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Multiwave, multicomplexiton, and positive multicomplexiton solutions to a (3 + 1)-dimensional generalized breaking soliton equation

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Abstract There are a lot of physical phenomena which their mathematical models are decided by nonlinear evolution (NLE) equations. Our concern in the present work is to study a special type of NLE equations called the (3 + 1)-dimensional generalized breaking soliton (3D-GBS) equation. To this end, the linear superposition (LS) method along with a series of specific techniques are utilized and as an achievement, multiwave, multicomplexiton, and positive multicomplexiton solutions to the 3D-GBS equation are formally constructed. The study confirms the efficiency of the methods in handling a wide variety of nonlinear evolution equations.

1. Introduction

During the last few decades, significant attention has been drawn to study nonlinear evolution equations especially their exact solutions. There are different types of exact solutions, for example, periodic solutions, soliton solutions, and so on. Many useful and well-organized techniques such as multiple exp-function method [1–3], simplified Hirota's method [4–8], Hirota's bilinear method [9–12], and linear superposition method [13–17] have been presented to handle NLE equations [18–37].

The linear superposition method which is an effective technique to extract multiwave solutions of nonlinear evolution

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equations has received much attention among academic researchers. For example, Zhang et al. [15] utilized the LS method to derive the N-wave solution of a (3 + 1)-dimensional generalized Kadomtsev– Petviashvili equation. The N-wave solution of the Hirota–Satsuma–Ito equation were obtained in [16] by Zhou and Manukure through the LS method. The LS method was adopted by Inc et al. [17] to look for the N-wave solution of the B-type Kadomtsev–Petviashvili equation [16,22,25,38–42].

In the present work, we deal with the following generalized (3 + 1)-dimensional generalized breaking soliton equation [18]

$$\frac{\partial w}{\partial t}+\alpha\frac{\partial^3 u}{\partial x^3}+\beta\frac{\partial^3 u}{\partial y\partial x^2}+6\alpha u\frac{\partial u}{\partial x}+3\beta u\frac{\partial u}{\partial y}+3\beta\frac{\partial u}{\partial x}v=0,$$

$$\frac{\partial w}{\partial x}-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}-\frac{\partial u}{\partial z}=0,$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=0,$$

or

$$\delta_x^{-1}\left(\frac{\partial^2 u}{\partial t\partial x}+\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial^2 u}{\partial t\partial z}\right)+\alpha\frac{\partial^3 u}{\partial x^3}+\beta\frac{\partial^3 u}{\partial y\partial x^2}+6\alpha u\frac{\partial u}{\partial x}+3\beta u\frac{\partial u}{\partial y}+3\beta\frac{\partial u}{\partial x}\delta_x^{-1}\frac{\partial u}{\partial y}=0, \quad (1)$$

and extracting multiwave, multicomplexiton, and positive multicomplexiton solutions of it using the linear superposition method along with a series of specific techniques. The above 3D-GBS equation is a generalization of the following (2 + 1)-dimensional generalized breaking soliton equation

$$\frac{\partial u}{\partial t}+\alpha\frac{\partial^3 u}{\partial x^3}+\beta\frac{\partial^3 u}{\partial y\partial x^2}+6\alpha u\frac{\partial u}{\partial x}+3\beta u\frac{\partial u}{\partial y}+3\beta\frac{\partial u}{\partial x}v=0,$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=0,$$

which has been studied via different techniques such as ansatz methods [19] and Hirota's bilinear method [20].

Gai et al. in [18] by considering the transformation

$$u=2(\ln\varphi)_{xx},\ v=2(\ln\varphi)_{xy},$$

and utilizing the symbolic computations provided the Hirota's bilinear form of the 3D-GBS Eq. (1) as follows

$$HBF_{3D-GBS}\equiv\left(\mathscr{D}_x\mathscr{D}_t+\mathscr{D}_y\mathscr{D}_t+\mathscr{D}_z\mathscr{D}_t+\alpha\mathscr{D}_x^4+\beta\mathscr{D}_x^3\mathscr{D}_y\right)\varphi.\varphi=0, \quad (2)$$

which $\mathscr{D}_x^4$, $\mathscr{D}_x^3\mathscr{D}_y$, $\mathscr{D}_x\mathscr{D}_t$, $\mathscr{D}_y\mathscr{D}_t$, and $\mathscr{D}_z\mathscr{D}_t$ are the Hirota's operators. Gai and his collaborators [18] found different wave structures of the 3D-GBS equation such as lump-type solutions, rogue wave type solutions, breather lump wave solutions, and interaction solutions. Owing to the exact solutions reported in [18], the current paper presents multiwave, multicomplexiton, and positive multicomplexiton solutions of the 3D-GBS equation by means of the linear superposition method along with a series of specific techniques.

2. 3D-GBS equation and its multiwave, multicomplexiton, and positive multicomplexiton solutions

In the currentsection, multiwave, multicomplexiton, and positive multicomplexiton solutions of the 3D-GBS equation are formally acquired. The dynamic and evolution behaviors for a positive complexiton solution are demonstrated by portrayingits three dimensional and density plots.

2.1. Multiwave solution

To construct the multiwave solution to the 3D-GBS equation, we seek $a_j, j=1,2,3,4$ and $\tau_j, j=1,2,3,4$ such that [15]

$$\alpha a_1^4(x^{\tau_1}-y^{\tau_1})^4+\beta a_1^3a_2(x^{\tau_1}-y^{\tau_1})^3(x^{\tau_2}-y^{\tau_2})+a_1a_4(x^{\tau_1}-y^{\tau_1})(x^{\tau_4}-y^{\tau_4})+a_2a_4(x^{\tau_2}-y^{\tau_2})(x^{\tau_4}-y^{\tau_4})+a_3a_4(x^{\tau_3}-y^{\tau_3})(x^{\tau_4}-y^{\tau_4})=0. \quad (3)$$

By simple analysis, one obtains $a_j, j=1,2,3,4$ if $(\tau_1,\tau_2,\tau_3,\tau_4)$ be considered as $(1,1,1,3)$. After substituting $\tau_j, j=1,2,3,4$ into Eq. (3) and equating the coefficients of the resulting expression to zero, one arrives at

$$\alpha a_1^4+\beta a_1^3a_2+a_1a_4+a_2a_4+a_3a_4=0,$$

$$4\alpha a_1^4+4\beta a_1^3a_2+a_1a_4+a_2a_4+a_3a_4=0,$$

$$6\alpha a_1^4+6\beta a_1^3a_2=0.$$

The nontrivial solution of the above nonlinear algebraic set is

$$a_2=-\frac{\alpha}{\beta}a_1,\ a_3=\frac{(\alpha-\beta)}{\beta}a_1.$$

Now, by assuming $a_1=1$, the following multiwave solution to the 3D-GBS equation is derived

$$u=2(\ln\varphi)_{xx},\ v=2(\ln\varphi)_{xy},$$

in which

$$\varphi=\sum_{j=1}^{N}c_je^{\theta_j},\ \theta_j=k_jx-\frac{\alpha}{\beta}k_jy+\frac{(\alpha-\beta)}{\beta}k_jz+a_4k_j^3t.$$

2.2. Multicomplexiton solution

Now to acquire the multicomplexiton solution of the 3D-GBS equation, let us consider [16]

$$k_j=k_{1j}+ik_{2j}, j=1,\ldots,N,\quad i^2=-1,$$

which k_{1j} and $k_{2j}(j=1,\ldots,N)$ are real parameters. Such an assumption results in

$$\theta_j = k_j x - \frac{\alpha}{\beta} k_j y + \frac{(\alpha-\beta)}{\beta} k_j z + a_4 k_j^3 t = \theta_{j,1} + i\theta_{j,2}, \bar{\theta}_j$$
$$= \bar{k}_j x - \frac{\alpha}{\beta} \bar{k}_j y + \frac{(\alpha-\beta)}{\beta} \bar{k}_j z + a_4 \bar{k}_j^3 t = \theta_{j,1} - i\theta_{j,2}.$$

It is clear that e^{θ_j} and $e^{\bar{\theta}_j}$ satisfy the bilinear form (2). Therefore, owing to the linear superposition principle, the new expression

$$\varphi = \sum_{j=1}^{N} \left(c_j e^{\theta_j} + \bar{c}_j e^{\bar{\theta}_j} \right)$$
$$= \sum_{j=1}^{N} e^{\theta_{j,1}} \left(c_{j,1} \cos(\theta_{j,2}) + c_{j,2} \sin(\theta_{j,2}) \right), c_{j,1}, c_{j,2} \in \mathbb{R},$$

is also a nontrivial solution of the bilinear form of the 3D-GBS equation. Finally, the multicomplexiton solution to the 3D-GBS equation can be written as

$$u = 2(\ln \varphi)_{xx}, v = 2(\ln \varphi)_{xy},$$

in which

$$\varphi = \sum_{j=1}^{N} e^{\theta_{j,1}} \left(c_{j,1} \cos(\theta_{j,2}) + c_{j,2} \sin(\theta_{j,2}) \right).$$

As a special case, suppose that $N = 2$, $k_1 = 1 + i$, and $k_2 = -1 - i$, then

$$\theta_1 = x - \frac{\alpha}{\beta} y + \frac{(\alpha-\beta)}{\beta} z - 2a_4 t$$
$$+ i\left(x - \frac{\alpha}{\beta} y + \frac{(\alpha-\beta)}{\beta} z + 2a_4 t \right)$$
$$= \theta_{1,1} + i\theta_{1,2},$$

$$\theta_2 = -x + \frac{\alpha}{\beta} y - \frac{(\alpha-\beta)}{\beta} z + 2a_4 t$$
$$+ i\left(-x + \frac{\alpha}{\beta} y - \frac{(\alpha-\beta)}{\beta} z - 2a_4 t \right)$$
$$= \theta_{2,1} - i\theta_{2,2}.$$

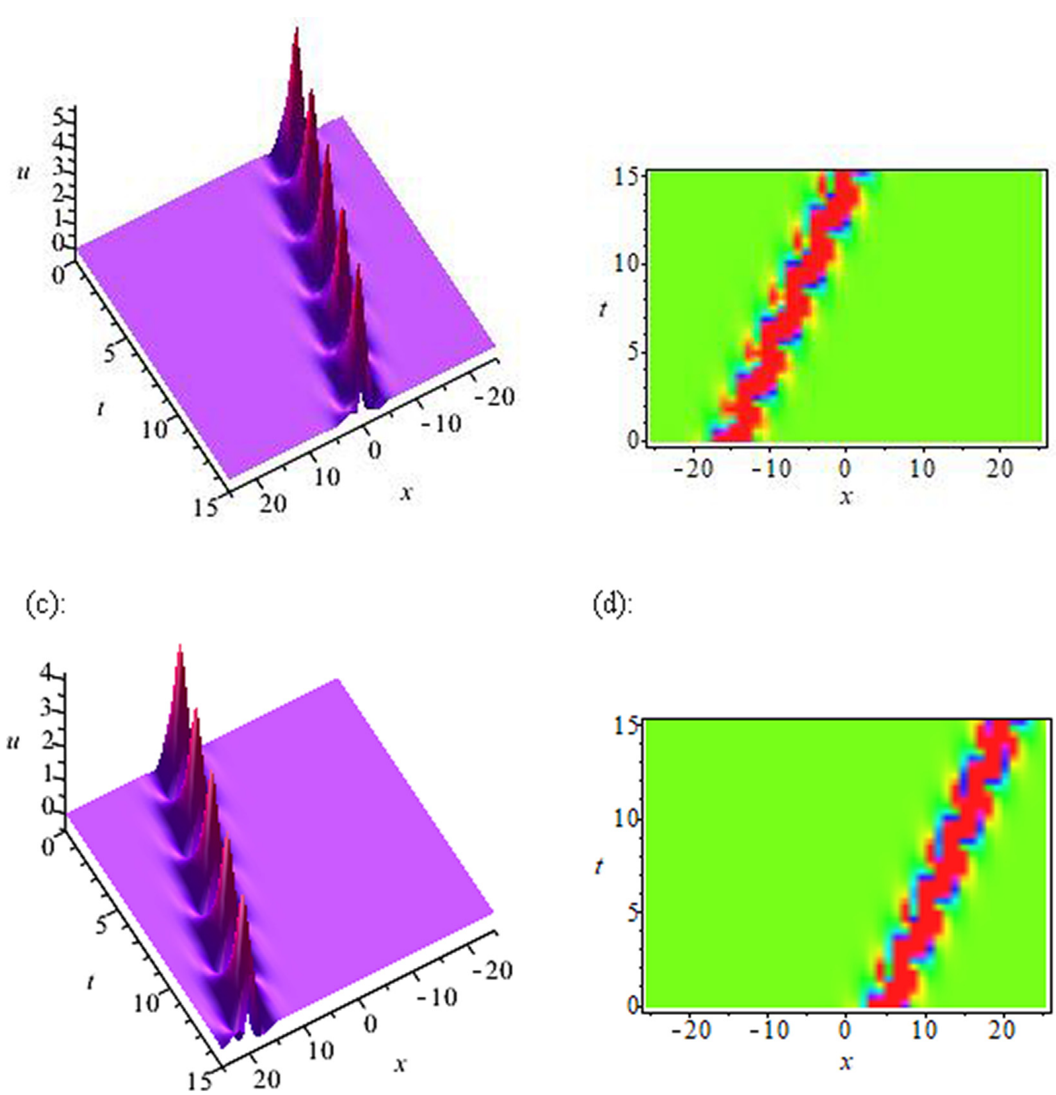

Fig. 1 The function u when $y = z = -15$: (a) Three-dimensional plot and (b) density plot; The function u when $y = z = 5$: (c) Three-dimensional plot and (d) density plot.

Now, the complexiton solution of the 3D-GBS equation is $u = 2(\ln\varphi)_{xx}, v = 2(\ln\varphi)_{xy}$,

in which

$$\varphi = e^{x-\frac{\alpha}{\beta}y+\frac{(\alpha-\beta)}{\beta}z-2a_4t}\left(c_{1,1}\cos\left(x-\frac{\alpha}{\beta}y+\frac{(\alpha-\beta)}{\beta}z+2a_4t\right)\right.$$
$$\left.+c_{1,2}\sin\left(x-\frac{\alpha}{\beta}y+\frac{(\alpha-\beta)}{\beta}z+2a_4t\right)\right)$$
$$+e^{-x+\frac{\alpha}{\beta}y-\frac{(\alpha-\beta)}{\beta}z+2a_4t}\left(c_{2,1}\cos\left(-x+\frac{\alpha}{\beta}y-\frac{(\alpha-\beta)}{\beta}z-2a_4t\right)\right.$$
$$\left.+c_{2,2}\sin\left(-x+\frac{\alpha}{\beta}y-\frac{(\alpha-\beta)}{\beta}z-2a_4t\right)\right).$$

2.3. Positive multicomplexiton solution

In a manner similar to that accomplished in [16], one can establish the following positive multicomplexiton solution to the 3D-GBS equation

$u = 2(\ln\varphi)_{xx}, v = 2(\ln\varphi)_{xy}$,

in which

$$\varphi = \sum_{j=1}^{N} c_j\cosh\left(k_jx-\frac{\alpha}{\beta}k_jy+\frac{(\alpha-\beta)}{\beta}k_jz+a_4k_j^3t\right)$$
$$+\sum_{j=N+1}^{N+M} c_j\cos\left(k_jx-\frac{\alpha}{\beta}k_jy+\frac{(\alpha-\beta)}{\beta}k_jz-a_4k_j^3t\right), c_j \in \mathbb{R}.$$

$$u = -2\frac{4\sinh(-x-y+2z+t)\sin(x+y-2z+t)-3}{4\cosh(-x-y+2z+t)^2+4\cosh(-x-y+2z+t)\cos(x+y-2z+t)+\cos(x+y-2z+t)^2},$$

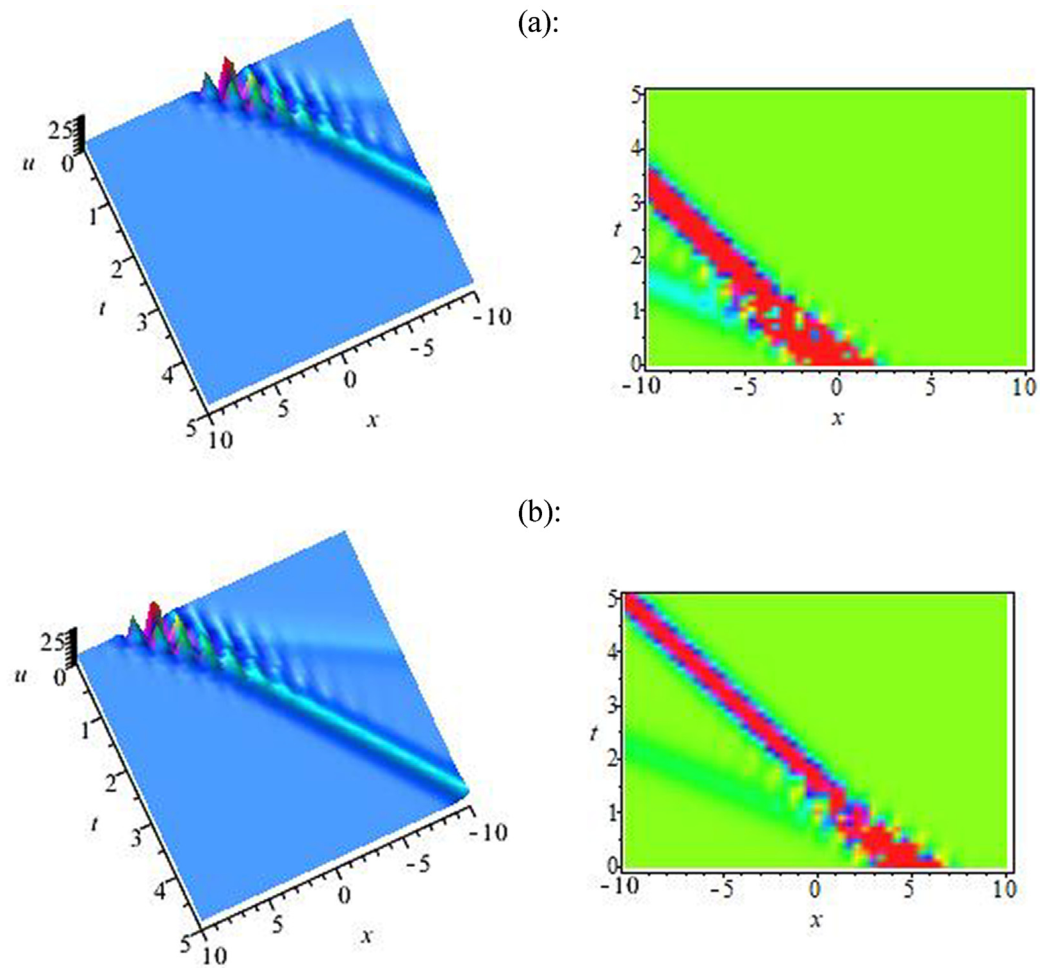

Fig. 2 The function u for $M = N = 2$, $c_1 = 1$, $c_2 = 1.8$, $c_3 = 1$, $c_4 = 1.5$, $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, $k_4 = 4$, $a_4 = 1$, $\alpha = -1$, $\beta = 1$, when (a) $y = z = 0$ and (b) $y = z = 5$.

$$v = -2\frac{4\sinh(-x-y+2z+t)\sin(x+y-2z+t)-3}{4\cosh(-x-y+2z+t)^2+4\cosh(-x-y+2z+t)\cos(x+y-2z+t)+\cos(x+y-2z+t)^2}.$$

It should be mentioned that if $c_j > 0$ for $j = 1, 2, ..., N$ and $\sum_{j=1}^{N} c_j > \sum_{j=N+1}^{N+M} |c_j|$, then the function φ is a positive function. For example, when $M = N = 1$, $c_1 = 2$, $c_2 = 1$, $k_1 = 1$, $k_2 = -1$, $a_4 = -1$, $\alpha = -1$, and $\beta = 1$, we find the following positive complexiton solution

The dynamical and evolutionary behaviors for the above positive complexiton solution have been demonstrated by portraying its three dimensional and density plots in Fig. 1.

Fig. 2 shows three dimensional and density plots of the positive complexiton solution for $M = N = 2$, $c_1 = 1$, $c_2 = 1.8$, $c_3 = 1$, $c_4 = 1.5$, $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, $k_4 = 4$, $a_4 = 1$, $\alpha = -1$, $\beta = 1$, when (a) $y = z = 0$ and (b) $y = z = 5$.

2.4. Other soliton and periodic solutions

Other soliton solutions of the 3D-GBS equation are extracted by considering the following hyperbolic functions [21,22]

$$u(x, y, z, t) = A_0 + A_1 \tanh(k_1x + l_1y + m_1z + w_1t),$$

$$v(x, y, z, t) = \frac{l_1}{k_1}(A_0 + A_1 \tanh(k_1x + l_1y + m_1z + w_1t)), \quad (4)$$

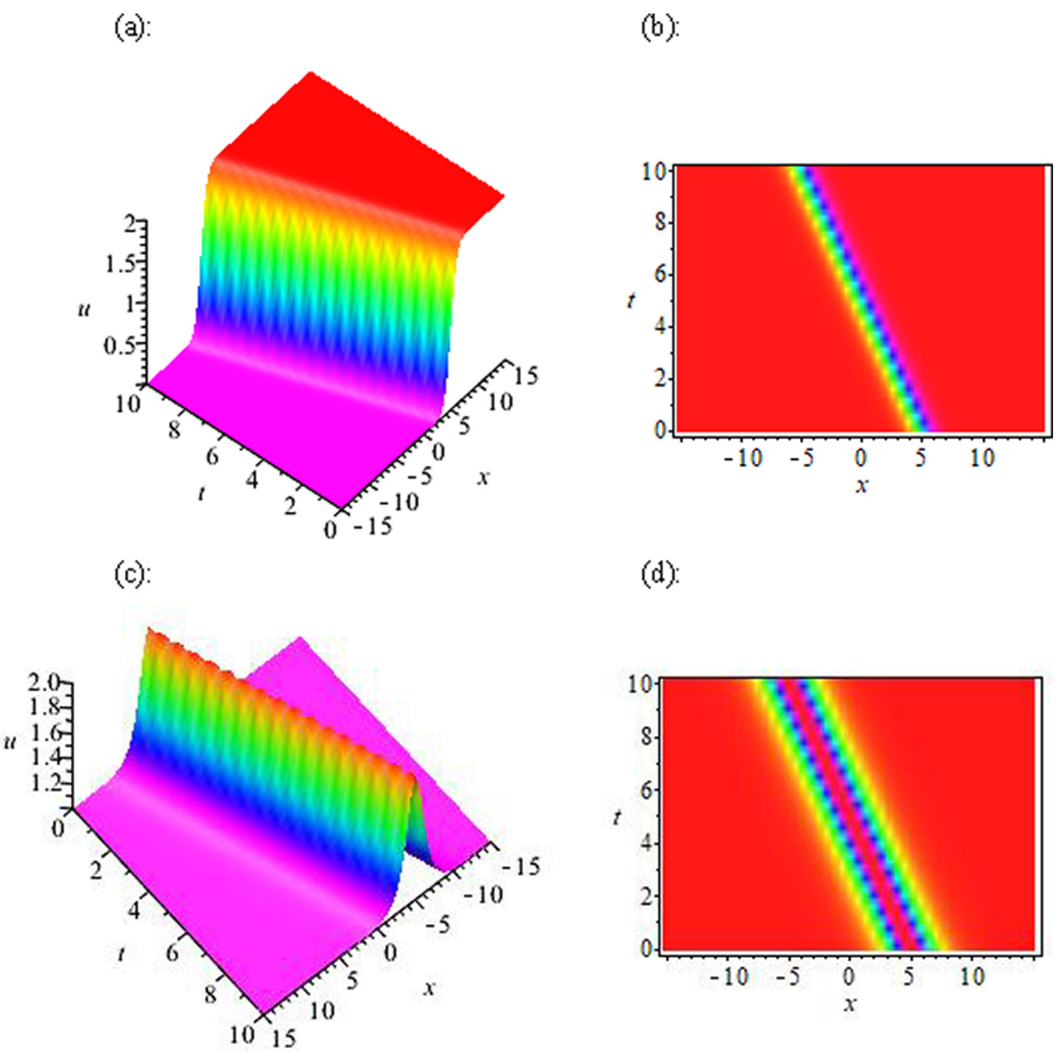

Fig. 3 The hyperbolic tangent function u when $A_0 = 1, A_1 = 1, k_1 = 1, w_1 = 1, \alpha = -1, \beta = 1$, and $y = z = 5$: (a) Three-dimensional plot and (b) density plot; The hyperbolic secant function u when $A_0 = 1, A_1 = 1, k_1 = 1, w_1 = 1, \alpha = -1, \beta = 1$, and $y = z = 5$: (c) Three-dimensional plot and (d) density plot.

and

$$u(x,y,z,t) = A_0 + A_1 \mathrm{sech}(k_1 x + l_1 y + m_1 z + w_1 t),$$

$$v(x,y,z,t) = \frac{l_1}{k_1}(A_0 + A_1 \mathrm{sech}(k_1 x + l_1 y + m_1 z + w_1 t)). \qquad (5)$$

Setting the solutions (4) and (5) in the 3D-GBS equation and solving the resulting system yields the following solution

$$l_1 = -\frac{\alpha}{\beta}k_1, m_1 = \frac{(\alpha-\beta)}{\beta}k_1.$$

Now, the following soliton solutions to the 3D-GBS equation can be obtained

$$u(x,y,z,t) = A_0 + A_1 \tanh\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right),$$

$$v(x,y,z,t) = -\frac{\alpha}{\beta}\left(A_0 + A_1 \tanh\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right)\right),$$

and

$$u(x,y,z,t) = A_0 + A_1 \mathrm{sech}\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right),$$

$$v(x,y,z,t) = -\frac{\alpha}{\beta}\left(A_0 + A_1 \mathrm{sech}\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right)\right).$$

Similarly, the following soliton solutions can be derived

$$u(x,y,z,t) = A_0 + A_1 \coth\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right),$$

$$v(x,y,z,t) = -\frac{\alpha}{\beta}\left(A_0 + A_1 \coth\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right)\right),$$

and

$$u(x,y,z,t) = A_0 + A_1 \mathrm{csch}\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right),$$

$$v(x,y,z,t) = -\frac{\alpha}{\beta}\left(A_0 + A_1 \mathrm{csch}\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right)\right).$$

The dynamical and evolutionary behaviors for a series of the obtained soliton solutions have been shown by representing their three dimensional and density plots in Fig. 3.

It is evident that Fig. 3(a) shows a kink-type soliton solution whereas Fig. 3(c) demonstrates a brigth soliton solution.

Finally, the following preiodic solutions to the 3D-GBS equation can be gained

$$u(x,y,z,t) = A_0 + A_1 \tan\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right),$$

$$v(x,y,z,t) = -\frac{\alpha}{\beta}\left(A_0 + A_1 \tan\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right)\right),$$

and

$$u(x,y,z,t) = A_0 + A_1 \cot\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right),$$

$$v(x,y,z,t) = -\frac{\alpha}{\beta}\left(A_0 + A_1 \cot\left(k_1 x - \frac{\alpha}{\beta}k_1 y + \frac{(\alpha-\beta)}{\beta}k_1 z + w_1 t\right)\right).$$

3. Conclusion

A (3 + 1)-dimensional generalized breaking soliton equation with diverse applications in physical sciences was considered in the present study. Through the bilinear form of the (3 + 1)-dimensional generalized breaking soliton equation and the linear superposition principle along with a series of specific techniques, distinct exact solutions including multiwave, multicomplexiton, and positive multicomplexiton solutions to the 3D-GBS equation were formally extracted. The results of the present work confirmed the efficiency of the schemes in handling a wide variety of nonlinear evolution equations.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Yeşim Sağlam Özkan, Emrullah Yaşar, Aly R. Seadawy, A third-order nonlinear Schrödinger equation: the exact solutions, group-invariant solutions and conservation laws, J. Taibah Univ. Sci. 14 (1) (2020) 585–597. https://www.tandfonline.com/doi/full/10.1080/16583655.2020.1760513, https://doi.org/10.1080/16583655.2020.1760513.

[2] J.G. Liu, L. Zhou, Y. He, Multiple soliton solutions for the new (2+1)-dimensional Korteweg-de Vries equation by multiple exp-function method, Appl. Math. Lett. 80 (2018) 71–78.

[3] A.R. Adem, M. Mirzazadeh, Q. Zhou, K. Hosseini, Multiple soliton solutions of the Sawada-Kotera equation with a nonvanishing boundary condition and the perturbed Korteweg de Vries equation by using the multiple exp-function scheme, Adv. Math. Phys. 2019 (2019) 3175213.

[4] A.M. Wazwaz, S.A. El-Tantawy, Solving the (3+1)-dimensional KP–Boussinesq and BKP–Boussinesq equations by the simplified Hirota's method, Nonlinear Dyn. 88 (2017) 3017–3021.

[5] A.H. Khater, D.K. Callebaut, A.R. Seadawy, General Soliton Solutions for Nonlinear Dispersive Waves in Convective Type Instabilities, Phys. Scr. 74 (2006) 384–393.

[6] R. Aly, Seadawy, Three-dimensional weakly nonlinear shallow water waves regime and its travelling wave solutions, Int. J. Comput. Methods 15 (03) (2018) 1850017.

[7] Aly Seadawy, Jalil Manafian, New soliton solution to the longitudinal wave equation in a magneto-electro-elastic circular rod, Results Phys. 8 (2018) 1158–1167.

[8] Ehab S. Selima, Aly R. Seadawy, Xiaohua Yao, The nonlinear dispersive Davey-Stewartson system for surface waves propagation in shallow water and its stability, Eur. Phys. J. Plus 131 (2016) 425.

[9] K. Hosseini, M. Aligoli, M. Mirzazadeh, M. Eslami, J.F. Gómez-Aguilar, Dynamics of rational solutions in a new generalized Kadomtsev-Petviashvili equation, Mod. Phys. Lett. B (2019), https://doi.org/10.1142/S0217984919504372.

[10] R. Hirota, The direct method in soliton theory, Cambridge University Press, 2004.

[11] Aly Seadawy Abdullah, Jun Wang, Mathematical methods and solitary wave solutions of three-dimensional Zakharov-Kuznetsov-Burgers equation in dusty plasma and its applications, Results Phys. 7 (2017) 4269–4277.

[12] R. Pouyanmehr, K. Hosseini, R. Ansari, S.H. Alavi, Different wave structures to the (2+1)-dimensional generalized Bogoyavlensky-Konopelchenko equation, Int. J. Appl. Comput. Math. 5 (2019) 149.

[13] Aly Seadawy, Two-dimensional interaction of a shear flow with a free surface in a stratified fluid and its a solitary wave solutions via mathematical methods, Eur. Phys. J. Plus 132 (2017) 518.

[14] Aly R. Seadawy, Sultan Z. Alamri, Mathematical methods via the nonlinear two-dimensional water waves of Olver dynamical equation and its exact solitary wave solutions, Results Phys. 8 (2018) 286–291.

[15] Ahmed H. Arnous, Aly R. Seadawy, Rubayyi T. Alqahtani, Anjan Biswas, Optical solitons with complex Ginzburg-Landau equation by modified simple equation method, Optik – Int. J. Light Electron Optics 144 (2017) 475–480.

[16] Asghar Ali, Aly R. Seadawy, Lu. Dianchen, Computational methods and traveling wave solutions for the fourth-Order nonlinear Ablowitz-Kaup-Newell-Segur water wave dynamical equation via two methods and its applications, Open Phys. J. 16 (2018) 219–226.

[17] Y. Zhou, S. Manukure, Complexiton solutions to the Hirota–Satsuma–Ito equation, Math. Meth. Appl. Sci. (2019), https://doi.org/10.1002/mma.5512.

[18] M. Inc, K. Hosseini, M. Samavat, M. Mirzazadeh, M. Eslami, M. Moradi, D. Baleanu, Wave and other solutions to the B-type Kadomtsev-Petviashvili equation, Therm. Sci. (2020) (in press).

[19] L. Gai, W.X. Ma, M. Li, Lump-type solutions, rogue wave type solutions and periodic lump-stripe interaction phenomena to a (3 +1)-dimensional generalized breaking soliton equation, Phys. Lett. A (2019), https://doi.org/10.1016/j.physleta.2019.126178.

[20] Y.L. Ma, B.Q. Li, Interactions between soliton and rogue wave for a (2+1)-dimensional generalized breaking soliton system: Hidden rogue wave and hidden soliton, Comput. Math. Appl. 78 (2019) 827–839.

[21] X.W. Yan, S.F. Tian, M.J. Dong, L. Zhou, T.T. Zhang, Characteristics of solitary wave, homoclinic breather wave and rogue wave solutions in a (2+1)-dimensional generalizedbreaking soliton equation, Comput. Math. Appl. 76 (2018) 179–186.

[22] Asghar Ali, Aly R. Seadawy, Lu. Dianchen, New solitary wave solutions of some nonlinear models and their Applications, Advances in Difference Equations 2018 (232) (2018) 1–12.

[23] Aly R. Seadawy, Asghar Ali, Wafaa A. Albarakati, Analytical wave solutions of the (2 + 1)-dimensional first integro-differential Kadomtsev-Petviashivili hierarchy equation by using modified mathematical methods, Results Phys. 15 (2019) 102775.

[24] A.R. Seadawy, New exact solutions for the KdV equation with higher order nonlinearity by using the variational method, Comput. Math. Appl. 62 (2011) 3741–3755.

[25] Aly R. Seadawy, D. Kumar, K. Hosseini, F. Samadani, The system of equations for the ion sound and Langmuir waves and its new exact solutions, Results Phys. 9 (2018) 1631–1634. https://linkinghub.elsevier.com/retrieve/pii/S2211379718307526, https://doi.org/10.1016/j.rinp.2018.04.064.

[26] R. Aly, Seadawy, Abdullah, Nonlinear complex physical models: Optical soliton solutions of the complex Hirota dynamical model, Indian J. Phys. (2020), https://doi.org/10.1007/s12648-019-01662-w.

[27] Nadia Cheemaa, Sheng Chen, Aly R. Seadawy, Propagation of isolated waves of coupled nonlinear (2+1)-dimensional Maccari System in Plasma Physics, Results Phys. 17 (2020) 102987.

[28] Hijaz Ahmad, Aly R. Seadawyand, Tufail A. Khan, Phatiphat Thounthong, Analytic approximate solutions Analytic approximate solutions for some nonlinear Parabolic dynamical wave equations, Taibah Univ. J. Sci. 14 (1) (2020) 346–358.

[29] Asghar Ali, Aly R. Seadawy, Dumitru Baleanu, Propagation of harmonic waves in a cylindrical rod via generalized Pochhammer-Chree dynamical wave equation, Results Phys. 17 (2020) 103039.

[30] Saviour Worlanyo Akuamoah, Aly R. Seadawy, Dianchen Lu, Stability analysis of convection diffusion equations of different finite-element spaces at discrete times, Int. J. Modern Phys. B 33 (28) (2019) 1950330 (11 pages).

[31] Mujahid Iqbal, Aly R. Seadawy, Omer H. Khalil, Lu. Dianchen, Propagation of long internal waves in density stratified ocean for the (2+1)-dimensional nonlinear Nizhnik-Novikov-Vesselov dynamical equation, Results Phys. 16 (2020) 102838.

[32] Aly R. Seadawy, Mujahid Iqbal, Lu Dianchen, Nonlinear wave solutions of the Kudryashov-Sinelshchikov dynamical equation in mixtures liquid-gas bubbles under the consideration of heat transfer and viscosity, J. Taibah Univ. Sci. 13 (1) (2019) 1060–1072.

[33] Aly R. Seadawy, Nadia Chemaa, Improved perturbed nonlinear Schrodinger dynamical equation with type of Kerr law Nonlinearity with optical soliton solutions, Phys. Scr. 95 (6) (2020) 065209.

[34] Aly R. Seadawy, Nadia Cheemaa, Propagation of nonlinear complex waves for the coupled nonlinear Schrödinger Equations in two core optical fibers, Physica A 529 (121330) (2019) 1–10.

[35] Aly R. Seadawy, Nadia Cheemaa, Applications of extended modified auxiliary equation mapping method for high order dispersive extended nonlinear schrodinger equation in nonlinear optics, Mod. Phys. Lett. B 33 (18) (2019) 1950203.

[36] Sultan Z. Alamri, Aly R. Seadawy, Haya M. Al-Sharari, Study of optical soliton fibers with power law model by means of higher order nonlinear Schrodinger dynamical system, Results Phys. 13 (102251) (2019) 1–10.

[37] Naila Nasreen, Aly R. Seadawy, Lu. Dianchen, Muhammad Arshad, Construction of Modulation Instability Analysis and Optical Soliton Solutions of pertubed nonlinear Schrödinger dynamical equation with power law nonlinearity in non-kerr medium, Results Phys. 13 (2019) 102263.

[38] Aly Seadawy, K. El-Rashidy, Dispersive Solitary wave solutions of Kadomtsev-Petviashivili and modified Kadomtsev-Petviashivili dynamical equations in unmagnetized dust plasma, Results Phys. 8 (2018) 1216–1222.

[39] Lu. Dianchen, Aly R. Seadawy, Asghar Ali, Applications of Exact Traveling wave solutions of Modified Liouville and the Symmetric Regularized Long Wave equations via two new techniques, Results Phys. 9 (2018) 1403–1410.

[40] Mujahid Iqbal, Aly R. Seadawy, Dianchen Lu, Construction of solitary wave solutions to the nonlinear modified Kortewege-de Vries dynamical equation in unmagnetized plasma via mathematical methods, Mod. Phys. Lett. A 33(32) (2018) 1850183: 1-13.

[41] Lu. Dianchen, Aly R. Seadawy, Asghar Ali, Dispersive traveling wave solutions of the Equal-Width and Modified Equal-Width equations via mathematical methods and its applications, Results Phys. 9 (2018) 313–320.

[42] M.A. Helal, A.R. Seadawy, M.H. Zekry, Stability analysis of solitary wave solutions for the fourth-order nonlinear Boussinesq water wave equation, Appl. Math. Comput. 232 (2014) 1094–1103.