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Modeling the dynamics of the novel coronavirus using Caputo-Fabrizio derivative

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Abstract The virus that begins from Wuhan China, known as COVID-19 or coronavirus is still a huge panic for humans around the globe. The elimination of this virus from our society needs proper attentions to follows the rule suggested by World Health Organization (WHO). A vast literature on the modeling of this infection in various perspective is available. In the present work, we design a new mathematical model for COVID-19 pandemic by utilizing the real infected cases reported from Kingdom of Saudi Arabia. Initially, we formulate the model with the help of classical integer order nonlinear differential equations. The treatment class is considered the model to analyze the impact of treatment on the disease dynamics. The Caputo-Fabrizio derivative with the non-singular exponential kernel is applied in order to reformulate the proposed COVID-19 transmission model with a fractional order. The biologically important parameter called the basic reproductive number is investigated both theoretically and numerically. The estimated values of $\mathscr{R}_0$ for the selected period are approximated to be 1.63. Further, by making use of the Picard Lindelöf theorem we provide the existence and uniqueness of the COVID-19 fractional epidemic model. Moreover, the fractional model is solved numerically and a number of simulation results are depicted using the real estimated parameters. The impact of various model parameters and memory index are shown graphically. We conclude that the fractional order epidemic models are more appropriate and provide deep insights into the disease dynamics.

1. Introduction

The humans faced in the fast many outbreaks and suffered a lot with many infected and deaths cases, such as Dengue, Influenza, Ebola, Malaria etc. The new virus that is identified first in province of China, called Hubei city Wuhan is known as coronavirus. This infection is a respiratory disease and its causing is the SARS-CoV-2 virus. It has emerged in November

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2019 in Hubei Wuhan city of China and spread to more than 215 countries in the world. After the outbreak of COVID-19 was noticed, the WHO announced the COVID-19 is a pandemic disease in March 2020. The novel COVID-19 can cause mild or severe illness and in some cases can lead to being fatal. Symptoms of this novel infection are still not precise but usually include fever, dry cough, body pain, loss of taste, shortness of breath, etc. Some people experience no symptoms at all and referred to as asymptomatic cases. However, early human cases infected with this new infection are caused by exposure to infected animals, but infected individuals may spread this virus to others. The COVID-19 virus is considered to be transmitted primarily from person to person via respiratory droplets as the infected person coughs, sneezes or talks. Another main way of virus transmission is through environmental viral load. This virus can remains in the environment or on different surfaces for few hours. As a result, an individual can contract COVID-19 by touching a surface or object that is covered with SARS-CoV-2. The statistics as on 20 November 2020 reported on WHO show that the total registered infected cases are more than 55.6 million with 1.34 million deaths while the recovered cases are 35.8 million [1]. Saudi Arabia is one of the countries most affected by COVID-19 and has the highest number of cases of infection among the Persian Gulf states [2].

The complex dynamics of the novel COVID-19 pandemic were explored through a variety of approaches. Mathematical modeling is one of the effective tools and has been instrumental in disease epidemiology. These models provide a comprehensive information about the transmission mechanisms of emerging and reemerging infectious diseases. Moreover, mathematical models help in suggesting effective controlling measures against a disease outbreak. Mathematical epidemic models usually lead to the system of ordinary, partial, stochastic or delay differential systems [3–7]. But, the mathematical epidemic models based on aforementioned ordinary classical differential operators are local in nature and do not possess the memory effects. On the other hand, the transmission models constructed via fractional differential operators are more appropriate and are able to provide more in-depth insight about disease dynamics. This is due to the fact that fractional operators possess memory effects and capture cross-over behavior found in numerous biological and physical phenomena. In the recent era, a number of non-integer differential operator models using the singular and the nonsingular kernels have been proposed by the researchers. Among these, that gains attention world-wide are Caputo [8], Caputo-Fabrizio [9] and Atangana-Baleanu-Caputo (ABC) [10]. The application of these new operators were found useful for exploring many problems [11–17,7,18].

In the recent few months, a number of transmission models based on various fractional order operators have been constructed and discussed to investigate the dynamical behavior and possible control of the novel COVID-19 infection. A novel transmission model on the COVID-19 dynamics is constructed via the well-known Caputo-Fabrizio derivative in [19]. A fractional order model based on ABC operator was formulated in [20]. The authors in [20] used the reported cases in Wuhan, China by predicting the disease control and its spread. The role of quarantine and isolation related to the transmission and incidence of COVID-19 in China is discussed in [21]. The authors in [21] formulated the proposed model using a novel fractal-fractional operator. A fractal-fractional operator is used to formulate a mathematical model to analyze the influence of lockdown on COVID-19 transmission [22]. A theoretical and numerical investigation of a novel COVID-19 transmission model with non-singular kernel is presented in [23]. The impact of commonly used non-pharmaceutical controlling measures is analyzed with the help of Caputo fractional mathematical COVID-19 model in [24]. Recently, the numerical and theoretical investigation of fractional COVID-19 epidemic models with non-singular kernel have been carried out in [25,26].

In the current manuscript, we formulate and analyze a new mathematical model for the coronavirus infection using the reported cases observed in the Kingdom of Saudi Arabia. Environmental viral concentration is taken into consideration. First, we construct the COVID-19 compartmental model by using classical integer order derivative and then the model is further extended to fractional order with the help of the Caputo-Fabrizio derivative. The remaining paper is arranged as: Section 2 contains some basic definitions regarding the fractional derivative. Section 3 presents the formulation of the COVID-19 model in classical integer case and contains the parameters estimation procedure. Section 4 initially presents the formulation of the model in fractional cases. In the same section, we further explore some basic mathematical aspects including the basic reproductive number, the existence and uniqueness of the fractional COVID-19 epidemic model. Section 5 contains numerical scheme of the proposed model and simulation results with a brief discussion. Finally, the conclusion regarding the present finding is given in Section 6.

2. Preliminaries

This section provides the necessary basic materials regarding the fractional order operators from [27,28].

Definition 1. The Caputo-type operator with order q in $(l-1, l)$, where $l \in \mathbb{N}$ of a function described by $Z(t) \in C^n$ is defined as:

$$ {}^{C}D_t^q(Z(t)) = \frac{1}{\Gamma(l-q)} \int_0^t \frac{Z^l(\zeta)}{(t-\zeta)^{q-l+1}} d\zeta. \tag{1}$$

Clearly, ${}^{C}D_t^q(Z(t))$ tends to $Z\prime(t)$ as $q \to 1$.

Definition 2. For $Z \in H^1(a_1, a_2), a_2 > a_1, q \in [0,1]$, the Caputo-Fabrizio operator [27] is described as follows:

$$ {}^{CF}_{a_1}D_t^q Z(t) = \frac{M(q)}{1-q} \int_{a_1}^t Z\prime(u) exp\left[-q\frac{(t-u)}{1-q}\right] du. \tag{2}$$

In Eq. (2), $M(q)$ shows the function normality that holds $M(0) = M(1) = 1$ [27]. If $Z\neg \in H^1(a_1, a_2)$, then the following expression is obtained:

$$ {}^{CF}_{a_1}D_t^q Z(t) = \frac{qM(q)}{1-q} \int_{a_1}^t (Z(t) - Z(u)) exp\left[-q\frac{(t-u)}{1-q}\right] du. \tag{3}$$

Remark 1. If $\upsilon = \frac{1-q}{q} \in [0, \infty)$, then $q = \frac{1}{1+\upsilon} \in [0,1]$ and hence Eq. (2) can be written as:

$$^{CF}{}_{a_1}D_t^{\upsilon}Z(t) = \frac{N(\upsilon)}{\upsilon}\int_{a_1}^{t} Z\prime(u)exp\left[-\frac{(t-u)}{\upsilon}\right]du, \tag{4}$$

where $N(\upsilon)$ represents the normalization term of $M(q)$ such that $N(0) = N(\infty) = 1$. Moreover,

$$\lim_{\upsilon\to 0}\frac{1}{\upsilon}exp\left[-\frac{t-u}{\upsilon}\right] = \delta(t-u). \tag{5}$$

Definition 3. For $0 < q < 1$, then the corresponding integral is defined as follows: [29]:

$$^{CF}{}_0I_t^qZ(t) = \frac{2(1-q)}{M(q)(2-q)}Z(t) + \frac{2q}{M(q)(2-q)}\int_0^t Z(\tau)d\tau,\ t > 0. \tag{6}$$

3. The classical integer-order COVID-19 model

The present section of the manuscript is based on the formulation of the proposed mathematical model describing the dynamical behavior of the novel COVID-19. Due to the fact that the environmental shedding could enhance the transmission of virus, therefore, the environmental viral concentration is taken in consideration. In order to construct the desire compartmental model, the total population of humans is denoted by $N(t)$, which is classified into seven mutually-exclusive groups of susceptible $S(t)$, early or newly exposed $E_1(t)$, pre-symptomatic infectious and are close to surviving the incubation period $E_2(t)$, infectious showing clinical COVID-19 symptoms $I(t)$, infectious with no clinical COVID-19 symptoms $I_A(t)$, hospitalized/treated $T(t)$ and finally the individuals who recovered at any time t are represented by $R(t)$. Thus,

$$N(t) = S(t) + E_1(t) + E_2(t) + I(t) + I_A(t) + T(t) + R(t).$$

The concentration of virus of the coronavirus in the environment or surfaces is shown by $E_n(t)$. The individuals in the pre-symptomatic infectious $E_2(t)$, in the symptomatically-infected class $I(t)$ and in the asymptomatically-infected class $I_A(t)$ with no or mild symptoms can transmit the infection to others and are contributing the virus to the environment. Using the above discussion, the evolutionary differential equations for the coronavirus infection is give by the following system:

$$\begin{cases} \frac{d}{dt}S &= \Delta - \left(\eta_eE_2 + \eta_II + \eta_AI_A + \eta_{e_n}E_n\right)\frac{S}{N} - \mu S, \\ \frac{d}{dt}E_1 &= \left(\eta_eE_2 + \eta_II + \eta_AI_A + \eta_{e_n}E_n\right)\frac{S}{N} - (\sigma_1 + \mu)E_1, \\ \frac{d}{dt}E_2 &= \sigma_1E_1 - (\sigma_2 + \mu)E_2, \\ \frac{d}{dt}I &= \sigma_2(1-\rho)E_2 - (\mu + \zeta_1 + r_1 + \kappa)I, \\ \frac{d}{dt}I_A &= \rho\sigma_2E_2 - (\mu + r_2)I_A, \\ \frac{d}{dt}T &= \kappa I - (\mu + \zeta_2 + r_3)T, \\ \frac{d}{dt}R &= r_1I + r_2I_A + r_3T - \mu R, \\ \frac{d}{dt}E_n &= \varpi_1E_2 + \varpi_2I + \varpi_3I_A - vE_n, \end{cases} \tag{7}$$

where the additional non-negative initial conditions (ICs) are:

$$\begin{aligned} S(0) = &\ S_0 \geqslant 0, E_1(0) = E_{10} \geqslant 0, E_2(0) = E_{20} \geqslant 0, I(0) = I_0 \geqslant 0, \\ I_A(0) = &\ I_{A0} \geqslant 0, T(0) = T_0 \geqslant 0, R(0) = R_0 \geqslant 0, E_n(0) = E_{n0} \geqslant 0. \end{aligned} \tag{8}$$

In the mathematical model (7) formulated above, Δ denotes the recruitment rate and the parameter μ denotes the natural mortality rate in all epidemiological classes. The susceptible population acquired the virus after the effective contacts with individuals in the E_2, I and I_A classes at the disease transmission rate η_e, η_I and η_A respectively. The expression η_{e_n} shows the disease transmission rate due to the viral load in the environment (or surfaces). The flow of the early-exposed individuals to the pre-symptomatic infectious compartment (i.e., E_2) is denoted by σ_1 and the incubation period is denoted by σ_2. After completion of the incubation period a fraction $\rho, (0 < \rho < 1)$ joins the asymptomatically-infected class $I_A(t)$, while the remaining enters to the symptomatically-infected stage $I(t)$. The parameter κ accounts the flow rate of symptomatically-infectious individuals to the treatment (or hospitalized) class. The parameters r_i for $i = 1, 2, 3$, show the recovery rates of symptomatically, asymptomatically and treated COVID-19 infected population. The expressions ϖ_j with $j = 1, 2, 3$, express the rate at which the pre-symptomatic infectious, symptomatically and asymptomatically infectious individuals contributed the virus to the environmental reservoir. The removal rate of the coronavirus infection from reservoir is shown by the parameter v.

3.1. Parameter estimations

This part presents the estimation procedure of parameters taken in the proposed model. The well-known non-linear least square curve fitting technique is implemented for this purpose and the confirmed infected cases from the Kingdom of Saudi Arabia from March 1 till 20 August 2020 is taken in the estimation process. We estimated the birth and the natural mortality rate which is shown respectively by Δ and μ are obtained using [30], while the remaining parameters values have been obtained using the adjustment to better fit to the cases. The aforementioned method of parameters estimation has applied in recent literature see [31] and the references therein. We have the Fig. 1 after using the curve fitting method and the resulting appropriate estimated or fitted numerical values of the parameters are presented in Table 1. It can be seen that the fitting to the reported cases of Saudi Arabia is well fitted to the proposed model. Based on the updated estimated values fitted parameters, the reproduction number evaluated based is $\mathscr{R}_0 \approx 1.63$, whereas the ICs are taken as $S(0) = 34811870, E_1(0) = 2000$, $E_2(0) = 200$, $I(0) = 1, I_A(0) = T(0) = R(0) = 0$ and $E_n(0) = 30000$.

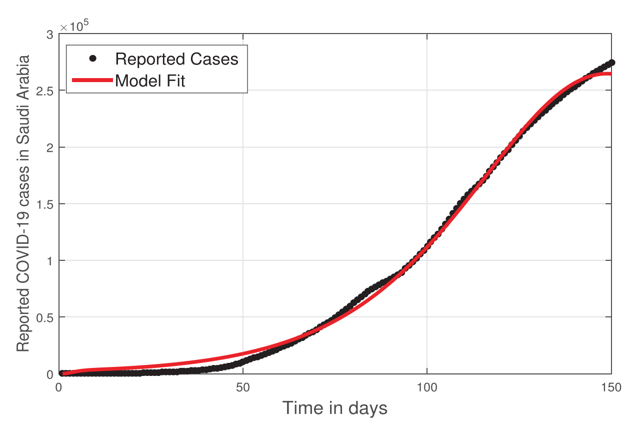

Fig. 1 The solid red line shows the model predicting infected curve to the reported COVID-19 infected cases.

Table 1 Parameters along with respective numerical values fitted from real data.

Symbol	Details	Numerical value/day	Ref
Δ	Recruitment rate	$N(0) \times \mu$	Estimated
μ	Natural mortality rate	$1/(74.87 \ast 365)$	[30]
η_e	Transmission rate due to exposed class	0.1001	Fitted
η_I	Transmission rate due relative to symptomatic class	0.5909	Fitted
η_A	Transmissibility due to asymptomatic class	0.5198	Fitted
η_{e_n}	Transmission rate due to environmental viral load	0.4666	Fitted
σ_1	Flow ratge from E_1 to E_2	0.1002	Fitted
σ_2	Incubation period	0.5559	Fitted
ρ	fraction of the asymptomatically-infected people	0.0180	Fitted
ζ_1	Death rate in I compartment due to COVID-19	0.0107	Fitted
ζ_2	Death rate in T compartment due to COVID-19	0.0253	Fitted
r_1	Recovery rate in symptomatically-infected population	0.4214	Fitted
r_2	Rate of recovery from I_A	0.2908	Fitted
r_3	Recovery rate from T	0.6990	Fitted
ϖ_1	Environmental viral contribution of E class	0.2927	Fitted
ϖ_2	Environmental viral contribution of I class	0.0982	Fitted
ϖ_3	Environmental viral contribution of I_A class	0.2098	Fitted
v	Rate of removal of the virus from environment	0.4121	Fitted

4. The fractional model in Caputo-Fabrizio sense

Mathematical epidemic models constructed using fractional order operator are helpful and give a better understanding of dynamics of a disease. We extend the classical COVID-19 model described in (7) by replacing the classical integer derivative with non-integer order operator in Caputo-Fabrizio sense. The resulting fractional order model is given as follows:

$$\begin{cases} {}^{CF}{}_0D_t^q S &= \Delta - \left(\eta_e E_2 + \eta_I I + \eta_A I_A + \eta_{e_n} E_n\right)\frac{S}{N} - \mu S, \\ {}^{CF}{}_0D_t^q E_1 &= \left(\eta_e E_2 + \eta_I I + \eta_A I_A + \eta_{e_n} E_n\right)\frac{S}{N} - (\sigma_1 + \mu)E_1, \\ {}^{CF}{}_0D_t^q E_2 &= \sigma_1 E_1 - (\sigma_2 + \mu)E_2, \\ {}^{CF}{}_0D_t^q I &= \sigma_2(1-\rho)E_2 - (\mu + \zeta_1 + r_1 + \kappa)I, \\ {}^{CF}{}_0D_t^q I_A &= \rho\sigma_2 E_2 - (\mu + r_2)I_A, \\ {}^{CF}{}_0D_t^q T &= \kappa I - (\mu + \zeta_2 + r_3)T, \\ {}^{CF}{}_0D_t^q R &= r_1 I + r_2 I_A + r_3 T - \mu R, \\ {}^{CF}{}_0D_t^q E_n &= \varpi_1 E_2 + \varpi_2 I + \varpi_3 I_A - vE_n. \end{cases} \tag{9}$$

In the above system, ${}^{CF}{}_0D_t^q$represents the fractional operator with exponential kernel having order $0 < q \le 1$. To express the COVID-19 model (9) in a simple form, let us denote

$$\lambda(t) = \frac{\left(\eta_e E_2 + \eta_I I + \eta_A I_A + \eta_{e_n} E_n\right)}{N},\ \mathrm{k}_1 = (\sigma_1 + \mu),\ \mathrm{k}_2 = (\sigma_2 + \mu),$$
$$\mathrm{k}_3 = (\mu + \zeta_1 + r_1 + \kappa), \mathrm{k}_4 = (\mu + r_2),\ \mathrm{k}_5 = (\mu + \zeta_2 + r_3).$$

$$\begin{cases} {}^{CF}{}_0D_t^q S &= \Delta - \lambda(t)S - \mu S, \\ {}^{CF}{}_0D_t^q E_1 &= \lambda(t)S - \mathrm{k}_1 E_1, \\ {}^{CF}{}_0D_t^q E_2 &= \sigma_1 E_1 - \mathrm{k}_2 E_2, \\ {}^{CF}{}_0D_t^q I &= \sigma_2(1-\rho)E_2 - \mathrm{k}_3 I, \\ {}^{CF}{}_0D_t^q I_A &= \rho\sigma_2 E_2 - \mathrm{k}_4 I_A, \\ {}^{CF}{}_0D_t^q T &= \kappa I - \mathrm{k}_5 T, \\ {}^{CF}{}_0D_t^q R &= r_1 I + r_2 I_A + r_3 T - \mu R, \\ {}^{CF}{}_0D_t^q E_n &= \varpi_1 E_2 + \varpi_2 I + \varpi_3 I_A - vE_n. \end{cases} \tag{10}$$

4.1. Analysis of the model in fractional case

We proceed to explore some basic mathematical aspects of the COVID-19 compartmental epidemic model in fractional as described in (10).

4.2. The basic reproductive number

Let $\mathscr{D}_0$ denotes disease free equilibrium of the system (10), then it is given as follows:

$$\mathscr{D}_0 = \left(\frac{\Delta}{\mu}, 0, 0, 0, 0, 0, 0, 0\right).$$

Moreover, to obtain the reproductive number $\mathscr{R}_0$ we utilize the most common next generation matrix approach. For this purpose, the necessary matrices obtained by considering only the specific compartments in the system shown in (10) are as follows:

$$F = \begin{pmatrix} 0 & \eta_e & \eta_I & \eta_A & 0 & \eta_{e_n} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$V = \begin{pmatrix} \mathrm{k}_1 & 0 & 0 & 0 & 0 & 0 \\ -\sigma_1 & \mathrm{k}_2 & 0 & 0 & 0 & 0 \\ 0 & -(1-\rho)\sigma_2 & \mathrm{k}_3 & 0 & 0 & 0 \\ 0 & -\rho\sigma_2 & 0 & \mathrm{k}_4 & 0 & 0 \\ 0 & 0 & -\kappa & 0 & \mathrm{k}_5 & 0 \\ 0 & -\varpi_1 & -\varpi_2 & -\varpi_3 & 0 & v \end{pmatrix}.$$

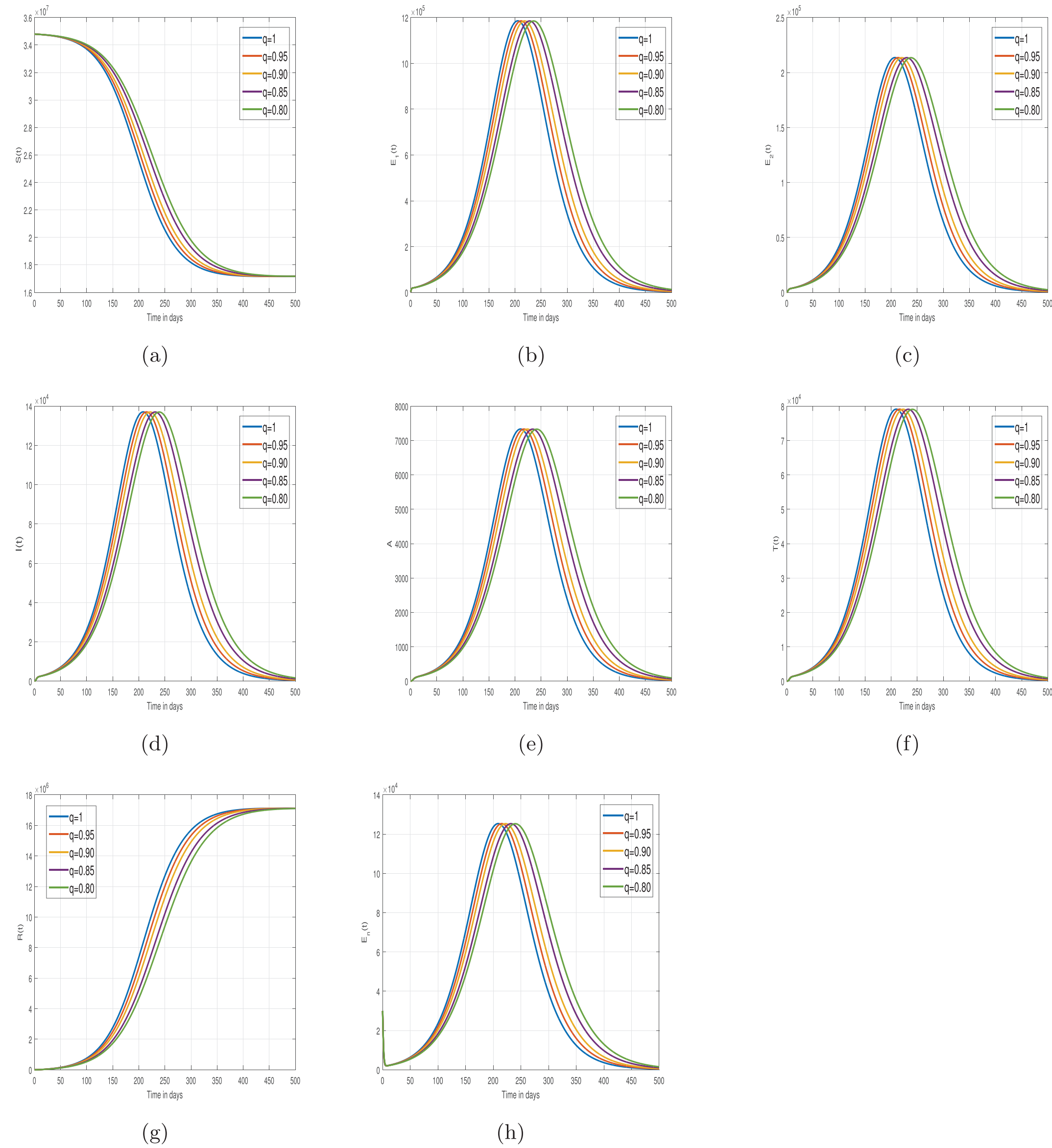

Fig. 2 Graphical interpretation of the COVID-19 epidemic model in fractional case (10) for various values of parameter q.

Thus, utilizing the concept of aforementioned procedure, the following is the representation of the basic reproduction number:

$$\mathscr{R}_0 = \underbrace{\frac{\sigma_1 \eta_e}{\mathrm{k}_1 \mathrm{k}_2}}_{\mathscr{R}_1} + \underbrace{\frac{\sigma_1 \sigma_2 (1-\rho) \eta_I}{\mathrm{k}_1 \mathrm{k}_2 \mathrm{k}_3}}_{\mathscr{R}_2} + \underbrace{\frac{\sigma_1 \sigma_2 \rho \eta_A}{\mathrm{k}_1 \mathrm{k}_2 \mathrm{k}_4}}_{\mathscr{R}_3} + \underbrace{\frac{\sigma_1 (\mathrm{k}_4 \varpi_2 \sigma_2 (1-\rho) + \mathrm{k}_3 \varpi_3 \sigma_2 \rho + \mathrm{k}_3 \mathrm{k}_4 \varpi_1) \eta_{e_n}}{\mathrm{k}_1 \mathrm{k}_2 \mathrm{k}_3 \mathrm{k}_4 v}}_{\mathscr{R}_4}.$$

4.3. Existence and uniqueness of solution of system (10)

We establish the existence and uniqueness results for the system (10) with ICs (8). The concept of fixed point theory coupled with the Picard Lindelöf approach is utilized for this purpose. To proceed further, the fractional integral operator in Caputo-Fabrizio sense defined by (2) is applied to both sides of equations in system (10) and using the definition in (6), we arrived

$$\begin{aligned}
S(t) - S(0) &= {}^{CF}{}_0I_t^q\{\Delta - \lambda(t)S - \mu S\},\\
E_1(t) - E_1(0) &= {}^{CF}{}_0I_t^q\{\lambda(t)S - \mathrm{k}_1 E_1\},\\
E_2(t) - E_2(0) &= {}^{CF}{}_0I_t^q\{\sigma_1 E_1 - \mathrm{k}_2 E_2\},\\
I(t) - I(0) &= {}^{CF}{}_0I_t^q\{\sigma_2(1-\rho)E_2 - \mathrm{k}_3 I\},\\
I_A(t) - I_A(0) &= {}^{CF}{}_0I_t^q\{\rho\sigma_2 E_2 - \mathrm{k}_4 I_A\},\\
T(t) - T(0) &= {}^{CF}{}_0I_t^q\{\kappa I - \mathrm{k}_5 T\},\\
R(t) - R(0) &= {}^{CF}{}_0I_t^q\{r_1 I + r_2 I_A + r_3 T - \mu R\},\\
E_n(t) - E_n(0) &= {}^{CF}{}_0I_t^q\{\varpi_1 E_2 + \varpi_2 I + \varpi_3 I_A - v E_n\}.
\end{aligned} \tag{11}$$

For simplicity, the system described in (11) can be expressed in term of kernels

$$
\begin{aligned}
S(t)-S(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_1(,S)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_1(r,S)dr,\\
E_1(t)-E_1(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_2(t,E_1)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_2(r,E_1)dr,\\
E_2(t)-E_2(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_3(t,E_2)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_3(r,E_2)dr,\\
I(t)-I(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_4(t,I)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_4(r,I)dr,\\
I_A(t)-I_A(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_5(t,A)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_5(r,I_A)dr,\\
T(t)-T(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_6(t,T)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_6(r,T)dr,\\
R(t)-R(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_7(t,R)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_7(r,R)dr,\\
E_n(t)-E_n(0) &= 2\tfrac{(1-q)}{(2-q)\mathtt{M}(q)}K_8(t,E_n)+\tfrac{2q}{\mathtt{M}(q)(2-q)}\int_0^t K_8(r,E_n)dr,
\end{aligned}
\tag{12}
$$

where, the kernels defined above are given as follows:

$$
\begin{aligned}
K_1(t,S) &= \Delta-\lambda(t)S-\mu S,\\
K_2(t,E_1) &= \lambda(t)S-\mathrm{k}_1E_1,\\
K_3(t,E_2) &= \sigma_1E_1-\mathrm{k}_2E_2,\\
K_4(t,I) &= \sigma_2(1-\rho)E_2-\mathrm{k}_3I,\\
K_5(t,I_A) &= \rho\sigma_2E_2-\mathrm{k}_4I_A,\\
K_6(t,T) &= \kappa I-\mathrm{k}_5T,\\
K_7(t,R) &= r_1I+r_2I_A+r_3T-\mu R,\\
K_8(t,E_n) &= \varpi_1E_2+\varpi_2I+\varpi_3I_A-vE_n.
\end{aligned}
\tag{13}
$$

Moreover, utilizing the Picard iterations we derived the following recursive equations

$$
\begin{aligned}
S^{n+1}(t) &= \tfrac{2(1-q)}{(2-q)\mathtt{M}(q)}K_1(t,S^n)+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_1(r,S^n)dr,\\
E_1^{n+1}(t) &= \tfrac{2(1-q)}{(2-q)\mathtt{M}(q)}K_2(t,E_1^n)+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_2(r,E^n)dr,\\
E_2^{n+1}(t) &= \tfrac{2(1-q)}{(2-q)\mathtt{M}(q)}K_3(t,E_2^n)+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_3(r,E_2^n)dr,\\
I^{n+1}(t) &= \tfrac{2(1-q)}{(q)(2-q)\mathtt{M}}K_4(t,I^n)+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_4(r,I^n)dr,\\
I_A^{n+1}(t) &= \tfrac{2(1-q)}{(q)(2-q)\mathtt{M}}K_5(t,I_{An})+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_5(r,I_A{}^n)dr,\\
T^{n+1}(t) &= \tfrac{2(1-q)}{(q)(2-q)\mathtt{M}}K_6(t,T^n)+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_6(r,T^n)dr,\\
R^{n+1}(t) &= \tfrac{2(1-q)}{(q)(2-q)\mathtt{M}}K_7(t,R^n)+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_7(r,R^n)dr,\\
E_n^{n+1}(t) &= \tfrac{2(1-q)}{(q)(2-q)\mathtt{M}}K_8(t,E_n^n)+2\tfrac{q}{(2-q)\mathtt{M}(q)}\int_0^t K_8(r,E_n^n)dr.
\end{aligned}
\tag{14}
$$

Next, we re-write the system described in (10) as below:

$$
\begin{cases}
{}^{CF}{}_0D_t^q\mathscr{Y}(t) = H(t,\mathscr{G}(t)),\\
\mathscr{G}(0)=\mathscr{G}_0 \quad 0<t<T,
\end{cases}
\tag{15}
$$

where, the vector variable $\mathscr{G}(t)=(S(t),E_1(t),E_2(t),I(t),I_A(t),T(t),R(t),E_n(t))^t$ and the vector function $H(t,\mathscr{G}(t))$ is shown as

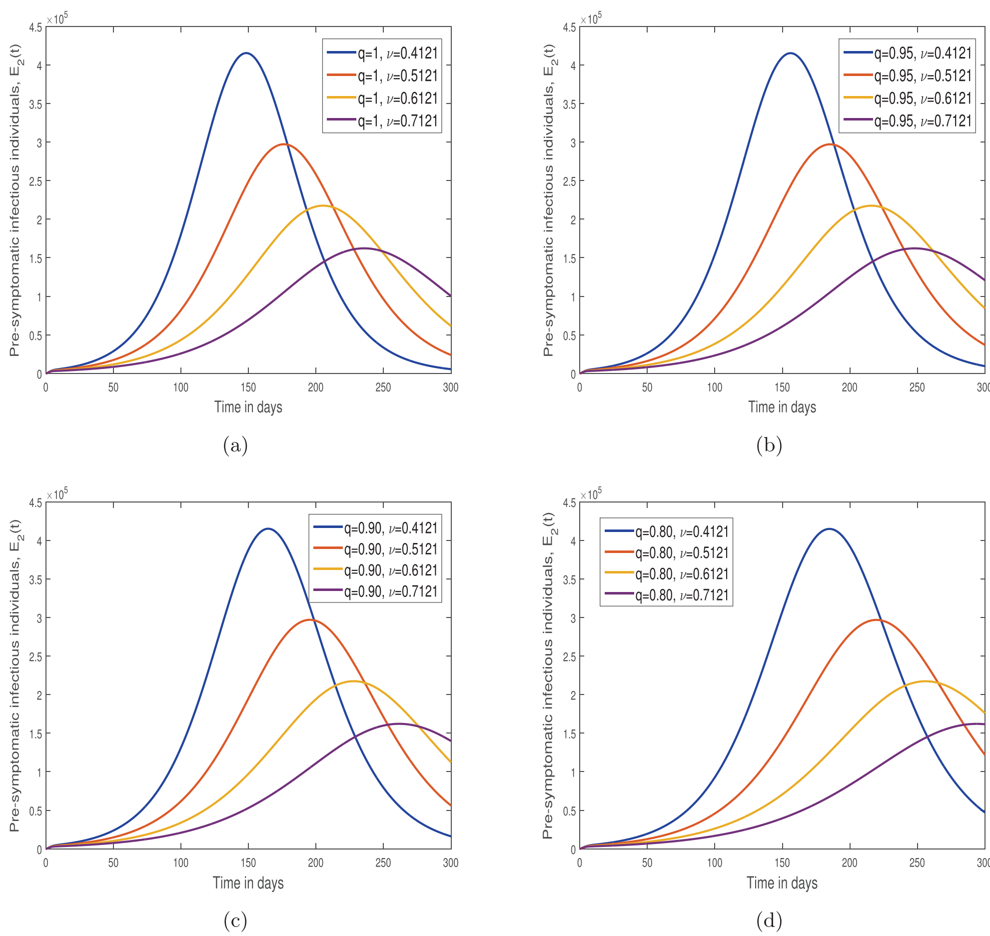

Fig. 3 Impact of the virus removal rate v on the pre-symptomatic infectious individuals, where the impact of q is shown in subplots (a) $q=1$, (b) $q=0.95$, (c) $q=0.90$, (d) $q=0.80$.

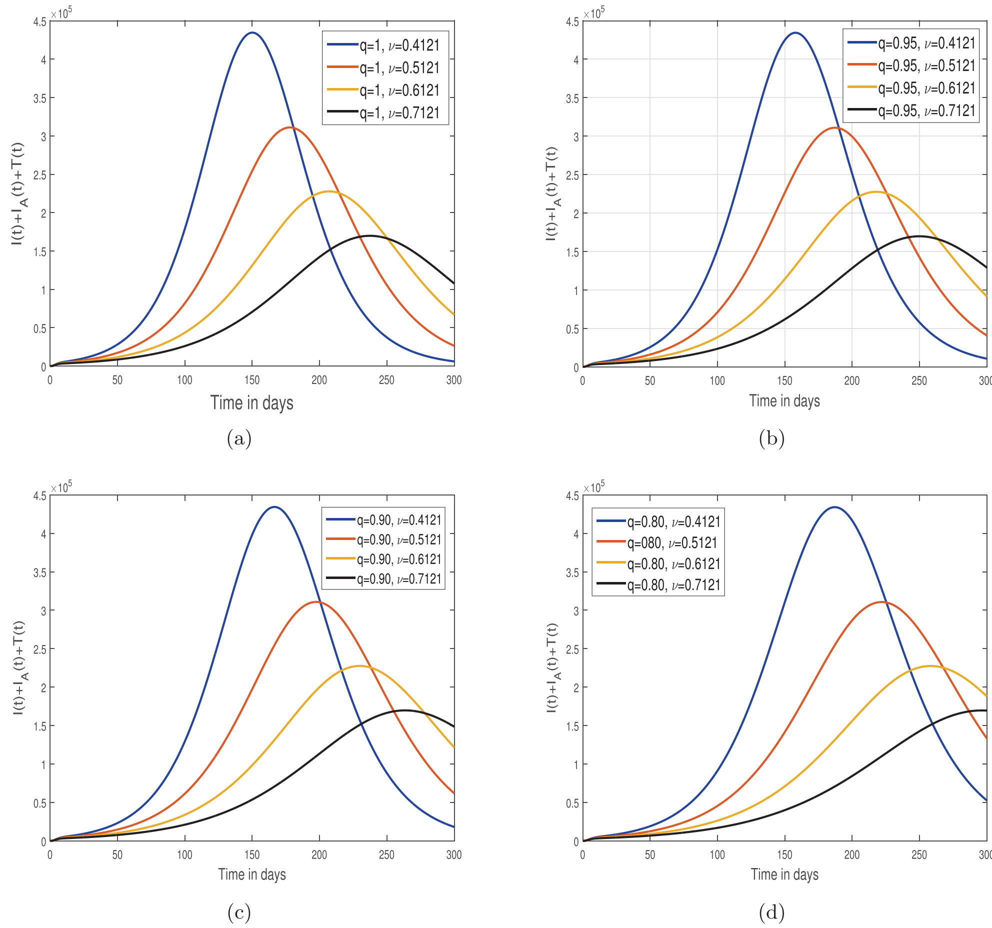

Fig. 4 Impact of the virus removal rate v on the total infected individuals in the symptomatic, asymptomatic and treated classes. Moreover, the impact of q is shown in subplots from (a-d) respectively by $1, 0.95, 0.90$, and 0.80.

$$H(t,\mathscr{G}(t)) = \begin{pmatrix} \Delta - \lambda(t)S - \mu S \\ \lambda(t)S - \mathrm{k}_1 E_1 \\ \sigma_1 E_1 - \mathrm{k}_2 E_2 \\ \sigma_2(1-\rho)E_2 - \mathrm{k}_3 I \\ \rho\sigma_2 E_2 - \mathrm{k}_4 I_A \\ \kappa I - \mathrm{k}_5 T \\ r_1 I + r_2 I_A + r_3 T - \mu R \\ \varpi_1 E_2 + \varpi_2 I + \varpi_3 I_A - v E_n \end{pmatrix}, \tag{16}$$

and the ICs are $\mathscr{G}_0 = (S(0), E_1(0), E_2(0), I(0), I_A(0), T(0), R(0), E_n(0))^t$.

Due to (12), and corresponding to (15), the integral equation can be written as

$$\mathscr{G}(t) = \mathscr{G}(0) + \phi(q)H(t,\mathscr{G}(t)) + \psi(q)\int_0^t H(s,\mathscr{G}(s))ds, \tag{17}$$

where, $\phi(q) = \frac{2(1-q)}{(2-q)M(q)}$ and $\psi(q) = \frac{2q}{(2-q)M(q)}$.

Proof 1. Let $\Omega = [0,T]$. Suppose that $\mathscr{G}(t) \in \mathfrak{C}(\Omega, \mathbb{R}^8)$, then the following condition holds. There exists $\Theta > 0$ such that

$$\|H(t,\mathscr{G}_1(t)) - H(t,\mathscr{G}_2(t))\| \leqslant \Theta\|\mathscr{G}_1(t) - \mathscr{G}_2(t)\| \tag{18}$$

for all $\mathscr{G}_1(t),\ \mathscr{G}_2(t) \in \mathfrak{C}(\Omega, \mathbb{R}^8)$, and $t \in \Omega$.

Proof. The definition of the kernels stated in (13) gives

$$\begin{aligned}
&\|H(t,\mathscr{G}_1(t)) - H(t,\mathscr{G}_2(t))\| \\
&= \begin{pmatrix} \|K_1(t,S_1) - K_1(t,S_2)\| \\ \|K_2(t,(E_1)_1) - K_2(t,(E_1)_2)\| \\ \|K_3(t,(E_2)_1) - K_3(t,(E_2)_2)\| \\ \|K_4(t,I_1) - K_4(t,I_2)\| \\ \|K_5(t,I_{A1}) - K_5(t,I_{A2})\| \\ \|K_6(t,T_1) - K_6(t,T_2)\| \\ \|K_7(t,R_1) - K_7(t,R_2)\| \\ \|K_8(t,E_{n1}) - K_8(t,E_{n2})\| \end{pmatrix} = \begin{pmatrix} (\|\lambda(t)\| + \mu)\|S_1(t) - S_2(t\|) \\ \mathrm{k}_1\|(E_1)_1(t) - (E_1)_2(t)\| \\ \mathrm{k}_2\|(E_1)_1(t) - (E_2)_2(t)\| \\ \mathrm{k}_3\|(E_2)_1(t) - (E_2)_2(t)\| \\ \mathrm{k}_4\|(I_1)_1(t) - (I_2)_2(t)\| \\ \mathrm{k}_5\|(I_A)_1(t) - (I_A)_2(t)\| \\ \mu\|R_1(t) - R_2(t)\| \\ v\|(E_n)_1(t) - (E_n)_2(t)\| \end{pmatrix} \\
&= \begin{pmatrix} \mathscr{B}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathscr{B}2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathscr{B}3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathscr{B}4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathscr{B}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathscr{B}6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathscr{B}7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathscr{B}8 \end{pmatrix} \times \|(\mathscr{G}_1(t) - \mathscr{G}_2(t))\| \\
&= \mathscr{B}\|(\mathscr{G}_1(t) - \mathscr{G}_2(t))\|,
\end{aligned} \tag{19}$$

where,

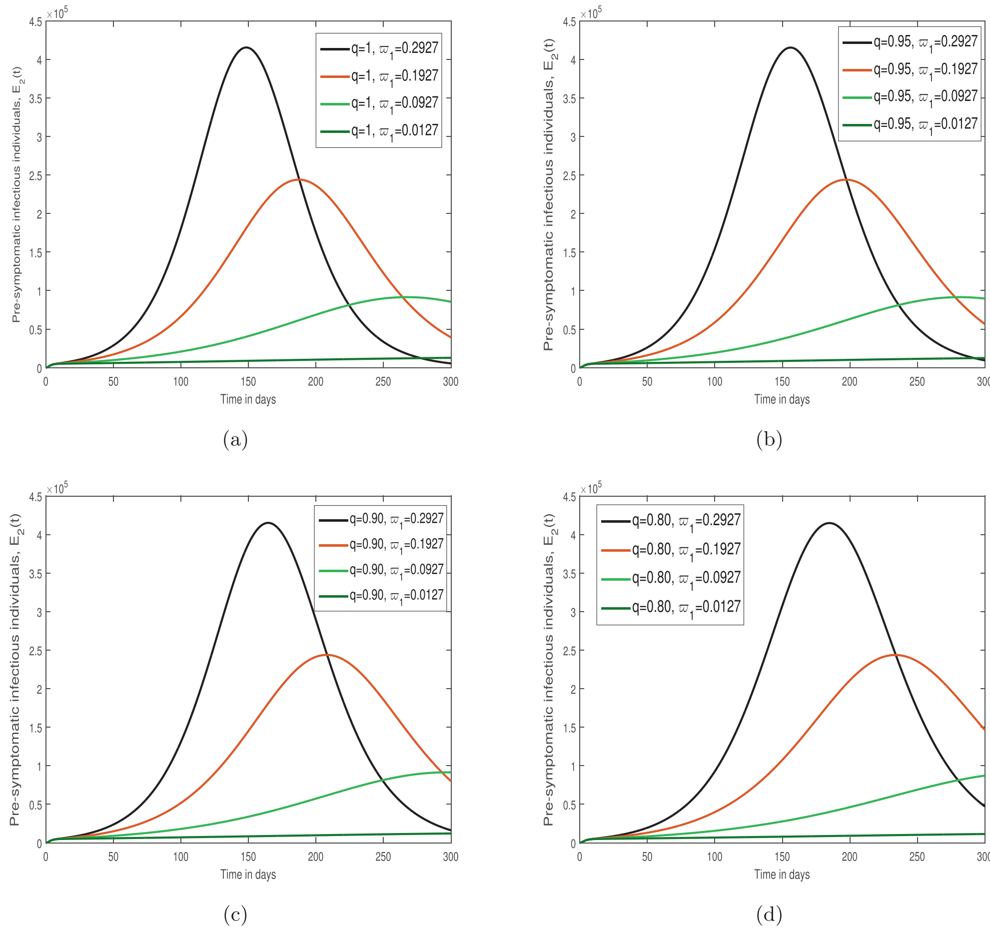

Fig. 5 Influence of the ϖ_1 on the pre-symptomatic infectious individuals $E_2(t)$. Moreover, the impact of q is shown in subplots (a-d) respectively by $q = 1, = 0.95, 0.90,$ and 0.80.

$$\mathscr{B}_1 = (\|\lambda(t)\| + \mu),\ \mathscr{B}_2 = \mathrm{k}_1,\ \mathscr{B}_3 = \mathrm{k}_2,\ \mathscr{B}_4 = \mathrm{k}_3,\ \mathscr{B}_5 = \mathrm{k}_4,$$
$$\mathscr{B}_6 = \mathrm{k}_5,\ \mathscr{B}_7 = \mu,\ \mathscr{B}_8 = \nu.$$

Thus,

$$\|H(t,\mathscr{G}_1(t)) - H(t,\mathscr{G}_2(t))\|t \leqslant \sup_{\|x\|<1} \|\mathscr{B}x\| \leqslant \Theta\|\mathscr{G}_1(t) - \mathscr{G}_2(t)\|, \tag{20}$$

where,

$$\Theta = \max(\sup \mathscr{B}_1, \sup \mathscr{B}_2, \sup \mathscr{B}_3, \sup \mathscr{B}_4, \sup \mathscr{B}_5, \sup \mathscr{B}_6, \sup \mathscr{B}_7, \sup \mathscr{B}_8). \tag{21}$$

Theorem 1. There exists a unique solution of system of Eqs. (10) if the following condition holds

$$(\phi(q) + \psi(q)T)\Theta < 1. \tag{22}$$

Proof. Let the map $\mathscr{P} : \mathfrak{C}(\Omega, \mathbb{R}^8) \to \mathfrak{C}(\Omega, \mathbb{R}^8)$ defined by

$$\mathscr{P}(\mathscr{G}(t)) = \mathscr{G}_0 + \phi(q)H(t,\mathscr{G}(t)) + \psi(q) \times \int_0^t H(s,\mathscr{G}(s))ds, \tag{23}$$

then the Eq. (17) becomes

$$\mathscr{G}(t) = \mathscr{P}(\mathscr{G}(t)). \tag{24}$$

We know the space $\mathfrak{C}(\Omega, \mathbb{R}^8)$ equipped with the norm $\|\varphi\|_{\mathfrak{C}} = \sup_{t\in\Omega}|\varphi(t)|$ is a Banach space.

Now, using the formula (17), we have

$$\begin{aligned}
&\|\mathscr{P}(\mathscr{G}_1(t)) - \mathscr{P}(\mathscr{G}_2(t))\|_{\mathfrak{C}} = \|\phi(q)(H(t,\mathscr{G}_1(t))\\
&-(H(t,\mathscr{G}_2(t)) + \psi(q)\int_0^t (H(s,\mathscr{G}_1(s)) - (H(s,\mathscr{G}_2(s)))ds\|_{\mathfrak{C}}\\
&\leqslant \phi(q)\|(H(t,\mathscr{G}_1(t)) - (H(t,\mathscr{G}_2(t))\|_{\mathfrak{C}}\\
&+\psi(q)\int_0^t \|(H(s,\mathscr{G}_1(s)) - (H(s,\mathscr{G}_2(s)))\|_{\mathfrak{C}}ds\\
&\leqslant \phi(q)\Theta\|\mathscr{G}_1(t) - \mathscr{G}_2(t)\|_{\mathfrak{C}} + \psi(q)\Theta\int_0^t \|\mathscr{G}_1(s)) - \mathscr{G}_2(s)\|_{\mathfrak{C}}ds\\
&\leqslant \phi(q)\Theta\|\mathscr{G}_1(t) - \mathscr{G}_2(t)\|_{\mathfrak{C}} + \psi(q)\Theta T\|\mathscr{G}_1(t)) - \mathscr{G}_2(t)\|_{\mathfrak{C}}\\
&\leqslant (\phi(q) + \psi(q)T)\Theta\|\mathscr{G}_1(t) - \mathscr{G}_2(t)\|_{\mathfrak{C}}.
\end{aligned}$$

Since $(\phi(q) + \psi(q)T)\Theta < 1$, the map $\mathscr{P}$ is a contraction, therefore the system of Eqs. (10) has a unique solution.

5. Numerical scheme

The present section provides the iterative scheme and the simulations results of the proposed fractional COVID-19 model

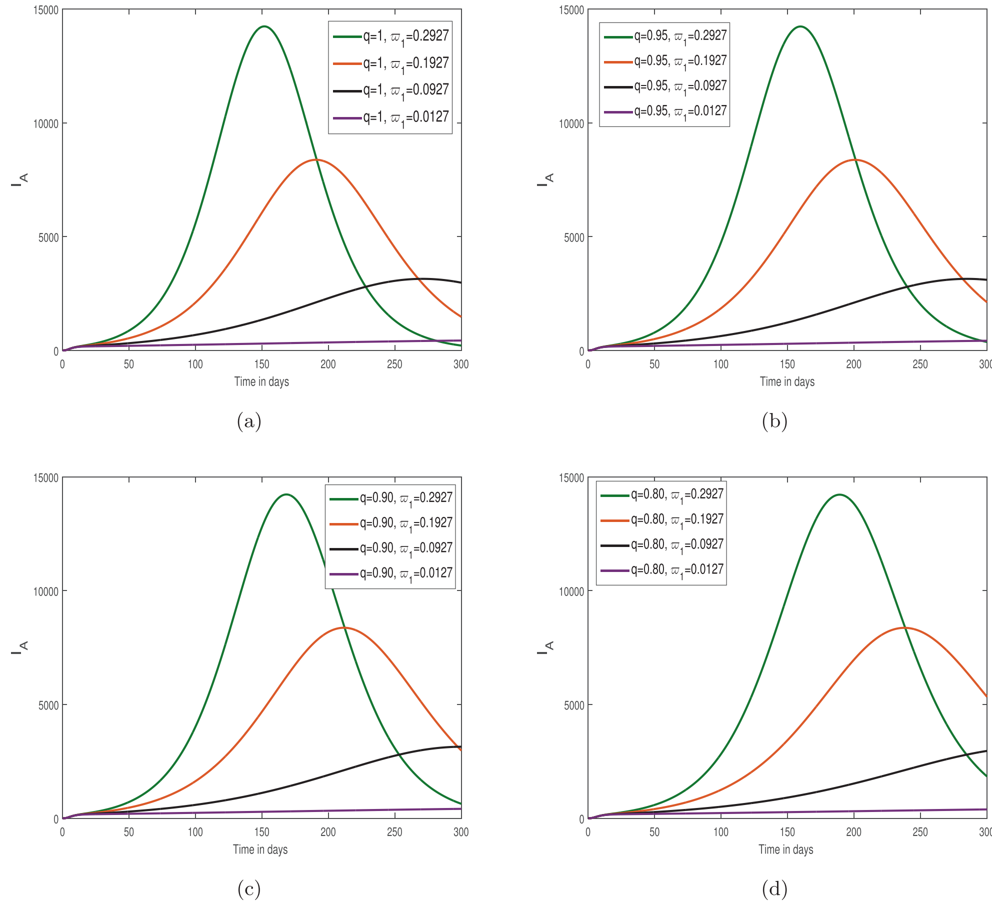

Fig. 6 Influence of the ϖ_1 on the symptomatically-infectious individuals $I_A(t)$. Moreover, the impact of q is shown in subplots (a-d) respectively by $q = 1, 0.95, 0.90$, and 0.80.

(10) to analyze the effect of fractional order q and other significant measures on the disease incidence and prevalence. Initially, we derive the recursive scheme using an efficient approach from recent literature. The derived iterative procedure is then utilized to perform the simulation results using Matlab. For the desired numerical solution of the proposed model (10), we use the technique of fractional Adams–Bashforth for Caputo-Fabrizio fractional order derivative [11,32,33]. To develop the desired iterative scheme, let us take only the first equation of the model (10) and then utilizing the fundamental principle of integration the following equation is obtained:

$$S(t) - S(0) = \frac{(1-q)}{M(q)} K_1(t, S) + \frac{q}{M(q)} \int_0^t K_1(\zeta, S) d\zeta. \tag{25}$$

At $t = t_{n+1}$ we have

$$S(t_{n+1}) - S(0) = \frac{(1-q)}{M(q)} K_1(t_n, S_n) + \frac{q}{M(q)} \int_0^{t_{n+1}} K_1(t, S) dt, \tag{26}$$

and similarly

$$S(t_n) - S(0) = \frac{(1-q)}{M(q)} K_1(t_{n-1}, S_{n-1}) + \frac{q}{M(q)} \int_0^{t_n} K_1(t, S) dt. \tag{27}$$

From these two Eqs. (26) and (27) we have

$$S_{n+1} - S_n = \frac{(1-q)}{M(q)} \{K_1(t_n, S_n) - K_1(t_{n-1}, S_{n-1})\} + \frac{q}{M(q)} \int_{t_n}^{t_{n+1}} K_1(t, S) dt. \tag{28}$$

Taking $h = t_{i+1} - t_i$, approximating $K_1(t, S)$ with the help of Lagrange interpolation and calculating the integral portion in Eq. (28) over $[t_n, t_{n+1}]$, we get

$$\int_{t_n}^{t_{n+1}} K_1(t, S) dt = \int_{t_n}^{t_{n+1}} \left[\frac{K_1(t_n, S_n)}{h}(t - t_{n-1}) - \frac{K_1(t_{n-1}, S_{n-1})}{h}(t - t_n)\right] dt = \frac{3h}{2} K_1(t_n, S_n) - \frac{h}{2} K_1(t_{n-1}, S_{n-1}).$$

$$= \frac{3h}{2} K_1(t_n, S_n) - \frac{h}{2} K_1(t_{n-1}, S_{n-1}). \tag{29}$$

Substituting this approximated value in Eq. (28) we have

$$S_{n+1} = S_n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\} K_1(t_n, S_n) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\} K_1(t_{n-1}, S_{n-1}). \tag{30}$$

Similarly, for the remaining equations of the proposed fractional COVID-19 model (10) we have

$$\begin{aligned}
E_1^{n+1} &= E_1^n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\}K_2\left(t_n, E_1^n\right) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\}K_2\left(t_{n-1}, E_1^{n-1}\right),\\
E_2^{n+1} &= E_2^n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\}K_3\left(t_n, E_2^n\right) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\}K_3\left(t_{n-1}, E_2^{n-1}\right),\\
I^{n+1} &= I^n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\}K_4\left(t_n, I^n\right) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\}K_4\left(t_{n-1}, I^{n-1}\right),\\
I_A^{n+1} &= I_A^n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\}K_5\left(t_n, I_A^n\right) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\}K_5\left(t_{n-1}, I_A^{n-1}\right), \quad (31)\\
T^{n+1} &= T^n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\}K_6\left(t_n, T^n\right) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\}K_6\left(t_{n-1}, T^{n-1}\right),\\
R^{n+1} &= R^n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\}K_7\left(t_n, R^n\right) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\}K_7\left(t_{n-1}, R^{n-1}\right),\\
E_n^{n+1} &= E_n^n + \left\{\frac{(1-q)}{M(q)} + \frac{3h}{2M(q)}\right\}K_8\left(t_n, E_n^n\right) - \left\{\frac{(1-q)}{M(q)} + \frac{qh}{2M(q)}\right\}K_8\left(t_{n-1}, E_n^{n-1}\right).
\end{aligned}$$

Now, we depict the graphical results of the model (10) in order to analyze the impact of fractional order q and of some parameters which are influential significantly on the disease transmission.The iterative scheme derived in (30) and (31) is implemented to carry the desired simulations.The numerical values used in the simulation are given in Table 1.To demonstrate the impact of order of the Caputo-Fabrizio operator, the time level is taken up to 500 days.The resulting graphical behavior is shown in Fig.2 for five various values of q.Fig.2 shows the stability of the model (10).The dynamics of pre-symptomatic COVID-19 infected individuals versus the variation in the rates of virus removal from the environment reservoir v and fractional order values are analyzed in Fig.3.The resulting graphical behavior for $q = 1, 0.95, 0.90, 80$, is shown in subplots 3(a-d) respectively.The population in E_2 class is decreased with the increase in v although the peaks move to the right.The decrease in the peaks is faster for the case of smaller values of q.The influence of the v on the total symptomatically, asymptomatically and under-treatment COVID-19 infected individuals for various four values of q is demonstrated in Fig.4 with subplots (a-d).From these graphical interpretations, we observed that with a decrease in the removal rate v with different rates, the cumulative population in symptomatic, asymptomatic and under-treatment compartments are significantly decreased.A similar analysis is observed for all values of the parameter q.Thus, these interpretations reveal that the removal of viruses from the surfaces via viral disinfection spray etc.is beneficial to reduce the infection.Moreover, we analyzed the influence of various rates of ϖ_1 (the viral contribution rate and the environment due to E_2) on the population in the pre-symptomatic symptomatic and asymptomatic infected classes. Fig.5, shows the dynamics of E_2 class versus ϖ_1.In Fig.6, we analyzed the dynamics of population in I_A compartment for various rates of ϖ_1. Finally, in Fig.7, we have studied the influence of the parameter ϖ_1 over the infected human population $I(t)$ for various values of q. It can be observed that the infected curves are decreasing very well with the decrease in the parameter ϖ_1. Similar behavior is found for the rest of values q (the fractional order) as can be seen in the subplots (a-d) of all Figs. 5–7.

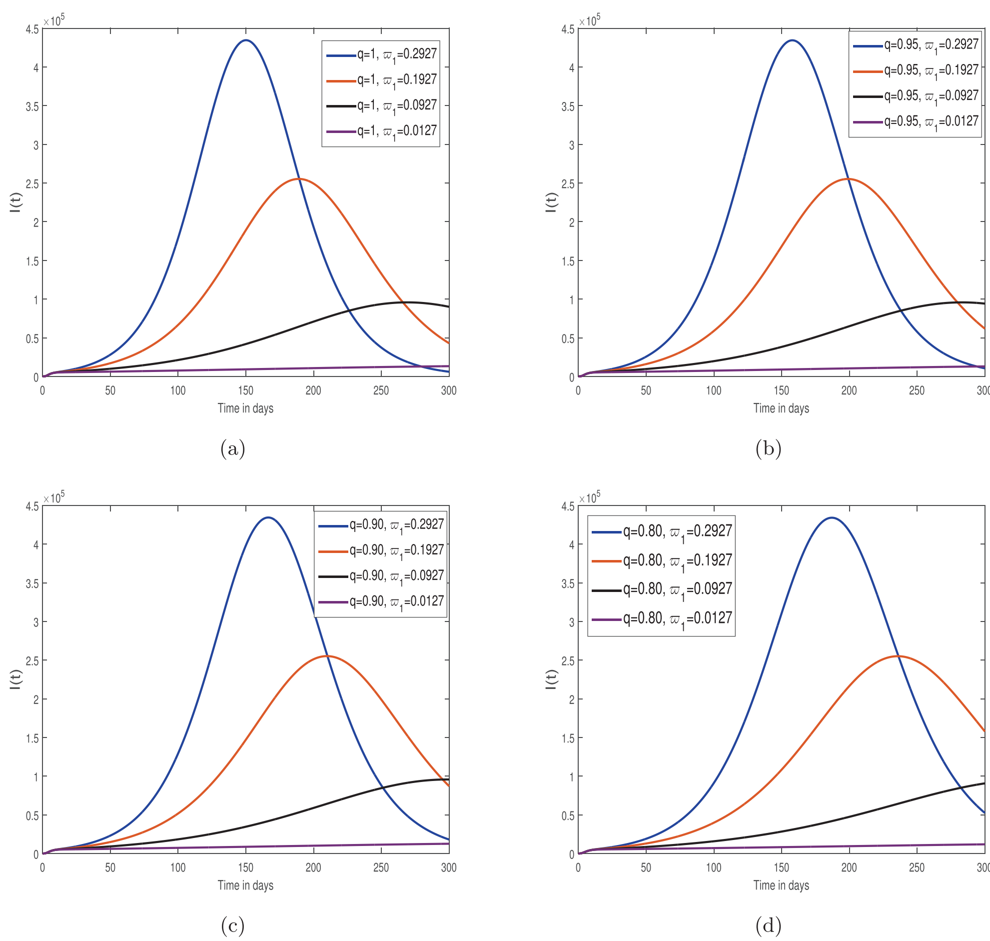

Fig. 7 Influence of the ϖ_1 on the asymptomatically-infectious populations $I(t)$ and the impact of q is shown in subplots (a-d) respectively by $q = 1, 0.95, 0.90$, and $q = 0.80$.

6. Conclusion

The world is facing a huge panic in the form of the novel coronavirus pandemic. The COVID-19 infection is serious threat to both humans health and economy. Although a number of papers and reports have been published in the last few months on the dynamics and possible control of this novel pandemic. In the present study, we formulated a new mathematical model with treatment to analyze the complex transmission dynamics of the ongoing novel COVID-19. The proposed compartmental model is firstly formulated through the classical differential equation having integer-order. The treatment class is considered in the model construction. The model has fitted to the reported COVID-19 detected cases in the Kingdom of Saudi Arabia using the well-known non-linear least square fitting approach. The best fitted and estimated parameters are obtained and the threshold parameter $\mathscr{R}_0$ is approximated for the selected country. The model is further extended to the fractional order using the Caputo-Fabrizio operator with exponential decay kernel in order to gain more insights into the disease transmission dynamics. The basic reproductive number of the fractional COVID-19 model is approximated both theoretically and numerically. Moreover, the existence and uniqueness of the fractional model are shown via the Picard Lindelöf theorem. Finally, the fractional model is solved numerically using the Adams–Bashforth scheme and the model simulations results are depicted for biologically significant parameters and various values of fractional order q.

Declaration of Competing Interest

None.

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