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Lump and Interaction solutions of a geophysical Korteweg–de Vries equation

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ABSTRACT

This manuscript retrieve lump soliton solution for geophysical Korteweg–de Vries equation (GKdVE) with the help of Hirota bilinear method (HBM). We will also obtain lump–kink soliton (which is interaction of lump with one kink soliton), lump-periodic solutions (which is formed by interaction between periodic waves and lump) and lump-kink-periodic solutions (which is formed by interaction of periodic waves and lump with one kink soliton). The dynamics of these solution are examined graphically by selecting significant parameters.

1. Introduction

In theory of soliton, exact solutions, Hamiltonian structure, integrable systems and Painleve analysis are highly concerned topics . And for the proper understanding of concerned processes and phenomenon in nonlinear sciences such as fluid dynamics, plasma physics, nonlinear optics and many more, exact solution of mathematical equations play a imperative role. Nonlinear evolution equations (NLEEs) have been given more consideration as a result of their colossal significance in the field of mathematics and physics for depicting the nonlinear physical phenomena [1–3]. Over the previous decades, various integration norms have been used to find the exact solution of NLEEs such as direct algebraic method [4], extended trial equation method [5], modified extended mapping method [6], HBM [7], modified $(\frac{G'}{G})$-expansion method [8], extended and modified direct algebraic method, extended mapping method and Seadawy techniques [9–18] and many others [19–29]. There are various type of soliton solutions e.g. bell type, kink wave, lump wave, peakons, S-shaped soliton, M-shaped soliton, compactons, cuspons and many others [30–33]. Lump solution have grabbed huge attention in mathematical physics. Lump soliton is localized in all space direction are rational function solutions to NLEE. Lump solutions are used to describe nonlinear physical phenomena like solitons. Lump solutions emerges in distinct scientific fields for instance water wave, plasma, Bose Einstein condensate, optic media and so on. [30,33–35]. Hao et al. studied generalized KP equation and obtained its soliton solutions and lump type solutions [36]. Manukure et al. studied extended KP equation and obtained its lump solution [37]. Yong et al. Jimbo–Miwa equation and obtained its general lump type solutions [38]. Li et al. studied generalized Bogoyavlensky–Konopelchenko equation and obtained its lump type solutions and lump solutions [39]. Pu and Hu studied soliton equation to obtain mixed lump–soliton solution [31]. Zhou et al. studied Hirota Satsuma Ito equation to obtain lump and lump–soliton solutions [35]. Rao et al. studied Fokas system to obtain lump–soliton solution [30]. The Korteweg–de Vries (KdV) equation and its different modified structures assume an exceptionally noteworthy job while contemplating solitary wave theory [40,41]. The investigation on proliferation of fluids making out of gas air pockets and disturbance is completed by [42,43]. Bruhl and Oumeraci completed examinations of long nonlinear cosine wave in shallow water [44]. Johnson studied the characteristics of nonlinear waves in an elastic tube [45]. Moreover, a lot of work has been done on KdV equation [46–49]. The $(1+1)$-dimensional geophysical KdV equation is given as [50]

$$u_t - \omega_0 u_x + \frac{3}{2} u u_x + \frac{1}{6} u_{xxx} = 0, \tag{1}$$

where u represents the free surface advancement and ω_0 is Coriolis effect parameter [51], u is function with respect to x, t.

In this paper, our goal is to find lump solution, lump–kink soliton solution (which is interaction of lump with one kink soliton), lump-periodic waves solution (interaction of lump with a periodic wave) and lump–kink-periodic (interaction of lump with one kink and periodic wave) of Eq. (1) with help of HBM.

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The paper is arranged as follows: In Section 2, we will discuss lump soliton. In Section 3, we obtain lump–kink soliton solutions, which is interaction of lump with one kink. In Section 4, we will study the lump periodic waves which is formed by interaction of lump with periodic waves. In Section 5, we obtain lump–kink-periodic which is formed by interaction of lump with one kink and periodic wave. We present our results in Section 6. In Section 7, conclusion will be given.

2. Lump soliton solution

For finding lump soliton solution, we use the following transformation [52].

$$u = 2(\ln \Omega)_{xx}. \tag{2}$$

The bilinear form of Eq. (1) can be obtained by using Eq. (2) to find lump soliton solution of Eq. (1). Hence, we get

$$\begin{aligned} &12\Omega^2\Omega_t\Omega_x^2 - 12\omega_0\Omega^2\Omega_x^3 - 12\Omega_x^5 - 12\Omega^3\Omega_x\Omega_{xt} \\ &- 6\Omega^3\Omega_t\Omega_{xx} + 18\omega_0\Omega_3\Omega_x\Omega_{xx} + 30\Omega\Omega_x^3\Omega_{xx} - 24\Omega^2\Omega_x\Omega_{xx}^2 \\ &+ 6\Omega_4\Omega_{xxt} - 6\omega_0\Omega_4\Omega_{xxx} + 2\Omega^2\Omega_x^2\Omega_{xxx} + 8\Omega^3\Omega_{xx}\Omega_{xxx} \\ &- 5\Omega^3\Omega_x\Omega_{xxxx} + \Omega^4\Omega_{xxxxx} = 0. \end{aligned} \tag{3}$$

Now by considering the function Π, Γ and Ω given as [52]:

$$\Pi = \alpha_1 x + \alpha_2 t + \alpha_3,\ \ \Gamma = \alpha_4 x + \alpha_5 t + \alpha_6, \tag{4}$$

and

$$\Omega = \alpha_7 + \Pi^2 + \Gamma^2. \tag{5}$$

and we have to find α_i, $1 \le i \le 7$ which all are real parameters.

We obtained the following sets of parameters by removing coefficients of x, t after substituting Ω into Eq. (3) .

Set 1. When $\alpha_1 = \alpha_6 = 0$, we get the following solutions:

$$\alpha_2 = \frac{3}{46}\sqrt{46}\,\alpha_5, \alpha_3 = 0, \alpha_4 = \frac{22}{23}\frac{\alpha_5}{\omega_0}, \alpha_5 = \alpha_5,\ \ \alpha_7 = \frac{85184}{3703}\frac{\alpha_5^2}{\omega_0^3}. \tag{6}$$

In order to make Eq. (2) well defined, $\omega_0 \neq 0$. By putting above values into Eq. (2), we get

$$u = 2\frac{(\Pi^2 + \Gamma^2 + \alpha_7)(2\alpha_4^2) - (2\alpha_4\Gamma)^2}{(\Pi^2 + \Gamma^2 + \alpha_7)^2}, \tag{7}$$

where Π and Γ are given as

$$\Pi = \frac{3}{46}\sqrt{46}\,\alpha_5 t,\ \ \Gamma = \frac{22}{23}\frac{\alpha_5}{\omega_0}\,x + \alpha_5 t. \tag{8}$$

Graph of solution set 1 is given below.

3. Lump–kink solution

We have the following transformation for lump with one kink soliton solution, Π, Γ, Y_1 and Ω given as [53]:

$$\Pi = \alpha_1 x + \alpha_2 t + \alpha_3,\ \ \Gamma = \alpha_4 x + \alpha_5 t + \alpha_6,\ \ Y_1 = k_1 x + k_2 t, \tag{9}$$

and

$$\Omega = \alpha_7 + \Pi^2 + \Gamma^2 + b_1\exp(Y_1). \tag{10}$$

and we have to find α_i, $1 \le i \le 7$ and k_j, $1 \le j \le 2$ which are all real parameters.

We obtained the following sets of parameters by removing coefficients of x, t after substituting Ω into Eq. (3) .

Set 1. When $\alpha_1 = \alpha_6 = k_1 = 0$, we get the following solutions:

$$\begin{aligned} &b_1 = b_1, \alpha_2 = \frac{1}{3}\sqrt{5}\sqrt{3}\,\alpha_5, \\ &\alpha_3 = \frac{1}{262440}\frac{\sqrt{301}\sqrt{262440}\sqrt{\frac{1}{\omega_0}}\alpha_5}{\omega_0}, \\ &\alpha_4 = -\frac{1}{9}\frac{\alpha_5}{\omega_0}, \alpha_5 = \alpha_5, \\ &\alpha_7 = \frac{19}{262440}\frac{\alpha_5^2}{\omega_0^3}, k_2 = 0. \end{aligned} \tag{11}$$

In order to make Eq. (2) well defined, $\omega_0 \neq 0$. After putting these values into Eq. (2), we get

$$u = 2\frac{(\Pi^2 + \Gamma^2 + \alpha_7 + b_1\exp(Y_1))(2\alpha_4^2) - (2\alpha_4\Gamma)^2}{(\Pi^2 + \Gamma^2 + \alpha_7 + b_1\exp(Y_1))^2}, \tag{12}$$

where Π and Γ, Y_1 are given as

$$\Pi = (\frac{1}{5}\sqrt{5}\sqrt{3}\,\alpha_5)t + \frac{1}{262440}\frac{\sqrt{301}\sqrt{262440}\sqrt{\frac{1}{\omega_0}}\alpha_5}{\omega_0},\ \ \Gamma = (-\frac{1}{9}\frac{\alpha_5}{\omega_0})x + \alpha_5 t, Y_1 = 0. \tag{13}$$

The graph of solution set 1 is given below.

4. Lump-periodic solution

We use the following transformation Π, Γ, Y_1 and Ω given as [53] for lump periodic solution:

$$\Pi = \alpha_1 x + \alpha_2 t + \alpha_3,\ \ \Gamma = \alpha_4 x + \alpha_5 t + \alpha_6,\ \ Y_1 = k_1 x + p_1 t, \tag{14}$$

and

$$\Omega = \alpha_7 + \Pi^2 + \Gamma^2 + b_1\cos(Y_1). \tag{15}$$

We have to find α_i, $1 \le i \le 7$ and k_1, p_1 which are all real parameters.

We obtained the following sets of parameters by removing coefficients of x, t after substituting Ω into Eq. (3).

Set 1. When $\alpha_1 = \alpha_5 = p_1 = 0$, we obtained the following solutions:

$$\begin{aligned} &b_1 = b_1, \alpha_2 = -\frac{1}{8}\frac{\sqrt{\frac{21\alpha_6^2k_1^4 + 18\alpha_6^2k_1^2\omega_0 + 70\alpha_4^2k_1^2 + 18\alpha_4^2\omega_0}{21k_1^2 + 18\omega_0}}\,k_1\alpha_6(7k_1^2 + 6\omega_0)}{\alpha_4}, \\ &\alpha_3 = \frac{\sqrt{\frac{21\alpha_6^2k_1^4 + 18\alpha_6^2k_1^2\omega_0 + 70\alpha_4^2k_1^2 + 18\alpha_4^2\omega_0}{21k_1^2 + 18\omega_0}}}{k_1}, \alpha_4 = \alpha_4, \alpha_6 = \alpha_6, \\ &\alpha_7 = -\frac{2}{3}\frac{42\alpha_6^2k_1^4 + 36\alpha_6^2k_1^2\omega_0 + 35\alpha_4^2k_1^2 + 9\alpha_4^2\omega_0}{k_1^2(7k_1^2 + 6\omega_0)}, k_1 = k_1. \end{aligned} \tag{16}$$

By putting above values into Eq. (2), we get

$$u = 2\frac{(\Pi^2 + \Gamma^2 + \alpha_7 + b_1\cos(Y_1))(2\alpha_4^2 - b_1k_1^2\cos(Y_1)) - (2\alpha_4\Gamma - b_1k_1 sin(Y_1))^2}{(\Pi^2 + \Gamma^2 + \alpha_7 + b_1\cos(Y_1))^2}, \tag{17}$$

where Π, Γ, and Y_1 are given as

$$\begin{aligned} \Pi = &-\frac{1}{8}\frac{\sqrt{\frac{21\alpha_6^2k_1^4 + 18\alpha_6^2k_1^2\omega_0 + 70\alpha_4^2k_1^2 + 18\alpha_4^2\omega_0}{21k_1^2 + 18\omega_0}}\,k_1\alpha_6(7k_1^2 + 6\omega_0)}{\alpha_4}t \\ &+ \frac{\sqrt{\frac{21\alpha_6^2k_1^4 + 18\alpha_6^2k_1^2\omega_0 + 70\alpha_4^2k_1^2 + 18\alpha_4^2\omega_0}{21k_1^2 + 18\omega_0}}}{k_1}, \\ \Gamma = &\ \alpha_4 x + \alpha_6,\ \ Y_1 = k_1 x. \end{aligned} \tag{18}$$

The graph of solution set 1 is given below.

5. Lump–kink-periodic solution

We use the following transformation Π, Γ, Y_1, Y_2 and Ω given as [53] for lump–kink periodic solution:

$$\Pi = \alpha_1 x + \alpha_2 t + \alpha_3,\ \Gamma = \alpha_4 x + \alpha_5 t + \alpha_6,\ Y_1 = k_1 x + p_1 t,\ Y_2 = k_2 x + p_2 t, \tag{19}$$

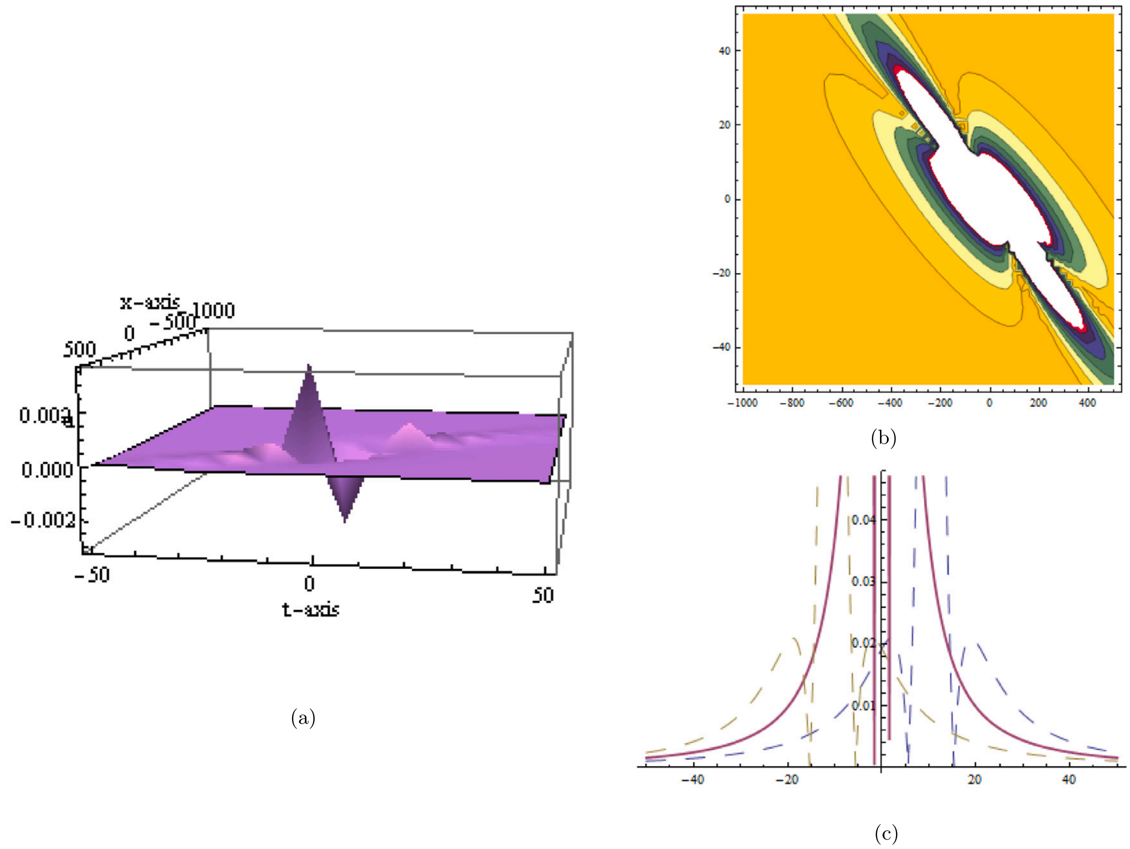


Fig. 1. The shape profile of $u(x,t)$ in Eq. (7) is given by these parameters $\alpha_5 = 100$, $\omega_0 = 10$. (a) 3D graph in interval [−1000, 500] and [−50, 50] (b) contour graph in interval [−1000, 500] and [−50, 50] and (c) 2D graph in interval [−50, 50].

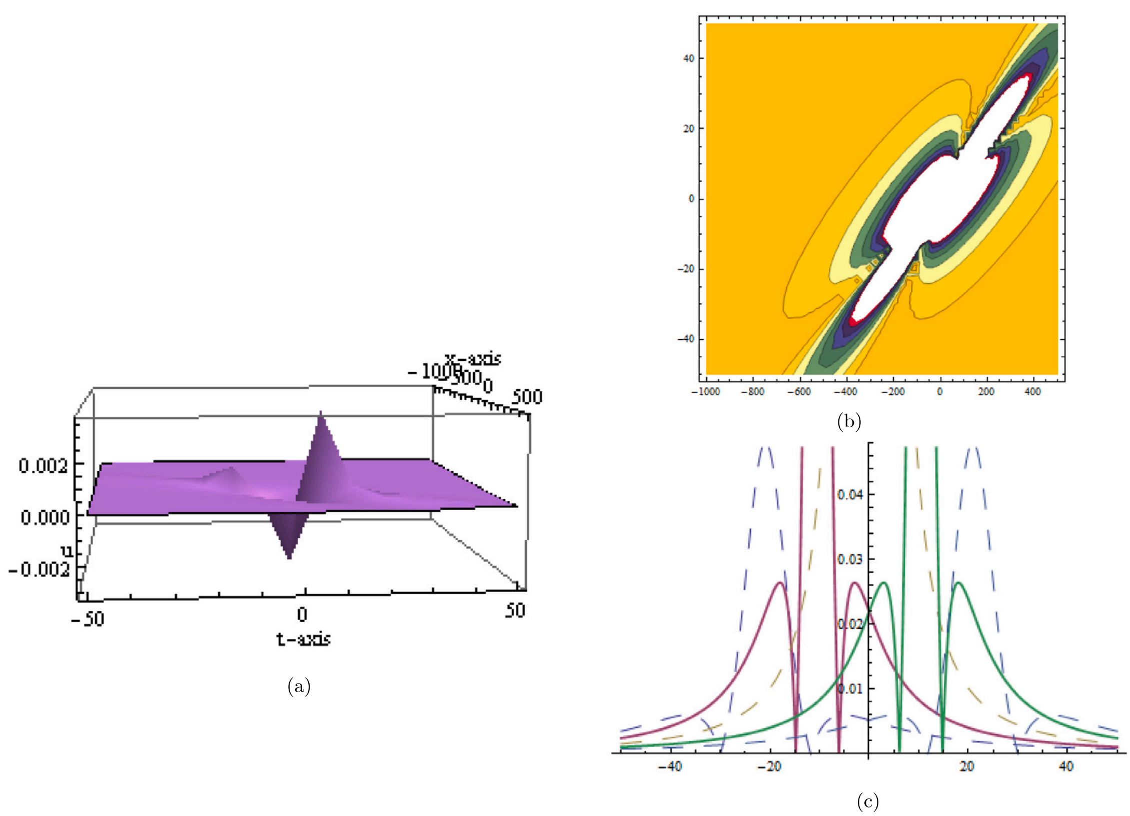


Fig. 2. The graphical description of $u(x,t)$ in Eq. (7) is given by these parameters $\alpha_5 = 5000$, $\omega_0 = -10$. (a) 3D graph in interval [−1000, 500] and [−50, 50] (b) contour graph in interval [−1000, 500] and [−50, 50] and (c) 2D graph in interval [−50, 50].

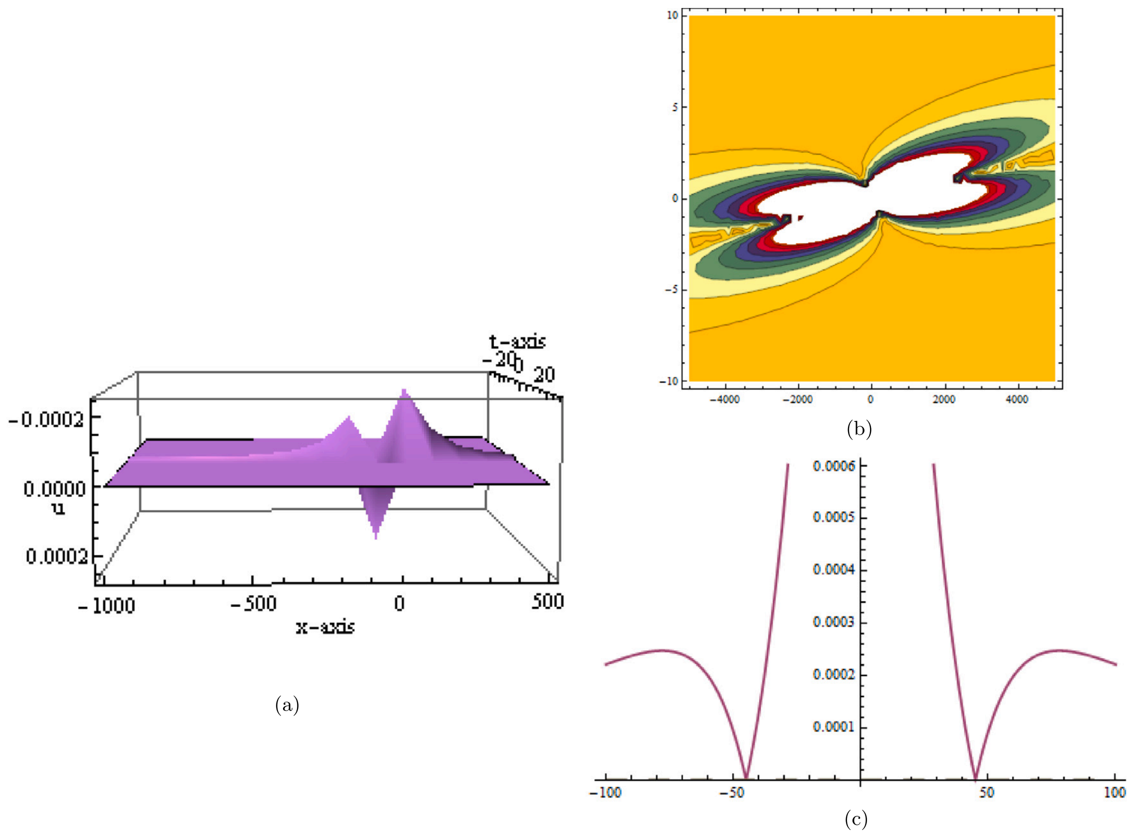


Fig. 3. The graphical description of $u(x,t)$ in Eq. (12) is given by these parameters $\alpha_5 = 200$, $b_1 = 100$, $\omega_0 = 100$. (a) 3D graph in interval [−1000, 500] and [30, 30] (b) contour graph in interval [−5000, 5000] and [−10, 10] and (c) 2D graph in interval [−100, 100].

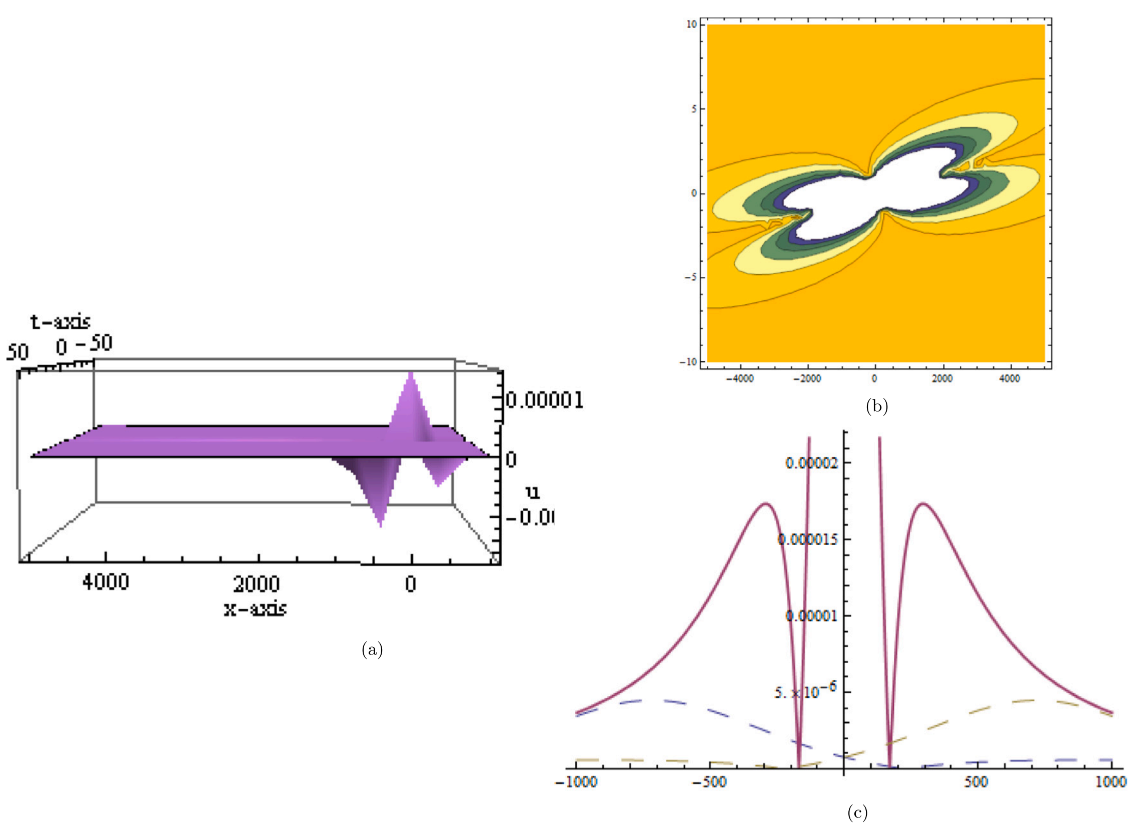
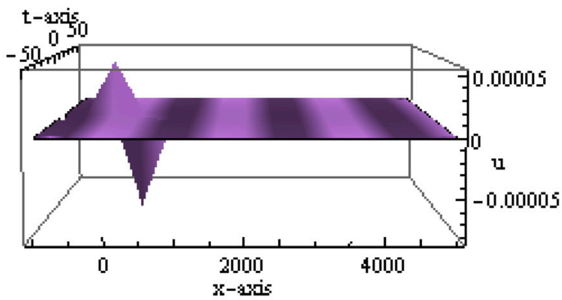
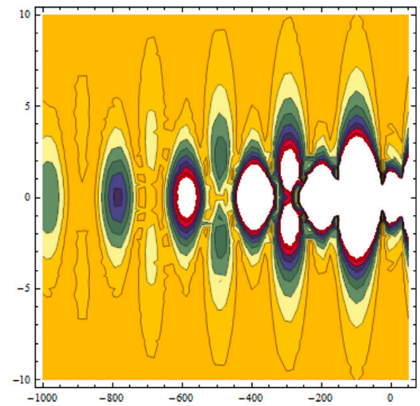


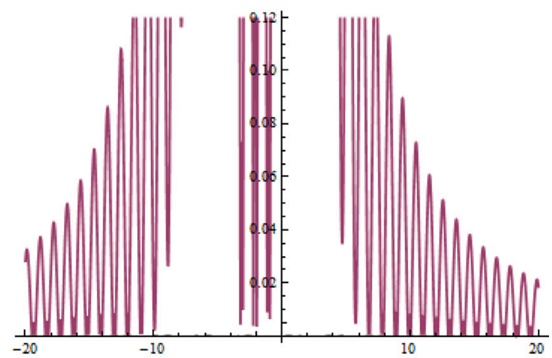
Fig. 4. The graphical description of $u(x,t)$ in Eq. (12) is given by these parameters $\alpha_5 = 5000$, $\omega_0 = -10$. (a) 3D graph in interval [−1000, 5000] and [−50, 50] (b) contour graph in interval [−5000, 5000] and [−10, 10] and (c) 2D graph in interval [−1000, 1000].



(a)



(b)



(c)

Fig. 5. The graphical description of $u(x,t)$ in Eq. (17) is given by these parameters $\alpha_4 = 10$, $\alpha_6 = 20$, $b_1 = 8$, $k_1 = 6\omega_0 = 15$. (a) 3D graph in interval [−1000, 5000] and [−50, 50] (b) contour graph in interval [−1000, 50] and [−10, 10] and (c) 2D graph in interval [−20, 20].

and

$$\Omega = \alpha_7 + \Pi^2 + \Gamma^2 + b_1\cos(Y_1) + b_2\exp(Y_2). \tag{20}$$

We have to find α_i, $1 \le i \le 7$, k_j, $1 \le j \le 2$ and p_m, $1 \le m \le 2$ which are all real parameters.

We obtained the following sets of parameters by removing coefficients of x, t after substituting Ω into Eq. (3).

Set 1. When $\alpha_1 = \alpha_6 = k_2 = 0$, we obtained the following solutions:

$$b_1 = b_1, b_2 = b_2, \alpha_2 = \alpha_2, \alpha_3 = \alpha_3, \alpha_4 = \frac{4(2\alpha_2^2 + \alpha_5^2)}{\alpha_5(21k_1^2 + 4\omega_0)}, \tag{21}$$

$$\alpha_5 = \alpha_5, \alpha_7 = \alpha_7, p_1 = \frac{7}{6}k_1^3 + k_1\omega_0, p_2 = 0, k_1 = k_1.$$

By putting above values into Eq. (2), we get

$$u = 2\frac{(\Pi^2 + \Gamma^2 + \alpha_7 + b_1\cos(Y_1) + b_2\exp(Y_2))(2\alpha_4^2 - b_1k_1^2\cos(Y_1)) - (2\alpha_4\Gamma - b_1k_1 sin(Y_1))^2}{(\Pi^2 + \Gamma^2 + \alpha_7 + b_1\cos(Y_1) + b_2\exp(Y_2))^2}, \tag{22}$$

where Π, Γ, Y_1 and Y_2 are given as

$$\Pi = \alpha_2 t + \alpha_3,\ \Gamma = \frac{4(2\alpha_2^2 + \alpha_5^2)}{\alpha_5(21k_1^2 + 4\omega_0)}x + \alpha_5 t, \tag{23}$$

$$Y_1 = k_1 x + (\frac{7}{6}k_1^3 + k_1\omega_0)t,\ Y_2 = 0.$$

The graph of solution set 1 is given below.

6. Result and discussion

In this section, we compare the newly obtained results with previous results in literature. Karunakar and Chakraverty studied geophysical KdV equation and also studied effect of Coriolis constant on this equation [54]. Johnson studied Camassa Holm, KdV and related models for water waves [55]. Wazzan [56] studied KdV and the KdV–Burgers equations and used modified tanh − coth method for solving these equations. Kudryashov [57] studied KdV and the KdV–Burgers equations and obtained its new travelling wave solution. Wazwaz [58] studied perturbed KdV equation and obtained its multiple-soliton solutions. Wang and Liu [59] studied two-component KdV systems by prolongation technique and Painlevé analysis. Brühl and Oumeraci [44] studied long-period cosine wave dispersion in very shallow water using nonlinear Fourier transform based on KdV equation.

We studied $(1+1)$-dimensional geophysical KdV equation (GKdVE) for lump solution, lump–kink solution (which is interaction of lump and one kink soliton),lump-periodic solution (which is interaction of lump with periodic waves) and lump–kink-periodic solution (which is interaction of lump with one kink soliton and periodic wave) using Eq. (2) with some parameters with help of HBM. Hirotas method is algebraic rather than analytic and it considered its big advantage over other methods. Lump solitons have been discussed in many physical phenomena such as optic media, plasma and shallow water wave and so on. We used Mathematica 7 to obtain lump soliton, lump–kink solution, lump-periodic solution and lump–kink-periodic solution for different values of parameters. Eq. (6) represents lump soliton solution. Eq. (11) represents lump–kink solution, Eq. (16) represent lump-periodic solution and Eq. (21) represents lump–kink-periodic solution. From Eq. (7) we observed that u approaches zero if and only if the sum of $\Pi^2 + \Gamma^2 + \alpha_7 \to \infty$. Figs. 1 and 2 represents lump soliton solutions. Fig. 1(a) represents 3D plot of u describing bright and dark lump solution. Fig. 1(b) describes contour plot of u. Fig. 1(c) illustrate 2D plot of u. Fig. 2 also define the similar properties of u. Figs. 3 and 4 represents lump–kink soliton solutions (which is interaction of lump with one kink soliton) in which there is interaction of lump with one kink soliton.

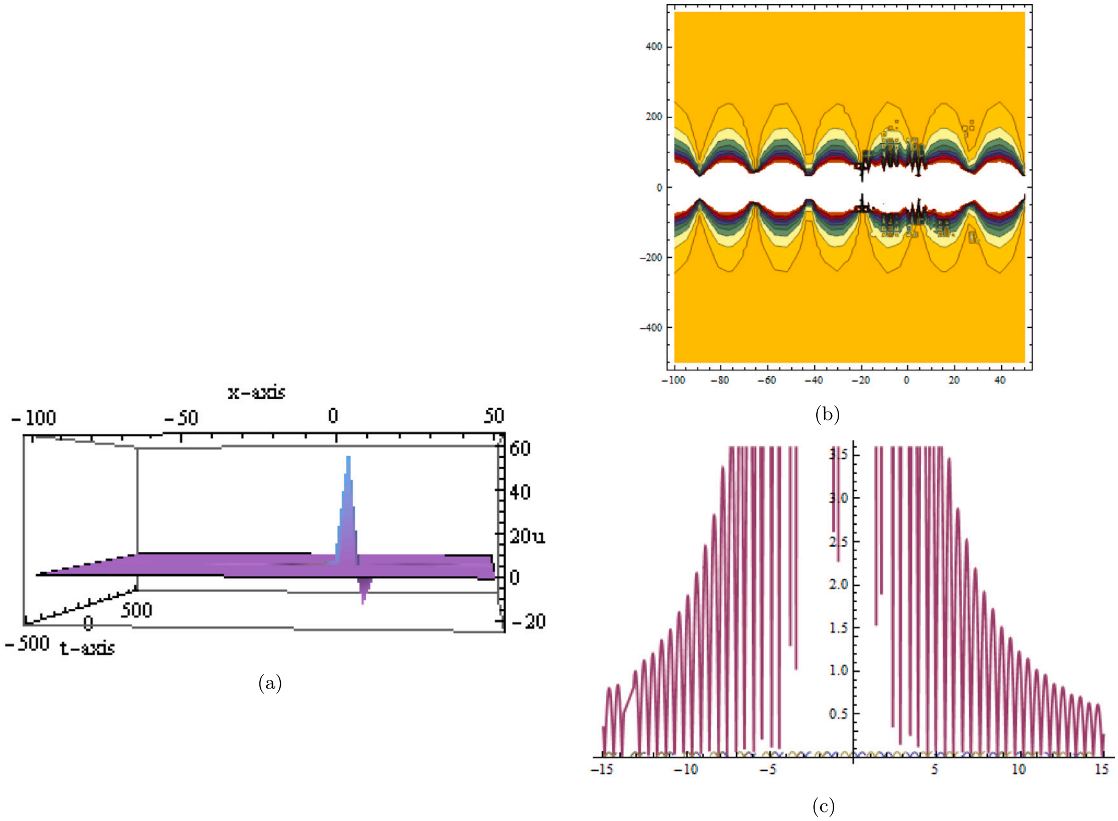


Fig. 6. The graphical description of $u(x,t)$ in Eq. (17) is given by these parameters $\alpha_4 = 2$, $\alpha_6 = 2$, $b_1 = 8$, $\omega_0 = 15$, $k_1 = 6$. (a) 3D graph in interval [−100, 50] and [−500, 500] (b) contour graph in interval [−100, 50] and [−500, 500] and (c) 2D graph in interval [−15, 15].

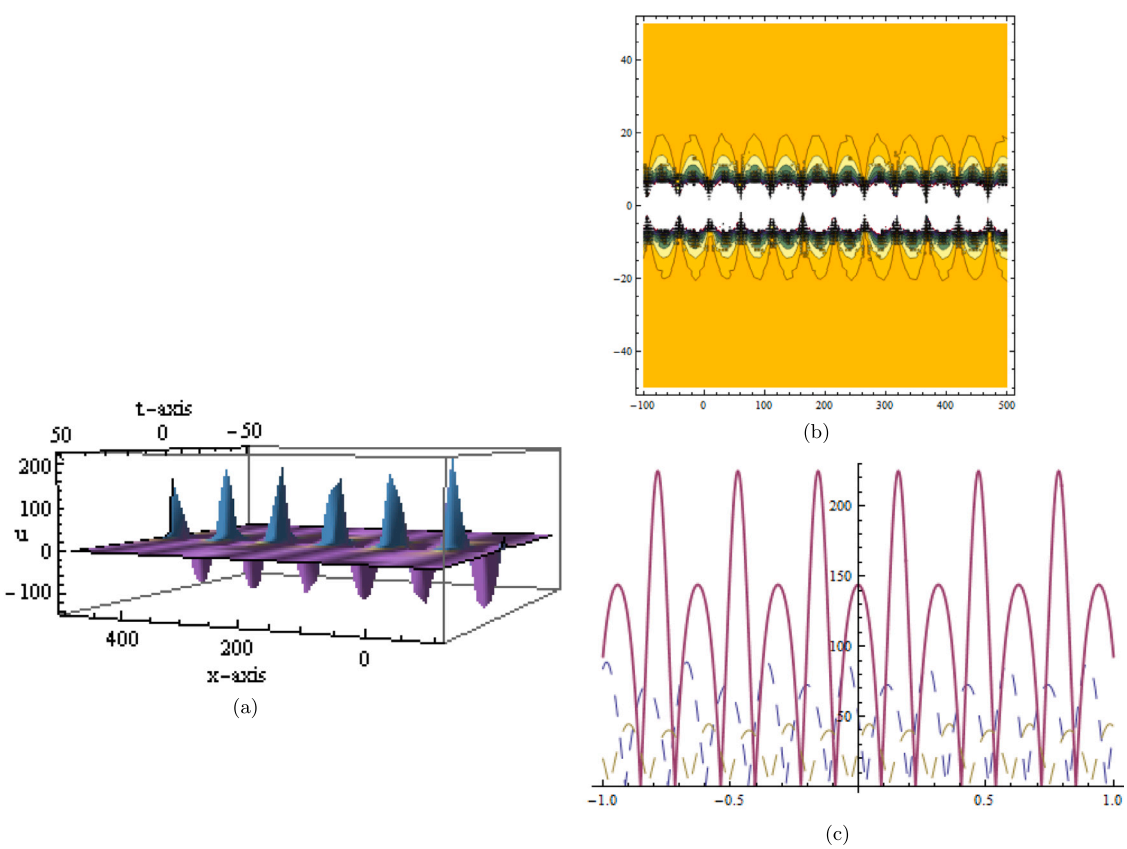


Fig. 7. The graphical description of $u(x,t)$ in Eq. (22) is given by these parameters $\alpha_2 = 5$, $\alpha_3 = 4$, $\alpha_4 = 20$, $\alpha_5 = 8$, $\alpha_7 = 10 b_1 = 9$, $b_2 = 15 k_1 = 20 \omega_0 = 5$. (a) 3D graph in interval [−100, 500] and [−50, 50] (b) contour graph in interval [−100, 500] and [−50, 50] and (c) 2D graph in interval [−1, 1].

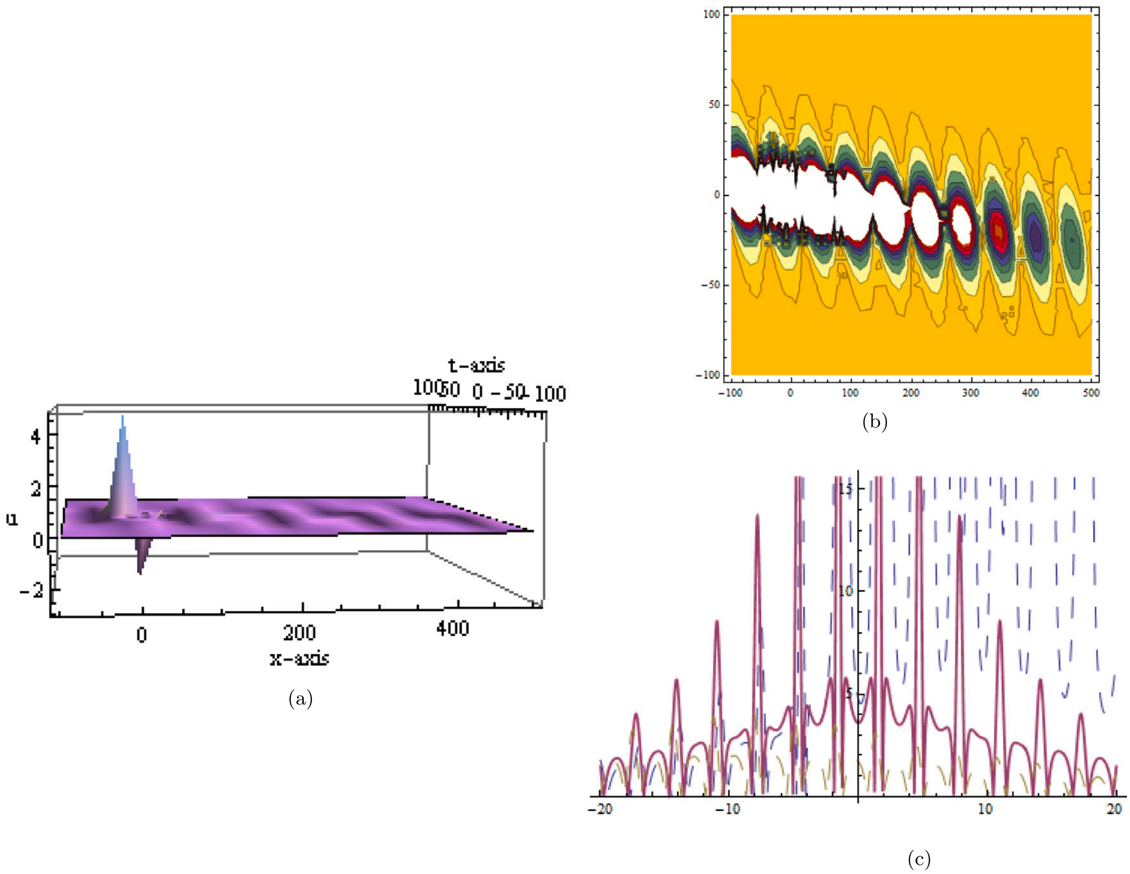


Fig. 8. The graphical description of $u(x, t)$ in Eq. (22) is given by these parameters $\alpha_2 = 2$, $\alpha_3 = 3$, $\alpha_4 = 4$, $\alpha_5 = 2$, $\alpha_7 = 1b_1 = 9$, $b_2 = 1k_1 = 2\omega_0 = 5$. (a) 3D graph in interval [−100, 500] and [−100, 100] (b) contour graph in interval [−100, 500] and [−100, 100] and (c) 2D graph in interval [−20, 20].

Fig. 3(a) represents 3D plot of u describing 2 bright part and 1 dark part of lump–kink soliton solution. Fig. 3(b) describes contour plot of u. Fig. 3(c) illustrate 2D plot of u. Similarly Fig. 4 describes similar properties of u. Figs. 5 and 6 shows lump-periodic solution(which is interaction of lump with periodic wave) . And Figs. 7 and 8 shows lump–kink-periodic solution (which is interaction of lump with one kink soliton and a periodic wave). And Figs. 7 and 8, 9 and 10 have similar properties of u as Figs. 3 and 4 have.

7. Conclusion

In this paper, we obtained lump soliton solutions of geophysical Korteweg–de Vries equation by using Hirota bilinear method (HBM). We have also obtained interactions solutions such as lump–kink solution (interaction of lump with one kink soliton), lump-periodic solution (interaction of lump with periodic wave) and lump–kink-periodic solution (formed by interaction of lump with one kink soliton and periodic wave). So as to demonstrate that our strategy is very compelling for finding nonlinear evolution equation solutions, we exhibit our solutions with graph.

CRediT authorship contribution statement

S.T.R. Rizvi: Visualization, Investigation. **Aly R. Seadawy:** Conceptualization, Methodology, Software, Supervision. **F. Ashraf:** Data curation, Writing - original draft. **M. Younis:** Software, Validation. **H. Iqbal:** Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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