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ON THE APPROXIMATE SOLUTIONS FOR A SYSTEM OF COUPLED KORTEWEG–DE VRIES EQUATIONS WITH LOCAL FRACTIONAL DERIVATIVE

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Abstract

In this paper, we utilize local fractional reduced differential transform (LFRDTM) and local fractional Laplace variational iteration methods (LFLVIM) to obtain approximate solutions for coupled KdV equations. The obtained results by both presented methods (the LFRDTM and the LFLVIM) are compared together. The results clearly show that those suggested algorithms are suitable and effective to handle linear and as well as nonlinear problems in engineering and sciences.

Keywords: Local Fractional Derivative Operators; Reduced Differential Transform Method; Coupled Korteweg–De Vries Equation; Laplace Variational Iteration Method.

1. INTRODUCTION

Assorted phenomena in biology, fluid flow, physical problems and other sciences can be described very successfully by nonlinear models. We know the theory of water waves in shallow channels can be described by the Korteweg–de Vries equation.[1] The aim of this paper is to consider the coupled KdV equations with LFDOs as follows:

$$\frac{\partial^{\vartheta}\varphi(\iota,\kappa)}{\partial\kappa^{\vartheta}}+\frac{\partial^{3\vartheta}\varphi(\iota,\kappa)}{\partial\iota^{3\vartheta}}+2\varphi(\iota,\kappa)\frac{\partial^{\vartheta}\varphi(\iota,\kappa)}{\partial\iota^{\vartheta}}+2\psi(\iota,\kappa)\frac{\partial^{\vartheta}\varphi(\iota,\kappa)}{\partial\iota^{\vartheta}}=0, \quad (1)$$

$$\frac{\partial^{\vartheta}\psi(\iota,\kappa)}{\partial\kappa^{\vartheta}}+\frac{\partial^{3\vartheta}\psi(\iota,\kappa)}{\partial\iota^{3\vartheta}}+2\psi(\iota,\kappa)\frac{\partial^{\vartheta}\psi(\iota,\kappa)}{\partial\iota^{\vartheta}}+2\varphi(\iota,\kappa)\frac{\partial^{\vartheta}\psi(\iota,\kappa)}{\partial\iota^{\vartheta}}=0, \quad (2)$$

subject to initial conditions

$$\varphi(\iota,0)=\phi_1(\iota),$$
$$\psi(\iota,0)=\phi_2(\iota),$$

where $\phi_1(\iota)$ and $\phi_2(\iota)$ are given functions.

During last two decades, many analytical methods and as well as numerical methods have been utilized to obtain analytical/approximate solution of local fractional PDEs such as local fractional function decomposition method,[2,3] local fractional Adomian decomposition method,[3,4] local fractional series expansion method,[5,6] local fractional Laplace transform method,[7,8] local fractional Laplace variational iteration method,[9,10] local fractional variational iteration method,[11–13] local fractional differential transform method,[14,15] local fractional Laplace decomposition method,[16] local fractional homotopy perturbation method,[17] local fractional Laplace homotopy perturbation method[18] and other methods.[19–23]

The LFLVIM and LFRDTM are powerful approximate methods for various kinds of linear and nonlinear PDEs on Cantor sets with LFDOs. For example, the LVIM has been applied to linear PDEs in physics mathematics. Jassim *et al.* applied this method to diffusion and wave equations,[24] Laplace equation.[25] Furthermore, Liu *et al.*[26] used LFLVIM for fractal vehicular traffic flow, and Li *et al.* applied it to fractal heat conduction problem.[27] Jafari *et al.* utilized the RDTM for solving PDEs on Cantor sets.[28,29]

2. THE METHODS

In this section, we briefly present those methods and apply to general PDEs include LFDOs.

2.1. The LFRDTM

The basic definitions and theorems of the RDT with LFDOs are given as follows.[28,29]

Definition 1. If $\varphi(\iota,\kappa)$ is a LF analytical function in the domain of interest, then the LF spectrum function

$$\Phi_{\xi}(\iota)=\frac{1}{\Gamma(1+\xi\vartheta)}\left[\frac{\partial^{\xi\vartheta}\varphi(\iota,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=\kappa_0} \quad (3)$$

is RDT of the function $\varphi(\iota,\kappa)$ via LFDOs.

Definition 2. For LFDOs, the inverse of RDTM of $\Phi_{\xi}(\iota)$ via is defined by the following formula:

$$\varphi(\iota,\kappa)=\sum_{\xi=0}^{\infty}\Phi_{\xi}(\iota)(\kappa-\kappa_0)^{\xi\vartheta}. \quad (4)$$

In view of Eqs. (3) and (4), we have

$$\varphi(\iota,\kappa)=\sum_{\xi=0}^{\infty}\frac{(\kappa-\kappa_0)^{\xi\vartheta}}{\Gamma(1+\xi\vartheta)}\left[\frac{\partial^{\xi\vartheta}\varphi(\iota,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=\kappa_0}. \quad (5)$$

Obviously, from Eq. (5), the LFRDT is derived from the well-known Taylor's theorem.

When $\kappa_0=0$, then we can rewrite Eqs. (3) and (4) as

$$\Phi_\xi(\iota)=\frac{1}{\Gamma(1+\xi\vartheta)}\left[\frac{\partial^{\xi\vartheta}\varphi(\iota,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=0},$$

$$\varphi(\iota,\kappa)=\sum_{\xi=0}^{\infty}\Phi_\xi(\iota)\kappa^{\xi\vartheta}.$$

In the following, we present few useful theorems, those might be concluded from (3) and (4).

Theorem 3. *If* $\varphi=\alpha\psi+\beta\theta$ *then*

$$\Phi_\xi(\iota)=\alpha\Psi_\xi(\iota)+\beta\Theta_\xi(\iota).$$

Theorem 4. *If* $\varphi=\psi\theta$ *then*

$$\Phi_\xi(\iota)=\sum_{l=0}^{\xi}\Psi_l(\iota)\Theta_{\xi-l}(\iota).$$

Theorem 5. *If* $\varphi=a\psi$, *then*

$$\Phi_\xi(\iota)=a\Psi_\xi(\iota).$$

Theorem 6. *If* $\varphi=\frac{\partial^{n\vartheta}\psi}{\partial\kappa^{n\vartheta}}$ *then*

$$\Phi_\xi(\iota)=\frac{\Gamma(1+(\xi+n)\vartheta)}{\Gamma(1+\xi\vartheta)}\Psi_{\xi+n}(\iota).$$

Theorem 7. *If* $\varphi=\frac{\iota^{n\vartheta}}{\Gamma(1+n\vartheta)}\frac{\kappa^{m\vartheta}}{\Gamma(1+m\vartheta)}$ *then*

$$\Phi_\xi(\iota)=\frac{\iota^{n\vartheta}}{\Gamma(1+n\vartheta)}\delta_\vartheta(\xi-m),$$

where $\delta_\vartheta(\xi-m)$ is 1 if $\xi=m$ otherwise 0.

Theorem 8. *If* $\varphi=\frac{\partial^{n\vartheta}\psi}{\partial\iota^{n\vartheta}}$ *then*

$$\Phi_\xi(\iota)=\frac{\partial^{n\vartheta}\Psi_\xi(\iota)}{\partial\iota^{n\vartheta}}.$$

Now, For expression of the method, consider the below PDE within LFDO:

$$L_\vartheta[\varphi]+R_\vartheta[\varphi]+N_\vartheta[\varphi]=\omega(\iota,\kappa),$$
$$\varphi(\iota,0)=\phi(\iota). \quad (6)$$

where $L_\vartheta=\frac{\partial^\vartheta}{\partial\kappa^\vartheta}$ and R_ϑ are linear LFDO, N_ϑ is nonlinear LFDO and the known function $\omega(\iota,\kappa)$ is called inhomogeneous term.

In view of the above theorems and definitions, we rewrite all terms of (6) in the following format:

$$\frac{\Gamma(1+(\xi+1)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+1}(\iota)=\Omega_\xi(\iota)-R_\vartheta[\Phi_\xi(\iota)+N_\vartheta[\Phi_\xi(\iota)],$$

where $\Phi_0(\iota)=\phi(\iota)$, $\Phi_\xi(\iota)$ and $\Omega_\xi(\iota)$ are LFRDT of φ and ω, respectively.

2.2. The LFLVIM

As stated by the LFVIM,[11–13] we can write the following local fractional correction functional for (6):

$$\varphi_{n+1}(\kappa)=\varphi_n(\kappa)+\frac{1}{\Gamma(1+\vartheta)}\int_0^\kappa\frac{\sigma(\xi-\kappa)^\vartheta}{\Gamma(1+\vartheta)}\times(L_\vartheta[\varphi_n(\xi)]+R_\vartheta[\widetilde{\varphi}_n(\xi)]+N_\vartheta[\widetilde{\varphi}_n(\xi)]-\omega(\xi))(d\xi)^\vartheta, \quad (7)$$

where $\frac{\sigma(\xi-\kappa)^\vartheta}{\Gamma(1+\vartheta)}$ is a fractal Lagrange multiplier.

We now take LF Laplace transform from (7), we get

$$\widetilde{L}_\vartheta\{\varphi_{n+1}(\kappa)\}=\widetilde{L}_\vartheta\{\varphi_n\}+\widetilde{L}_\vartheta\left\{\frac{\sigma(\kappa)^\vartheta}{\Gamma(1+\vartheta)}\right\}$$
$$\widetilde{L}_\vartheta\{L_\vartheta[\varphi_n(\xi)]+R_\vartheta[\widetilde{\varphi}_n(\xi)]+N_\vartheta[\widetilde{\varphi}_n(\xi)]-\omega(\xi)\}. \quad (8)$$

Taking the LF variation of (8), we obtain

$$\delta^\vartheta\left(\widetilde{L}_\vartheta\{\varphi_{n+1}(\kappa)\}\right)=\delta^\vartheta\left(\widetilde{L}_\vartheta\{\varphi_n(\kappa)\}\right)+\delta^\vartheta\left(\widetilde{L}_\vartheta\left\{\frac{\sigma(\kappa)^\vartheta}{\Gamma(1+\vartheta)}\right\}\times\widetilde{L}_\vartheta\big\{\big(L_\vartheta\big[\varphi_n(\kappa)\big]+R_\vartheta\big[\widetilde{\varphi}_n(\kappa)\big]\big)+N_\vartheta[\widetilde{\varphi}_n(\kappa)]-\omega(\kappa)\big\}\right). \quad (9)$$

By using computation of (9), we get

$$\delta^\vartheta(\widetilde{L}_\vartheta\{\varphi_{n+1}(\kappa)\})=\delta^\vartheta(\widetilde{L}_\vartheta\{\varphi_n(\kappa)\})+\widetilde{L}_\vartheta\left\{\frac{\sigma(\kappa)^\vartheta}{\Gamma(1+\vartheta)}\right\}\delta^\vartheta\times(\widetilde{L}_\vartheta\{L_\vartheta[\varphi_n(\kappa)]\})=0. \quad (10)$$

Hence, from (10), we get

$$1+\widetilde{L}_\vartheta\left\{\frac{\sigma(\kappa)^\vartheta}{\Gamma(1+\vartheta)}\right\}s^\vartheta=0, \quad (11)$$

where

$$\delta^{\vartheta}\big(\widetilde{L}_{\vartheta}\{L_{\vartheta}[\varphi_n(\kappa)]\}\big) = \delta^{\vartheta}\big(s^{\vartheta}\widetilde{L}_{\vartheta}\{\varphi_n(\kappa)\} - \varphi_n(0)\big) = s^{\vartheta}\delta^{\vartheta}\big(\widetilde{L}_{\vartheta}\{\varphi_n(\kappa)\}\big).$$

Hence, we get

$$\widetilde{L}_{\vartheta}\left\{\frac{\sigma(\kappa)^{\vartheta}}{\Gamma(1+\vartheta)}\right\} = -\frac{1}{s^{\vartheta}}. \tag{12}$$

Therefore, we have

$$\begin{aligned}\widetilde{L}_{\vartheta}\{\varphi_{n+1}(\kappa)\} &= \widetilde{L}_{\vartheta}\{\varphi_n(\kappa)\} - \frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\{L_{\vartheta}[\varphi_n] + R_{\vartheta}[\varphi_n] + N_{\vartheta}[\varphi_n] - \omega\}\\ &= \widetilde{L}_{\vartheta}\{\varphi_n(\kappa)\} - \frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\{s^{\vartheta}\varphi_n(\kappa) - \varphi_n(0)\} - \frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\{R_{\vartheta}[\varphi_n(\kappa)] + N_{\vartheta}[\varphi_n(\kappa)] - \omega(\kappa)\}\\ &= \frac{1}{s^{\vartheta}}\varphi_n(0) - \frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\{R_{\vartheta}[\varphi_n] + N_{\vartheta}[\varphi_n] - \omega\},\end{aligned} \tag{13}$$

where the initial value reads as

$$\widetilde{L}_{\vartheta}\{\varphi_0(\kappa)\} = \frac{1}{s^{\vartheta}}\varphi(0). \tag{14}$$

Therefore, the solution of (6) is

$$\varphi(\iota,\kappa) = \lim_{n\to\infty}\widetilde{L}_{\vartheta}^{-1}\big(\widetilde{L}_{\vartheta}\{\varphi_n(\iota,\kappa)\}\big). \tag{15}$$

3. APPLICATIONS

Example 9. Consider the following coupled of KdV:

$$\frac{\partial^{\vartheta}\varphi(\iota,\kappa)}{\partial\kappa^{\vartheta}} + \frac{\partial^{3\vartheta}\varphi}{\partial\iota^{3\vartheta}} + 2\varphi\frac{\partial^{\vartheta}\varphi}{\partial\iota^{\vartheta}} + 2\psi\frac{\partial^{\vartheta}\varphi}{\partial\iota^{\vartheta}} = 0, \tag{16}$$

$$\frac{\partial^{\vartheta}\psi(\iota,\kappa)}{\partial\kappa^{\vartheta}} + \frac{\partial^{3\vartheta}\psi}{\partial\iota^{3\vartheta}} + 2\psi\frac{\partial^{\vartheta}\psi}{\partial\iota^{\vartheta}} + 2\varphi\frac{\partial^{\vartheta}\psi}{\partial\iota^{\vartheta}} = 0, \tag{17}$$

with initial values

$$\varphi(\iota,0) = E_{\vartheta}(-\iota^{\vartheta}),$$
$$\psi(\iota,0) = -E_{\vartheta}(-\iota^{\vartheta}).$$

(I) Using LFRDTM.

Applying the LFRDT on both sides of (16) and (17), we have

$$\frac{\Gamma(1+(\xi+1)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+1}(\iota) + \frac{\partial^{3\vartheta}\Phi_{\xi}(\iota)}{\partial\iota^{3\vartheta}} + 2\sum_{l=0}^{\xi}\Phi_l(\iota)\frac{\partial^{\vartheta}\Phi_{\xi-l}(\iota)}{\partial\iota^{\vartheta}} + 2\sum_{l=0}^{\xi}\psi_l(\iota)\frac{\partial^{\vartheta}\Phi_{\xi-l}(\iota)}{\partial\iota^{\vartheta}} = 0,$$

$$\frac{\Gamma(1+(\xi+1)\vartheta)}{\Gamma(1+\xi\vartheta)}\Psi_{\xi+1}(\iota) + \frac{\partial^{3\vartheta}\Psi_{\xi}(\iota)}{\partial\iota^{3\vartheta}} + 2\sum_{l=0}^{\xi}\Psi_l(\iota)\frac{\partial^{\vartheta}\Psi_{\xi-l}(\iota)}{\partial\iota^{\vartheta}} + 2\sum_{l=0}^{\xi}\Phi_l(\iota)\frac{\partial^{\vartheta}\Psi_{\xi-l}(\iota)}{\partial\iota^{\vartheta}} = 0$$

which reduces to

$$\Phi_{\xi+1}(\iota) = -\frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+1)\vartheta)} \times \left[\Phi_{\xi}^{(3\vartheta)} + 2\sum_{l=0}^{\xi}\Phi_l\Phi_{\xi-l}^{(\vartheta)} + 2\sum_{l=0}^{\xi}\Psi_l\Phi_{\xi-l}^{(\vartheta)}\right], \tag{18}$$

$$\Psi_{\xi+1}(\iota) = -\frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+1)\vartheta)} \times \left[\Psi_{\xi}^{(3\vartheta)} + 2\sum_{l=0}^{\xi}\Psi_l\Psi_{\xi-l}^{(\vartheta)} + 2\sum_{l=0}^{\xi}\Phi_l\Psi_{\xi-l}^{(\vartheta)}\right], \tag{19}$$

where

$$\Phi_0(\iota) = E_{\vartheta}(-\iota^{\vartheta}), \tag{20}$$

$$\Psi_0(\iota) = -E_{\vartheta}(-\iota^{\vartheta}). \tag{21}$$

By iterative calculations on (18)–(21), we obtain

$$\Phi_1(\iota) = \frac{1}{\Gamma(1+\vartheta)}E_{\vartheta}(-\iota^{\vartheta}),$$

$$\Psi_1(\iota) = -\frac{1}{\Gamma(1+\vartheta)}E_{\vartheta}(-\iota^{\vartheta}),$$

$$\Phi_2(\iota) = \frac{1}{\Gamma(1+2\vartheta)}E_\vartheta(-\iota^\vartheta),$$

$$\Psi_2(\iota) = -\frac{1}{\Gamma(1+2\vartheta)}E_\vartheta(-\iota^\vartheta),$$

$$\Phi_3(\iota) = \frac{1}{\Gamma(1+3\vartheta)}E_\vartheta(-\iota^\vartheta),$$

$$\Psi_3(\iota) = -\frac{1}{\Gamma(1+3\vartheta)}E_\vartheta(-\iota^\vartheta),$$

$$\vdots$$

$$\Phi_\xi(\iota) = \frac{1}{\Gamma(1+\xi\vartheta)}E_\vartheta(-\iota^\vartheta),$$

$$\Psi_\xi(\iota) = -\frac{1}{\Gamma(1+\xi\vartheta)}E_\vartheta(-\iota^\vartheta).$$

Hence, $\varphi(\iota,\kappa)$ and $\psi(\iota,\kappa)$ are

$$\varphi(\iota,\kappa) = \sum_{\xi=0}^{\infty}\Phi_\xi(\iota)\kappa^{\xi\vartheta} = \sum_{\xi=0}^{\infty}\frac{1}{\Gamma(1+\xi\vartheta)}E_\vartheta(-\iota^\vartheta) = E_\vartheta(\kappa^\vartheta-\iota^\vartheta), \tag{22}$$

$$\psi(\iota,\kappa) = \sum_{\xi=0}^{\infty}\Psi_\xi(\iota)\kappa^{\xi\vartheta} = -\sum_{\xi=0}^{\infty}\frac{1}{\Gamma(1+\xi\vartheta)}E_\vartheta(-\iota^\vartheta) = -E_\vartheta(\kappa^\vartheta-\iota^\vartheta). \tag{23}$$

(II) Using LFLVIM.

In view of Eqs. (13), (16) and (17), we have

$$\widetilde{L}_\vartheta\{\varphi_{n+1}\} = \widetilde{L}_\vartheta\{\varphi_n\} - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta\varphi_n}{\partial\kappa^\vartheta} + \frac{\partial^{3\vartheta}\varphi_n}{\partial\iota^{3\vartheta}} + 2\varphi_n\frac{\partial^\vartheta\varphi_n}{\partial\iota^\vartheta} + 2\psi_n\frac{\partial^\vartheta\varphi_n}{\partial\iota^\vartheta}\right\},$$

$$\widetilde{L}_\vartheta\{\psi_{n+1}\} = \widetilde{L}_\vartheta\{\psi_n\} - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta\psi_n}{\partial\kappa^\vartheta} + \frac{\partial^{3\vartheta}\psi_n}{\partial\iota^{3\vartheta}} + 2\psi_n\frac{\partial^\vartheta\psi_n}{\partial\iota^\vartheta} + 2\varphi_n\frac{\partial^\vartheta\psi_n}{\partial\iota^\vartheta}\right\},$$

which leads to

$$\widetilde{L}_\vartheta\{\varphi_{n+1}(\iota,\kappa)\} = \frac{1}{s^\vartheta}\varphi_n(\iota,0) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^{3\vartheta}\varphi_n}{\partial\iota^{3\vartheta}} + 2\varphi_n\frac{\partial^\vartheta\varphi_n}{\partial\iota^\vartheta} + 2\psi_n\frac{\partial^\vartheta\varphi_n}{\partial\iota^\vartheta}\right\}, \tag{24}$$

$$\widetilde{L}_\vartheta\{\psi_{n+1}(\iota,\kappa)\} = \frac{1}{s^\vartheta}\psi_n(\iota,0) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^{3\vartheta}\psi_n}{\partial\iota^{3\vartheta}} + 2\psi_n\frac{\partial^\vartheta\psi_n}{\partial\iota^\vartheta} + 2\varphi_n\frac{\partial^\vartheta\psi_n}{\partial\iota^\vartheta}\right\}, \tag{25}$$

where the initial values read

$$\widetilde{L}_\alpha\{\varphi_0(\iota,\kappa)\} = \frac{1}{s^\vartheta}E_\vartheta(-\iota^\vartheta), \tag{26}$$

$$\widetilde{L}_\alpha\{\psi_0(\iota,\kappa)\} = -\frac{1}{s^\vartheta}E_\vartheta(-\iota^\vartheta). \tag{27}$$

Using (24)–(27), we get the first approximation, namely

$$\widetilde{L}_\vartheta\{\varphi_1(\iota,\kappa)\} = \frac{1}{s^\vartheta}\varphi_0(\iota,0) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^{3\vartheta}\varphi_0}{\partial\iota^{3\vartheta}} + 2\varphi_0\frac{\partial^\vartheta\varphi_0}{\partial\iota^\vartheta} + 2\psi_0\frac{\partial^\vartheta\varphi_0}{\partial\iota^\vartheta}\right\},$$

$$\widetilde{L}_\vartheta\{\psi_1(\iota,\kappa)\} = \frac{1}{s^\vartheta}\psi_0(\iota,0) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^{3\vartheta}\psi_0}{\partial\iota^{3\vartheta}} + 2\psi_0\frac{\partial^\vartheta\psi_0}{\partial\iota^\vartheta} + 2\varphi_0\frac{\partial^\vartheta\psi_0}{\partial\iota^\vartheta}\right\}.$$

Hence, we have

$$\widetilde{L}_\vartheta\{\varphi_1(\iota,\kappa)\} = \frac{1}{s^\vartheta}E_\vartheta(-\iota^\vartheta) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{-E_\vartheta(-\iota^\vartheta)\right\} = \frac{1}{s^\vartheta}E_\vartheta(-\iota^\vartheta) + \frac{1}{s^{2\vartheta}}E_\vartheta(-\iota^\vartheta),$$

$$\widetilde{L}_\vartheta\{\psi_1(\iota,\kappa)\} = -\frac{1}{s^\vartheta}E_\vartheta(-\iota^\vartheta) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{E_\vartheta(-\iota^\vartheta)\right\} = -\frac{1}{s^\vartheta}E_\vartheta(-\iota^\vartheta) - \frac{1}{s^{2\vartheta}}E_\vartheta(-\iota^\vartheta).$$

The second approximation reads

$$\widetilde{L}_\vartheta\{\varphi_2(\iota,\kappa)\} = \frac{1}{s^\vartheta}\varphi_1(\iota,0) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^{3\vartheta}\varphi_1}{\partial\iota^{3\vartheta}} + 2\varphi_1\frac{\partial^\vartheta\varphi_1}{\partial\iota^\vartheta} + 2\psi_1\frac{\partial^\vartheta\varphi_1}{\partial\iota^\vartheta}\right\},$$

$$\widetilde{L}_\vartheta\{\psi_2(\iota,\kappa)\} = \frac{1}{s^\vartheta}\psi_1(\iota,0) - \frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^{3\vartheta}\psi_1}{\partial\iota^{3\vartheta}} + 2\psi_1\frac{\partial^\vartheta\psi_1}{\partial\iota^\vartheta} + 2\varphi_1\frac{\partial^\vartheta\psi_1}{\partial\iota^\vartheta}\right\}.$$

Therefore, we obtain

$$\widetilde{L}_\vartheta\{\varphi_2(\iota,\kappa)\} = \left(\frac{1}{s^\vartheta} + \frac{1}{s^{2\vartheta}} + \frac{1}{s^{3\vartheta}}\right) \times E_\vartheta(-\iota^\vartheta),$$

$$\widetilde{L}_\vartheta\{\psi_2(\iota,\kappa)\} = \left(-\frac{1}{s^\vartheta} - \frac{1}{s^{2\vartheta}} - \frac{1}{s^{3\vartheta}}\right) \times E_\vartheta(-\iota^\vartheta),$$

$$\vdots$$

$$\widetilde{L}_\vartheta\{\varphi_n\} = \sum_{k=0}^{n} \frac{1}{s^{(k+1)\vartheta}} E_\vartheta(-\iota^\vartheta),$$

$$\widetilde{L}_\vartheta\{\psi_n\} = -\sum_{k=0}^{n} \frac{1}{s^{(k+1)\vartheta}} E_\vartheta(-\iota^\vartheta)$$

and so on.

In view of Eq. (15), the solution is

$$\varphi(\iota,\kappa) = \lim_{n\to\infty} \widetilde{L}_\vartheta^{-1}\big(\widetilde{L}_\vartheta\{\varphi_n(\iota,\kappa)\}\big) = \lim_{n\to\infty} \widetilde{L}_\vartheta^{-1} \times \left(\sum_{k=0}^{n} \frac{1}{s^{(k+1)\vartheta}} E_\vartheta(-\iota^\vartheta)\right),$$

$$\psi(\iota,\kappa) = \lim_{n\to\infty} \widetilde{L}_\vartheta^{-1}\big(\widetilde{L}_\vartheta\{\psi_n(\iota,\kappa)\}\big) = -\lim_{n\to\infty} \widetilde{L}_\vartheta^{-1} \times \left(\sum_{k=0}^{n} \frac{1}{s^{(k+1)\vartheta}} E_\vartheta(-\iota^\vartheta)\right)$$

and in a closed from by

$$\varphi(\iota,\kappa) = E_\vartheta(\kappa^\vartheta - \iota^\vartheta), \tag{28}$$

$$\psi(\iota,\kappa) = -E_\vartheta(\kappa^\vartheta - \iota^\vartheta). \tag{29}$$

From Eqs. (22), (23), (28) and (29), the approximate solution of (16) and (17) by using LFRDTM is the same results as that obtained by LFLVIM.

4. CONCLUSIONS

In this work, we have considered the coupled KdV equations with LF derivatives. Two methods which are called the LFLVIM and the LFRDTM have been successfully used to obtain the solutions . The obtained solutions were in the form of infinite power series which can be written in a closed form. The example shows that the results of LFRDTM are in excellent agreement with the results given by LFLVIM. In view of the results, we can say that these two techniques are powerful mathematical tools for solving system of coupled KdV equations. Also, we can use them to obtain approximate (or even analytical) solution of other problems containing LFDOs.

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