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Research Article

Numerical Study of MHD Third-Grade Fluid Flow through an Inclined Channel with Ohmic Heating under Fuzzy Environment

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The uncertainties or fuzziness occurs due to insufficient knowledge, experimental error, operating conditions, and parameters that give the imprecise information. In this article, we discuss the combined effects of the gravitational and magnetic parameters for both crisp and fuzzy cases in the three basic flow problems (namely, Couette flow, Poiseuille flow, and Couette–Poiseuille flow) of a third-grade fluid over an inclined channel with heat transfer. The dimensionless governing equations with the boundary conditions are converted into coupled fuzzy differential equations (FDEs). The fuzzified forms of the governing equations along with the boundary conditions are solved by employing the numerical technique bvp4c built in MATLAB for both cases, which is very efficient and has a less computational cost. In the first case, proposed problems are analyzed in a crisp environment, while in the second case, they are discussed in a fuzzy environment with the help of α-cut approach, which controls the fuzzy uncertainty. It is observed that the fuzzy gravitational and magnetic parameters are less sensitive for a better flow and heat situation. Using triangular fuzzy numbers (TFNs), the left, right, and mid values of the velocity and temperature profile are presented due to various values of the involved parameters in tabular form. For validation, the present results are compared with existing results for some special cases, viz., crisp case, and they are found to be in good agreement.

1. Introduction

The differential type fluids [1, 2] have gained considerable attention. The third-grade fluid is a subclass of differential type fluids, which has been examined successfully in different types of flow situations [3] and is known to capture the non-Newtonian effects such as shear thinning or shear thickening as well as normal stresses. The study of the flow of third-grade fluids through an inclined channel with heat transfer has important applications in engineering, technology, and science. Some of these applications can be found in materials manufactured by the extraction procedure particularly in polymer processing, the flow of synovial fluid in human joints, geological flows inside the earth's mantle, microfluids, drilling of oil and gas wells, etc. Heat transfer is important in industrial areas to launch transportation of energy in the system. Generally, the flowing mixtures are made of solid particles in a fluid, so coal-based slurries show non-Newtonian features, which is why these mixtures are important in industrial applications. Also, heat transfer plays a vital role in processing and handling these mixtures. For example, it has been shown that substantial performance benefits can be obtained if the coal-water mixture is preheated [4, 5].

In fluid dynamics, the study of three fundamental flow problems (namely, Couette flow, Poiseuille flow, and Couette–Poiseuille flow) attracts the researchers because of their use in technology, engineering, and industry. The unidirectional flows are used in polymer engineering such as die flow, injection molding, extrusion, plastic forming, continuous casting, and asthenosphere flows [6–9]. Magnetohydrodynamics (MHD) deals with the study of the motion of electrically conducting fluids in the presence of magnetic field. MHD flow has significant important applications in an inclined channel such as geophysical, astrophysical, metallurgical processing, MHD generators, pumps, geothermal reservoirs, polymer technology, and mineral industries. MHD fluid is used as a lubricant in industrial and other applications, for stopping the unexpected variation of lubricant viscosity with temperatures under certain norms. In this regard, there are a large number of works, such as Khan et al. [10], Hayat et al. [7, 11, 12], and Islam [13]. Kamran and Siddique [14] calculated the analytical solutions for MHD flow between infinite parallel plates. Siddiqui et al. [15] deliberated the flow of third-grade fluid between two inclined parallel plates with heat transfer using the homotopy perturbation method (HPM). Later on, Aiyesimi et al. [16] studied the solution of MHD Couette flow and Poiseuille flow problems of temperature and velocity profile by employing the perturbation method.

Fluid flow plays the main role in the field of science and engineering. There is a rise in an extensive range of problems such as chemical diffusion, magnetic effect, and heat transfer. After governing, these physical problems are converted into linear or nonlinear DEs. In general, the physical problems with involved geometry, coefficients, parameters, and initial and boundary conditions greatly affect the solutions of DEs. Then, the coefficients, parameters, and initial and boundary conditions are not crisp due to the mechanical defect, experimental error, measurement error, etc. Therefore, in this situation, fuzzy set theory is an effective tool for a better understanding of the considered phenomena, and it is more accurate than assuming the crisp or classical physical problems. More precisely, the FDEs play a major role in reducing the uncertainty and are a proper way to describe the physical problem which arises in uncertain parameters and initial and boundary conditions.

The fuzzy set theory (FST) was presented by Zadeh [17] in 1965. FST is a very valuable tool to define the situation in which information is imprecise, vague, or uncertain. FST is completely defined by its membership function or belongingness. In FST, the membership function describes each element of the universe of discourse by a number from [0, 1] interval. On the other hand, the degree of nonbelongingness is a complement to "one" of the membership degree or belongingness. Fuzzy number (FN) can be expected as a function whose range is specified from zero to one. Every numerical value in the range is allocated a definite grade of membership function, where 0 signifies the minimum possible grade and 1 is the maximum possible grade. FST is the generalization of crisp (classical) set theory, and similarly, FNs are the generalization of real intervals. Arithmetic operations on FNs were developed by Dubois and Prade [18]. Different types of FNs can be categorized into triangular, trapezoidal, and Gaussian fuzzy numbers. Here, we consider TFNs for the sake of completeness.

The information of dynamical systems modelled by partial or ordinary differential equations is commonly incomplete, vague, or uncertain, while fuzzy differential equations (FDEs) represent a proper way to model the dynamical systems under vagueness or uncertainty. This impreciseness or vagueness can be defined mathematically using FNs or TFNs. In recent years, there have been many studies revolving around the concept of "FDEs." Seikala [19] introduced the fuzzy differentiability concept. Later on, Kaleva [20] presented fuzzy differentiation and integration. Kandel and Byatt [21] introduced the FDEs in 1987. Buckley et al. [22] used two methods, extension principle and FNs, for the solution of FDEs. Nieto [23] studied the Cauchy problem for continuous FDEs. Lakshmikantham and Mohapatra studied the initial-value problems for FDEs [24]. Park and Hyo [25] used the successive approximation method for the existence and uniqueness solution of FDE. Gasilov et al. [26] developed the geometric method to solve the system of FDEs. Gasilov et al. [27] studied the system of FDEs with the help of TFNs. Salahsour et al. [28] studied the fuzzy logistic equation and alley effect using FDEs with the help of TFNs.

The fuzzy boundary value problems (FBVPs) depend on the concept of the solution of FDE. The scholars utilize the derivative in the FDE such as Hukuhara derivative (H-derivative) or generalized Hukuhara derivative (GH-derivative). Chalco-Cano et al. [29, 30] established the idea of fuzzy H-derivative. Bede and Gal [31] introduced the idea of the fuzzy generalized H-derivative. Khastan and Nieto [32, 33] proposed a new solution concept for a two-point FBVP for a second-order FDE using a generalized differentiability concept. Also, many scholars have applied FST to obtain well-known results in the field of commerce and science, such as in bank account model [34], HIV model [35], bacteria culture model [36], population dynamics model [37, 38], predator-prey model [39], computational biology [40], growth model [41], decay model [42], quantum optics and gravity [43], modeling hydraulic [44], model of friction [45], application in Laplace transform [46], civil engineering [47], integrodifferential equation [48], giving up smoking model [49], chemostat model [50], dengue virus model [51], and many others [52–55].

In the review of the literature, an attempt has been made to describe the three fundamental flow problems, namely, plane Couette, fully developed plane Poiseuille, and plane Couette–Poiseuille flow of a third-grade fluid through inclined parallel plates in an imprecise environment. Here, we discuss two cases: one is crisp and the other is fuzzy for a better flow situation. In this regard, the gravitational and magnetic parameters are taken as TFNs. The uncertainty of TFNs is controlled by α-cut ($0 \leq \alpha \leq 1$) with the help of the membership functions. The membership function provides a better understanding of uncertain parameters and flow situations. The basic purpose of this article is to show the uncertain flow mechanism through FDEs.

The rest of the article is structured as follows. Some basic definitions related to the fuzzy numbers, TFN, and fuzzy valued functions are discussed in Section 2, to convert the FDEs to parametric forms. In Section 3, the mathematical formulation of the problem is given. The FDEs of the considered problem have been presented in Section 4, which are solved by a numerical scheme bvp4c built in MATLAB. In Section 5, some fuzzy plots of velocity and temperature profiles are presented. Also, the present results have been compared with the existing results for some special cases, viz., crisp case. Finally, in Section 6, conclusions have been made.

2. Preliminaries

In this section, some basic notations and dentitions are given.

Definition 1 (see [17]). Fuzzy set is defined as a set of ordered pairs such that $\widetilde{U} = \{(x, \mu_{\widetilde{U}}(x)): x \in X, \mu_{\widetilde{U}}(x) \in [0,1]\}$, where X is the universal set and $\mu_{\widetilde{U}}(x)$ is the membership function of $\widetilde{U}$ and its mapping is defined as $\mu_{\widetilde{U}}(x): X \longrightarrow [0,1]$.

Definition 2 (see [18]). α-cut or α-level of a fuzzy set $\widetilde{U}$is a crisp set U_α and defined by $U_\alpha = \{x/\mu_{\widetilde{U}}(x) \geq \alpha\}$, where $0 \leq \alpha \leq 1$.

Definition 3 (see [17]). Convex fuzzy set $\widetilde{U}$ is defined as $\widetilde{U} = \{x, \mu_{\widetilde{U}}(x)\} \subseteq X$. It is called convex fuzzy set if all U_α for every $\alpha \in [0,1]$ are convex fuzzy set, i.e., for every $y_1, y_2 \in U_\alpha$ and $\alpha y_1 + (1-\alpha) y_2 \in U_\alpha$ for all $\alpha \in [0,1]$; otherwise, it is a nonconvex fuzzy set.

Definition 4 (see [17]). The fuzzy set $\widetilde{U}$ defined on the universal set of real number R is said to be an FN, which satisfies the following properties: (i) $\mu_{\widetilde{U}}(x)$ is piecewise continuous, (ii) $\widetilde{U}$ is convex, (iii) $\widetilde{U}$ is normal, i.e., $\exists x_0 \in R$ such that $\mu_{\widetilde{U}}(x) = 1$, and (iv) support of $\widetilde{U}$ must be bounded.

U_α must be closed interval for every $0 \leq \alpha \leq 1$, where α is called the level of credibility or presumption. Membership function or grade is also named as a grade of possibility or grade of credibility for a given number.

Definition 5 (see [18]). Let $\widetilde{U} = (a_1, a_2, a_3)$ with membership function $\mu_{\widetilde{U}}(x)$ is called a TFN if

$$\mu_{\widetilde{U}}(x) = \begin{cases} \dfrac{a_1 - x}{a_2 - a_2}, & \text{for } x \in [a_1, a_2], \\[2ex] \dfrac{x - a_3}{a_3 - a_3}, & \text{for } x \in [a_2, a_3], \\[2ex] 0, & \text{otherwise}. \end{cases} \quad (1)$$

The TFNs with peak (or centre) a_2, left width $a_2 - a_1 > 0$, and right width $a_3 - a_2 > 0$ are transformed into interval numbers through α-cut approach, which is written as $\widetilde{U} = [u(x;\alpha), v(x;\alpha)] = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$, where $\alpha \in [0,1]$. A TFN $\widetilde{U} = (a_1, a_2, a_3)$ and α-cut of membership function are shown in Figure 1. An arbitrary TFN satisfies the following conditions: (i) $u(x;\alpha)$ is an increasing function on [0, 1]; (ii) $v(x;\alpha)$ is a decreasing function on [0, 1]; (iii) $u(x;\alpha) \leq v(x;\alpha)$ on [0, 1]; (iv) $u(x;\alpha)$ and $v(x;\alpha)$ are bounded on left continuous and right continuous at [0, 1] respectively; and (v) $u(x;\alpha) = v(x;\alpha) = u(x)$, where $u(x)$ is the crisp number.

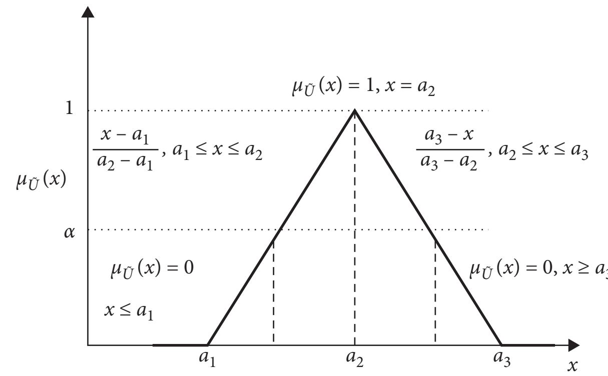


Figure 1: Membership functions of a TFN.

Definition 6 (see [19–21]). Let I be a real interval. A mapping $\tilde{u}: I \longrightarrow F$ is called a fuzzy process, defined as $\tilde{u}(x;\alpha) = [u(x;\alpha), v(x;\alpha)]$, $x \in I$ and $\alpha \in [0,1]$ The derivative $d\tilde{u}(x;\alpha)/dx \in F$ of a fuzzy process $\tilde{u}(x;\alpha)$ is defined by $d\tilde{u}(x;\alpha)/dx = [du(x;\alpha)/dx, dv(x;\alpha)/dx]$.

Definition 7 (see [19–21]). Let $I \subseteq R$, $\tilde{u}$ be a fuzzy valued function defined on I. Let $\tilde{u}(x;\alpha) = [u(x;\alpha), v(x;\alpha)]$ for all α-cut. Assume that $u(x;\alpha)$ and $v(x;\alpha)$ have continuous derivatives or differentiable, for all $x \in I$ and α, then $[d\tilde{u}(x;\alpha)/dx]_\alpha = [du(x;\alpha)/dx, dv(x;\alpha)/dx]_\alpha$. Similarly, we can define higher-order ordinary derivatives in the same way. An FN by an ordered pair of functions $[d\tilde{u}(x;\alpha)/dx]_\alpha$ satisfies the following conditions: (i) $du(x;\alpha)/dx$ and $dv(x;\alpha)/dx$ are continuous on [0, 1]; (ii) $du(x;\alpha)/dx$ is an increasing function on [0, 1]; (iii) $dv(x;\alpha)/dx$ is a decreasing function on [0, 1]; and (iv) $du(x;\alpha)/dx \leq dv(x;\alpha)/dx$ on [0, 1].

3. Mathematical Formulation of the Problem

The basic equations which govern the MHD flow of an incompressible, third-grade electrically conducting fluid are as follows:

$$\text{div} V = 0, \quad (2)$$

$$\rho \frac{dV}{dt} = \text{div}\widehat{S} - \nabla p + J \times B + \rho f, \quad (3)$$

where V is the velocity vector, ρ is the constant density, d/dt is the material derivative, p is the pressure, f is the external body force, B is the total magnetic field, $B = B_0 + b$, in which B_0 represents the imposed magnetic field and b denotes the induced magnetic field, and

$$J = \sigma[V \times B + E], \quad (4)$$

which is the current density, σ is the electrical conductivity, E is the electric field which is not considered (i.e., $E = 0$), and $\widehat{S}$ is the Cauchy stress tensor which for a third-grade fluid satisfies the following constitutive equation:

$$\widehat{S} = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 A_1 (\mathrm{tr} A_2), \tag{5}$$

$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1}(\mathrm{grad}V) + (\mathrm{grad}V)^T A_{n-1}, \quad n \geq 1, \tag{6}$$

where pI is the isotropic stress due to constraint incompressibility, μ is the dynamic viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2$, and β_3 are material constants, T indicates the matrix transpose, A_1, A_2, and A_3 are the first three Rivlin–Ericksen tensors, and $A_\circ = I$ is the identity tensor [13–16].

3.1. Couette Flow. Consider a steady MHD flow of a third-grade fluid between two infinite inclined horizontally parallel plates of distance $2H$ apart, by angle α. The upper and lower plates are at $y = H$ and $y = -H$ of a rectangular system with the x-axis as flow direction. The upper plate is moving with constant speed U while the lower plate is fixed as shown in Figure 2. The temperature of the lower plate is maintained at T_0 and that of the upper plate at T_1. A homogeneous magnetic field B_0 is applied in positive y-direction and is expected to be undisturbed as the induced magnetic field is neglected under the assumption of small magnetic Reynolds number. The ambient air is assumed stationary so that the flow is due to the movement of the upper plate and gravity alone.

Consider a velocity field of the form

$$V = [u(y), 0, 0]. \tag{7}$$

In the absence of modified pressure gradient, equations (2) and (3) along with equations (5)–(7), after introducing the following nondimensional parameters

$$u = \overline{u}U, \quad y = \overline{y}H, \quad \overline{\theta} = \frac{T - T_0}{T_1 - T_0}, \tag{8}$$

yield

$$\frac{d^2u}{dy^2} + 6\beta\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} - \mathrm{M}u + k = 0, \tag{9}$$

with boundary conditions

$$u(-1) = 0, \quad u(1) = 1. \tag{10}$$

Also, the thermal boundary layer equation for the thermodynamically compatible third-grade fluid with viscous dissipation, work done due to deformation and Joule heating in a nondimensional form, is given as

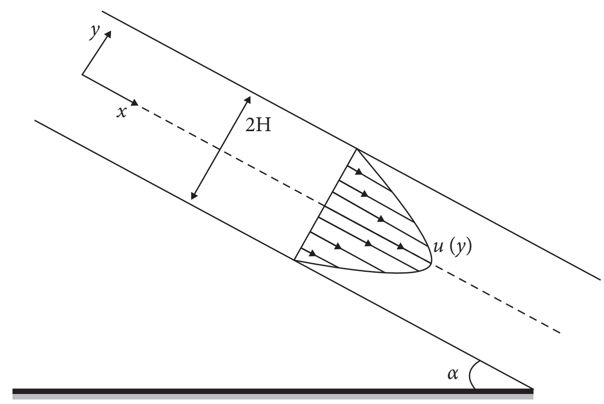


Figure 2: Geometry of the problem.

$$\frac{d^2\theta}{dy^2} + B_r\left(\frac{du}{dy}\right)^2 + 2\beta B_r\left(\frac{du}{dy}\right)^4 + B_r \mathrm{M}u^2 = 0, \tag{11}$$

with boundary conditions

$$\theta(-1) = 0, \quad \theta(1) = 1, \tag{12}$$

where $\beta = (U/H)^2(\beta_2 + \beta_3)/\mu$ is the third-grade fluid parameter, $k = \rho g H^2/\mu U \sin\phi$ is the gravitational parameter, $M = H^2\sigma B_0^2/\mu$ is the magnetic parameter, and $B_r = \mu U^2/k(T_1 - T_0)$ is the Brinkman number.

3.2. Poiseuille Flow. In this case, both the upper and lower plates are kept stationary and we assumed that the fluid motion is due to gravity alone. Consequently, in the absence of pressure gradient, equations (9) and (10) remain the same after scaling given by equation (8), while the boundary conditions (10) and (12) become

$$u(-1) = 0, \quad u(1) = 0, \tag{13}$$

$$\theta(-1) = 0, \quad \theta(1) = 1. \tag{14}$$

3.3. Couette–Poiseuille Flow. Because we considered that the flow is due to gravity and the movement of upper plate while the pressure gradient is neglected, the momentum and energy equations with their boundary conditions for Couette–Poiseuille flow will result to those of the Couette flow.

4. Fuzzification of the Problem

The velocity of fluid depends upon involved engineering parameters such as β, k, and M. Some researchers take fixed crisp values; the point is that the flow of fluid just depends on these fixed crisp values. Then, uncertainty arises due to the fixed crisp values of parameters. So, it is better to take these parameters as an uncertain or fuzzy parameter. In this study, uncertain parameters k and M are taken as FN.

To handle this problem, we have taken TFN and discretization in the form of (a_1, a_2, a_3) (see Table 1). This discretization is used in fluid parameters for certain flow behaviour because these parameters are taken as FNs. Using the α-cut approach, the fuzzy fluid parameters can be decomposed into an interval form regarding the α-cut.

Table 1: TFNs of the fuzzy parameters.

Fuzzy parameters	Crisp value	TFN
k (gravitational parameter)	3	[1, 3, 5]
M (magnetic parameter)	15	[5, 14, 24]

Therefore, the governing equations (9) and (11) are converted into coupled FDEs:

$$\frac{d^2\overline{u}(y;\alpha)}{dy^2} + 6\beta\frac{d^2\overline{u}(y;\alpha)}{dy^2}\left(\frac{d\overline{u}(y;\alpha)}{dy}\right)^2 + \overline{k} - \overline{M}\overline{u}(y,\alpha) = 0, \tag{15}$$

$$\frac{d^2\overline{\theta}(y,\alpha)}{dy^2} + B_r\left(\frac{d\overline{u}(y,\alpha)}{dy}\right)^2 + 2\beta B_r\left(\frac{d\overline{u}(y,\alpha)}{dy}\right)^4 + B_r\overline{M}\overline{u}^2(y,\alpha) = 0, \tag{16}$$

subject to boundary conditions

$$\begin{aligned} u(-1) &= 0,\\ u(1) &= 1, \end{aligned} \tag{17}$$

$$\begin{aligned} \theta(-1) &= 0,\\ \theta(1) &= 1, \end{aligned} \tag{18}$$

where "—" stands for the fuzzy form and the lower and upper bounds of fuzzy velocity profiles are $\overline{u}(y,\alpha) = [u_1(y,\alpha), u_2(y,\alpha)]$, $0 \le \alpha \le 1$. Similarly, the fuzzy temperature profiles are $\overline{\theta}(y,\alpha) = [\theta_1(y,\alpha), \theta_2(y,\alpha)]$, $0 \le \alpha \le 1$.

The crisp values and TFNs of these parameters are listed in Table 1. The TFNs are used to describe the triangular membership functions of the FNs. The investigated ranges are generally used to build up the aforesaid problem.

Now, we use a numerical method for boundary value problem solver MATLAB built-in function bvp4c in the governing crisp differential equations and FDEs (equations (15)–(17)) with boundary conditions. It is a finite difference algorithm that uses the three-stage Lobatto IIIA formula. It is a collocation polynomial, and the collocation formula yields a $C1$-continuous solution in $[a, b]$ that is fourth-order accurate uniformly. The residual of the continuous solution is used for error control and mesh selection. To use this approach, transform the system of nonlinear ODEs and its boundary conditions to the system of the first-order ODEs and initial conditions as [56, 57].

Let

$$\begin{aligned}
&u(y) = f(1), u'(y) = f'(1) = f(2), \text{and}\, u'' = f'(2),\\
&f'(2) = \frac{\mathrm{M}f(1) - k}{1 + 6\beta(f(2))^2},\\
&\theta(y) = f(3),\\
&\theta'(y) = f'(3) = f(4),\\
&\theta''(y) = f'(4),\\
&f'(4) = -\mathrm{Br}(f(2))^2 - 2\beta\mathrm{Br}(f(2))^4 - \mathrm{Br}M(f(1))^2.
\end{aligned} \tag{19}$$

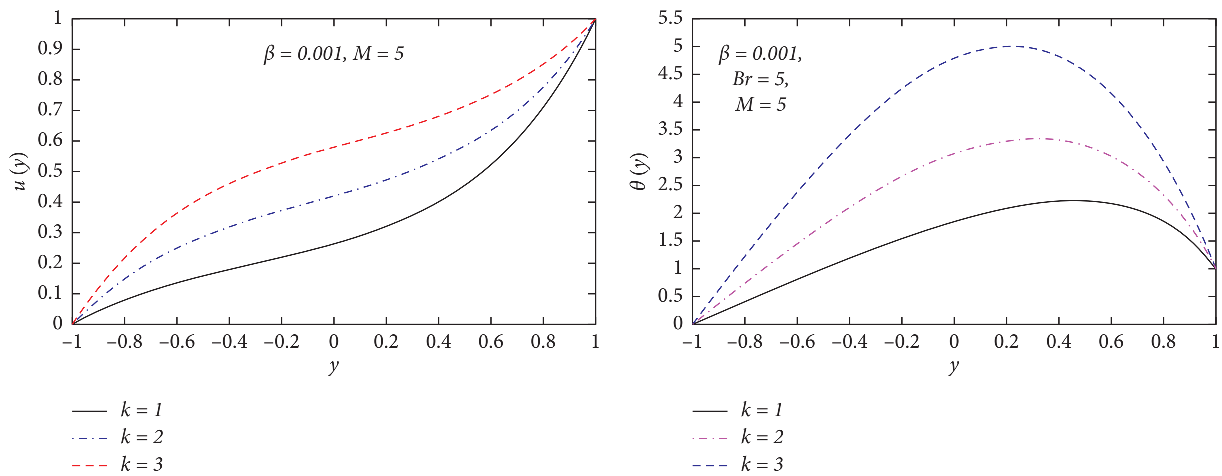
And, boundary conditions are

$$\left.\begin{aligned} f_a(1) = 0, f_b(1) = 1, &\quad \text{at } y = -1,\\ f_a(1) = 0, f_b(1) = 1, &\quad \text{at } y = 1. \end{aligned}\right\} \tag{20}$$

All computations are performed with a tolerance of $\varepsilon = 10^{-6}$ and a validated MATLAB code.

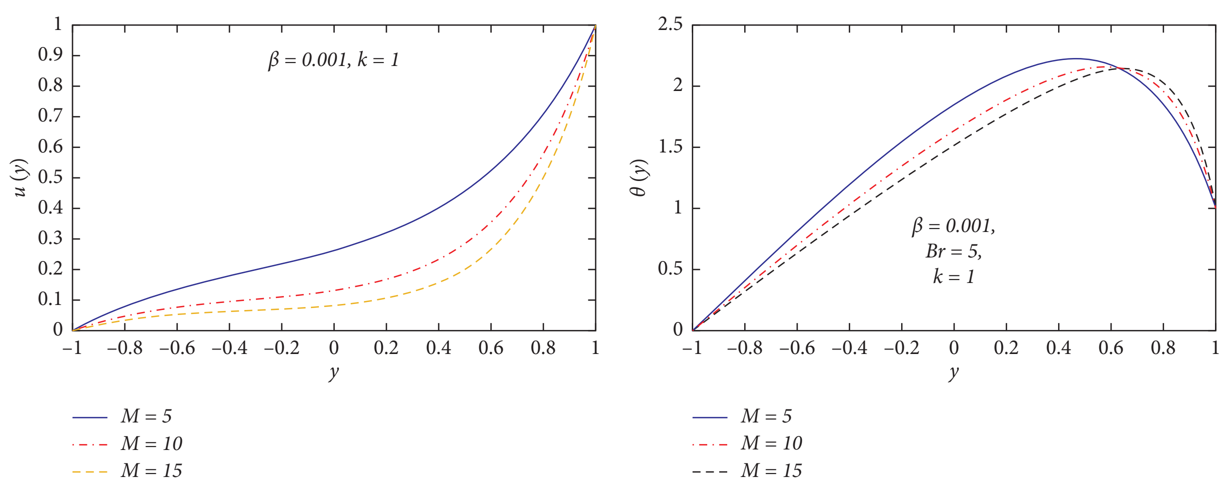
5. Results and Discussions

5.1. Crisp Case. In this section, we solve the proposed system of the crisp ODEs (9) and (11) together with the boundary conditions (10) and (12) numerically by MATLAB built-in technique bvp4c. Furthermore, we discuss the effects of gravitational parameter k and magnetic parameter M on the



(a) (b)

Figure 3: Effect of k on crisp velocity and temperature profiles of Couette flow.



(a) (b)

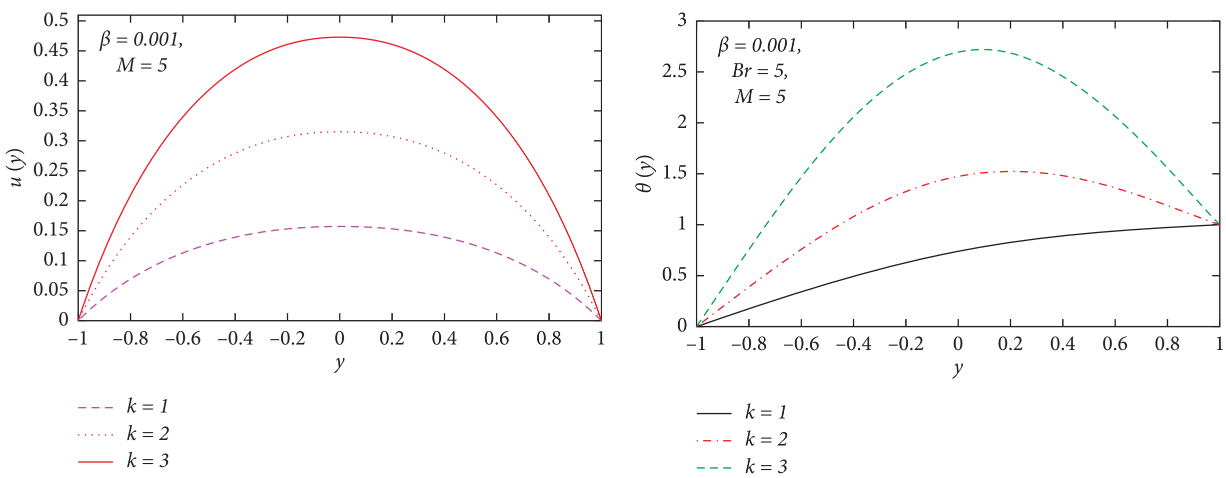
Figure 4: Effect of M on crisp velocity and temperature profiles of Couette flow.

Table 2: Comparison of numerical results with analytical results for the velocity of Couette flow when $k = 1$ and $M = 5$.

	Velocity at $\beta = 0.001$		Velocity at $\beta = 0.1$	
y	Aiyesimi et al. [16] (PM)	Present result (bvp4c)	Aiyesimi et al. [16] (PM)	Present result (bvp4c)
−1	0	0	0	0
−0.8	0.0805992	0.080607	0.0.81260	0.083115
−0.6	0.136936	0.1369396	0.140203	0.142375
−0.4	0.180453	0.1804493	0.187494	0.189643
−0.2	0.219997	0.2199837	0.232351	0.233923
0	0.263605	0.263602	0.283677	0.283364
0.2	0.320143	0.320104	0.351506	0.347445
0.4	0.401095	0.400987	0.448301	0.436373
0.6	0.522872	0.522653	0.578833	0.563876
0.8	0.710055	0.709739	0.753411	0.746170
1	1	0.999997	1	1

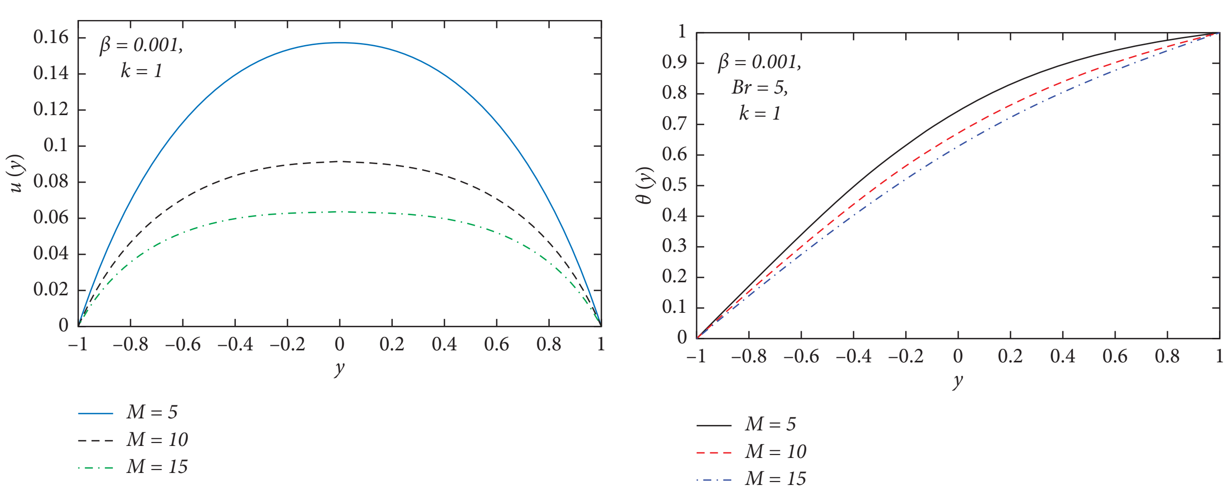
Table 3: Comparison of numerical results with analytical results for the temperature of Couette flow when $k = 1$ and $M = 5$.

	Temperature at $\beta = 0.001$		Temperature at $\beta = 0.1$	
y	Aiyesimi et al. [16] (PM)	Present result (bvp4c)	Aiyesimi et al. [16] (PM)	Present result (bvp4c)
−1	0	0	0	0
−0.8	1.22328	0.533675	1.402449	0.596488
−0.6	2.34181	1.037249	2.404120	1.159899
−0.4	3.31125	1.509760	3.151470	1.688806
−0.2	4.09503	1.941188	3.982890	2.171262
0	4.65215	2.315419	4.776961	2.587627
0.2	4.91100	2.606950	5.081522	2.906360
0.4	4.81112	2.770656	4.783381	3.072202
0.6	4.33810	2.718227	4.162820	2.980832
0.8	3.17165	2.264215	3.194671	2.426644
1	1	1	1	1



(a) (b)

Figure 5: Effect of k on velocity and temperature profiles of Poiseuille flow.



(a) (b)

Figure 6: Effect of M on velocity and temperature profiles of Poiseuille flow.

Table 4: Comparison of numerical results with analytical results for the velocity of Poiseuille flow when $k = 1$, $\mathrm{Br} = 5$, and $M = 5$.

	Velocity at $\beta = 0.001$		Velocity at $\beta = 0.1$	
y	Aiyesimi et al. [16] (PM)	Present result (bvp4c)	Aiyesimi et al. [16] (PM)	Present result (bvp4c)
−1	0	0	0	0
−0.8	0.070019	0.069038	0.069009	0.068269
−0.6	0.113620	0.113095	0.112676	0.113143
−0.4	0.139659	0.139406	0.138934	0.139295
−0.2	0.153428	0.153341	0.152856	0.140774
0	0.157725	0.157745	0.157206	0.159183
0.2	0.153428	0.153517	0.152856	0.154794
0.4	0.139659	0.139493	0.138934	0.140774
0.6	0.113620	0.113072	0.112676	0.113143
0.8	0.0700194	0.069038	0.069009	0.068269
1	0	0	0	0

Table 5: Comparison of numerical results with analytical results for the temperature of Poiseuille flow when $k = 1$, $\mathrm{Br} = 5$, and $M = 5$.

	Temperature at $\beta = 0.001$		Temperature at $\beta = 0.1$	
y	Aiyesimi et al. [16] (PM)	Present result (bvp4c)	Aiyesimi et al. [16] (PM)	Present result (bvp4c)
−1	0	0	0	0
−0.8	0.67471	0.197694	0.67412	0.199630
−0.6	1.26537	0.374075	1.26427	0.377380
−0.4	1.73772	0.531557	1.73622	0.535884
−0.2	20.6938	0.667706	2.06763	0.672683
0	2.24792	0.780066	2.24609	0.785268
0.2	2.26938	0.867706	2.26763	0.872683
0.4	2.13772	0.931557	2.13622	0.935884
0.6	1.86537	0.974075	1.86427	0.977380
0.8	1.47471	0.997694	1.47412	0.999630
1	1	1	1	1

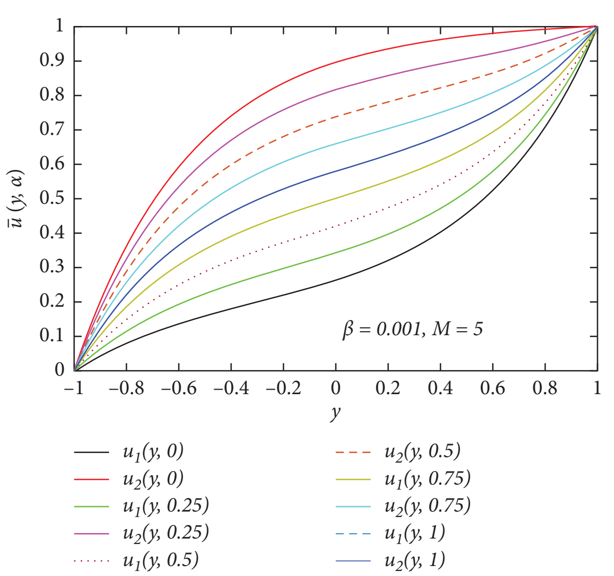


Figure 7: Effect of TFN k on fuzzy velocity profile of Couette flow.

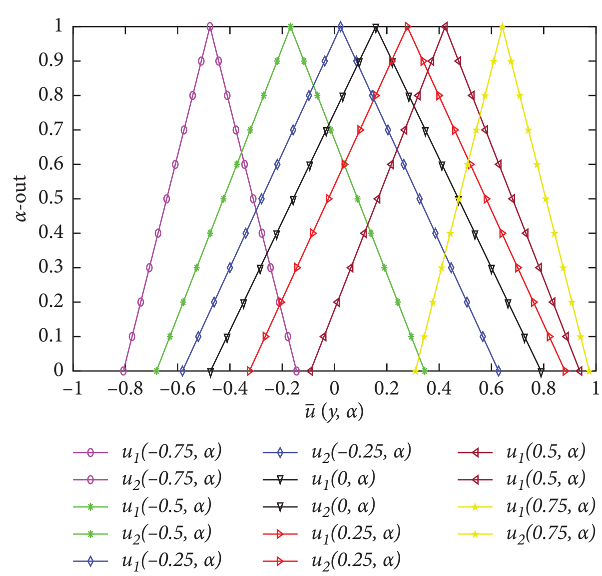


Figure 8: Membership functions of TFN if k is fuzzy (Couette flow).

velocity and temperature profiles of Couette and Poiseuille flow graphically. Furthermore, for validation, the present results are compared with the existing results in some special cases, viz., crisp case.

For Couette flow, Figures 3(a) and 3(b) show the effect of gravitational parameter k on crisp velocity and temperature profile with a fixed magnetic parameter M. It is seen that the

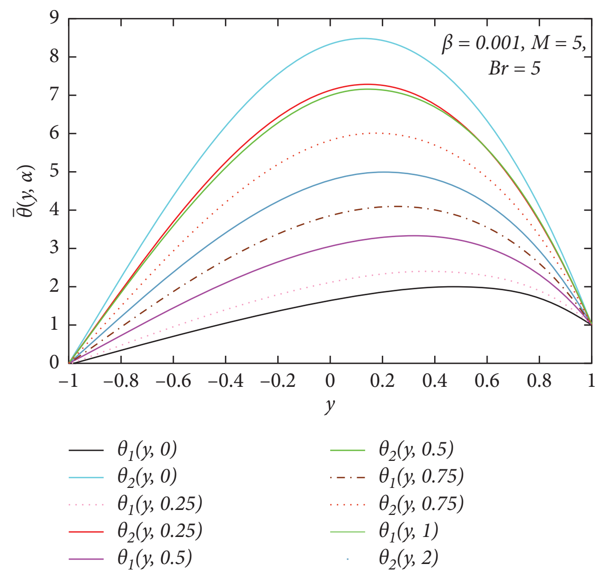


Figure 9: Effect of TFN k on fuzzy temperature profile of Couette flow.

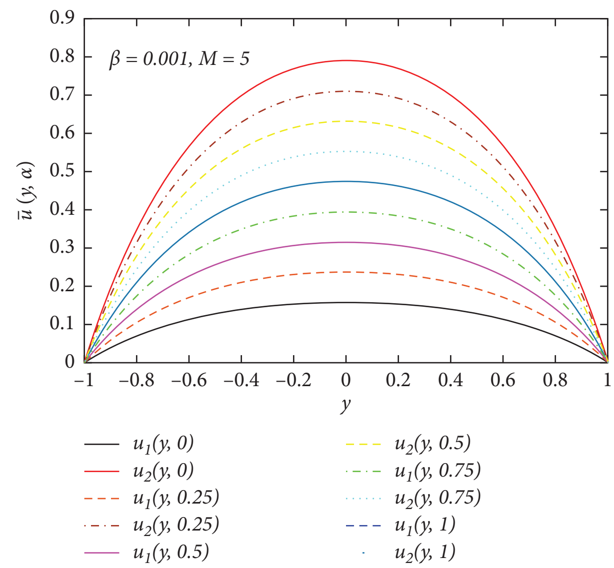


Figure 11: Effect of TFN k on fuzzy velocity profile of Poiseuille flow.

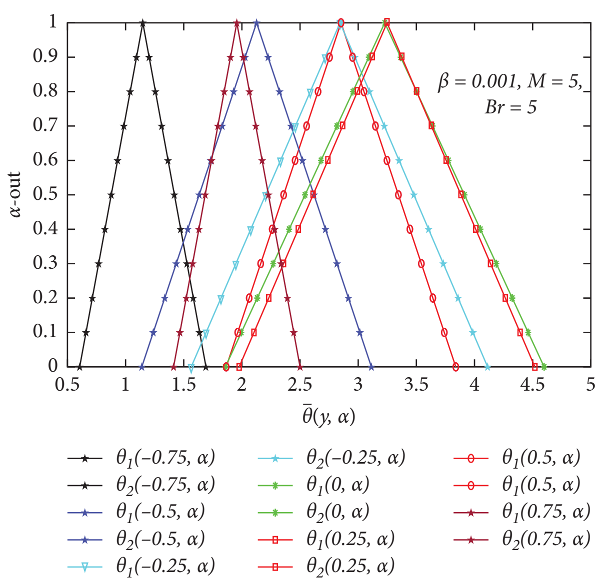


Figure 10: Membership functions of TFN if k is fuzzy (Couette flow).

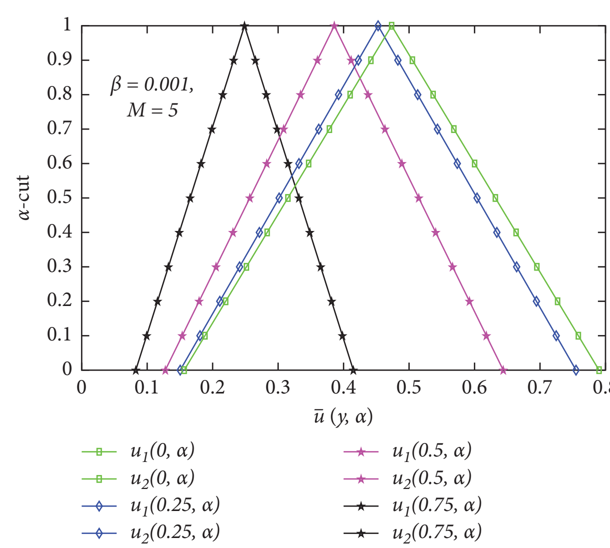


Figure 12: Membership functions of TFN if k is fuzzy (Poiseuille flow).

velocity and temperature profiles increase rapidly in the centre of the plates with increasing the values of k. Figures 4(a) and 4(b) show the effect of M on crisp velocity and temperature profiles with a fixed value of k. It is perceived that velocity and temperature profile decreases with increasing the value of k. Particularly in Figure 4(b), the temperature decreases in the region $-1 < y \leq 0.6$ and it increases in region $0.6 < y \leq 1$. Tables 2 and 3 show the comparison of velocity and temperature profiles for different values of β with [16]. The validated results of the present study are found to be in excellent agreement.

For Poiseuille flow, Figures 5(a) and 5(b) show the effect of gravitational parameter k on crisp velocity and temperature profiles with fixed magnetic parameter M. It is observed that the velocity and temperature profile increase rapidly in the centre of the plates with increasing the values of k. Figures 6(a) and 6(b) show the effect of M on crisp velocity and temperature profile with a fixed value of k. It is noted that velocity and temperature profile decrease rapidly in the centre of the plates with increasing the value of k. Tables 4 and 5 show the comparison of velocity and temperature profiles for different values of β with [16]. The validated results of the present study are found to be in excellent agreement.

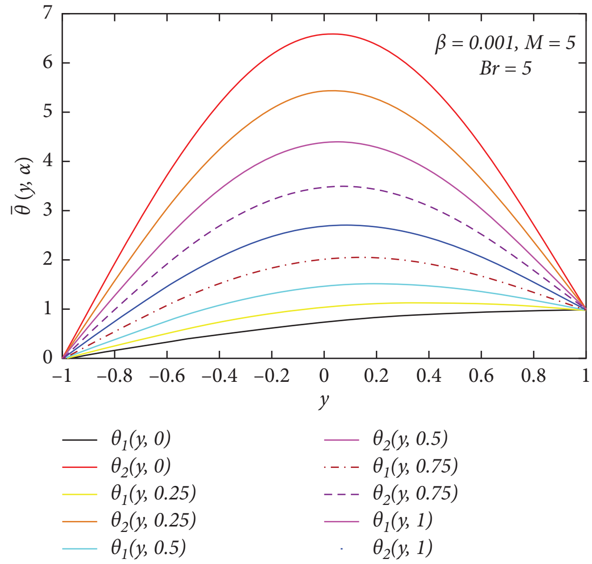


FIGURE 13: Effect of TFN k on fuzzy temperature profile of Poiseuille flow.

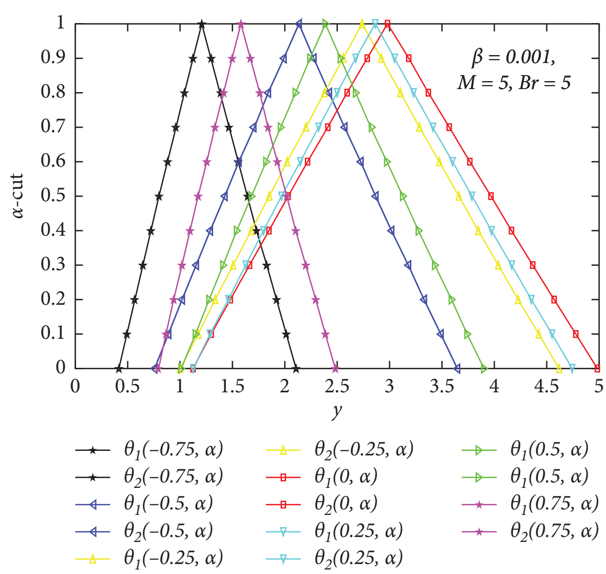


FIGURE 14: Membership functions of TFN if k is fuzzy (Poiseuille flow).

5.2. Fuzzy Case. In this section, we solve the proposed system of the fuzzy environmental problems (15) and (16) together with the boundary conditions (17) and (18) numerically by using MATLAB built-in technique bvp4c. Furthermore, we analyze the effects of the uncertain gravitational parameter k and uncertain magnetic parameter M through α-cut approach $(0 \leq \alpha \leq 1)$ as discussed in detail in Section 4, on velocity and temperature profiles for Couette and Poiseuille flow graphically and tabularly. Here, $\alpha - \text{cut}$ controls the fuzzy term, for example, if $\alpha - \text{cut} = 0$, it will cover the whole interval in the form of lower and upper bounds of fuzzy velocity or temperature profiles. If $\alpha - \text{cut}$increases from 0.05 to 0.95, the lower and upper bound fuzzy velocity or temperature profiles decrease; when $\alpha - \text{cut} = 1$, the lower and upper bounds fuzzy velocity or temperature profiles cohere with each other, so they provide crisp results. It is important to note that if the width between lower and upper bounds of velocity or temperature profile is less, then the uncertainty is less. The fuzzy velocity and temperature profiles are plotted in Figures 7–22 for some particular values of $\alpha - \text{cut}$ $(\alpha = 0, 0.25, 0.5, 0.75, 1)$ with different values of y. The triangular membership functions are depicted in Figures 8, 10, 12, 14, 16, 18, 20, and 22. In the triangular membership functions, if $\alpha = 0$, we can notice that the crisp solutions are limited by the lower and upper

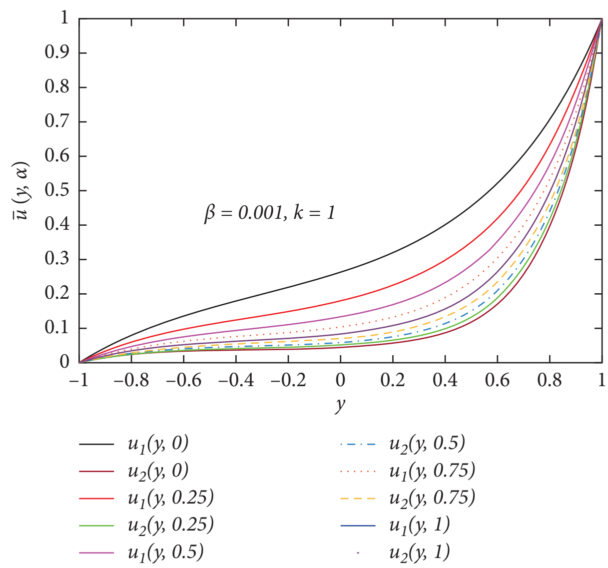


Figure 15: Effect of TFN M on fuzzy velocity profile of Couette flow.

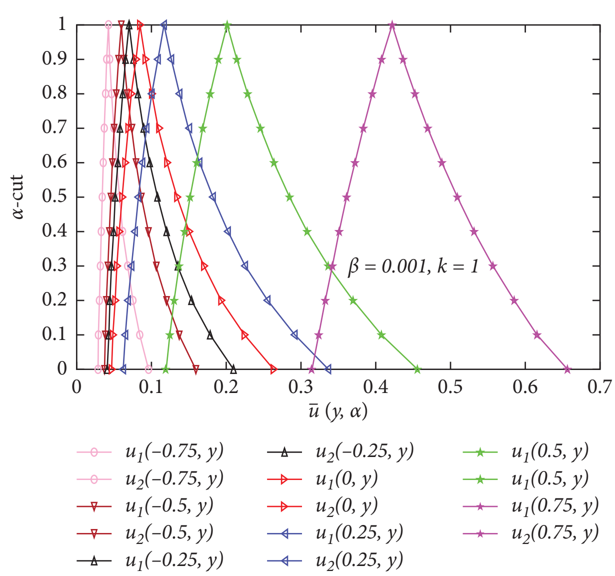


Figure 16: Membership functions of TFN if M is fuzzy (Couette flow).

outlets of the dependent parameters. Furthermore, if we take $\alpha = 1$, the projection of the peaks of the triangles coincides with the crisp solution.

In Figures 7–14, the gravitational parameter k is taken as a TFN (see Table 1), and the fuzzy velocity and temperature profiles are controlled by $\alpha -$ cut. The crisp or classical solution lies among the fuzzy solutions; when α increases, the width between lower and upper bounds of fuzzy velocity and temperature profiles decreases and when $\alpha = 1$, they cohere with one another (see Figures 7, 9, 11, and 13). Furthermore, crisp or classical solutions behave the single flow situation, while fuzzy solution behaves the lower and upper flow situation. Figures 8, 10, 12, and 14 represent the membership functions of fuzzy velocity and temperature profiles when k is taken as TFN. In Figures 7 and 8, it is seen that the width of the fuzzy velocity profile at the centre of the plates is less, so uncertainty is less, while in Figures 9 and 10, the width of fuzzy temperature becomes more at the centre of the plates, therefore the uncertainty is maximum. In Figures 11 and 12, it is seen that triangular membership functions of fuzzy velocity show the same behaviour in the region $-1 \leq y \leq 0$ and $0 \leq y \leq 1$. The width of the triangular membership functions is high, so the uncertainty is high, which means that the uncertain parameter k is sensitive. Now, in Figures 13 and 14, the width of fuzzy temperature from the centre value of each TFN is less, so there is

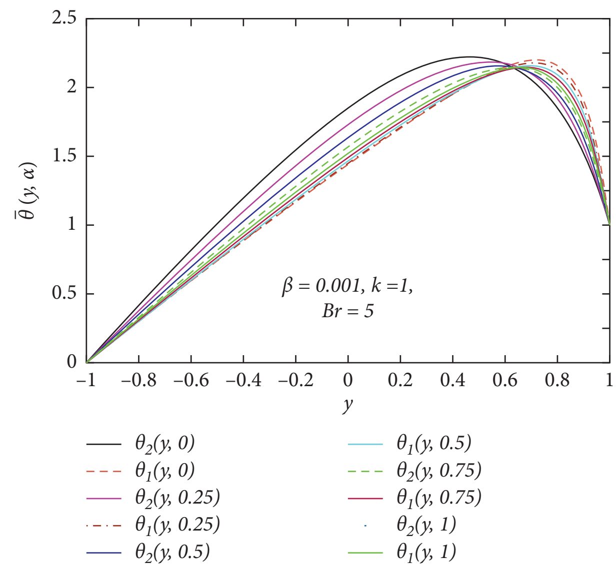


FIGURE 17: Effect of TFN M on the fuzzy temperature profile of Couette flow.

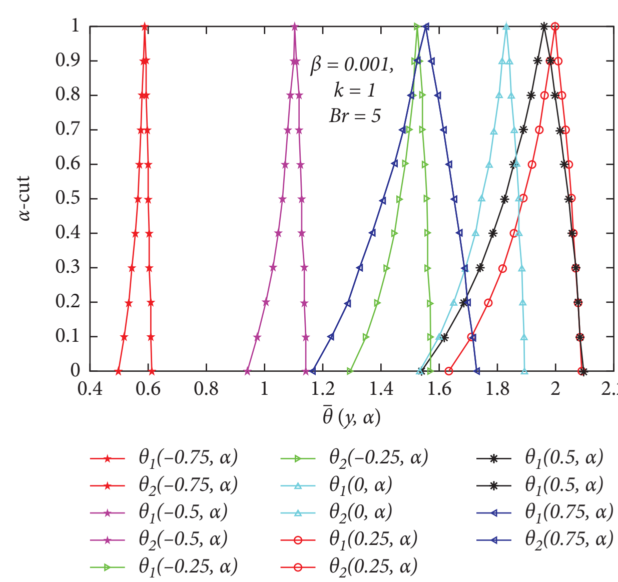


FIGURE 18: Membership functions of TFN if M is fuzzy (Couette flow).

minimum possibility of the uncertainty of fuzzy temperature.

In Figures 15–22, the magnetic parameter M is taken as a TFN (see Table 1), and the fuzzy velocity and temperature profiles are controlled by $\alpha -$ cut. It is seen that $\alpha -$ cut increases from 0 to 1, and the lower bound of fuzzy velocity and temperature is a increasing set-valued function whereas an upper bound is a decreasing one, which shows that the results are fuzzy numbers.

Figures 16, 18, 20, and 22 represent the triangular membership functions of fuzzy velocity and temperature profiles when M is taken as a TFN. In Figures 15 and 16, the widths of fuzzy velocity are less, so the uncertain parameter M is less sensitive, while in Figures 17 and 18, the width of fuzzy temperature is less at the centre of the plate, so the uncertain parameter M is less sensitive. In Figures 19 and 20, it is seen that triangular membership functions of fuzzy velocity show the same behaviour in the region $-1 \le y \le 0$ and $0 \le y \le 1$. The width of the triangular membership functions is very less, so uncertain parameter M is less sensitive. While in Figures 21 and 22, the width of fuzzy temperature from the centre value of each TFN is large, so there is maximum possibility of the uncertainty of fuzzy temperature and M is more sensitive.

The uncertain velocity and temperature values are presented in Tables 6–9. Using TFN, the lower, upper, and crisp or mid values of the uncertain velocity and temperature profiles are presented for different involved uncertain parameters. In Tables 6 and 7, the gravitational parameter k is taken as a TFN, and in Tables 8 and 9, the magnetic parameter M is taken as a

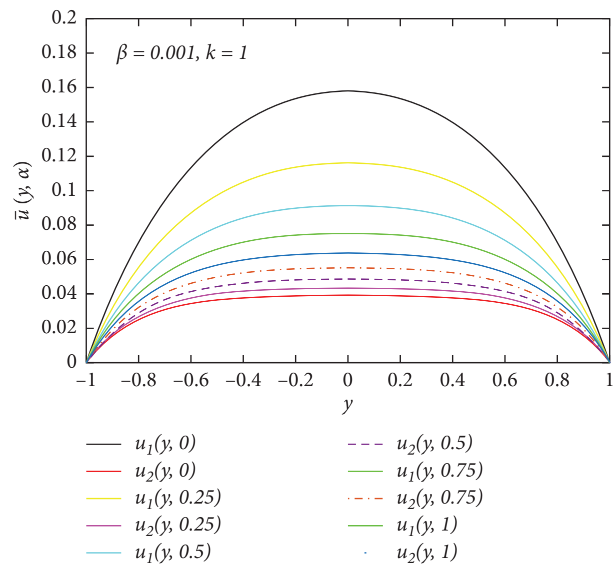


FIGURE 19: Effect of TFN M on fuzzy velocity profile of Poiseuille flow.

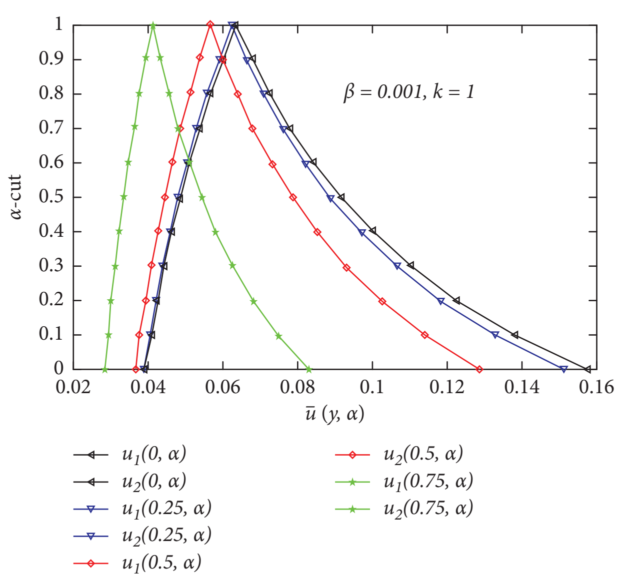


FIGURE 20: Membership functions of TFN plot if M is fuzzy (Poiseuille flow).

TFN. The lower and upper bounds of the uncertain velocity and temperature are calculated at $\alpha - \text{cut} = 0$. It is noted that the lower bound defines the minimum uncertain velocity and temperature while the upper bound defines the maximum uncertain velocity and temperature. The mid values of uncertain velocity and temperature are calculated at $\alpha - \text{cut} = 1$. It has been observed that the lower and upper uncertain velocity and temperature values coincide at $\alpha - \text{cut} = 1$. Here, it is observed that the uncertainties in physical parameters have nonnegligible effect on the fuzzy velocity and temperature profiles. Also, it may be observed that, as α increases from 0 to 1, we have a narrow width of fuzzy velocity and temperature profiles and uncertainty decreases drastically which finally provides crisp results for $\alpha = 1$. Finally, it can be seen that the fuzzy velocity and temperature profiles of the fluid are a better option as compared to the crisp or classical case. The crisp or classical velocity profile of fluid gives the single flow situation of the fluid, while the fuzzy velocity profile of fluid gives the interval flow situation like lower and upper bounds of the velocity profiles.

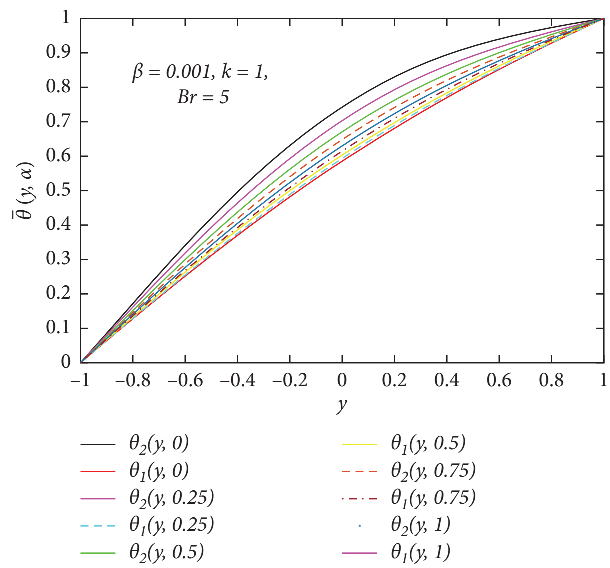


Figure 21: Effect of TFN M on the fuzzy temperature profile of Poiseuille flow.

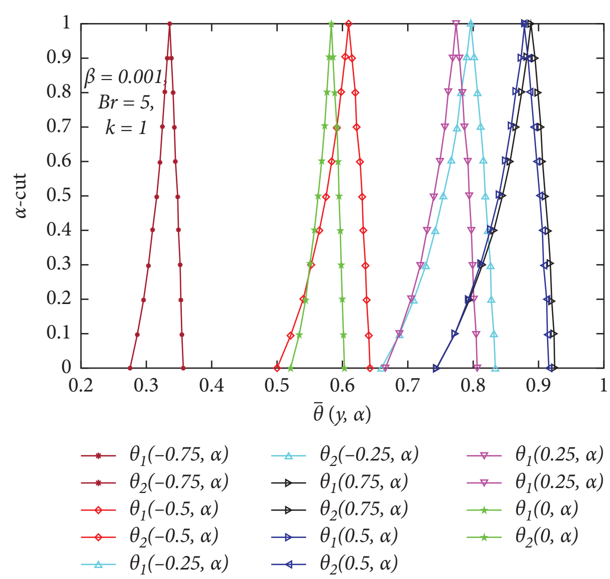


Figure 22: Membership functions of TFN plot if M is fuzzy (Poiseuille flow).

Table 6: Effect of fuzzy parameter k on fuzzy velocity and temperature of Couette flow when $M = 5$, Br = 5, and $\beta = 0.001$.

y	$u_1(y, \alpha)$	Crisp values	$u_2(y, \alpha)$	$\theta_1(y, \alpha)$	Crisp values	$\theta_2(y, \alpha)$
−1	0	0	0	0	0	0
−0.8	0.0793910383	0.2171491139	0.3541804031	0.3572109537	1.1940552414	2.1969200270
−0.6	0.1361377460	0.3620192484	0.5871865522	0.7159346247	2.3621707042	4.0864124256
−0.4	0.1798509668	0.4584108744	0.7364366065	1.0586808094	3.3992050827	6.1568171199
−0.2	0.2194526664	0.5259231666	0.8320410278	1.3735796018	4.2263529512	7.5284885661
0	0.2630274365	0.5783186895	0.8934076205	1.6468577300	4.7765309259	8.3100412353
0.2	0.3194707436	0.6262889631	0.9330297085	1.8600434054	4.9909331263	8.4100027173
0.4	0.4003009777	0.6796248866	0.9589849659	1.9840433370	4.8105538479	7.7656469133
0.6	0.5219955851	0.74920928316	0.9765685868	1.9670368365	4.1639897083	6.3343194207
0.8	0.7092930641	0.84922492764	0.9893692322	1.7084185223	2.9489067699	4.0864124256
1	1	1	1	1	1	1

Table 7: Effect of fuzzy parameter M on fuzzy velocity and temperature of Couette flow when $k = 1$, $\mathrm{Br} = 5$, and $\beta = 0.001$.

y	$u_1(y, \alpha)$	Mid values	$u_2(y, \alpha)$	$\theta_1(y, \alpha)$	Mid values	$\theta_2(y, \alpha)$
−1	0	0	0	0	0	0
−0.8	0.0251183255	0.0361886680	0.0794004671	0.2932734573	0.3145774346	0.4021269400
−0.6	0.0348150703	0.0541349796	0.1361596251	0.5894568242	0.6316208361	0.8056886060
−0.4	0.0388557462	0.0641457277	0.1798887488	0.8796771002	0.9399561571	1.1905214684
−0.2	0.0416481841	0.0725600838	0.2195125756	1.1623486661	1.2359097774	1.5427072376
0	0.0462385847	0.0847064491	0.2631194954	1.4362432704	1.5158252942	1.8460902450
0.2	0.0576344273	0.1082760594	0.3196093565	1.6991065685	1.7735086688	2.0789963435
0.4	0.0882654730	0.1581887046	0.4005032222	1.9440774756	1.9938534158	2.2072806878
0.6	0.1715120242	0.2659915363	0.5222692537	2.1437790808	2.1314185792	2.1701181990
0.8	0.3974990501	0.4993373042	0.7095850362	2.1577592359	2.0254091924	1.8493906825
1	1	1	1	1	1	1

Table 8: Effect of fuzzy parameter k on fuzzy velocity and temperature of Poiseuille flow when $M = 5$, $\mathrm{Br} = 5$, and $\beta = 0.001$.

y	$u_1(y, \alpha)$	Crisp/mid values	$u_2(y, \alpha)$	$\theta_1(y, \alpha)$	Crisp/mid values	$\theta_2(y, \alpha)$
−1	0	0	0	0	0	0
−0.8	0.0690238311	0.2068138971	0.3438429272	0.1697422956	0.7417557187	1.8870799070
−0.6	0.1130780838	0.3389955158	0.5642044944	0.3379379389	1.4534208777	3.6832778722
−0.4	0.1393877654	0.4179808104	0.6960314849	0.4934237771	2.0508411412	5.1627306592
−0.2	0.1533221660	0.4598230806	0.7658975816	0.6296208486	2.4748142954	6.1612132925
0	0.1577257957	0.4730473134	0.7879820816	0.7424429911	2.6885465757	6.5764285891
0.2	0.1534976786	0.4603501420	0.7667777480	0.8305122187	2.6796276828	6.3738573373
0.4	0.1397746217	0.4191423902	0.6979707849	0.8951523920	2.4599804768	5.5866630488
0.6	0.1137552488	0.3410281075	0.5675957317	0.9403959798	2.0659154647	4.3158009846
0.8	0.0701294721	0.2101298414	0.3493661793	0.9727790572	1.5562503437	2.7244461600
1	0	0	0	1	1	1

Table 9: Effect of fuzzy parameter M on fuzzy velocity and temperature of Poiseuille flow when $k = 1$, $\mathrm{Br} = 5$, and.$\beta = 0.001$.

y	$u_1(y, \alpha)$	Crisp/mid values	$u_2(y, \alpha)$	$\theta_1(y, \alpha)$	Crisp/mid values	$\theta_2(y, \alpha)$
−1	0	0	0	0	0	0
−0.8	0.0690238311	0.0354590058	0.0250124563	0.1697422953	0.1379807796	0.1253350726
−0.6	0.1130780838	0.0521999379	0.0344859841	0.3379379385	0.2750045627	0.2501840381
−0.4	0.1393877654	0.0597798909	0.0379444484	0.4934237765	0.4039839461	0.3691961681
−0.2	0.1533221660	0.0629987330	0.0391605600	0.6296208481	0.5223124983	0.4810440916
0	0.1577257957	0.0638948132	0.0394609454	0.7424429906	0.6287559082	0.5852215968
0.2	0.1534976786	0.0630355833	0.0391732884	0.8305122181	0.7229570372	0.6816004849
0.4	0.1397746217	0.0598769275	0.0379837901	0.8951523915	0.8052584530	0.7703035479
0.6	0.1137552488	0.0524186091	0.0345948561	0.9403959794	0.8768729386	0.8518263495
0.8	0.0701294721	0.0359377764	0.0253096216	0.9727790569	0.9403737363	0.9274730619
1	0	0	0	1	1	1

6. Conclusion

In this work, we have studied the three fundamental flow problems that frequently arise in the field of fluid dynamics, namely, Couette flow, Poiseuille flow, and Couette–Poiseuille flow of a third-grade fluid through the inclined channel with heat transfer. The dimensionless nonlinear governing equations are converted into FDEs and then solved numerically by MATLAB built-in technique bvp4c. In the fuzzy case, the uncertain gravitational parameter k and magnetic parameter M are taken as the TFNs. Some findings of this work are given as follows:

(i) The obtained results indicate that the ranges of calculated lower and upper velocity profiles depend upon $\alpha -$ cut approach. Also, the obtained results are an envelope of solutions with a crisp solution, between the upper and lower solutions.

(ii) Furthermore, it is observed that, in triangular membership functions, if the width of fuzzy or uncertain velocity and temperature becomes more, then the parameters are more sensitive, while for less width of fuzzy or uncertain velocity and temperature, the assumed uncertain parameters are less sensitive.

(iii) Also, for validation, the present results are compared with the existing results in some special cases, viz., crisp case, which have good agreement.

(iv) In future, the concept provided here can be simply applied to other fuzzy numbers as well.

Data Availability

No data were used in this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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