta_2(t) = \mathfrak{f}^*$ s.t for any bounded subset $D \subset \mathfrak{X}$,

$$\mu(\mathfrak{f}(t, u)) \le \Theta_2(t)[\sup_{t\in J} \mu(D(t))].$$

(H2) The function $\mathfrak{g} : J \times \mathfrak{X} \to \mathcal{L}_2^0$ satisfies
(i) $u \to \mathfrak{g}(t, u)$ is continuous for a.e $t \in J$ and $t \to \mathfrak{g}(t, u)$ is strongly measurable for each $u \in \mathfrak{X}$.
(ii) $\exists$ a function $m_{\mathfrak{g}}(t) \in \mathcal{L}(J, \mathbb{R}^+)$ and a continuous non-decreasing function $\Theta_3 : [0, \infty) \to (0, \infty)$ s.t for any $u \in \mathfrak{X}$ and each $t \in J$,

$$\mathbb{E}\,\|\mathfrak{g}(t, u(t))\|^p_{\mathcal{L}_2^0} \le m_{\mathfrak{g}}(t)\Theta_3(\|u(t)\|^p_{\mathbb{H}}).$$

(iii) $\exists$ a function $\Theta_4 \in \mathcal{L}(J, \mathbb{R}^+)$ and a constant $\mathfrak{g}^* > 0$ with $\sup_{t\in J} \Theta_4(t) = \mathfrak{g}^*$ s.t for any bounded subset $D \subset \mathfrak{X}$,

$$\mu(\mathfrak{g}(t, u)) \le \Theta_4(t)[\sup_{t\in J} \mu(D(t))].$$

(H3) The functions $\Im_i : (t_i, s_i] \times \mathfrak{X} \to \mathfrak{X}$, $i = 1, 2, \cdots, \mathbb{N}$ are continuous and satisfy the following conditions:
(i) For $r > 0$, $\exists$ +ve functions $\rho_i(r)$, $i = 1, 2, \cdots, \mathbb{N}$ dependent on r s.t

$$\mathbb{E}\,\|\Im_i(t, u(t))\|^p_{\mathfrak{X}} \le \rho_i(r).$$

(ii) $\exists$ constants $\bar{\rho}_i > 0$ s.t for each bounded subset $D \subset \mathfrak{X}$,

$$\mu(\Im_i(t, D)) \le \bar{\rho}_i \sup_{t\in(t_i, s_i]} \mu(D(t)),\ i = 1, 2, \cdots, \mathbb{N}.$$

(H4) (i) The functions $\mathfrak{h} : J \times \mathfrak{X} \to \mathfrak{X}$ is continuous, and $\exists$ a $m_{\mathfrak{h}} > 0$ s.t $\forall$ $t \in J$, $u, v \in \mathfrak{X}$

$$\mathbb{E}\,\|\mathfrak{h}(t, u(t)) - \mathfrak{h}(t, v(t))\|^p \le m_{\mathfrak{h}}(\|u - v\|^p_{\mathbb{H}}),$$

$$\mathbb{E}\,\|\mathfrak{h}(t, u(t))\|^p \le m_{\mathfrak{h}}(1 + \|u\|^p_{\mathbb{H}}).$$

(ii) $\exists$ a function $\Theta_5 \in \mathcal{L}(J, \mathbb{R}^+)$ and a constant $\mathfrak{h}^* > 0$ with $\sup_{t\in J} \Theta_5(t) = \mathfrak{h}^*$ s.t for any bounded subset $D \subset \mathfrak{X}$,

$$\mu(\mathfrak{h}(t, u)) \le \Theta_5(t)[\sup_{t\in J} \mu(D(t))].$$

(**H6**)

$$\Lambda^* = \Big\{\mathfrak{h}^* + 2\left[\frac{M_T}{\Gamma(\beta)}\right]\left[\frac{t_1^\beta}{\beta}\right](\mathfrak{f}^* + \sqrt{t_1}c_p\mathfrak{g}^*) + \max_{i=1,2,\dots,\mathbb{N}}(\bar{\rho}_i)$$
$$+\left[\frac{M_T}{\Gamma(\gamma)}\right](t_{i+1} - s_i)^{p(1-\gamma)}[\bar{\rho}_i] + \mathfrak{h}^* + 4\left[\frac{M_T}{\Gamma(\beta)}\right]\left[\frac{b^\beta}{\beta}\right]\Big(\mathfrak{f}^* + c_p\mathfrak{g}^*\sqrt{b}\Big)\Big\} < 1.$$

Theorem 3.1. *If assumptions* (**H1**) – (**H6**) *holds. Then NIHFNSDEs of the Eq (1.1) has a mild solution on* J.

Proof: Define an operator $\Phi : \mathbb{H} \to \mathbb{H}$ as follows:

$$(\Phi x)(t) = \begin{cases} S_{\alpha,\beta}(t)u_0 + \mathfrak{h}(t, u(t)) + \int_0^t P_\beta(t-s)\mathfrak{f}(s, u(s))ds \\ + \int_0^t P_\beta(t-s)\left[\int_0^s \mathfrak{g}(\tau, u(\tau))dw(\tau)\right]ds, \quad \text{for } t \in [0, t_1],\ i = 0. \\ \mathfrak{I}_i(t, u(t)), \quad \text{for } t \in (t_i, s_i],\ i \geq 1. \\ S_{\alpha,\beta}(t - s_i)\left[\mathfrak{I}_i(t, u(s_i))\right] + \mathfrak{h}(t, u(t)) + \int_0^{s_i} P_\beta(s_i - s)\mathfrak{f}(s, u(s))ds \\ + \int_0^{s_i} P_\beta(s_i - s)\left[\int_0^s \mathfrak{g}(\tau, u(\tau))dw(\tau)\right]ds + \int_0^t P_\beta(t-s)\mathfrak{f}(s, u(s))ds \\ + \int_0^t P_\beta(t-s)\left[\int_0^s \mathfrak{g}(\tau, u(\tau))dw(\tau)\right]ds, \quad \text{for } t \in (s_i, t_{i+1}], i \geq 1. \end{cases}$$

By using M*ö*nch fixed point theorem, we prove that Φ has a fixed point which is a mild solution of (1.1). The proof is given in the following four steps.

Step 1: Φ maps bounded set into bounded set in $\mathbb{H}$.

Indeed, it is sufficient to prove for any $r > 0$, $\exists$ a $L > 0$ s.t for each $u \in \mathbb{B}_r = \Big\{u \in \mathbb{H},\ \|u\|_{\mathbb{H}}^p < r\Big\}$, we have $\|\Phi u\|_{\mathbb{H}}^p \leq L$

For $t \in [0, t_1]$,

$$\sup_{t\in[0,t_1]} t_1^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u)(t)\|^p \leq 4^{p-1}\sup_{t\in[0,t_1]} t_1^{p(1-\gamma)}\Big\{\mathbb{E}\left\|S_{\alpha,\beta}(t)u_0\right\|^p + \mathbb{E}\,\|\mathfrak{h}(t, u(t))\|^p$$
$$+\mathbb{E}\left\|\int_0^t P_\beta(t-s)\mathfrak{f}(s, u(s))ds\right\|^p$$
$$+\mathbb{E}\left\|\int_0^t P_\beta(t-s)[\int_0^s \mathfrak{g}(\tau, x(\tau))dw(\tau)]ds\right\|^p\Big\}$$
$$\leq 4^{p-1}\sum_{i=1}^4 \mathcal{G}_i. \tag{3.1}$$

By Lemma 2.3, we get,

$$\mathcal{G}_1 = \mathbb{E}\left\|S_{\alpha,\beta}(t)u_0\right\|^p$$
$$\leq \left[\frac{M_T}{\Gamma(\gamma)}t_1^{\gamma-1}\right]^p\mathbb{E}\,\|u_0\|^p.$$

By using Lemma 2.3, and (**H4**), we have,

$$\mathcal{G}_2 = \mathbb{E}\,\|\mathfrak{h}(t, u(t))\|^p$$

$$\leq m_{\mathfrak{h}}(1+\|u\|_{\mathbb{H}}^{p})$$
$$\leq m_{\mathfrak{h}}(1+r).$$

Using Hölder inequality, Lemma (**H1**)(**ii**) we get,

$$\begin{aligned}
\mathcal{G}_3 &= \mathbb{E}\left\|\int_0^t P_\beta(t-s)\mathfrak{f}(s,u(s))ds\right\|^p \\
&\leq \left[\int_0^t \left(\frac{M_T}{\Gamma(\beta)}\right)(t-s)^{\beta-1}ds\right]^p \mathbb{E}\,\|\mathfrak{f}(s,u(s))\|^p \\
&\leq \left[\frac{M_T}{\Gamma(\beta)}\right]^p \left[\frac{t_1^\beta}{\beta}\right]^{p-1} \left(\int_0^t m_{\mathfrak{f}}(s)\Theta_1(\|u(t)\|_{\mathbb{H}}^p)ds\right) \\
&\leq \left[\frac{M_T}{\Gamma(\beta)}\right]^p \left[\frac{t_1^\beta}{\beta}\right]^{p-1} \left(\int_0^t m_{\mathfrak{f}}(s)ds\right)\Theta_1(r).
\end{aligned}$$

By Lemma 2.3, and (**H2**)(**ii**) we have,

$$\begin{aligned}
\mathcal{G}_4 &= \mathbb{E}\left\|\int_0^t P_\beta(t-s)[\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)]ds\right\|^p \\
&\leq \left[\frac{M_T}{\Gamma(\beta)}\right]^p c_p\left[\int_0^t (t-s)^{\beta-1}\left(\int_0^s \mathbb{E}\,\|\mathfrak{g}(\tau,u(\tau))\|^p d\tau\right)^{\frac{2}{p}} ds\right]^{\frac{p}{2}} \\
&\leq \left[\frac{M_T}{\Gamma(\beta)}\right]^p c_p\left[\int_0^t (t-s)^{\beta-1}ds\right]^{\frac{p}{2}} \left(\int_0^s \mathbb{E}\,\|\mathfrak{g}(\tau,u(\tau))\|^p ds\right) \\
&\leq \left[\frac{M_T}{\Gamma(\beta)}\right]^p c_p\left[\frac{t_1^\beta}{\beta}\right]^{\frac{p}{2}} \left(\int_0^t m_{\mathfrak{g}}(s)ds\right)\Theta_3(r).
\end{aligned}$$

From the above, (3.1) becomes,

$$\begin{aligned}
\sup_{t\in[0,t_1]} t_1^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u)(t)\|^p \leq 4^{p-1} \sup_{t\in[0,t_1]} t_1^{p(1-\gamma)}&\left\{\left[\frac{M_T}{\Gamma(\gamma)}t_1^{\gamma-1}\right]^p \mathbb{E}\,\|u_0\|^p + m_{\mathfrak{h}}(1+r)\right. \\
&+\left[\frac{M_T}{\Gamma(\beta)}\right]^p \left[\frac{t_1^\beta}{\beta}\right]^{p-1}\left(\int_0^t m_{\mathfrak{f}}(s)ds\right)\Theta_1(r) \\
&\left.+\left[\frac{M_T}{\Gamma(\beta)}\right]^p c_p\left[\frac{t_1^\beta}{\beta}\right]^{\frac{p}{2}}\left(\int_0^t m_{\mathfrak{g}}(s)ds\right)\Theta_3(r)\right\} \\
:= L_1.&
\end{aligned}$$

Next, for any $t \in (t_i, s_i]$, $i = 1, 2, \cdots, \mathbb{N}$,

$$\begin{aligned}
\sup_{t\in[t_i,s_i]} \mathbb{E}\,\|(\Phi u)(t)\|^p &\leq \sup_{t\in[t_i,s_i]}\left\{\mathbb{E}\,\|\Im_i(u(t_i))\|^p\right\} \\
&\leq \{\rho_i(r)\}
\end{aligned}$$

$$:= \quad L_2.$$

ll^y for any $t \in (s_i, t_{i+1}]$, $i = 1, 2, \cdots, \mathbb{N}$ one can estimate,

$$\begin{aligned}
&\sup_{t\in[s_i,t_{i+1}]} (t-s_i)^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u)(t)\|^p \\
&\le 6^{p-1}\sup_{t\in[s_i,t_{i+1}]} (t-s_i)^{p(1-\gamma)}\Big\{\mathbb{E}\,\big\|S_{\alpha,\beta}(t-s_i)[\Im_i(s_i,u(s_i))]\big\|^p + \mathbb{E}\,\|\mathfrak{h}(t,u(t))\|^p \\
&\quad + \mathbb{E}\Big\|\int_0^{s_i} P_\beta(s_i-s)\mathfrak{f}(s,u(s))ds\Big\|^p + \mathbb{E}\Big\|\int_0^{s_i} P_\beta(s_i-s)[\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)]ds\Big\|^p \\
&\quad + \mathbb{E}\Big\|\int_0^{t} P_\beta(t-s)\mathfrak{f}(s,u(s))ds\Big\|^p + \mathbb{E}\Big\|\int_0^{t} P_\beta(t-s)\Big[\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)\Big]ds\Big\|^p\Big\} \\
&\le 6^{p-1}\sup_{t\in[s_i,t_{i+1}]} (t-s_i)^{p(1-\gamma)}\Big\{\Big[\frac{M_T}{\Gamma(\gamma)}\Big]^p[(t-s_i)^{p(1-\gamma)}]\rho_i(r) + m_{\mathfrak{h}}(1+r) \\
&\quad + \Big[\frac{M_T}{\Gamma(\beta)}\Big]^p s_i^{p\beta-1}\Big(\int_0^{s_i} m_{\mathfrak{f}}(s)ds\Big)\Theta_1(r) \\
&\quad + \Big[\frac{M_T}{\Gamma(\beta)}\Big]^p c_p s_i^{p\beta-1}\Big(\int_0^{s_i} m_{\mathfrak{g}}(s)ds\Big)\Theta_3(r) \\
&\quad + \Big[\frac{M_T}{\Gamma(\beta)}\Big]^p t^{p\beta-1}\Big(\int_0^{t} m_{\mathfrak{f}}(s)ds\Big)\Theta_1(r) \\
&\quad + \Big[\frac{M_T}{\Gamma(\beta)}\Big]^p c_p t^{p\beta-1}\Big(\int_0^{t} m_{\mathfrak{g}}(s)ds\Big)\Theta_3(r)\Big\} \\
&\le 6^{p-1}b^{p(1-\gamma)}\Big\{\Big[\frac{M_T}{\Gamma(\gamma)}\Big]^p b^{p(\gamma-1)}\rho_i(r) + m_{\mathfrak{h}}(1+r) \\
&\quad + 2\Big[\frac{M_T}{\Gamma(\beta)}\Big]^p\Big(b^{p\beta-1}(\int_0^b m_{\mathfrak{f}}(s)ds)\Theta_1(r) + c_p b^{p\beta-1}(\int_0^b m_{\mathfrak{g}}(s)ds)\Theta_3(r)\Big)\Big\} \\
&:= L_3.
\end{aligned}$$

Let $L = \max\{L_1, L_2, L_3\}$ then for each $u \in \mathbb{B}_r$, we have $\|(\Phi u)(t)\|_{\mathbb{H}}^p \le L$.
Step 2: Φ is continuous on $\mathbb{B}_r$.
Let $\{u^n(t)\}_{n=1}^\infty \subset \mathbb{B}_r$ with $t^n \to u$, $(n \to \infty)$ in $\mathbb{B}_r$. Therefore, the continuous functions are $\mathfrak{h}, \mathfrak{f}$ and $\mathfrak{g}$ $\forall$ $\epsilon > 0$, $\exists\, \mathbb{N}$ s.t for each $n \in \mathbb{N}$,

$$\begin{aligned}
\mathbb{E}\,\|\mathfrak{h}(s,u^n(s)) - \mathfrak{h}(s,u(s))\|^p &< \epsilon, \\
\mathbb{E}\,\|\mathfrak{f}(s,u^n(s)) - \mathfrak{f}(s,u(s))\|^p &< \epsilon, \\
\mathbb{E}\,\|\mathfrak{g}(s,u^n(s)) - \mathfrak{g}(s,u(s))\|^p &< \epsilon.
\end{aligned}$$

For each $t \in J$, we get

$$\begin{aligned}
\mathbb{E}\,\|\mathfrak{f}(s,u^n(s)) - \mathfrak{f}(s,u(s))\|^p &\le 3^{p-1}m_{\mathfrak{f}}(t)\Theta_1(r), \\
\mathbb{E}\Big\|\int_0^s [\mathfrak{g}(\tau,u^n(\tau)) - \mathfrak{g}(\tau,u(\tau))]dw(\tau)\Big\|^p &\le 3^{p-1}c_p\Big(\int_0^s m_{\mathfrak{g}}(t)\Theta_3(r)dr\Big).
\end{aligned}$$

From (**H1**) − (**H5**) and dominated convergence theorem, for $t \in [0, t_1]$

$$\begin{aligned}
&\sup_{t\in J} t^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u^n)(t) - (\Phi u)(t)\|^p \\
&\quad \le 3^{p-1} \sup_{t\in[0,t_1]} t_1^{p(1-\gamma)}\Big\{\mathbb{E}\,\|\mathfrak{h}(s, u^n(s)) - \mathfrak{h}(s, u(s))\|^p \\
&\qquad + \mathbb{E}\left\|\int_0^t P_\beta(t-s)[\mathfrak{f}(s, u^n(s)) - \mathfrak{f}(s, u(s))]ds\right\|^p \\
&\qquad + \mathbb{E}\left\|\int_0^t P_\beta(t-s)[\int_0^s [\mathfrak{g}(\tau, u^n(\tau)) - \mathfrak{g}(\tau, u(\tau))]dw(\tau)]ds\right\|^p\Big\} \\
&\quad \to 0 \text{ as } n \to \infty.
\end{aligned}$$

Next, for any $t \in (t_i, s_i]$, $i = 1, 2, \cdots, \mathbb{N}$,

$$\begin{aligned}
\sup_{t\in J} t^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u^n)(t) - (\Phi u)(t)\|^p &\le \sup_{t\in T_i}\mathbb{E}\,\|\Im_i(t, u^n(t)) - \Im_i(t, u(t))\|^p \\
&\to 0 \text{ as } n \to \infty.
\end{aligned}$$

For any $t \in (s_i, t_{i+1}]$, $i = 1, 2, \cdots, \mathbb{N}$,

$$\begin{aligned}
&\sup_{t\in J} t^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u^n)(t) - (\Phi u)(t)\|^p \\
&\le 6^{p-1}\sup_{t\in J_k}(t-s_i)^{p(1-\gamma)}\Big\{\left\|S_{\alpha,\beta}(t-s_i)\{\Im_i(s_i, u^n(s_i)) - \Im_i(s_i, u(s_i))\}\right\|^p \\
&\quad + \|\mathfrak{h}(t, u^n(t)) - \mathfrak{h}(t, u(t))\|^p \\
&\quad + \left\|\int_0^{s_i} P_\beta(s_i - s)[\mathfrak{f}(t, u^n(s)) - \mathfrak{f}(t, u(s))]ds\right\|^p \\
&\quad + \left\|\int_0^{s_i} P_\beta(s_i - s)[\int_0^s [\mathfrak{g}(\tau, u^n(\tau)) - \mathfrak{g}(\tau, u(\tau))]dw(\tau)]ds\right\|^p \\
&\quad + \left\|\int_0^{t} P_\beta(t - s)[\mathfrak{f}(t, u^n(s)) - \mathfrak{f}(t, u(s))]ds\right\|^p \\
&\quad + \left\|\int_0^{t} P_\beta(t - s)[\int_0^s [\mathfrak{g}(\tau, u^n(\tau)) - \mathfrak{g}(\tau, u(\tau))]dw(\tau)]ds\right\|^p\Big\} \\
&\to 0 \text{ as } n \to \infty.
\end{aligned}$$

Then,

$$\sup_{t\in J} t^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u^n)(t) - (\Phi u)(t)\|^p \to 0 \text{ as } n \to \infty.$$

Thus Φ is continuous.
Step 3: Φ maps bounded sets into equicontinuous sets of $\mathbb{B}_r$.
Let $0 < \tau_1 < \tau_2 < t_1$. For each $u \in \mathbb{B}_r$,

$$\sup_{t\in[0,t_1]} t_1^{p(1-\gamma)}\,\|(\Phi u)(\tau_2) - (\Phi u)(\tau_1)\|^p$$

$$
\begin{aligned}
&\leq 4^{p-1}\sup_{t\in[0,t_1]} t_1^{p(1-\gamma)}\Big\{\mathbb{E}\left\|[S_{\alpha,\beta}(\tau_2)-S_{\alpha,\beta}(\tau_1)]u_0\right\|^p\\
&\quad+\mathbb{E}\,\|\mathfrak{h}(\tau_2,u(\tau_2))-\mathfrak{h}(\tau_1,u(\tau_1))\|^p\\
&\quad+\mathbb{E}\left\|\int_0^{\tau_1}[P_\beta(\tau_2-s)-P_\beta(\tau_1-s)]\mathfrak{f}(s,u(s))ds\right\|^p+\mathbb{E}\left\|\int_{\tau_1}^{\tau_2}P_\beta(\tau_2-s)\mathfrak{f}(s,u(s))ds\right\|^p\\
&\quad+\mathbb{E}\left\|\int_0^{\tau_1}[P_\beta(\tau_2-s)-P_\beta(\tau_1-s)][\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)]ds\right\|^p\\
&\quad+\mathbb{E}\left\|\int_{\tau_1}^{\tau_2}P_\beta(\tau_2-s)[\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)]ds\right\|^p\Big\}.
\end{aligned}
$$

For any $\tau_1,\tau_2\in(t_i,s_i]$, $\tau_1<\tau_2$, $i=1,2,\cdots,\mathbb{N}$,

$$
\begin{aligned}
\mathbb{E}\,\|(\Phi u)(\tau_2)-(\Phi u)(\tau_1)\|^p &= \sup_{t\in T_i}[\mathbb{E}\,\|\Im_i(\tau_2,u(\tau_2))-\Im_i(\tau_1,u(\tau_1))\|^p]\\
&= \sup_{t\in T_i}\mathbb{E}\,\|\Im_i(\tau_2,u(\tau_2))-\Im_i(\tau_1,u(\tau_1))\|^p\,.
\end{aligned}
$$

ll^y for any $\tau_1,\tau_2\in(s_i,t_{i+1}]$, $\tau_1<\tau_2$, $i=1,2,\cdots,\mathbb{N}$,

$$
\begin{aligned}
&\sup_{t\in J_k}(t-s_i)^{p(1-\gamma)}\mathbb{E}\,\|(\Phi u)(\tau_2)-(\Phi u)(\tau_1)\|^p\\
&\leq 6^{p-1}\sup_{t\in J_k}(t-s_i)^{p(1-\gamma)}\Big\{\mathbb{E}\left\|[S_{\alpha,\beta}(\tau_2-s_i)-S_{\alpha,\beta}(\tau_1-s_i)]\times[\Im_i(s_i,u(s_i))]\right\|^p\\
&\qquad+\mathbb{E}\,\|\mathfrak{h}(\tau_2,u(\tau_2))-\mathfrak{h}(\tau_1,u(\tau_1))\|^p\\
&\qquad+\mathbb{E}\left\|\int_0^{\tau_1}[P_\beta(\tau_2-s)-P_\beta(\tau_1-s)]\mathfrak{f}(s,u(s))ds\right\|^p\\
&\qquad+\mathbb{E}\left\|\int_{\tau_1}^{\tau_2}P_\beta(\tau_2-s)\mathfrak{f}(s,u(s))ds\right\|^p\\
&\qquad+\mathbb{E}\left\|\int_0^{\tau_1}[P_\beta(\tau_2-s)-P_\beta(\tau_1-s)][\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)]ds\right\|^p\\
&\qquad+\mathbb{E}\left\|\int_{\tau_1}^{\tau_2}P_\beta(\tau_2-s)[\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)]ds\right\|^p\Big\}.
\end{aligned}
$$

Right hand side of the above inequalities tends to zero as $\tau_2\to\tau_1$, since the definitions of $S_{\alpha,\beta}(\cdot)$, $P_\beta(\cdot)$ imply the continuity, one can see that $\|(\Phi u)(t_2)-(\Phi u)(t_1)\|^2_{\mathbb{H}}$ tends to zero independently of $u\in\mathbb{B}_r$ as $\tau_2\to\tau_1$, for ϵ sufficiently small. Further, Φu, $u\in\mathbb{B}_r$ is equicontinuous. Thus, Φ maps $\mathbb{B}_r$ into a family of equicontinuous.

Step 4: M$\ddot{o}$nch conditions holds. Let us consider an arbitrary bounded subset $D\subset\mathbb{B}_r$ which is countable and $D\subset\overline{co}(\{0\}\cup\Phi(D))$. We prove that $\mu(D)=0$, where $\mu(\cdot)$ is Hausdorff measure of non compactness. Without loss of generality we assume that $D=\{u^n\}_{n=1}^{\infty}$, from Step 3 it is easy to verify that D is bounded and equicontinuous.

Now, Define

$$\Phi(D)=\begin{cases}\mathfrak{h}(t,u(t))+\int_0^t P_\beta(t-s)\mathfrak{f}(s,u(s))ds\\ +\int_0^t P_\beta(t-s)\left[\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)\right]ds, \quad \text{for } t\in[0,t_1],\ i=0.\\ \Im_i(t,u(t)), \quad \text{for } t\in(t_i,s_i],\ i\geq 1.\\ S_{\alpha,\beta}(t-s_i)\left[\Im_i(t,u(s_i))\right]+\mathfrak{h}(t,u(t))+\int_0^{s_i} P_\beta(s_i-s)\mathfrak{f}(s,u(s))ds\\ +\int_0^{s_i} P_\beta(s_i-s)\left[\int_0^\tau \mathfrak{g}(s,u(s))dw(s)\right]ds\\ +\int_0^t P_\beta(t-s)\mathfrak{f}(s,u(s))ds\\ +\int_0^t P_\beta(t-s)\left[\int_0^\tau \mathfrak{g}(s,u(s))dw(s)\right]ds, \quad \text{for } t\in(s_i,t_{i+1}],\ i\geq 1.\end{cases}$$

Let $\Phi(D)=\Phi_1(D)+\Phi_2(D)+\Phi_3(D)$.
First, we estimate $\Phi_1(D)$, for $t\in[0,t_1]$, we get,
Let

$$\begin{aligned}\{\Phi_1(D(t))\}&=\Big\{\mathfrak{h}(t,u(t))+\int_0^t P_\beta(t-s)\mathfrak{f}(s,u(s))ds\\ &\quad+\int_0^t P_\beta(t-s)\left[\int_0^s \mathfrak{g}(\tau,u(\tau))dw(\tau)\right]ds\Big\}\\ &\leq \Phi_{11}(D(t))+\Phi_{12}(D(t))+\Phi_{13}(D(t)).\end{aligned}$$

By assumptions (**H4**)(**ii**), the estimate of $\Phi_{11}(D(t))$ can be derived as

$$\begin{aligned}\mu\left[\{\Phi_{11}(D(t))\}\right]&\leq \mu\left[\mathfrak{h}(t,D(t))\right]\\ &\leq \Theta_5(t)\big[\sup_{t\in[0,t_1]}\mu(D(t))\big].\end{aligned}$$

By assumptions (**H1**)(**iii**), the estimate of $\Phi_{12}(D(t))$, we have

$$\begin{aligned}\mu\left[\{\Phi_{12}(D(t))\}\right]&\leq \mu\left[\int_0^t P_\beta(t-s)\mathfrak{f}(s,D(s))ds\right]\\ &\leq 2\left[\frac{M_T}{\Gamma(\beta)}\right]\left[\frac{t_1^\beta}{\beta}\right]\Theta_2(t)\big[\sup_{t\in[0,t_1]}\mu(D(t))\big].\end{aligned}$$

ll^y, by assumptions (**H2**)(**iii**), the estimate of $\Phi_{13}(D(t))$, we have

$$\begin{aligned}\mu\left[\{\Phi_{13}(D(t))\}\right]&\leq \mu\left[\int_0^t P_\beta(t-s)\left[\int_0^s \mathfrak{g}(\tau,D(\tau))dw(\tau)\right]ds\right]\\ &\leq \mu\left[\int_0^t P_\beta(t-s)\left[\left(\int_0^s \mathfrak{g}(\tau,D(\tau))dw(\tau)\right]^2\right)^{\frac{1}{2}}ds\right]\\ &\leq 2\left[\frac{M_T}{\Gamma(\beta)}\right]c_p\left[\frac{t_1^\beta}{\beta}\right]\sqrt{t_1}\Theta_4(t)\big[\sup_{t\in[0,t_1]}\mu(D(t))\big].\end{aligned}$$

By using the above estimates, becomes

$$\{\Phi_1(D(t))\}=\left\{\mathfrak{h}^*+2\left[\frac{M_T}{\Gamma(\beta)}\right]\left[\frac{t_1^\beta}{\beta}\right](\mathfrak{f}^*+\sqrt{t_1}c_p\mathfrak{g}^*)\right\}\mu(D(t))$$

$$\leq \Lambda_1^* \mu(\mathrm{D}(\mathrm{t})),$$

where

$$\Lambda_1^* = \left\{ \mathfrak{h}^* + 2\left[\frac{\mathrm{M_T}}{\Gamma(\beta)}\right]\left[\frac{\mathrm{t}_1^{\beta}}{\beta}\right](\mathfrak{f}^* + \sqrt{\mathrm{t}_1} c_{\mathrm{p}} \mathfrak{g}^*) \right\}$$

For $\mathrm{t} \in (\mathrm{t}_i, \mathrm{s}_i]$, $i = 1, 2, \cdots, \mathbb{N}$, we have

$$\begin{aligned} \mu\left[\{\Phi_2(\mathrm{D}(\mathrm{t}))\}\right] &\leq \mu\left[\Im(\mathrm{t}, \mathrm{D}(\mathrm{t}))\right] \\ &\leq \bar{\rho}_i \mu(\mathrm{D}(\mathrm{t})) \\ &\leq \Lambda_2^* \mu(\mathrm{D}(\mathrm{t})). \end{aligned}$$

where $\Lambda_2^* = \bar{\rho}_i$
For $\mathrm{t} \in (\mathrm{s}_i, \mathrm{t}_{i+1}]$, $i = 1, 2, \cdots, \mathrm{N}$, we have

$$\begin{aligned} \mu\left[\{\Phi_3(\mathrm{D}(\mathrm{t}))\}\right] \leq \mu\Big\{ & \mathrm{S}_{\alpha,\beta}(\mathrm{t} - \mathrm{s}_i)[\Im(\mathrm{s}_i, \mathrm{u}(\mathrm{s}_i))] + \mathfrak{h}(t, \mathrm{u}(\mathrm{t})) + \int_0^{\mathrm{s}_i} \mathrm{P}_\beta(\mathrm{s}_i - \mathrm{s}) \mathfrak{f}(\mathrm{s}, \mathrm{u}(\mathrm{s})) d\mathrm{s} \\ & + \int_0^{\mathrm{s}_i} \mathrm{P}_\beta(\mathrm{s}_i - \mathrm{s}) \left[\int_0^{\mathrm{s}} \mathfrak{g}(\tau, \mathrm{u}(\tau)) dw(\tau)\right] d\mathrm{s} + \int_0^{\mathrm{t}} \mathrm{P}_\beta(\mathrm{t} - \mathrm{s}) \mathfrak{f}(\mathrm{s}, \mathrm{u}(\mathrm{s})) d\mathrm{s} \\ & + \int_0^{\mathrm{t}} \mathrm{P}_\beta(\mathrm{t} - \mathrm{s}) \left[\int_0^{\mathrm{s}} \mathfrak{g}(\tau, \mathrm{u}(\tau)) dw(\tau)\right] d\mathrm{s} \Big\} \\ \leq \Phi_{31}(\mathrm{D}(\mathrm{t})) &+ \Phi_{32}(\mathrm{D}(\mathrm{t})) + \Phi_{33}(\mathrm{D}(\mathrm{t})) + \Phi_{34}(\mathrm{D}(\mathrm{t})) + \Phi_{35}(\mathrm{D}(\mathrm{t})) + \Phi_{36}(\mathrm{D}(\mathrm{t})). \end{aligned}$$

By assumptions (**H3**)(**ii**), the estimate of $\Phi_{31}(\mathrm{D}(\mathrm{t}))$ can be derived as

$$\begin{aligned} \mu\left[\{\Phi_{31}(\mathrm{D}(\mathrm{t}))\}\right] &\leq \mu\left[\mathrm{S}_{\alpha,\beta}(\mathrm{t} - \mathrm{s}_i)[\Im(\mathrm{s}_i, \mathrm{u}(\mathrm{s}_i))]\right] \\ &\leq \mu\left[\mathrm{S}_{\alpha,\beta}(\mathrm{t} - \mathrm{s}_i)[\Im(\mathrm{s}_i, \mathrm{D}(\mathrm{s}_i))]\right] \\ &\leq \left[\frac{\mathrm{M_T}}{\Gamma(\gamma)}\right](\mathrm{t} - \mathrm{s}_i)^{(1-\gamma)}[\bar{\rho}_i] \sup_{\mathrm{t} \in (\mathrm{s}_i, \mathrm{t}_{i+1}]} \mu(\mathrm{D}(\mathrm{t})). \end{aligned}$$

By assumptions (**H4**)(**ii**), the estimate of $\Phi_{32}(\mathrm{D}(\mathrm{t}))$ can be derived as

$$\begin{aligned} \mu\left[\{\Phi_{32}(\mathrm{D}(\mathrm{t}))\}\right] &\leq \mu\left[\mathfrak{h}(\mathrm{t}, \mathrm{u}(\mathrm{t}))\right] \\ &\leq \mu\left[\mathfrak{h}(\mathrm{t}, \mathrm{D}(\mathrm{t}))\right] \\ &\leq \Theta_5(\mathrm{t}) \sup_{\mathrm{t} \in (\mathrm{s}_i, \mathrm{t}_{i+1}]} \mu(\mathrm{D}(\mathrm{t})). \end{aligned}$$

By assumptions (**H1**)(**iii**), the estimate of $\Phi_{33}(\mathrm{D}(\mathrm{t}))$ can be derived as

$$\begin{aligned} \mu\left[\{\Phi_{33}(\mathrm{D}(\mathrm{t}))\}\right] &\leq \mu\left[\int_0^{\mathrm{s}_i} \mathrm{P}_\beta(\mathrm{t} - \mathrm{s}_i) \mathfrak{f}(\mathrm{s}, \mathrm{u}(\mathrm{s})) d\mathrm{s}\right] \\ &\leq 2\left[\frac{\mathrm{M_T}}{\Gamma(\beta)}\right]\frac{(\mathrm{s}_i)^{\beta}}{\beta} \Theta_2(\mathrm{t})[\sup_{t \in (\mathrm{s}_i, \mathrm{t}_{i+1}]} \mu(\mathrm{D}(\mathrm{t}))]. \end{aligned}$$

ll^y by assumptions (**H2**)(**iii**), the estimate of $\Phi_{34}(\mathrm{D}(\mathrm{t}))$ can be derived as

$$\mu\left[\{\Phi_{34}(\mathrm{D}(\mathrm{t}))\}\right] \leq \mu\left[\int_0^{\mathrm{s}_i} \mathrm{P}_\beta(\mathrm{t} - \mathrm{s}_i) \left[\int_0^{\mathrm{s}} \mathfrak{g}(\tau, \mathrm{u}(\tau)) dw(\tau)\right] d\mathrm{s}\right]$$

$$
\begin{aligned}
&\leq \mu\left[\int_0^{s_i} P_\beta(t-s)\left[\left(\int_0^s g(\tau, D(\tau))dw(\tau)\right]^2\right)^{\frac{1}{2}} ds\right] \\
&\leq 2\left[\frac{M_T}{\Gamma(\beta)}\right] c_p\left(\frac{s_i^\beta}{\beta}\right)\sqrt{s_i}\Theta_5(t)[\sup_{t\in(s_i,t_{i+1}]}\mu(D(t))].
\end{aligned}
$$

By assumptions (**H1**)(**iii**), the estimate of $\Phi_{35}(D(t))$ can be derived as

$$
\begin{aligned}
\mu\left[\{\Phi_{35}(D(t))\}\right] &\leq \mu\left[\int_0^t P_\beta(t-s)f(s, u(s))ds\right] \\
&\leq 2\left[\frac{M_T}{\Gamma(\beta)}\right]\frac{(t)^\beta}{\beta}\Theta_2(t)[\sup_{t\in[s_i,t_{i+1}]}\mu(D(t))].
\end{aligned}
$$

Similarly by assumptions (**H2**)(**iii**), the estimate of $\Phi_{36}(D(t))$ can be derived as

$$
\begin{aligned}
\mu\left[\{\Phi_{36}(D(t))\}\right] &\leq \mu\left[\int_0^t P_\beta(t-s)\left[\int_0^s g(\tau, u(\tau))dw(\tau)\right]ds\right] \\
&\leq \mu\left[\int_0^t P_\beta(t-s)\left[\left(\int_0^s g(\tau, D(\tau))dw(\tau)\right]^2\right)^{\frac{1}{2}} ds\right] \\
&\leq 2\left[\frac{M_T}{\Gamma(\beta)}\right] c_p\left(\frac{t^\beta}{\beta}\right)\sqrt{t_1}\Theta_5(t)[\sup_{t\in[s_i,t_{i+1}]}\mu(D(t))].
\end{aligned}
$$

By using the above estimates, becomes

$$
\begin{aligned}
\mu\left[\{\Phi_3(D(t))\}\right] \leq \Bigg\{&\left[\frac{M_T}{\Gamma(\gamma)}\right](t_{i+1}-s_i)^{p(1-\gamma)}[\bar{\rho}_i] + \mathfrak{h}^* \\
&+ 4\left[\frac{M_T}{\Gamma(\beta)}\right]\left[\frac{b^\beta}{\beta}\right]\left(\mathfrak{f}^* + c_p\mathfrak{g}^*\sqrt{b}\right)\Bigg\}\mu(D(t)) \\
\leq{}& \Lambda_3^*\mu(D(t)).
\end{aligned}
$$

where

$$
\Lambda_3^* = \left\{\left[\frac{M_T}{\Gamma(\gamma)}\right](t_{i+1}-s_i)^{p(1-\gamma)}[\bar{\rho}_i] + \mathfrak{h}^* + 4\left[\frac{M_T}{\Gamma(\beta)}\right]\left[\frac{b^\beta}{\beta}\right]\left(\mathfrak{f}^* + c_p\mathfrak{g}^*\sqrt{b}\right)\right\}.
$$

$$
\begin{aligned}
\{\Phi(D(t))\} &= \mu\left[\Phi_1(D) + \Phi_2(D) + \Phi_3(D)\right] \\
&\leq \left[\Lambda_1^* + \Lambda_2^* + \Lambda_3^*\right]\mu(D(t)) \\
&\leq \Lambda^*\mu(D(t)).
\end{aligned}
$$

where Λ^* is a constant given in (**H5**), and $\Lambda^* \in (0, 1)$.
By using Lemma 2.3, we have

$$
\begin{aligned}
\mu(D) &\leq \mu(\overline{co}(\{0\}\cup\Phi(\{D\}))) \\
&= \mu(\Phi(D)) \\
&\leq \Lambda^*\mu(D),
\end{aligned}
$$

which implies that $\mu(D) = 0$, D is relatively compact set. Therefore, by Lemma 2.5, Φ has a fixed point in D. Thus, the NIHFNSDEs of the system (1.1) has a fixed point on J, which is a mild solutions.

4. An example

Consider the following partial NIHFNSDEs, system of the form

$$\begin{aligned}
\mathfrak{D}_{0^+}^{\frac{1}{2},\frac{1}{8}}\left[\mathrm{u}(\mathrm{t},\zeta)-\frac{\sin(\mathrm{u}(\mathrm{t},\zeta))}{40}\right] &= \frac{\partial^2}{\partial \mathrm{u}^2}\left[\mathrm{u}(\mathrm{t},\zeta)-\frac{sin(\mathrm{u}(\mathrm{t},\zeta))}{40}\right]+\frac{e^{-\mathrm{t}}}{1+e^{-\mathrm{t}}}\sin(\mathrm{u}(\mathrm{t},\zeta))\\
&+ e^{-\mathrm{t}}\sin \mathrm{t}dw(\mathrm{t}), \quad \mathrm{t}\in(0,1/3]\cup(2/3,1],\\
\mathrm{u}(\mathrm{t},\zeta) &= \frac{\cos \mathrm{t}|\mathrm{u}(\mathrm{t},\zeta)|}{25+|\mathrm{u}(\mathrm{t},\zeta)|}, \quad \mathrm{t}\in(1/3,2/3],\\
\mathrm{u}(\mathrm{t},0) &= \mathrm{u}(\mathrm{t},1)=0, \quad \mathrm{t}\in[0,1],\\
\mathfrak{I}_{0^+}^{(1-\gamma)}[\mathrm{u}(0)-\mathfrak{h}(0,\mathrm{u}(0))] &= \mathrm{u}_0, \qquad (4.1)
\end{aligned}$$

where $\mathfrak{D}^{\frac{1}{2},\frac{1}{8}}$ is the Hilfer fractional derivative of order $1/2$ and degree $1/8$. Take the Hilbert space $\mathfrak{X}=\mathfrak{Y}=\mathcal{L}^p([0,1])$ and the operators $\mathfrak{A}:\mathscr{D}(\mathfrak{A})\subset\mathfrak{X}\to\mathfrak{X}$ and defined by $\mathfrak{A}=\frac{\partial^2}{\partial \mathrm{u}^2}$ with $\mathscr{D}(\mathfrak{A})=\{\mathrm{u}\in\mathfrak{X}:\mathrm{u},\mathrm{u}'$ are absolutely continuous, $\mathrm{u}''\in\mathfrak{X},\mathrm{u}(0)=0\}$. Thus $\mathfrak{A}$ can be written as $\mathfrak{A}\mathrm{u}=\sum_{n=1}^{\infty}n^2<\mathrm{u},\mathrm{u}_n>\mathrm{u}_n$, $\mathrm{u}\in\mathscr{D}(\mathfrak{A})$ where $\mathrm{u}_n(s)=\sqrt{\frac{2}{\pi}}\sin ns$, $n=1,2,\cdots$, is an orthogonal set of eigenvectors of $\mathfrak{A}$. Moreover, for $\mathrm{u}\in\mathfrak{X}$, we have $\mathrm{u}=\sum_{n=1}^{\infty}\frac{1}{1+n^2}<\mathrm{u},\mathrm{u}_n>\mathrm{u}_n$, $\mathfrak{A}\mathrm{u}=\sum_{n=1}^{\infty}\frac{n^2}{1+n^2}<\mathrm{u},\mathrm{u}_n>\mathrm{u}_n$.

It is known that $\mathfrak{A}$ is self adjoint and infinitesimal generator of an analytic semigroup $\{\mathfrak{T}(t):t\geq 0\}$ in $\mathfrak{X}$ which is given by

$$\mathfrak{T}(\mathrm{t})\mathrm{u}=\sum_{n=1}^{\infty}e^{-n^2t}<\mathrm{u},\mathrm{u}_n>, \quad \mathrm{u}\in\mathfrak{X}.$$

Therefore, $\|\mathfrak{T}(\mathrm{t})\|\leq e^{-1}<1=\mathrm{M},\ \mathrm{t}\geq 0$.

Now, D is any bounded subset $\mathbb{B}_r$ in $\mathfrak{X}$. Define

$$\begin{aligned}
\mathfrak{f}(\mathrm{t},\mathrm{u}(\mathrm{t}))(\zeta) &= \mathfrak{f}(\mathrm{t},\mathrm{u}(\mathrm{t},\zeta))=\frac{e^{-\mathrm{t}}}{1+e^{-\mathrm{t}}}\sin(\mathrm{u}(\mathrm{t},\zeta)),\\
\mathfrak{g}(\mathrm{t},\mathrm{u}(\mathrm{t}))(\zeta) &= \mathfrak{g}(\mathrm{t},\mathrm{u}(\mathrm{t},\zeta))=e^{-\mathrm{t}}\sin\mathrm{t},\\
\mathfrak{h}(\mathrm{t},\mathrm{u}(\mathrm{t}))(\zeta) &= \mathfrak{h}(\mathrm{t},\mathrm{u}(\mathrm{t},\zeta))=\frac{\sin(\mathrm{u}(\mathrm{t},\zeta))}{40},\\
\mu(\mathfrak{f}(\mathrm{t},\mathrm{D})) &= \mu(\mathfrak{f}(\mathrm{t},\mathrm{D}(\mathrm{t},\zeta)))\leq\Theta_2(\mathrm{t})[\sup_{\mathrm{t}\in\mathrm{J}}\mu(\mathrm{D}(\mathrm{t}))],\\
\mu\left(\int_0^{\mathrm{t}}\mathfrak{g}(s,\mathrm{D})ds\right) &= \mu\left(\int_0^{\mathrm{t}}\mathfrak{g}(s,\mathrm{D}(\mathrm{t},\zeta))ds\right)\leq\Theta_4(\mathrm{t})[\sup_{\mathrm{t}\in\mathrm{J}}\mu(\mathrm{D}(\mathrm{t}))],\\
\mu(\mathfrak{h}(\mathrm{t},\mathrm{D})) &= \mu(\mathfrak{f}(\mathrm{t},\mathrm{D}(\mathrm{t},\zeta)))\leq\Theta_5(\mathrm{t})[\sup_{\mathrm{t}\in\mathrm{J}}\mu(\mathrm{D}(\mathrm{t}))],
\end{aligned}$$

and $\mathrm{t},\mathrm{u}\in(\mathrm{t}_i,\mathrm{s}_i]\times\mathfrak{X}$, $i=1,2,\cdots,\mathbb{N}$, one can estimate,

$$\mathbb{E}\,\|\mathfrak{I}_i(\mathrm{t},\mathrm{u})\|^{\mathrm{p}}=\frac{\cos\mathrm{t}|\mathrm{u}(\mathrm{t},\zeta)|}{25+|\mathrm{u}(\mathrm{t},\zeta)|}\mathbb{E}\,\|\mathrm{u}(s)\|^{\mathrm{p}}$$

and for any bounded subset $\mathrm{D}\subset\mathfrak{X}$, $\mathrm{t}\in(\mathrm{t}_i,\mathrm{s}_i]$, $i=1,2,\cdots,\mathbb{N}$, we get

$$\mu(\mathfrak{I}_i(\mathrm{t},\mathrm{u}))^{\mathrm{p}}\leq\bar{\rho}_i\sup_{\mathrm{t}\in(\mathrm{t}_i,\mathrm{s}_i]}\mu(\mathrm{D}(\mathrm{t})).$$

with the above system (4.1) can be formulated in the abstract form of (1.1), since, the functions $\mathfrak{f},\mathfrak{g},\mathfrak{h}$ and $\mathfrak{I}$ are uniformly bounded. It is easy to verify that conditions of Theorem 3.1. holds, partial NIHFNSDEs, admits a mild solution.

5. Conclusions

The aim of this manuscript is to investigate the existence of mild solution of non-instantaneous impulsive neutral Hilfer fractional stochastic differential equation (NIHFNSDEs). We establish a new criteria to guarantee the sufficient conditions for a class of NIHFNSDEs of order $0 < \beta < 1$ and type $0 \leq \alpha \leq 1$ is derived with the help of fractional calculus, stochastic theory, fixed point theorem and semigroup theory. M$\ddot{o}$nch fixed point theorem is adopted to prove the existence of solution. In addition, a numerical example is provided to validate the theoretical result. Further, this result could be extended to investigate the optimal controllability of NIHFNSDEs in future.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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