N-WAVE AND OTHER SOLUTIONS TO THE B-TYPE KADOMTSEV-PETVIASHVILI EQUATION

by

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The present article studies a B-type Kadomtsev-Petviashvili equation with certain applications in the fluids. Stating with the Hirota's bilinear form and adopting reliable methodologies, a group of exact solutions such as the N-wave and other solutions to the B-type Kadomtsev-Petviashvili equation is formally derived. Some figures in two and three dimensions are given to illustrate the characteristics of the obtained solutions. The results of the current work actually help to complete the previous studies about the B-type Kadomtsev-Petviashvili equation.

Key words: *B-type Kadomtsev-Petviashvili equation, Hirota's bilinear form, reliable methodologies, N-wave and other solutions*

Introduction

During the last two decades, the topic of numerous studies has been about the integrable equations. Many researchers have focused on studying the integrable equations and their exact solutions. Because, the integrable equations describe many real phenomena in the broad branches of science and engineering. There is a wide range of reliable methods that can be used to handle the integrable equations; for instance, Kudryashov methods [1-6], ansatz methods [7-11], simplified Hirota's method [12-16], linear superposition method [17-19], and multiple -function method [20-22].

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The B-type Kadomtsev-Petviashvili (KP) equations are considered as non-linear models in the fluids or the plasmas which have been investigated using different methods [23-37]. In this paper, a B-type KP equation with certain applications in the fluids is studied such that its mathematical model can be expressed [36, 37]:

$$\begin{gathered}\frac{\partial}{\partial x}\left\{\frac{\partial^5 u(x,y,t)}{\partial x^5}+60\left[\frac{\partial u(x,y,t)}{\partial x}\right]^3+5\frac{\partial^3 u(x,y,t)}{\partial t\partial x^2}+\frac{\partial u(x,y,t)}{\partial y}+\right.\\ \left.+30\frac{\partial^3 u(x,y,t)}{\partial x^3}\frac{\partial u(x,y,t)}{\partial x}+30\frac{\partial u(x,y,t)}{\partial x}\frac{\partial u(x,y,t)}{\partial t}\right\}-\\ -5\frac{\partial^2 u(x,y,t)}{\partial t^2}=0\end{gathered} \tag{1}$$

The B-type KP eq. (1) passes the Painleve test and in the context of Painleve feature and Hirota's formalism is an integrable equation [36]. Such an integrable equation has been investigated by reliable methods. Singh and Gupta [36] obtained the soliton solutions and Du *et al*. [37] derived the lump and other wave solutions of the B-type KP equation. The reader can see [38-50].

The Hirota's bilinear form of the B-type KP equation [36, 37]:

$$(D_x^6+5D_x^3D_t-5D_t^2+D_xD_y)ff=0,\ u=(\ln f)_x \tag{2}$$

where f is an unknown function and D is the Hirota's operator.

The *N*-wave and other solutions

To acquire the *N*-wave solutions of the B-type KP equation, we search a_j, j = 1, 2, 3 and c_j, j = 1, 2,3 such that:

$$a_1^6(x^{c_1}-y^{c_1})^6+5a_1^3(x^{c_1}-y^{c_1})^3a_3(x^{c_3}-y^{c_3})-5a_3^2(x^{c_3}-y^{c_3})^2+a_1(x^{c_1}-y^{c_1})a_2(x^{c_2}-y^{c_2})=0 \tag{3}$$

It is easy to show that a_j, j = 1,2,3 can be obtained if c_1 = 1, c_2 = 5, and c_3 = 3. By inserting c_j, j = 1, 2, 3 into eq. (3) and using some operations, one can obtain:

$$\begin{gathered}a_1^6+5a_1^3a_3-5a_3^2+a_1a_2=0,\ -6a_1^6-15a_1^3a_3-a_1a_2=0\\ a_1^6+a_1^3a_3=0,\quad -2a_1^6-a_1^3a_3+a_3^2=0\end{gathered}$$

The aforementioned non-linear system can be solved for deriving:

$$a_2=9a_1^5,\quad a_3=-a_1^3$$

Now, the following *N*-wave, complexiton, and positive complexiton solutions to the eq. (1) can be constructed:

$$u=(\ln f)_x,\ f=\sum_{j=1}^N d_je^{\theta j},\theta_j=k_jx+9k_j^5y-k_j^3t$$

$$u=(\ln f)_x,\ f=\sum_{j=1}^N e^{\theta_{j,1}}(d_{j,1}\cos(\theta_{j,2})+d_{j,2}\sin(\theta_{j,2}))$$

$$\theta_j=k_jx+9k_j^5y-k_j^3t=\theta_{j,1}+i\theta_{j,2},\quad d_{j,1},d_{j,2}\in\mathbb{R},\quad i^2=-1$$

$$u=\left(\ln f\right)_x,\ f=\sum_{j=1}^{N}d_j\cosh\left(k_jx+9k_j^5y-k_j^3t\right)+$$

$$+\sum_{j=N+1}^{N+M}d_j\cos\left(k_jx+9k_j^5y+k_j^3t\right),\ d_j>0\ \text{ for }\ j=1,2,\ldots,N\ \text{ and }\ \sum_{j=1}^{N}d_j>\sum_{j=N+1}^{N+M}\left|d_j\right|$$

Particularly, if we select $N = M = 1$, then the positive complexiton solution can be written:

$$u=(\ln f)_x \tag{4}$$

in which:

$$f=d_1\cosh\left(k_1x+9k_1^5y-k_1^3t\right)+d_2\cos\left(k_2x+9k_2^5y+k_2^3t\right),\ \ d_1,d_2>0,\ \ d_1>\left|d_2\right|$$

Figure 1 shows the positive complexiton solution (4) on the x-y plane when $d_1 = 1.5$, $d_2 = 1$, $k_1 = 1$, $k_2 = 2.5$, and $t = 0$.

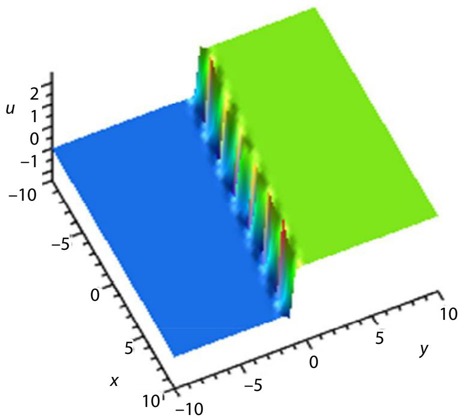

Figure 1. The positive complexiton solution (4) on the *x*-*y* plane when $d_1 = 1.5$, $d_2 = 1$, $k_1 = 1$, $k_2 = 2.5$, and $t = 0$

Other rational solutions

In the current section, a series of ansatz methods are formally utilized to derive other rational solutions of the B-type KP equation.

Rational tanh method

Let us consider the solution of the B-type KP equation as:

$$u\left(x,y,t\right)=\frac{\tanh\left(\kappa x+\tau y-\omega t\right)}{\rho+\sigma\tanh\left(\kappa x+\tau y-\omega t\right)} \tag{5}$$

By setting eq. (5) in eq. (1) and using a number of operations, we find:

$$\kappa=\frac{\rho}{2(\rho-\sigma)(\rho+\sigma)},\quad \tau=\frac{9\rho^5}{2(\rho-\sigma)^5(\rho+\sigma)^5},\quad \omega=\frac{\rho^3}{2(\rho-\sigma)^3(\rho+\sigma)^3}$$

Now, a rational solution the eq. (1) can be constructed as:

$$u\left(x,y,t\right)=\frac{\tanh\left[\dfrac{\rho}{2(\rho-\sigma)(\rho+\sigma)}x+\dfrac{9\rho^5}{2(\rho-\sigma)^5(\rho+\sigma)^5}y-\dfrac{\rho^3}{2(\rho-\sigma)^3(\rho+\sigma)^3}t\right]}{\rho+\sigma\tanh\left[\dfrac{\rho}{2(\rho-\sigma)(\rho+\sigma)}x+\dfrac{9\rho^5}{2(\rho-\sigma)^5(\rho+\sigma)^5}y-\dfrac{\rho^3}{2(\rho-\sigma)^3(\rho+\sigma)^3}t\right]} \tag{6}$$

It is easy to show that:

$$u\left(x,y,t\right)=\frac{\coth\left[\dfrac{\rho}{2(\rho-\sigma)(\rho+\sigma)}x+\dfrac{9\rho^5}{2(\rho-\sigma)^5(\rho+\sigma)^5}y-\dfrac{\rho^3}{2(\rho-\sigma)^3(\rho+\sigma)^3}t\right]}{\rho+\sigma\coth\left[\dfrac{\rho}{2(\rho-\sigma)(\rho+\sigma)}x+\dfrac{9\rho^5}{2(\rho-\sigma)^5(\rho+\sigma)^5}y-\dfrac{\rho^3}{2(\rho-\sigma)^3(\rho+\sigma)^3}t\right]}$$

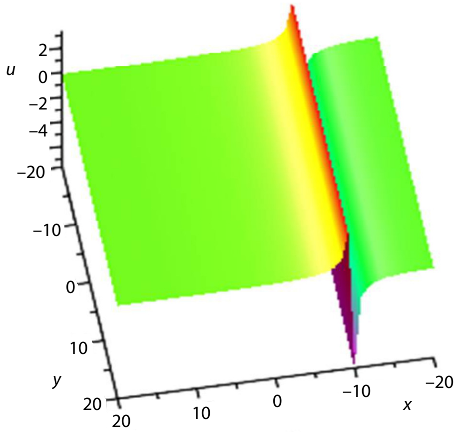

Figure 2. The rational solution (6) on the *x-y* plane when $\rho = -1$, $\sigma = -5$, and $t = 0$

is another rational solution of eq. (1). Figure 2 illustrates the rational solution (6) on the *x-y* plane when $\rho = -1$, $\sigma = -5$, and $t = 0$.

Rational cosh-sinh method

The rational cosh-sinh approach considers the solution of the B-type KP equation:

$$u(x,y,t) = \frac{\rho\cosh(\kappa x+\tau y-\omega t)}{\sigma\cosh(\kappa x+\tau y-\omega t)+\psi\sinh(\kappa x+\tau y-\omega t)} \tag{7}$$

As before, by setting eq. (7) in eq. (1) and using a number of operations, we acquire:

$$\rho = \frac{2\kappa(-\sigma^2+\psi^2)}{\psi}, \quad \tau = 144\kappa^5, \quad \omega = 4\kappa^3$$

Now, the following rational solution the eq. (1) can be derived:

$$u(x,y,t) = \frac{2\kappa(-\sigma^2+\psi^2)\cosh(\kappa x+144\kappa^5 y-4\kappa^3 t)}{\psi\left[\sigma\cosh(\kappa x+144\kappa^5 y-4\kappa^3 t)+\psi\sinh(\kappa x+144\kappa^5 y-4\kappa^3 t)\right]}$$

Rational tan method

Starting with:

$$u(x,y,t) = \frac{\tan(\kappa x+\tau y-\omega t)}{\rho+\sigma\tan(\kappa x+\tau y-\omega t)}$$

as the solution of the B-type KP equation, substituting it into eq. (1), and using a number of operations:

$$\kappa = -\frac{\rho}{2(\rho^2+\sigma^2)}, \quad \tau = -\frac{9\rho^5}{2(\rho^2+\sigma^2)^5}, \quad \omega = \frac{\rho^3}{2(\rho^2+\sigma^2)^3}$$

Now, a rational solution the eq. (1) can be derived:

$$u(x,y,t) = \frac{\tan\left(-\frac{\rho}{2(\rho^2+\sigma^2)}x-\frac{9\rho^5}{2(\rho^2+\sigma^2)^5}y-\frac{\rho^3}{2(\rho^2+\sigma^2)^3}t\right)}{\rho+\sigma\tan\left(-\frac{\rho}{2(\rho^2+\sigma^2)}x-\frac{9\rho^5}{2(\rho^2+\sigma^2)^5}y-\frac{\rho^3}{2(\rho^2+\sigma^2)^3}t\right)}$$

It is easy to show that:

$$u\left(x,y,t\right) \quad \frac{\cot\left(\dfrac{\rho}{\left(\rho^2 \quad \sigma^2\right)}x+\dfrac{\rho^5}{2\left(\rho^2+\sigma^2\right)^5}y+\dfrac{\rho^3}{2\left(\rho^2+\sigma^2\right)^3}t\right)}{\rho+\sigma\cot\left(\dfrac{\rho}{\left(\rho^2 \quad \sigma^2\right)}x+\dfrac{\rho^5}{2\left(\rho^2+\sigma^2\right)^5}y+\dfrac{\rho^3}{2\left(\rho^2+\sigma^2\right)^3}t\right)}$$

is another rational solution of eq. (1).

Rational cos-sin method

The rational cos-sin approach considers the solution of the B-type KP equation:

$$u\left(x,y,t\right)=\frac{\rho\cos\left(\kappa x+\tau y-\omega t\right)}{\sigma\cos\left(\kappa x+\tau y-\omega t\right)+\psi\sin\left(\kappa x+\tau y-\omega t\right)} \tag{8}$$

By inserting eq. (8) into eq. (1) and using a number of operations:

$$\rho=\frac{2\kappa\left(\sigma^2+\psi^2\right)}{\psi},\quad \tau=144\kappa^5,\quad \omega=-4\kappa^3$$

Now, the following rational solution the eq. (1):

$$u\left(x,y,t\right)=\frac{2\kappa\left(\sigma^2+\psi^2\right)\cos\left(\kappa x+144\kappa^5 y+4\kappa^3 t\right)}{\psi\left[\sigma\cos\left(\kappa x+144\kappa^5 y+4\kappa^3 t\right)+\psi\sin\left(\kappa x+144\kappa^5 y+4\kappa^3 t\right)\right]}$$

Multiple exp-function method

It is easy to show that through the use of the multiple-function method, the following 1-, 2-, and 3-wave solutions to the B-type KP equation can be constructed:

$$u\left(x,y,t\right)=2\frac{k_1\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}}{1+\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}}$$

$$u(x,y,t)=2\frac{k_1e^{k_1x+9k_1^5y-k_1^3t}+k_2e^{k_2x+9k_2^5y-k_2^3t}+a_{12}(k_1+k_2)e^{k_1x+9k_1^5y-k_1^3t}e^{k_2x+9k_2^5y-k_2^3t}}{1+e^{k_1x+9k_1^5y-k_1^3t}+e^{k_2x+9k_2^5y-k_2^3t}+a_{12}e^{k_1x+9k_1^5y-k_1^3t}e^{k_2x+9k_2^5y-k_2^3t}}$$

and

$$\begin{aligned}
u(x,y,t)&=2(k_1\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}+k_2\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}+a_{12}(k_1+k_2)\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}+\\
&+a_{13}(k_1+k_3)\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}\mathrm{e}^{k_3x+9k_3^5y-k_3^3t}+a_{23}(k_2+k_3)\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}\mathrm{e}^{k_3x+9k_3^5y-k_3^3t}+\\
&+a_{12}a_{13}a_{23}(k_1+k_2)(k_1+k_3)(k_2+k_3)\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}\mathrm{e}^{k_3x+9k_3^5y-k_3^3t})/\\
&/(1+\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}+\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}+a_{12}\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}+a_{13}\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}\mathrm{e}^{k_3x+9k_3^5y-k_3^3t}+\\
&+a_{23}\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}\mathrm{e}^{k_3x+9k_3^5y-k_3^3t}+a_{12}a_{13}a_{23}\mathrm{e}^{k_1x+9k_1^5y-k_1^3t}\mathrm{e}^{k_2x+9k_2^5y-k_2^3t}\mathrm{e}^{k_3x+9k_3^5y-k_3^3t})
\end{aligned}$$

which

$$a_{ij} = \frac{\left(k_i - k_j\right)^2}{\left(k_i + k_j\right)^2}, \; 1 \le i, \; j \le 3$$

Figures 3-5 show, respectively, 1-, 2-, and 3-wave solutions on the *x*-*y* plane when $k_1 = 1$, $k_2 = -2$, $k_3 = 3$, and $t = 0$.

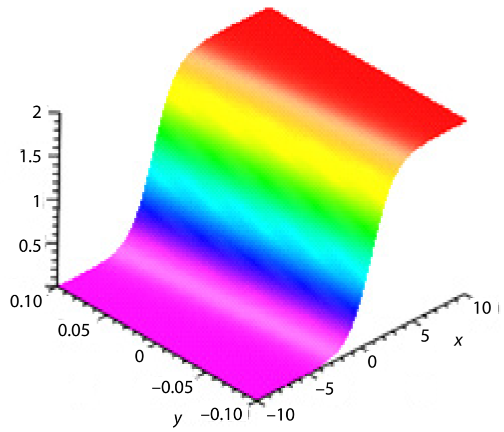

Figure 3. The 1-wave solution on the *x*-*y* plane when $k_1 = 1$ and $t = 0$

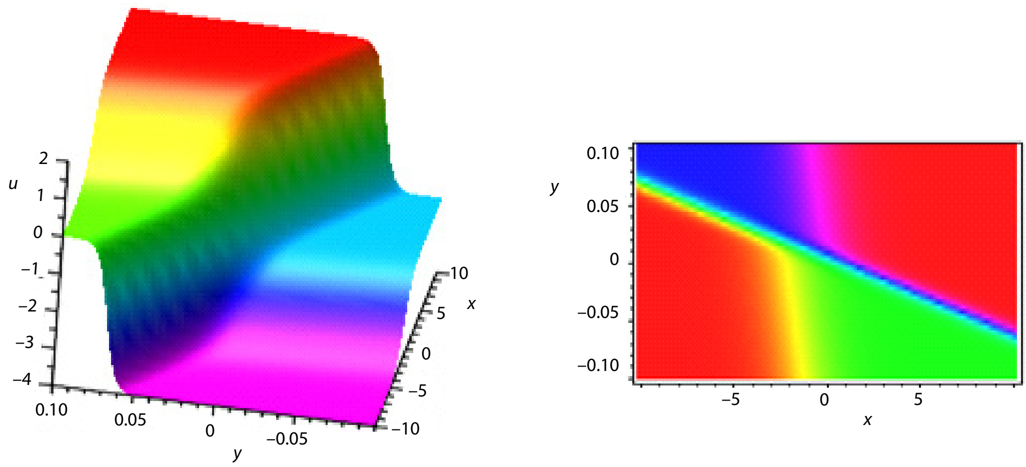

Figure 4. The 2-wave solution on the *x*-*y* plane when $k_1 = 1$, $k_2 = -2$, and $t = 0$

Conclusion

In summary, a B-type Kadomtsev-Petviashvili equation presented as a non-linear model in the fluids was investigated in this work. A group of exact solutions including the *N*-wave and other solutions to the B-type KP equation was formally extracted by considering the B-type KP equation, its bilinear expression, and exerting capable techniques. Figures in two and three dimensions were provided to demonstrate the characteristics of the solutions. The current research certainly helped to complete the previous studies about the B-type KP equation.

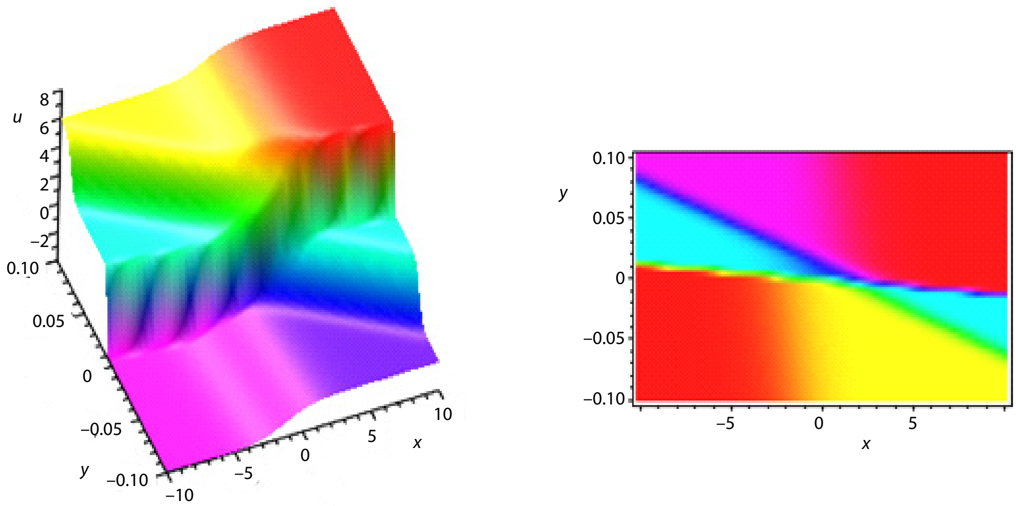

Figure 5. The 3-wave solution on the *x*-*y* plane when $k_1 = 1$, $k_2 = -2$, $k_3 = 3$, and $t = 0$

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