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Original Article

Lump, its interaction phenomena and conservation laws to a nonlinear mathematical model

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A B S T R A C T

We solve the Ostrovsky equation in the absence of the rotation effect using the Hirota bilinear method and symbolic calculation. Some unique interaction phenomena have been obtained between lump solution, breather wave, periodic wave, kink soliton, and two-wave solutions. All the obtained solutions are validated by putting them into the original problem using the Wolfram Mathematica 12. The physical characteristics of the solutions have been visually represented to shed additional light on the acquired results. Furthermore, using the novel conservation theory, the conserved vectors of the governing equation have been generated. The discovered results are helpful in understanding particular physical phenomena in fluid dynamics as well as the dynamics of nonlinear higher dimensional wave fields in computational physics and ocean engineering and related disciplines.

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1. Introduction

Lump solitons are utilized in a wide range of applied disciplines, including mathematics, chemistry, communication, biology, and, in particular, all aspects of engineering and physics [1–3]. Although some researchers used numerical simulation or analytical methods to research the output of such structures, the theoretical analysis for such systems needs to be further explored [4–6]. Rogue wave (RW) is an instinctive ocean waves that has based increasingly on the theoretical and experimental aspects [7]. In the simplest form, nonlinear Schrodinger equation's RW was proposed in [8]. It is of worth noting that, wave phenomena in different areas, such as Bose-Einstein condensates, plasmas, nonlinear mechanics, biophysics and finance can be depicted [9–11].

In order to create a new mix of functions using the Hirota bilinear apprach, several writers used lump solutions and their interaction phenomena to get some novel solutions. Literature has generated several important works on lump solutions. A number of lump techniques have been presented from various perspectives, including Zakharov [12], lump wave solution [13], and lump solution using the Hirota bilinear method [14–17]. When several important features of lump solutions are considered, it can be observed that solitons' forms, amplitudes, and speeds are retained after collision with another soliton, which is the elastic property of collision. The interaction between the kink solitary wave and the rouge wave solution was also described in [18]. And many more [19–52].

For every given partial differential equation, symmetries are transformations that make the whole space of the problem's solutions invariant. Symmetries can be utilized to produce reductions and precise group-invariant solutions. Other analytical features, such as asymptotic and blow-up behavior, rely heavily on invariant solutions. Furthermore, explicit solutions may be utilized to verify the correctness and reliability of numerical solution methods. The Lie technique may be used to identify all acceptable Lie symmetries for a particular PDE. Furthermore, the conservation law of a particular equation of evolution is a continuity equation that gives basic values preserved for all solutions. Among other important uses, they allow the detection and construction of mappings to linear equations of nonlinear evolution equations. In addition, they can be used to test integrability [53–60].

The interaction between internal waves and ocean topography has been an active field of research for long. Driving mechanism

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of waves solutions impacts the propagation of surface and internal gravity waves. These waves are quite common in oceans, lakes and the atmosphere, the mechanism of their derivation could play a significant part in ocean engineering. When waves enter areas of shallow water, they are influenced by the ocean floor. The water's free orbital motion is interrupted, and water particles in orbital motion are no longer able to return to their original position. The swell gets bigger and steeper as the water gets shallower, eventually taking on the typical sharp-crested wave shape. After the wave breaks, it transforms into a tidal wave[61]. The model equation for the unidirectional propagation of weakly nonlinear long surface and internal waves of small amplitude in a rotating fluid is known as the Ostrovsky equation [62] and is given by

$$\left(\chi_t - \beta\chi_{xxx} + (\chi^2)_x\right)_x = \gamma\chi, \tag{1}$$

where χ denotes the free surface of the in-compressible and inviscid liquid and the parameter γ measures the effect of rotation. When $\gamma = 0$, Eq 1 changes to

$$\left(\chi_t - \beta\chi_{xxx} + (\chi^2)_x\right)_x = 0. \tag{2}$$

Here, we employ the Hirota bilinear technique [63] to establish some RW, lump solutions and their interaction for the Ostrovsky equation appearing in Eq 2. In addition, we will investigate its Lie symmetry, symmetry reduction, and conservation laws. Placing the ColeHopf transformation

$$\chi(x,t) = -6\beta\left(\ln f(x,t)\right)_{xx} \tag{3}$$

into Eq 2, we get the following bilinear form:

$$6\beta\left(3\beta f_{xx}^2 - f f_{xt} + f_x(f_t - 4\beta f_{xxx}) + \beta f_{xxxx}\right) = 0. \tag{4}$$

Using Eq 4, we reach the lump, lump-soliton, lump-kink, lump-periodic, breather wave and other interaction phenomena to Eq 2 that will be discussed in the next section.

2. Lump and its interaction phenomena

This section presents the lump and its interaction solutions to the Ostrovsky equation given in Eq 2.

2.1. Lump solution

The lump solution to Eq 2 will be reported in this portion. Suppose that the positive quadratic solutions to Eq 4 to be

$$\eta = a_2 t + a_1 x + a_3, \quad \zeta = a_5 t + a_4 x + a_6, \quad f = a_7 + \eta^2 + \zeta^2. \tag{5}$$

Substituting Eq 5 into Eq 4, provides a polynomial in x and t. Collecting the coefficients of the same power, and equating each collection to zero, gives a system of equations. The values of the parameter are obtained by solving the obtained system of equations. Inserting the obtained values of the coefficients into Eq 3, gives For

$$a_4 = -ia_1, \quad a_5 = -\frac{a_2 a_4}{a_1}, \quad a_7 = 0, \quad i = \sqrt{-1},$$

one reaches

$$f(x,t) = (a_2 t + a_1 x + a_3)^2 + (ia_2 t - ia_1 x + a_6)^2. \tag{6}$$

Thus,

$$\chi(x,t) = \frac{6\beta\left(2a_1(a_2 t + a_1 x + a_3) - 2ia_1(ia_2 t - ia_1 x + a_6)\right)^2}{\left((a_2 t + a_1 x + a_3)^2 + (ia_2 t - ia_1 x + a_6)^2\right)^2}. \tag{7}$$

2.2. Lump-soliton solution

Herein, the lump-soliton to Eq 2 is reported. Consider the hyperbolic test function to be the solution of

$$f(x,t) = \eta^2 + \zeta^2 + \sinh(\xi) + a_{10}, \tag{8}$$

Eq 4

where $\eta = a_2 t + a_1 x + a_3$, $\zeta = a_5 t + a_4 x + a_6$, $\xi = a_8 t + a_7 x + a_9$. Substituting Eq 8 into Eq 4, we generates a polynomial in x, t and hyperbolic sine function. Collecting the same power coefficients, and equating each collection to zero, gives a system of equations. The values of the parameter are attained by solving the obtained system of equations. Inserting the obtained values into Eq 3, gives For

$$a_2 = 3a_1 a_7^2\beta, \quad a_4 = -ia_1, \quad a_5 = 3a_4 a_7^2\beta,$$

$$a_6 = \frac{2a_1 a_3 a_4 - \sqrt{4a_3^2 a_1^4 + 4a_3^2 a_4^2 a_1^2 - a_7^2 a_1^2}}{2a_1^2},$$

$a_8 = a_7^3\beta$, $i = \sqrt{-1}$, one attains

$$f(x,t) = a_{10} - \phi_1^2 + \phi_2^2 + \sinh(\phi_2). \tag{9}$$

Thus,

$$\chi(x,t) = -\frac{6\beta\left(a_7^2\sinh(\phi_2)\left(a_{10} - \phi_1^2 + \phi_3^2 + \sinh(\phi_2)\right) - (2a_1\phi_1 + 2a_1\phi_3 + a_7\cosh(\phi_2))^2\right)}{\left(a_{10} - \phi_1^2 + \phi_3^2 + \sinh(\phi_2)\right)^2}, \tag{10}$$

where $\phi_1 = -3a_1 a_7^2\beta t - a_1 x + \frac{-2a_3 a_1^2 - a_7 a_1}{2a_1^2}$, $\phi_2 = a_7^3\beta t + a_7 x + a_9$, $\phi_3 = 3a_1 a_7^2\beta t + a_1 x + a_3$.

2.3. Lump-kink solution

Herein, the lump-kink solution to Eq 2 is provided. Taking into account the exponential test function as a solution to

$$f(x,t) = (a_1 x + a_2 t + a_3)^2 + (a_4 x + a_5 t + a_6)^2 + e^{a_7 x + a_8 t + a_9} + a_{10}. \tag{11}$$

Substituting Eq 11 into Eq 4, produces a polynomial in x, t and exponential function. Collecting the same power coefficients, and equating each collection to zero, gives a system of equations. The values of the parameter are obtained by solving the obtained system of equations. Inserting the obtained values into Eq 3, gives: For

$$a_2 = 3a_1 a_7^2\beta, \; a_4 = -ia_1, \; a_5 = \frac{a_2 a_4}{a_1}, \; a_6 = \frac{-\sqrt{a_2^2 a_3^2 + a_5^2 a_3^2} - a_2 a_3}{a_5},$$

$a_8 = a_7^3\beta$,

we obtain

$$f(x,t) = e^{a_7^3\beta t + a_7 x + a_9} + a_{10}. \tag{12}$$

Thus,

$$\chi(x,t) = -\frac{6\beta\left(a_7^2 e^{a_7^3\beta t + a_7 x + a_9}\left(e^{a_7^3\beta t + a_7 x + a_9} + a_{10}\right) - a_7^2 e^{2a_7^3\beta t + 2a_7 x + 2a_9}\right)}{\left(e^{a_7^3\beta t + a_7 x + a_9} + a_{10}\right)^2}. \tag{13}$$

2.4. Lump-periodic solution

Herein, the lump-periodic solution to Eq 2 is given. Taking into account the trigonometric test function as a solution to

$$f(x,t) = (a_1 x + a_2 t + a_3)^2 + (a_4 x + a_5 t + a_6)^2 + \sin(a_7 x + a_8 t + a_9) + a_{10}. \tag{14}$$

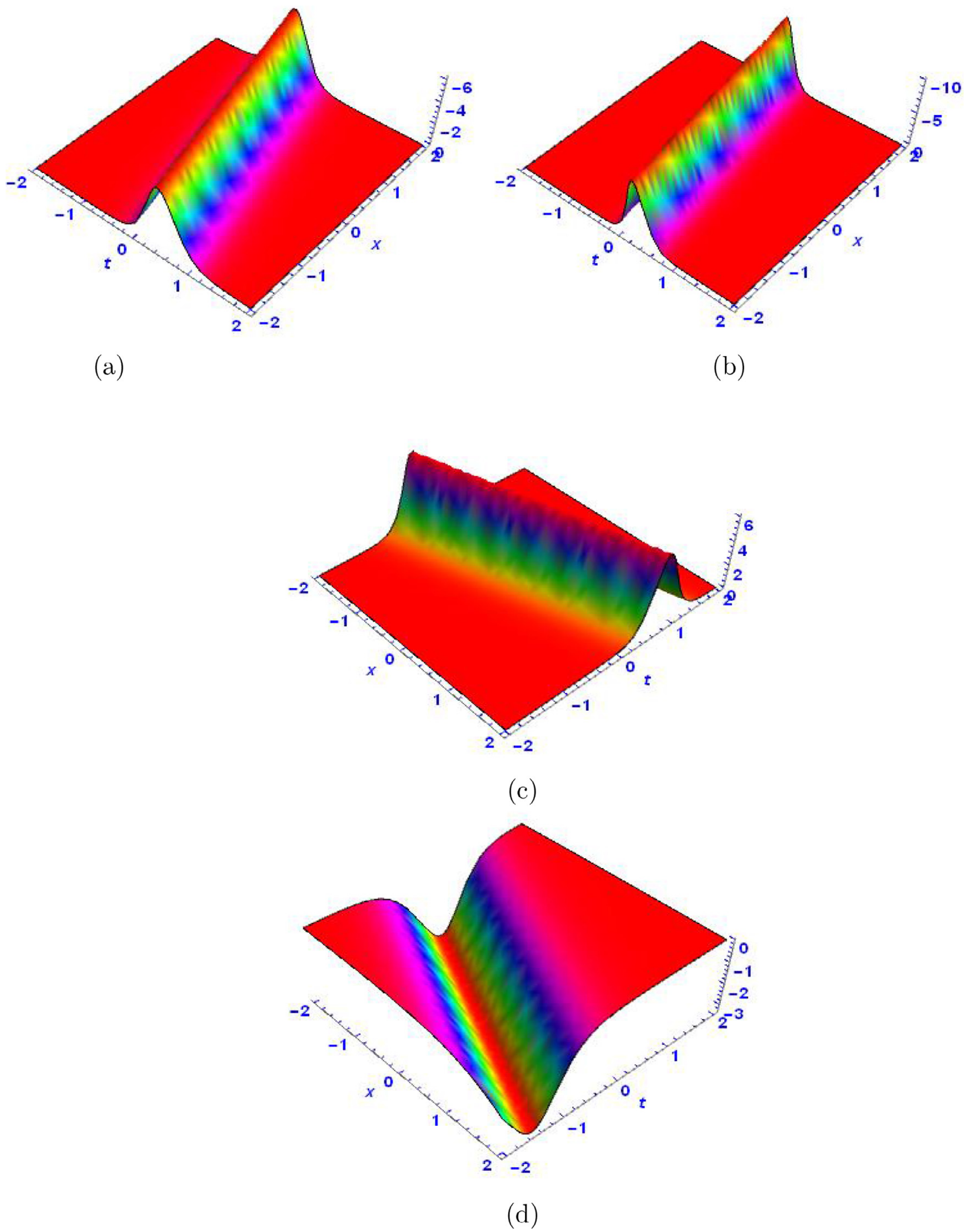

Fig. 1. The (a) 3D profile at $\beta = -6.7$ (b) 3D profile at $\beta = 6.7$ of Eq 10 and (c) 3D profile at $\beta = -1.5$ (d) 3D profile at $\beta = 0.4$ of Eq 13.

Eq 4
Substituting Eq 14 into Eq 4, produces a polynomial in x, t and trigonometric function. Collecting coefficients of the same power, and equating each collection to zero, gives a system of algebraic equations. The values of the coefficients are obtained by solving the obtained system equations. Inserting the obtained values of the coefficients into Eq 3, gives: For

$a_2 = -3a_1a_7^2\beta,\ a_4 = -ia_1;\ a_5 = -3a_4a_7^2\beta,$

$$a_6 = \frac{2a_1a_3a_4 - \sqrt{4a_3^2a_1^4 + 4a_3^2a_4^2a_1^2 - a_7^2a_1^2}}{2a_1^2},$$

$a_8 = a_7^3(-\beta)$, we have

$$f(x,t) = a_{10} - \phi_4^2 + \phi_6^2 + \sin(\phi_5). \tag{15}$$

Thus,

$$\chi(x,t) = -\frac{6\beta\left(a_7^2(-\sin(\phi_5))\left(a_{10} - \phi_4^2 + \phi_6^2 + \sin(\phi_5)\right) - (2a_1\phi_4 + 2a_1\phi_6 + a_7\cos(\phi_5))^2\right)}{\left(a_{10} - \phi_4^2 + \phi_6^2 + \sin(\phi_5)\right)^2}, \tag{16}$$

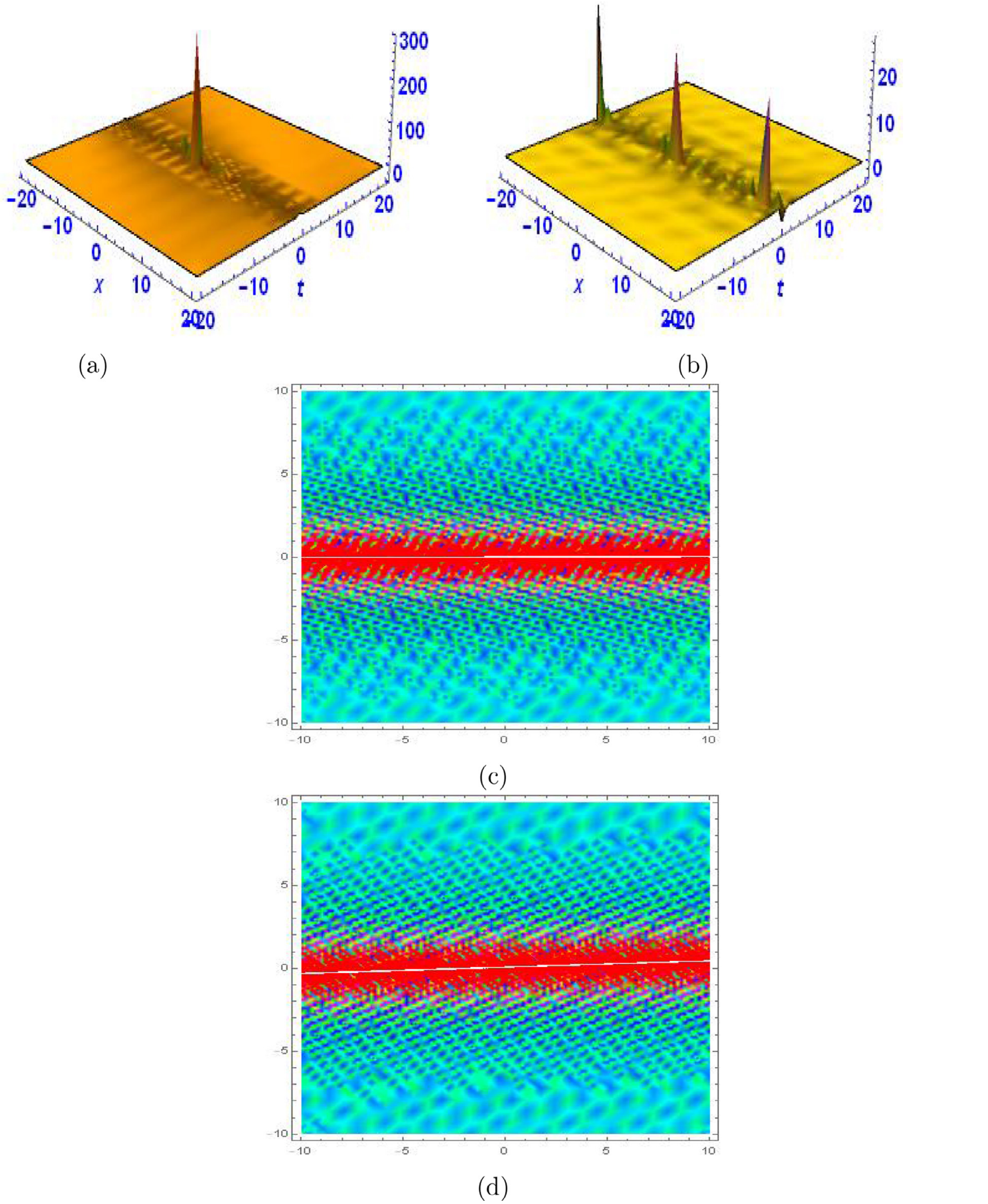

Fig. 2. The (a) 3D profile at $\beta = -2$ (b) 3D profile at $\beta = 9.02$ (c) density profile at $\beta = -2$ (d) density profile at $\beta = 9.02$ of Eq 16.

where $\phi_4 = 3a_1 a_7^2 \beta t - a_1 x - \frac{2a_3 a_1^2 + a_7 a_1}{2a_1^2}$, $\phi_5 = a_7^3 \beta(-t) + a_7 x + a_9$, $\phi_6 = -3a_1 a_7^2 \beta t + a_1 x + a_3$.

2.5. Breather wave solutions

Herein, the breather wave solutions to Eq 2 is reported. Consider the following test function to be a trial solution to Eq 4:

$$f(x,t) = e^{-p_1(a_0 t + x)} + m_1 \cos(p_0(b_0 t + x)) + m_2 e^{p_1(a_0 t + x)}. \tag{17}$$

Substituting Eq 17 into Eq 4, produces a polynomial in x, t exponential and trigonometric functions. Collecting the coefficients of the same power, and equating each collection to zero, gives a system of equations. The values of the coefficients are obtained by solving the obtained system of equations. Inserting the obtained values of the coefficients into Eq 3, gives: **Case-1:** For

$$a_0 = \beta p_1^2 - 3\beta p_0^2, \; b_0 = -a_0 - 4\beta p_0^2 + 4\beta p_1^2,$$

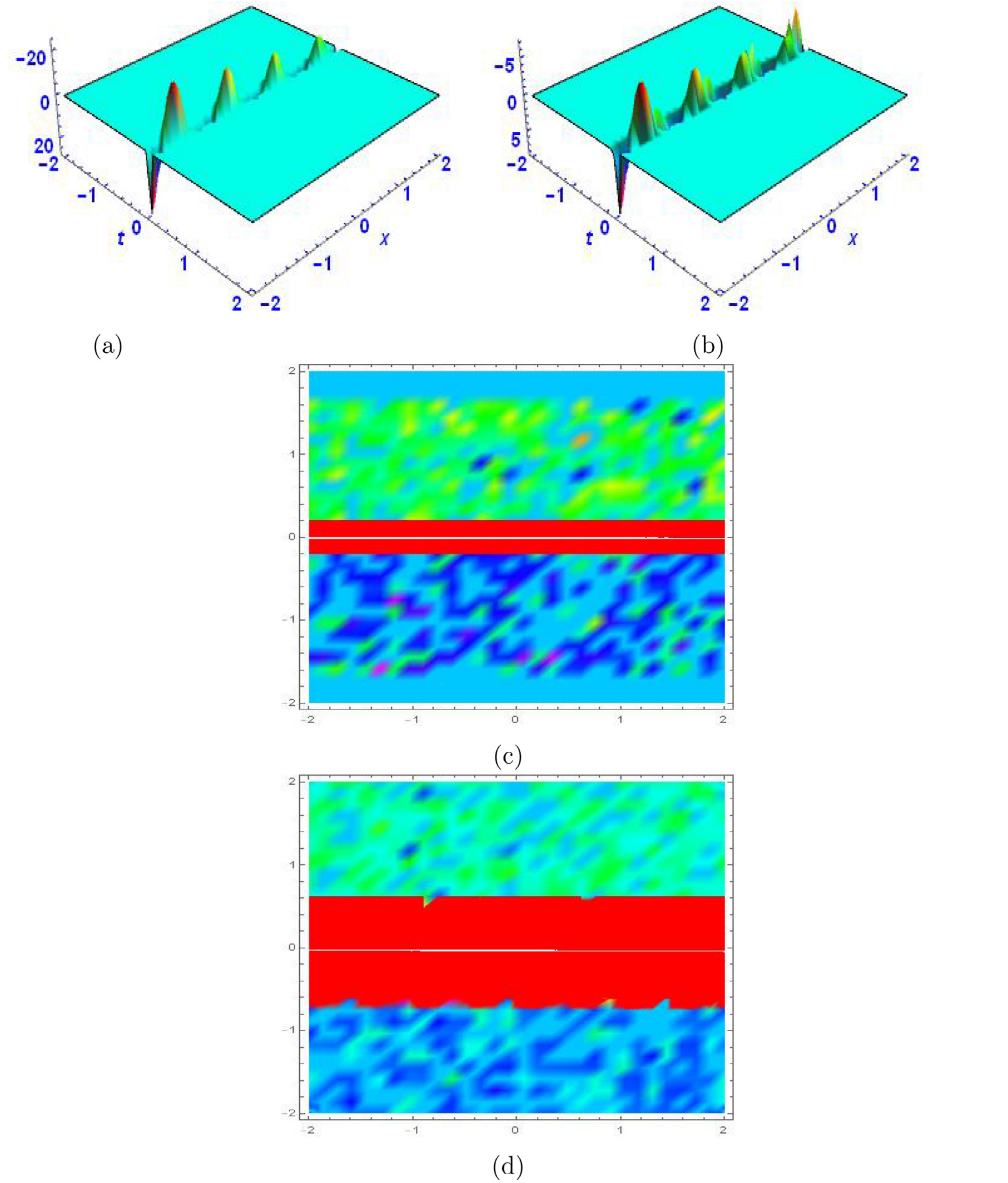

Fig. 3. The (a) 3D profile at $\beta = -1.2$ (b) 3D profile at $\beta = -0.3$ (c) density profile at $\beta = -1.2$ (d) density profile at $\beta = -0.3$ of Eq 19.

$$m_2 = \frac{m_1^2\left(a_0 p_1^2 + 3\beta p_0^4 + 6\beta p_1^2 p_0^2 - \beta p_1^4\right)}{4p_1^2\left(a_0 - 4\beta p_1^2\right)},$$

we get

$$f(x,t) = m_1 \cos(\phi_9) + \frac{\phi_7}{4p_1^2} + \phi_8. \tag{18}$$

Thus,

$$\chi(x,t) = -\frac{6\beta\left(\psi\left(m_1\cos(\phi_9)+\frac{\phi_7}{4p_1^2}+\phi_8\right)-\left(m_1 p_0(-\sin(\phi_9))+\frac{\phi_7}{4p_1}-p_1\phi_8\right)^2\right)}{\left(m_1\cos(\phi_9)+\frac{\phi_7}{4p_1^2}+\phi_8\right)^2}, \tag{19}$$

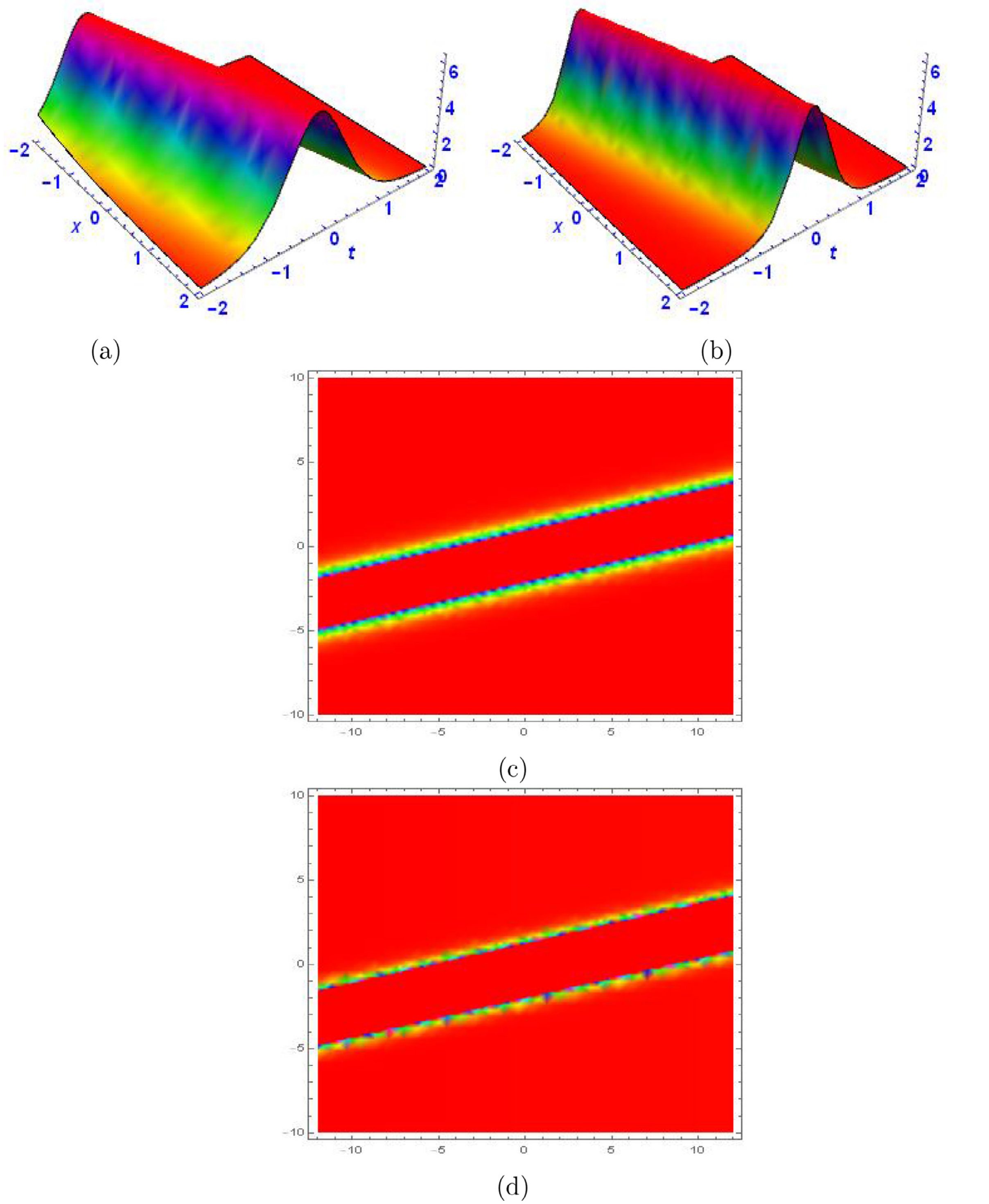

Fig. 4. The (a) 3D profile at $\beta=-4.3$ (b) 3D profile at $\beta=-0.5$ (c) density profile at $\beta=-4.3$ (d) density profile at $\beta=-0.5$ of Eq 21.

where $\phi_7=\frac{m_1^2(3\beta p_0^4+6\beta p_1^2p_0^2-\beta p_1^4+p_1^2(\beta p_1^2-3\beta p_0^2))e^{p_1(t(\beta p_1^2-3\beta p_0^2)+x)}}{-3\beta p_0^2-3\beta p_1^2}$, $\phi_8=e^{-p_1(t(\beta p_1^2-3\beta p_0^2)+x)}$, $\phi_9=p_0\left(t\left(3\beta p_1^2-\beta p_0^2\right)+x\right)$, $\psi=m_1p_0^2(-\cos(\phi_9))+p_1^2\phi_8+\frac{\phi_7}{4}$.

Case-2: When

$$p_1=-\frac{\sqrt{3b_0-a_0}}{2\sqrt{2}\sqrt{\beta}},\ p_0=-\frac{\sqrt{b_0-3a_0}}{2\sqrt{2}\sqrt{\beta}},\ m_2=\frac{b_0m_1^2-3a_0m_1^2}{4(a_0-3b_0)},$$

we have

$$f(x,t)=\frac{(b_0m_1^2-3a_0m_1^2)e^{-\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}}{4(a_0-3b_0)}+m_1\cos\left(\frac{\sqrt{b_0-3a_0}(b_0t+x)}{2\sqrt{2}\sqrt{\beta}}\right)+e^{\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}. \tag{20}$$

Thus,

$$\chi(x,t) = -\frac{6\beta\left((\phi_{11}+\phi_{18})(\phi_{12}+\phi_{13}-\phi_{17})-(-\phi_{10}+\phi_{14}-\phi_{16})^2\right)}{\left(e^{\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}+m_1\cos(\phi_{15})+\phi_{11}\right)^2}, \tag{21}$$

where $\phi_{10} = \frac{\sqrt{3b_0-a_0}\left(b_0m_1^2-3a_0m_1^2\right)e^{-\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}}{8\sqrt{2}\sqrt{\beta}(a_0-3b_0)}$, $\phi_{11} = \frac{\left(b_0m_1^2-3a_0m_1^2\right)e^{-\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}}{4(a_0-3b_0)}$, $\phi_{12} = \frac{(3b_0-a_0)\left(b_0m_1^2-3a_0m_1^2\right)e^{-\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}}{32\beta(a_0-3b_0)}$, $\phi_{13} = \frac{(3b_0-a_0)e^{\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}}{8\beta}$, $\phi_{14} = \frac{\sqrt{3b_0-a_0}e^{\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}}{2\sqrt{2}\sqrt{\beta}}$, $\phi_{15} = \frac{\sqrt{b_0-3a_0}(b_0t+x)}{2\sqrt{2}\sqrt{\beta}}$, $\phi_{16} = \frac{m_1\sqrt{b_0-3a_0}\sin(\phi_{15})}{2\sqrt{2}\sqrt{\beta}}$, $\phi_{17} = \frac{m_1(b_0-3a_0)\cos(\phi_{15})}{8\beta}$, $\phi_{18} = e^{\frac{\sqrt{3b_0-a_0}(a_0t+x)}{2\sqrt{2}\sqrt{\beta}}}+m_1\cos(\phi_{15})$.

2.6. Some new interaction solutions

Here, we report some new interaction solutions to Eq 2. Consider the following test function to be a trial solution to Eq 4:

$$f(x,t) = c_1e^{(b_2t+b_1x)}+c_2e^{(-(b_2t+b_1x))}+c_3\sin(b_4t+b_3x) + c_4\sinh(b_6t+b_5x), \tag{22}$$

Plugging Eq 22 into Eq 4, produces a polynomial in trigonometric, hyperbolic and exponential functions. Collecting the coefficients of the same power, and equating each collection to zero, we reach a system of equations. The values of the parameter are obtained by solving the obtained system of equations. Inserting the obtained values of the coefficients into Eq 3, gives: **Case-1:** When

$$b_4 = \frac{4\beta b_3b_1^3-4\beta b_3^3b_1-b_2b_3}{b_1},$$

$$c_3 = -\frac{2\sqrt{b_1b_2c_1c_2-4\beta b_1^4c_1c_2}}{\sqrt{-\beta b_1^4+6\beta b_3^2b_1^2+3\beta b_3^4+b_2b_1}},\ c_4 = 0$$

$b_3 = -\frac{\sqrt{\beta b_1^3-b_2}}{\sqrt{3}\sqrt{\beta}\sqrt{b_1}}$, we have

$$f(x,t) = \frac{2\sin(\phi_{19})\sqrt{b_1b_2c_1c_2-4\beta b_1^4c_1c_2}}{\sqrt{-\beta b_1^4+2b_1\left(\beta b_1^3-b_2\right)+\frac{(\beta b_1^3-b_2)^2}{3\beta b_1^2}+b_2b_1}} + c_1e^{b_2t+b_1x}+c_2e^{b_1(-x)-b_2t}. \tag{23}$$

Thus,

$$\chi(x,t) = -\frac{6\beta\left((\phi_{25}+\phi_{21}\sin(\phi_{19}))(\phi_{26}-\phi_{22}\sin(\phi_{19}))-(\phi_{24}+\phi_{20}\cos(\phi_{19}))^2\right)}{\left(c_1e^{b_2t+b_1x}+c_2e^{b_1(-x)-b_2t}+\phi_{23}\sin(\phi_{19})\right)^2}, \tag{24}$$

where $\phi_{19} = \frac{x\sqrt{\beta b_1^3-b_2}}{\sqrt{3}\sqrt{\beta}\sqrt{b_1}} - \frac{t\left(\frac{4\left(\beta b_1^3-b_2\right)^{3/2}}{3\sqrt{3}\sqrt{\beta}\sqrt{b_1}}+\frac{b_2\sqrt{\beta b_1^3-b_2}}{\sqrt{3}\sqrt{\beta}\sqrt{b_1}}-\frac{4\sqrt{\beta}b_1^{5/2}\sqrt{\beta b_1^3-b_2}}{\sqrt{3}}\right)}{b_1}$,

$\phi_{20} = \frac{2\sqrt{\beta b_1^3-b_2}\sqrt{b_1b_2c_1c_2-4\beta b_1^4c_1c_2}}{\sqrt{3}\sqrt{\beta}\sqrt{b_1}\sqrt{-\beta b_1^4+2b_1\left(\beta b_1^3-b_2\right)+\frac{(\beta b_1^3-b_2)^2}{3\beta b_1^2}+b_2b_1}}$, $\phi_{21} = \frac{2\sqrt{b_1b_2c_1c_2-4\beta b_1^4c_1c_2}}{\sqrt{-\beta b_1^4+2b_1\left(\beta b_1^3-b_2\right)+\frac{(\beta b_1^3-b_2)^2}{3\beta b_1^2}+b_2b_1}}$,

$\phi_{22} = \frac{2\left(\beta b_1^3-b_2\right)\sqrt{b_1b_2c_1c_2-4\beta b_1^4c_1c_2}}{3\beta b_1\sqrt{-\beta b_1^4+2b_1\left(\beta b_1^3-b_2\right)+\frac{(\beta b_1^3-b_2)^2}{3\beta b_1^2}+b_2b_1}}$, $\phi_{23} = \frac{2\sqrt{b_1b_2c_1c_2-4\beta b_1^4c_1c_2}}{\sqrt{-\beta b_1^4+2b_1\left(\beta b_1^3-b_2\right)+\frac{(\beta b_1^3-b_2)^2}{3\beta b_1^2}+b_2b_1}}$, $\phi_{24} = b_1c_1e^{b_2t+b_1x} - b_1c_2e^{b_1(-x)-b_2t}$, $\phi_{25} = c_1e^{b_2t+b_1x}+c_2e^{b_1(-x)-b_2t}$, $\phi_{26} = b_1^2c_1e^{b_2t+b_1x}+b_1^2c_2e^{b_1(-x)-b_2t}$.

Case-2: When

$$b_6 = \frac{b_5\left(4\beta b_1^3+4\beta b_5^2b_1-b_2\right)}{b_1},\ c_3 = 0,$$

$$c_4 = -\frac{2\sqrt{-b_1}\sqrt{c_1}\sqrt{c_2}\sqrt{b_2-4\beta b_1^3}}{\sqrt{-\beta b_1^4-6\beta b_5^2b_1^2+3\beta b_5^4+b_2b_1}},$$

$b_5 = -\frac{\sqrt{b_2-\beta b_1^3}}{\sqrt{3}\sqrt{\beta}\sqrt{b_1}}$, we have

$$f(x,t) = \frac{2\sqrt{-b_1}\sqrt{c_1}\sqrt{c_2}\sqrt{b_2-4\beta b_1^3}\sinh(\phi_{27})}{\sqrt{-\beta b_1^4-2b_1\left(b_2-\beta b_1^3\right)+\frac{(b_2-\beta b_1^3)^2}{3\beta b_1^2}+b_2b_1}} + c_1e^{b_2t+b_1x}+c_2e^{b_1(-x)-b_2t}. \tag{25}$$

Thus,

$$\chi(x,t) = -\frac{6\beta\left((\phi_{32}+\phi_{28}\sinh(\phi_{27}))(\phi_{33}+\phi_{30}\sinh(\phi_{27}))-(\phi_{31}+\phi_{29}\cosh(\phi_{27}))^2\right)}{\left(c_1e^{b_2t+b_1x}+c_2e^{b_1(-x)-b_2t}+\phi_{28}\sinh(\phi_{27})\right)^2}, \tag{26}$$

where $\phi_{27} = \frac{t\left(4\beta b_1^3+\frac{4}{3}\left(b_2-\beta b_1^3\right)-b_2\right)\sqrt{b_2-\beta b_1^3}}{\sqrt{3}\sqrt{\beta}b_1^{3/2}} + \frac{x\sqrt{b_2-\beta b_1^3}}{\sqrt{3}\sqrt{\beta}\sqrt{b_1}}$,

$\phi_{28} = \frac{2\sqrt{-b_1}\sqrt{c_1}\sqrt{c_2}\sqrt{b_2-4\beta b_1^3}}{\sqrt{-\beta b_1^4-2b_1\left(b_2-\beta b_1^3\right)+\frac{(b_2-\beta b_1^3)^2}{3\beta b_1^2}+b_2b_1}}$, $\phi_{29} = \frac{2\sqrt{-b_1}\sqrt{c_1}\sqrt{c_2}\sqrt{b_2-4\beta b_1^3}\sqrt{b_2-\beta b_1^3}}{\sqrt{3}\sqrt{\beta}\sqrt{b_1}\sqrt{-\beta b_1^4-2b_1\left(b_2-\beta b_1^3\right)+\frac{(b_2-\beta b_1^3)^2}{3\beta b_1^2}+b_2b_1}}$, $\phi_{30} = \frac{2\sqrt{-b_1}\sqrt{c_1}\sqrt{c_2}\sqrt{b_2-4\beta b_1^3}\left(b_2-\beta b_1^3\right)}{3\beta b_1\sqrt{-\beta b_1^4-2b_1\left(b_2-\beta b_1^3\right)+\frac{(b_2-\beta b_1^3)^2}{3\beta b_1^2}+b_2b_1}}$, $\phi_{31} = b_1c_1e^{b_2t+b_1x} - b_1c_2e^{b_1(-x)-b_2t}$, $\phi_{32} = c_1e^{b_2t+b_1x}+c_2e^{b_1(-x)-b_2t}$, $\phi_{33} = b_1^2c_1e^{b_2t+b_1x}+b_1^2c_2e^{b_1(-x)-b_2t}$.

Case-3: When

$$b_1 = -\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2},\ b_4 = -\frac{2b_1b_2}{b_3},\ c_3 = -\frac{b_4\sqrt{-c_1}\sqrt{c_2}}{b_2},$$

$$c_4 = 0,\ \beta = \frac{-3b_4c_3^2-14b_4c_1c_2}{32b_3^3c_1c_2},$$

we have

$$f(x,t) = c_1e^{b_2t-\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x}+c_2e^{\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x-b_2t} - \frac{\sqrt{3b_3^2-\sqrt{17}b_3^2}\sqrt{-c_1}\sqrt{c_2}\sin(\phi_{34})}{b_3}. \tag{27}$$

Thus,

$$\chi(x,t) = \frac{-3\phi_{35}\left((\phi_{37}-\phi_{41}\sin(\phi_{34}))(\phi_{38}+\phi_{42}\sin(\phi_{34}))-(\phi_{36}-\phi_{40}\cos(\phi_{34}))^2\right)}{16b_3^3c_1c_2\left(\phi_{39}-\frac{\sqrt{3b_3^2-\sqrt{17}b_3^2}\sqrt{-c_1}\sqrt{c_2}\sin(\phi_{34})}{b_3}\right)^2}, \tag{28}$$

where $\phi_{34} = \frac{\sqrt{3b_3^2-\sqrt{17}b_3^2}b_2t}{b_3}+b_3x$, $\phi_{35} = \frac{3b_2\left(3b_3^2-\sqrt{17}b_3^2\right)^{3/2}c_1c_2}{b_3^3} - \frac{14b_2\sqrt{3b_3^2-\sqrt{17}b_3^2}c_1c_2}{b_3}$, $\phi_{36} = \frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}c_2e^{\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x-b_2t} - \frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}c_1e^{b_2t-\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x}$, $\phi_{38} = \frac{1}{4}\left(3b_3^2-\sqrt{17}b_3^2\right)c_1e^{b_2t-\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x}+\frac{1}{4}\left(3b_3^2-\sqrt{17}b_3^2\right)c_2e^{\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x-b_2t}$, $\phi_{37} = c_1e^{b_2t-\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x} +$

$c_2 e^{\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x-b_2 t}$, $\phi_{39} = c_1 e^{b_2 t-\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x} + c_2 e^{\frac{1}{2}\sqrt{3b_3^2-\sqrt{17}b_3^2}x-b_2 t}$, $\phi_{40} = \sqrt{3b_3^2-\sqrt{17}b_3^2}\sqrt{-c_1}\sqrt{c_2}$, $\phi_{41} = \frac{\sqrt{3b_3^2-\sqrt{17}b_3^2}\sqrt{-c_1}\sqrt{c_2}}{b_3}$, $\phi_{42} = b_3\sqrt{3b_3^2-\sqrt{17}b_3^2}\sqrt{-c_1}\sqrt{c_2}$.

3. Symmetry analysis

Here we report the symmetry analysis, nonlinear self-adjointness and conservation laws for Eq 2. For Eq 2 we have the vector fields as

$$\mathcal{X} = \eta_1(x,t,\chi)\frac{\partial}{\partial x} + \eta_2(x,t,\chi)\frac{\partial}{\partial t} + \eta_3(x,t,\chi)\frac{\partial}{\partial \chi}. \quad (29)$$

Eq. Eq 2 admits the followings infinitesimals:

$$\begin{aligned}\eta_1 &= \frac{xc_1}{2} + F_1(t),\\ \eta_2 &= -\frac{3tc_1}{2} + c_2,\\ \eta_3 &= \chi c_1 + \frac{F_1'}{2},\end{aligned} \quad (30)$$

where $F_1(t)$ is an arbitrary function of t. And the symmetries are

$$\begin{aligned}\mathcal{X}_1 &= \partial_t,\\ \mathcal{X}_2 &= \partial\chi + 2F_1'(t)\partial x,\\ \mathcal{X}_3 &= \left(-3t\partial_t + 2\chi\partial_\chi\right) - x\partial_x.\end{aligned} \quad (31)$$

3.1. Adjoint system and conditions for nonlinear self-adjointness

Theorem 3.1. *A symmetry such as Lie point, Lie-Bäcklund, nonlocal symmetry etc is given by*

$$\mathcal{X} = \xi_i\frac{\partial}{\partial \bar{x}^i} + \eta_{\bar{\alpha}}\frac{\partial}{\partial \chi^{\bar{\alpha}}}, \quad (32)$$

of a nonlinear partial differential equations

$$F_{\bar{\alpha}}(\bar{x},\chi,\dots,\chi_s) = 0,\ \bar{\alpha} = 1,2,\dots,\bar{m}, \quad (33)$$

with an m dependent variables will have an adjoint equation

$$\mathcal{F}^*_{\bar{\alpha}}(\bar{x},\chi,\dots,\chi_s) = \frac{\delta(\upsilon^{\bar{\beta}}\mathcal{F}_{\bar{\beta}})}{\delta\chi^{\bar{\alpha}}},\ \bar{\alpha} = 1,2,\dots,\bar{m}, \quad (34)$$

and Lagrangian given by

$$\mathcal{L} = \upsilon^{\bar{\beta}}\mathcal{F}_{\bar{\beta}}(\bar{x},\chi,\chi_{(1)}\dots,\chi_{(s)}), \quad (35)$$

with $\upsilon = \upsilon(\bar{x},t)$ depicting a nonlocal dependent variables.

Considering Eq 2, the formal Lagrangian can be given by

$$\mathcal{L} = \upsilon(x,t)\left(-\beta\chi_{xxxx} + 2\chi_x^2 + \chi_{xt} + 2\chi\chi_{xx}\right), \quad (36)$$

where υ is the new-dependent variables called the nonlocal variables. The adjoint system can be obtained using

$$\mathcal{F}^* = \frac{\delta\mathcal{L}}{\delta\chi} = 0, \quad (37)$$

where

$$\frac{\delta\mathcal{L}}{\delta\chi} = \frac{\partial\mathcal{L}}{\partial u} - D_t\frac{\partial\mathcal{L}}{\partial u_t} - D_x\frac{\partial\mathcal{L}}{\partial u_x} + (D_x)^2\frac{\partial\mathcal{L}}{\partial u_{xx}} - (D_{xxx})^3\frac{\partial\mathcal{L}}{\partial u_{xxx}} + (D_x)^4\frac{\partial\mathcal{L}}{\partial u_{xxxx}}. \quad (38)$$

On the basis of Lagrangian reported in Eq 28, one can get the adjoint equation as

$$\mathcal{F}^* = \upsilon_{xt} + 2\chi\upsilon_{xx} - \beta\upsilon_{xxxx} = 0. \quad (39)$$

Now, we want establish a differential substitution of the form

$$\upsilon = \Phi(x,t,\chi), \quad (40)$$

so that (2) will become nonlinear self-adjointness. To this aim, we insert Eq 40 into Eq 39 and by expressing χ_{xt} from (2) we reach

$$\begin{aligned}&\chi_t\Phi_{xu} + \Phi_{xt} + 2\chi\Phi_{xx} + \chi_x\left(\chi_t\Phi_{\chi\chi} - 4\beta\chi_{xxx}\Phi_{\chi\chi} + \Phi_{t\chi}\right.\\ &\left.-12\beta\chi_{xx}\Phi_{x\chi\chi} - 4\beta\Phi_{xxx\chi}\right) = \beta\chi_x^4\Phi_{\chi\chi\chi\chi} + 4\beta\chi_x^3\Phi_{x\chi\chi\chi}\\ &+2\chi_x^2\left(\Phi_u - \chi\Phi_{\chi\chi} + 3\beta(\chi_{xx}\Phi_{\chi\chi\chi} + \Phi_{xx\chi\chi})\right)\\ &+\beta\left(3\chi_{xx}^2\Phi_{\chi\chi} + 4\chi_{xxx}\Phi_{x\chi} + 6\chi_{xx}\Phi_{xx\chi} + \Phi_{xxxx}\right).\end{aligned} \quad (41)$$

By equating to zero the coefficients of the derivatives $\chi_t, \chi_x, \chi_{xx}, \chi_{xxx}$, we attain

$$\begin{aligned}&\Phi_{\chi\chi} = 0,\ -4\beta\Phi_{\chi\chi} = 0,\ -3\beta\Phi_{\chi\chi} = 0,\ -6\beta\Phi_{\chi\chi\chi},\\ &-\beta\Phi_{\chi\chi\chi,\chi} = 0,\\ &\Phi_{x\chi} = 0,\ -4\beta\Phi_{x\chi} = 0,\ -12\beta\Phi_{x\chi\chi} = 0,\ -4\beta\Phi_{x\chi\chi\chi} = 0,\\ &-6\beta\Phi_{x,x,\chi} = 0\\ &-2\Phi_\chi + 2\chi\Phi_{\chi\chi} - 6\beta\Phi_{x,x\chi\chi} = 0,\ \Phi_{t,\chi} + 4\chi\Phi_{t,\chi}\\ &-4\beta\Phi_{x,x,x,\chi} = 0, \Phi_{x,t} + 2\chi\Phi_{x,x} - \beta\Phi_{x,x,x,x} = 0.\end{aligned} \quad (42)$$

And the corresponding solution becomes

$$\Phi = xc_1 + F_1(t), \quad (43)$$

where c_1 and $F_1(t)$ are arbitrary constant and function of t, respectively.

3.2. Conservation laws

The fact that Eq 2 is a nonlinear self-adjointness, with the help of its point symmetries, we use the Noether operator $\mathcal{N}$ [29]–[31] to get (C^1, C^2). The conserved vectors will satisfy

$$D_i(C^i)|_{(25)=0} = 0, \quad (44)$$

where

$$\begin{aligned}C^i &= \xi_i\mathcal{L} + W^{\bar{\alpha}}\left[\frac{\partial\mathcal{L}}{\partial u_i^{\bar{\alpha}}} - D_j\left(\frac{\partial\mathcal{L}}{\partial u_{ij}^{\bar{\alpha}}}\right) + D_jD_k\left(\frac{\partial\mathcal{L}}{\partial u_{ijk}^{\bar{\alpha}}}\right) - \dots\right] +\\ &D_j\left(W^{\bar{\alpha}}\right)\left[\frac{\partial\mathcal{L}}{\partial u_{ij}^{\bar{\alpha}}} - D_k\left(\frac{\partial\mathcal{L}}{\partial u_{ijk}^{\bar{\alpha}}}\right) + \dots\right] + D_jD_k\left(W^{\bar{\alpha}}\right)\left[\frac{\partial\mathcal{L}}{\partial u_{ijk}^{\bar{\alpha}}} + \dots\right],\end{aligned} \quad (45)$$

and $W^{\bar{\alpha}} = \eta_{\bar{\alpha}} - \xi_j u_j^{\bar{\alpha}}$. To this aim, the non-local variables in the equation must be replaced in compliance with Eq 43. We now present the conserved vectors for each one of the symmetries Eq 31 follows:

- The symmetry $\mathcal{X}_1 = \partial_t$ admits the following conserved vectors:

$$\begin{aligned}C_1^t &= \frac{1}{2}(c_1\chi_t + (xc_1 + F_1(t)))\left(4\chi_x^2 + \chi_{xt} + 4\chi\chi_{xx} - 2\beta\chi_{xxxx}\right),\\ C_1^x &= \frac{1}{2}\left(\chi_t(4c_1\chi + F_1' - 4(xc_1 + F_1)\chi_x) - 2\beta c_1\chi_{xxt}\right.\\ &\left.-(xc_1 + F_1)(\chi_{tt} + 4\chi\chi_{xt} - 2\beta\chi_{xxxt})\right).\end{aligned}$$

- The symmetry $\mathcal{X}_2 = \partial\chi + 2F_1'(t)\partial x$ admits the following conserved vectors:

$$\begin{aligned}C_2^t &= -\frac{1}{2}c_1F_1' + F_1(c_1\chi_x - (xc_1 + F_1)\chi_{xx}),\\ C_2^x &= -\frac{1}{2}F_1'(4c_1u + F_1') + (xc_1 + F_1)\left(\frac{F_1''}{2} + F_1'\chi_x + F_1\chi_{xt}\right)\\ &+F_1\left(4c_1\chi + F_1'\right)\chi_x - 2\beta c_1u_{xxx}\big).\end{aligned}$$

- The symmetry $\mathcal{X}_3 = \left(-3t\partial_t + 2\chi\partial_\chi\right) - x\partial_x$ admits the conserved vectors:

$$\begin{aligned}C_3^t &= \frac{1}{2}\left(-c_1(2\chi + 3t\chi_t + x\chi_x) + (xc_1 + F_1)(3\chi_x - 12tu_x^2 - 3tu_{xt}\right.\\ &+(x - 12t\chi)\chi_{xx} + 6t\beta\chi_{xxxx}),\end{aligned}$$

$$\begin{aligned}C_3^x = &-\frac{1}{2}(2\chi+3t\chi_t+xu_x)(4c_1\chi+F_1'-4(xc_1+F_1)\chi_x)\\&+\frac{1}{2}(xc_1+F_1)(5\chi_t+3t\chi_{tt}+x\chi_{xt})\\&+2\chi(xc_1+F_1)(3\chi_x+3t\chi_{xt}+x\chi_{xx})\\&+\beta c_1(4\chi_{xx}+3t\chi_{xxt}+x\chi_{xxx})\\&-\beta(xc_1+F_1)(5\chi_{xxx}+3t\chi_{xxxt}+x\chi_{xxxx})\\&-x(xc_1+F_1)(2\chi_x^2+\chi_{xt}+2\chi\chi_{xx}-\beta\chi_{xxxx}).\end{aligned}$$

4. Conclusion

It is a common knowledge that many science and engineering aspects that may be represented by nonlinear equations have an empirical parameters. Lump solutions therefore allow researchers the freedom to design and run experiments in order to decide certain parameters by creating convenient or natural conditions. In nonlinear sciences, therefore, analyzing and obtaining lump solutions is becoming more desirable. In this work, the Ostrovsky equation in the absence of rotational effect has been investigated by means of the Hirota bilinear approach and symbolic calculation. Ostrovsky equation is an equation for the unidirectional propagation of weakly nonlinear long surface and internal waves of small amplitude in a rotating fluid. As a consequence, various novel interaction phenomena between lump solution with two-wave, periodic wave, breather wave, periodic wave and kink solution have been acquired. A periodic wave has a wavelength and frequency determined by a repeating continuous pattern. Breathers are pulsating localized structures that have been used to mimic extreme waves in a variety of nonlinear dispersive media with a narrow banded starting process. Several recent investigations, on the other hand, imply that breathers can survive in more complex habitats, such as random seas, despite the attributed physical restrictions.

All the acquired solutions are verified by inserting them into the original equation with the help of the Wolfram Mathematica 12 package. The solution's physical features were graphically depicted to shed more light on the results obtained (see Fig. 1, Fig. 2, Fig. 3 and Fig. 4). In addition, the governing equation's conserved vectors are developed using a new conservation theorem. The results obtained are useful in understanding the basic nonlinear scenarios in fluid dynamics as well as the dynamics related to computational physics and engineering sciences in nonlinear higher dimensional wave fields. Our future study will be on the bifurcation analysis and stochastic fractional solitons to the studied nonlinear model.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

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