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Rogue, multi-wave, homoclinic breather, M-shaped rational and periodic-kink solutions for a nonlinear model describing vibrations

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ABSTRACT

Our aim in this paper is to determine rogue-wave solutions for Maccari-system. We also construct multi-waves, homoclinic breathers, M-shaped rational and periodic cross kink solutions with the combination of exponential, rational, trigonometric functions and various bilinear forms. We will also draw graphical structures of our newly attained results and explain their physique.

Introduction

Rogue or killer or freak waves with heights, transcend about 18.3 m, are famous nonlinear waves that are capable to produced giant damages even for gigantic ships. They are much bigger as compared with normal waves for a given sea phase. Now rogue waves are treated as remarkable and distressing water waves. His existence in the scientific community has been acknowledged since 1995 when a freak wave raged on the Draupner oil platform in the North Sea. It was recorded with a maximum wave height of 25.6 meter with a peak top of 18.5 meters [1–5]. After a lot of experimental work, there is no harmony when and how rogue waves appear. They are still unpredictable and nowhere to be seen and disappear without a trace [6].

In this scenario, many papers considered the so-called Peregrine or breather solitons with an isolated high peak. They are called breathers because they grow first and then disappear [7]. They are somehow compatible with the behavior of rogue waves, but theory and experiment have yielded opposite results. Different approaches are used to investigate the solitary wave solutions such as, lie-symmetry method [8], F- expansion scheme [9], projective Riccati equation technique [10], $exp(-\phi(\xi))$ -expansion scheme [11], $exp((-\psi'/\psi)\eta)$ -expansion approach [12], Weierstrass elliptic-function approach [13], homogeneous balance technique [14], nonlinear wave solutions are systematically analyzed [15–24] and many others [25–30].

The Maccari-system, which Maccari derived from the reduction scheme based on spatio-temporal resealing in 1996, is a nonlinear system model that allows nonlinear vibrations of (2 + 1) dimensions not only in water waves but also in many other fields, like Environmental physics, plasma, Bose–Einstein condensate, superconductivity, nonlinear optics and so on [31–41]. Here, we consider Maccari-system as [42],

$$ir_t + r_{xx} + rs = 0, \quad s_t + s_y + (|r|^2)_x = 0. \tag{1}$$

Our aim in this paper is to determine rogue waves, multi-waves, homoclinic breathers, M-shaped rational, and periodic cross kink solutions for Maccari-system.

Rogue wave solutions

To transform Eq. (1) we use the following rational function alteration [43],

$$r = \frac{\beta\ exp(i\ \alpha\ t)\ g}{f}, \qquad s = 2(\ln f)_{xx} - \alpha. \tag{2}$$

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Bilinear forms,

$$2f_xg_x + ig(f_t + if_{xx}) + f(2ag - ig_tg_{xx}) = 0,$$
$$-b^2g^2f_x + 2f_yf_x^2 + b^2fgg_x - f(2f_xf_{xt} + 2f_xf_{xy} + f_yf_{xx})$$
$$+f_t(2f_x^2 - ff_{xx}) + f^2f_{xxt} + f^2f_{xxy} = 0. \quad (3)$$

For rogue wave solution, we use the hypotheses as [44],

$$f = \mathfrak{P}_1^2 + \mathfrak{P}_2^2 + e_9 + n_1\cosh(\zeta), \qquad g = \mathfrak{P}_1^2 + \mathfrak{P}_2^2 + e_0 + n_2\cosh(\zeta). \quad (4)$$

where $\mathfrak{P}_1 = e_1x + e_2y + e_3t + e_4$, $\mathfrak{P}_2 = e_5x + e_6y + e_7t + e_8$ and $\zeta = d_1x + d_2y + d_3t$ all are actual parameters to be set up. Then, inserting f and g into Eq. (3), and get all coefficients of x, y, t, $\cosh(\zeta)$ and $\sinh(\zeta)$. After that we get a system of equations, then by solving them we achieved the following results:

Set I. When $e_5 = e_8 = 0$, we get the following nonzero solutions:

$$\alpha = \frac{1}{2}d_1^2, \beta = \beta, d_1 = d_1, d_2 = d_2, d_3 = \frac{2id_1e_1(n_1+n_2)}{e_4(n_1-n_2)},$$
$$e_1 = e_1, e_2 = ie_6, e_4 = e_4, e_6 = e_6, \quad (5)$$
$$e_9 = e_9, e_0 = e_0, n_1 = n_1, n_2 = n_2,$$

and

$$\alpha = \alpha, \beta = \beta, d_1 = d_1, d_2 = -\frac{1}{3}\frac{-d_1d_3n_1^2 + \beta^2n_1n_2 - \beta^2n_2^2}{d_1n_1^2},$$
$$d_3 = d_3, e_2 = ie_6, e_3 = ie_7, e_6 = e_6,$$
$$e_7 = e_7, e_9 = e_9, e_0 = e_0, n_1 = n_1, n_2 = n_2.$$

By using these Parameters into Eq. (4), and then using the transformation Eq. (2) we get the following solutions Eq. (6a) is given in Box I, and

$$r(x,y,t) = \frac{\beta e^{i\alpha t}(e_0 + n_2\cosh(d_3t + d_1x + \frac{d_1d_3n_1^2+\beta^2n_2(n_2-n_1)y}{3d_1n_1^2}))}{\Lambda_2}, \quad (6b)$$

$$s(x,y,t) = -\frac{2\alpha e_9^2 + \alpha n_1^2 - 4d_1^2n_1^2 + 4(\alpha - d_1^2)\Lambda_2 + \alpha n_1^2\cosh(\frac{2}{3}(3d_1x + \frac{\beta^2n_2(n_2-n_1)y}{d_1n_1^2} + d_3(3t+y)))}{2(\Lambda_2)^2}.$$

where $\Lambda_1 = e_9 + e_6^2y^2 + (e_4 + e_1x + ie_6y)^2 + n_1\cos(\frac{2d_1e_1(n_1+n_2)t}{e_4(n_1-n_2)} - id_1x - id_2y)$, $\Omega = \frac{2d_1e_1(n_1+n_2)t}{e_4(n_1-n_2)} - id_1x - id_2y$ and $\Lambda_2 = e_9 + n_1\cosh(d_3t + d_1x + \frac{(d_1d_3n_1^2+\beta^2n_2(n_2-n_1))y}{3d_1n_1^2})$.

Set II. When $e_4 = e_6 = 0$, we get the following nonzero solutions:

$$\alpha = \frac{1}{2}d_1^2, \beta = \beta, d_1 = d_1, d_2 = d_2, d_3 = d_3, e_1 = ie_5, e_3 = ie_7,$$
$$e_5 = e_5, e_7 = e_7, e_8 = e_8, e_9 = e_9, \quad (7)$$
$$e_0 = e_0, n_1 = n_1, n_2 = n_2,$$

and

$$\alpha = \alpha, \beta = \beta, d_1 = d_1, d_2 = -\frac{1}{3}\frac{-d_1d_3n_1^2 + \beta^2n_1n_2 - \beta^2n_2^2}{d_1n_1^2}, d_3 = d_3,$$
$$e_2 = e_2, e_3 = e_3, e_5 = e_5, e_7 = e_7,$$
$$e_8 = e_8, e_9 = e_9, n_1 = n_1, n_2 = n_2.$$

Using parameters from Eq. (7) into Eq. (4), and then using the transformation Eq. (2) to get the following solutions Eq. (8a) is given in Box II, and Eq. (8b) is given in Box III where $\Delta_1 = e_8^2 + e_9 + 2e_8(e_7t + e_5x) + n_1\cosh(d_3t + d_1x + d_2y)$, $\Delta = e_9 + (e_8 + e_7t + e_5x)^2 + (e_3t + e_2y)^2 + n_1\cosh(\Omega)$ and $\Omega = d_3t + d_1x + \frac{(d_1d_3n_1^2+\beta^2n_2(n_2-n_1))y}{3d_1n_1^2}$.

Multi-waves solutions

For this purpose, first of all we convert our system into an ODE, for this we use the following transformations [42],

$$r(x,y,t) = R(\eta)e^{i\phi}, \qquad s(x,y,t) = S(\eta), \qquad \eta = x + y + et, \quad \phi = bx + cy + dt. \quad (9)$$

Using this transformation into Eq. (1) we get,

$$R'' - (d + b^2)R + RS = 0, \quad (10)$$
$$(e+1)S' + 2RR'' = 0.$$

Integrating 2nd eq. in Eq. (10) and ignoring the integration constant, we get,

$$-(e+1)S = R^2. \quad (11)$$

Using Eq. (11) into the 1st eq. of Eq. (10) and integrating, we get,

$$(e+1)R'' - (e+1)(d - b^2)R - R^3 = 0. \quad (12)$$

where R is a function of η. To convert Eq. (12) into bilinear form we use the log transformation $R = 2(\ln p)_\eta$, now Eq. (12) becomes,

$$2(e-1)p_\eta^3 - 3(e+1)pp_\eta p_{\eta\eta} + (e+1)p^2((b^2 - d)p_\eta + p_{\eta\eta\eta}) = 0. \quad (13)$$

Now, to attain multi-wave solutions we used p as following [45],

$$p = m_0\cosh(a_1\eta + a_2) + m_1\cos(a_3\eta + a_4) + m_2\cosh(a_5\eta + a_6), \quad (14)$$

where $a_i (1 \le i \le 6)$ and $m_i's$ all are real parameters to be investigated. Substituting Eq. (14) into Eq. (13) via allegorical calculation and computing all coefficients of $\sinh(a_1\eta + a_2)$ and $\sinh(a_5\eta + a_6)$, $\cos(a_3\eta + a_4)$, $\cosh(a_1\eta + a_2)$, $\cosh(a_5\eta + a_6)$, $\sin(a_3\eta + a_4)$, then we get an algebraic system of equations. After solving equations, we obtain some parametric values as:

Set I. For $a_2 = 0$,

$$b = b, c = c, d = -2a_5^2 + b^2, e = 1, a_1 = -a_5, a_3 = ia_5, a_4 = a_4,$$
$$a_5 = a_5, a_6 = a_6, m_0 = m_0, m_1 = m_1, m_2 = m_2. \quad (15)$$

By using these Parameters into Eq. (14), and then by using $R = 2(\ln p)_\eta$, to get the required sol. for Eq. (12),

$$R(\eta) = \frac{2a_5(-im_1\sin(a_4 + ia_5\eta) + m_0\sinh(a_5\eta) + m_2\sinh(a_6 + a_5\eta))}{m_1\cos(a_4 + ia_5\eta) + m_0\cosh(a_5\eta) + m_2\cosh(a_6 + a_5\eta)}. \quad (16)$$

Using Eqs. (16) and (11), into Eq. (9) to get required multi waves solution for Eq. (1),

$$r(x,y,t) = \frac{2a_5e^{i(-2a_5^2t + b^2t + bx + cy)}(-im_1\sin(a_4 + ia_5(t + x + y)) + \Delta)}{m_1\cos(a_4 + ia_5(t + x + y)) + m_0\cosh(a_5(t + x + y)) + m_2\cosh(a_6 + a_5(t + x + y))}, \quad (17)$$

$$s(x,y,t) = -\frac{2a_5^2(-im_1\sin(a_4 + ia_5(t + x + y)) + \Delta)^2}{m_1\cos(a_4 + ia_5(t + x + y)) + m_0\cosh(a_5(t + x + y)) + m_2\cosh(a_6 + a_5(t + x + y))}.$$

where $\Delta = m_0\sinh(a_5(t + x + y)) + m_2\sinh(a_6 + a_5(t + x + y))$.

Set II. For $a_5 = a_3 = 0$,

$$b = b, c = c, d = -a_1^2 + b^2, e = e, a_1 = a_1, a_2 = a_2, a_4 = a_4, a_6 = a_6,$$
$$m_0 = m_0, m_1 = m_1, m_2 = m_2. \quad (18)$$

By using Eq. (18) into Eq. (14), and then by using $R = 2(\ln p)_\eta$, to get the required sol. for Eq. (12),

$$R(\eta) = \frac{2a_1m_0\sinh(a_2 + a_1\eta)}{m_1\cos(a_4) + m_2\cosh(a_6) + m_0\cosh(a_2 + a_1\eta)}. \quad (19)$$

Using Eqs. (19) and (11), into Eq. (9) to get required multi waves solution for Eq. (1),

$$r(x,y,t) = \frac{2a_1e^{i((a_1^2+b^2)t + bx + cy)}m_0\sinh(a_2 + a_1(et + x + y))}{m_1\cos(a_4) + m_2\cosh(a_6) + m_0\cosh(a_2 + a_1(et + x + y))}, \quad (20)$$

$$s(x,y,t) = -\frac{4a_1^2m_0^2\sinh(a_2 + a_1(et + x + y))^2}{(1+e)(m_1\cos(a_4) + m_2\cosh(a_6) + m_0\cosh(a_2 + a_1(et + x + y)))^2}.$$

$$r(x,y,t)=\frac{\beta e^{0.5id_1^2t}(e_0+(e_4+e_1x)(e_4+e_1x+2ie_6y)+n_2\cos(\frac{2d_1e_1(n_1+n_2)t}{e_4(n_1-n_2)}-id_1x-id_2y))}{\Lambda_1},\quad (6a)$$
$$s(x,y,t)=\frac{-(d_1\Lambda_1)^2+4(\Lambda_1(2e_1^2+d_1^2n_1\cos(\frac{2d_1e_1(n_1+n_2)t}{e_4(n_1-n_2)}-id_1x-id_2y))-(2e_1(e_4+e_1x+ie_6y)+id_1n_1\sin(\Omega))^2)}{2(\Lambda_1)^2},$$

Box I.

$$r(x,y,t)=\frac{\beta e^{0.5id_1^2t}(e_0+e_8(e_8+2e_7t+2e_5x)+n_2\cosh(d_3t+d_1x+d_2y))}{\Delta_1},\quad (8a)$$
$$s(x,y,t)=\frac{-(d_1\Delta_1)^2+4(-4e_5^2e_8^2+d_1^2n_1^2+d_1^2n_1(e_8^2+e_9+2e_8(e_7t+e_5x))\cosh(d_3t+d_1x+d_2y)-4d_1e_5e_8n_1\sinh(d_3t+d_1x+d_2y))}{2(\Delta_1)^2},$$

Box II.

$$r(x,y,t)=\frac{\beta e^{i\alpha t}((e_8+e_7t+e_5x)^2+(e_3t+e_2y)^2+n_2\cosh(d_3t+d_1x+\frac{(d_1d_3n_1^2+\beta^2n_2(n_2-n_1))y}{3d_1n_1^2}))}{\Delta},\quad (8b)$$
$$s(x,y,t)=\frac{-\alpha\Delta^2+2(\Delta(2e_5^2+d_1^2n_1\cosh(d_3t+d_1x+\frac{(d_1d_3n_1^2+\beta^2n_2(n_2-n_1))y}{3d_1n_1^2}))-(2e_5(e_8+e_7t+e_5x)+d_1n_1\sinh(\Omega))^2)}{(\Delta)^2}.$$

Box III.

Homoclinic breather approach

For breather solution, we use the following form [45],

$$p=e^{-q(a_1\eta+a_2)}+m_1e^{q(a_3\eta+a_4)}+m_2\cos\left(q_1\left(a_5\eta+a_6\right)\right),\quad (21)$$

where m_1, m_2, q, q_1, and $a_i's$ all are real constants to be investigate. Inserting p into Eq. (13) via computational Mathematica and computing the coefficients of $e^{q(a_4+a_3\eta)}$, $\sin(q_1(a_6+a_5\eta))$, $\cos(q_1(a_6+a_5\eta))$, $e^{q(a_4+a_3\eta)}\sin(q_1(a_6+a_5\eta))$, $e^{-q(a_2+a_1\eta)+q(a_4+a_3\eta)}\cos(q_1(a_6+a_5\eta))$, $\cos(q_1(a_6+a_5\eta))\sin(q_1(a_6+a_5\eta))$, then we get a system of equations. After solving the system of equations, we attain some parametric values:

Set I. For $a_2=0$,

$$b=b,c=c,d=b^2+2a_5^2q_1^2,e=1,a_1=\frac{iq_1a_5}{q},a_3=\frac{iq_1a_5}{q},$$
$$a_4=a_4,a_5=a_5,a_6=a_6,q=q,q_1=q_1,\quad (22)$$
$$m_1=m_1,m_2=m_2.$$

By using these Parameters into Eq. (21), to get the required sol. for Eq. (12) by using $R=2(\ln p)_\eta$,

$$R(\eta)=\frac{2ia_5q_1(-1+e^{a_4q+2ia_5q_1\eta}m_1+ie^{ia_5q_1\eta}m_2\sin(q_1(a_6+a_5\eta)))}{1+e^{a_4q+2ia_5q_1\eta}m_1+e^{ia_5q_1\eta}m_2\cos(q_1(a_6+a_5\eta))}.\quad (23)$$

By using Eqs. (23), (9) and (11) to get required breather solution for Eq. (1),

$$r(x,y,t)=\frac{2ia_5e^{i(b^2t+2a_5^2q_1^2t+bx+cy)}q_1(\Delta)}{1+e^{a_4q+2ia_5q_1(t+x+y)}m_1+e^{ia_5q_1(t+x+y)}m_2\cos(q_1(a_6+a_5(t+x+y)))},\quad (24)$$
$$s(x,y,t)=\frac{2a_5^2q_1^2(\Delta)^2}{(1+e^{a_4q+2ia_5q_1(t+x+y)}m_1+e^{ia_5q_1(t+x+y)}m_2\cos(q_1(a_6+a_5(t+x+y))))^2}.$$

where $\Delta=-1+e^{a_4q+2ia_5q_1(t+x+y)}m_1+ie^{ia_5q_1(t+x+y)}m_2\sin(q_1(a_6+a_5(t+x+y)))$.

Set II. For $a_1=a_3=0$,

$$b=b,c=c,d=b^2-a_5^2q_1^2,e=e,a_4=a_4,a_5=a_5,a_6=a_6,$$
$$q=q,q_1=q_1,m_1=m_1,m_2=m_2.\quad (25)$$

By using these Parameters into Eq. (21), to get the required sol. for Eq. (12) by using $R=2(\ln p)_\eta$,

$$R(\eta)=\frac{2a_5m_2q_1\sin(q_1(a_6+a_5\eta))}{1+e^{a_4q}m_1+m_2\cos(q_1(a_6+a_5\eta))}.\quad (26)$$

By using Eqs. (26), (9) and (11) to get required breather solution for Eq. (1),

$$r(x,y,t)=-\frac{2a_5e^{i(b^2t-a_5^2q_1^2t+bx+cy)}m_2q_1\sin(q_1(a_6+a_5(et+x+y)))}{1+e^{a_4q}m_1+m_2\cos(q_1(a_6+a_5(et+x+y)))},\quad (27)$$
$$s(x,y,t)=-\frac{4a_5^2m_2^2q_1^2\sin(q_1(a_6+a_5(et+x+y)))^2}{(1+e)(1+e^{a_4q}m_1+m_2\cos(q_1(a_6+a_5(et+x+y))))^2}.$$

M-shaped rational solitons

For this, we make a positive quadratic solution p as following [46],

$$p=(a_1\eta+a_2)^2+(a_3\eta+a_4)^2+a_5,\quad (28)$$

where $a_i(1\le i\le 5)$, are all real parameters to be set up. Putting p into Eq. (13) and comparing all coefficients of power η, to obtain suitable results as following:

Set I. For $a_5=0$, the nonzero parameters are,

$$b=b,c=c,d=\frac{-6a_3^2+a_2^2b^2}{a_2^2},e=\frac{1}{7},a_2=a_2,a_3=a_3.\quad (29)$$

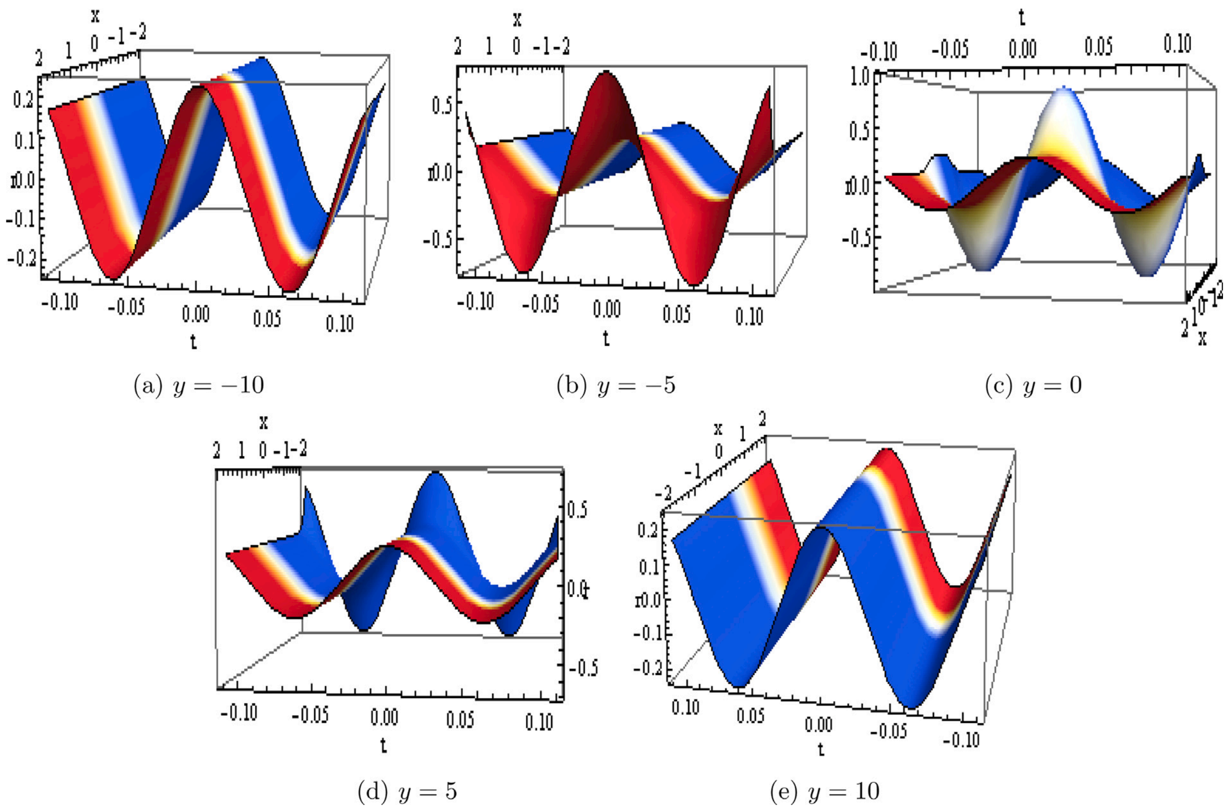


Fig. 1. Graphs of $r(x, y, t)$ in Eq. (6a) at $\beta = 1, e_0 = -6, e_1 = 1, e_4 = 20, e_6 = -2, e_9 = 8, n_1 = 5, n_2 = 1.2, d_1 = 10, d_2 = 5$ successively.

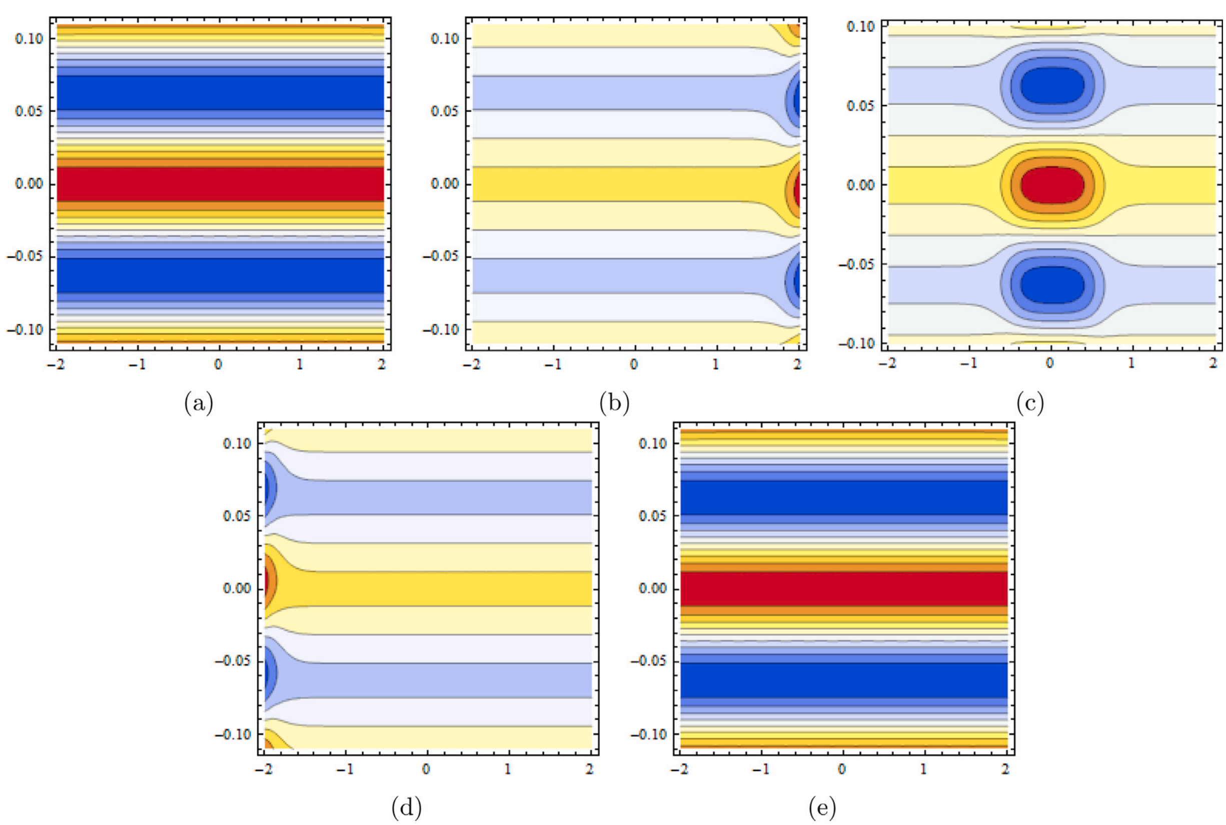


Fig. 2. Contour graphs of Fig. 1 respectively.

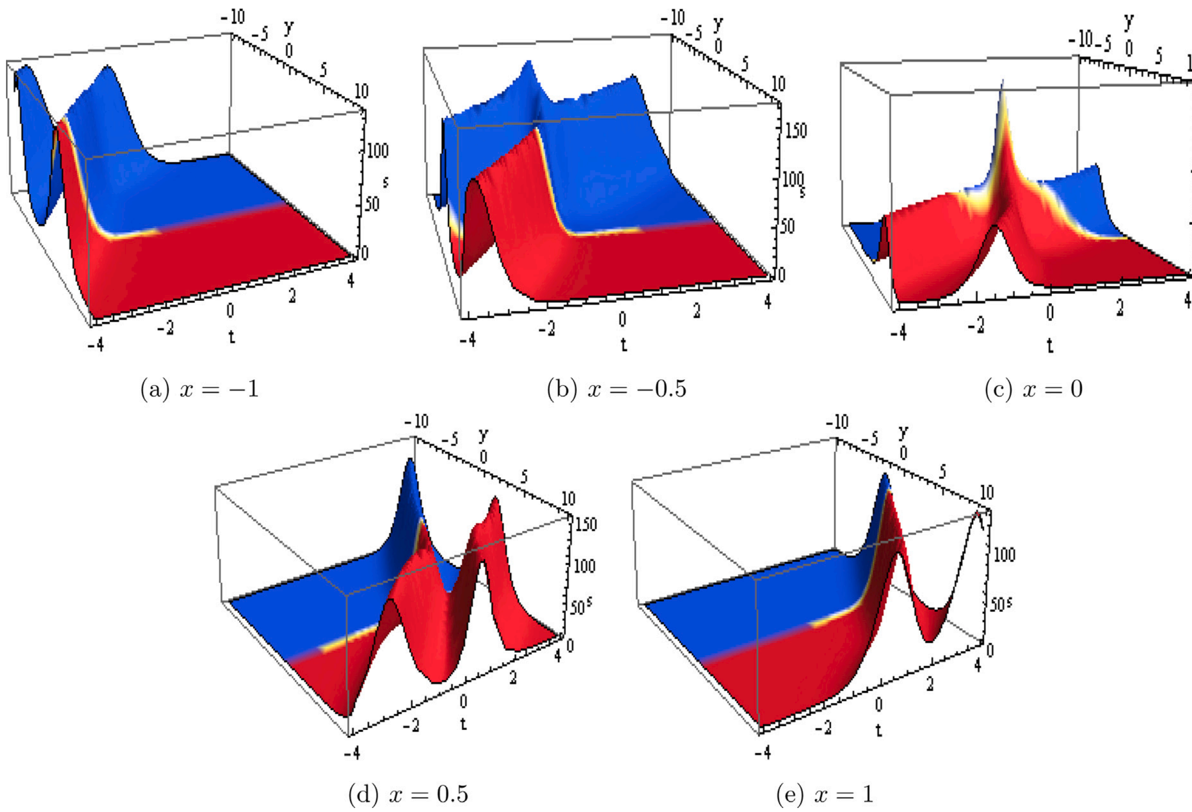


Fig. 3. Graphs of $s(x, y, t)$ in Eq. (8b) at $\alpha = -0.3, \beta = 0.8, e_2 = -1, e_3 = 5, e_5 = 3, e_7 = 2, e_8 = 1, e_9 = 0.09, n_1 = 2.5, n_2 = 0.08, d_1 = 16, d_3 = -3$ successively.

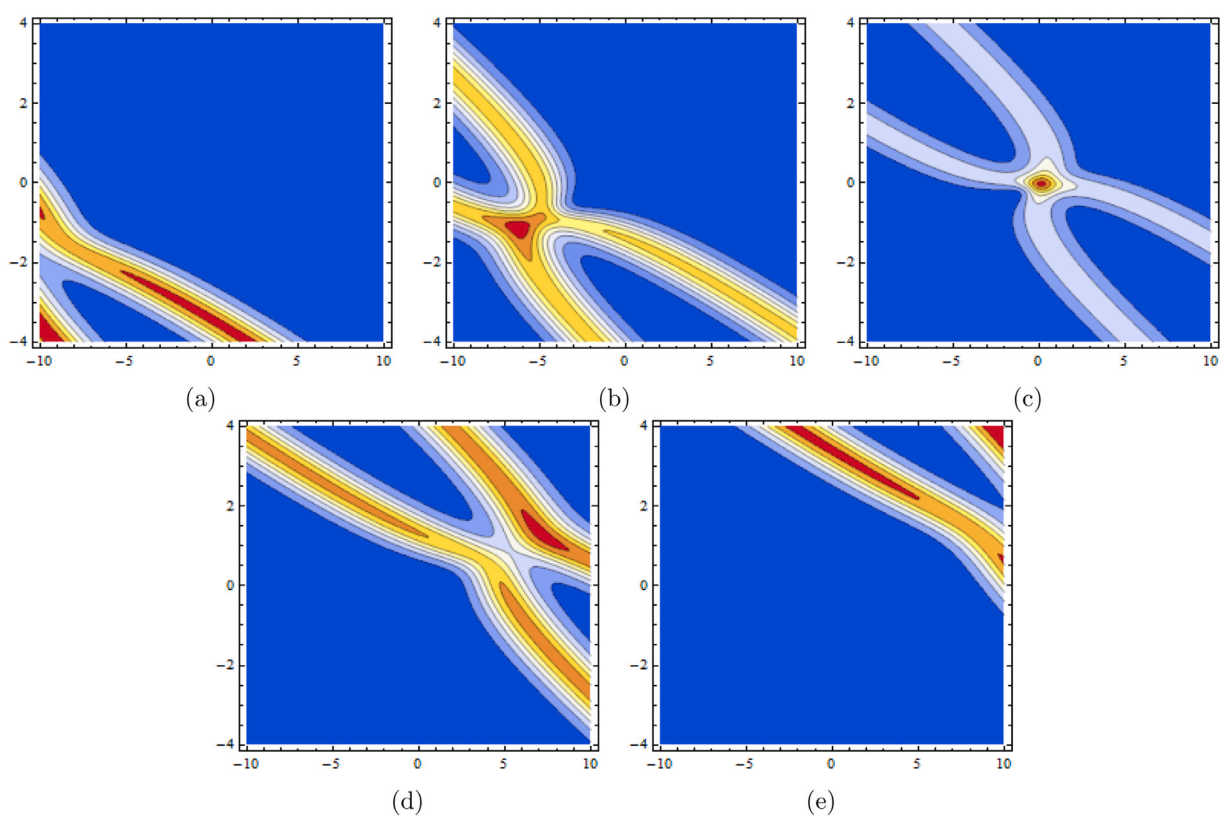


Fig. 4. Contour plots of Fig. 3 respectively.

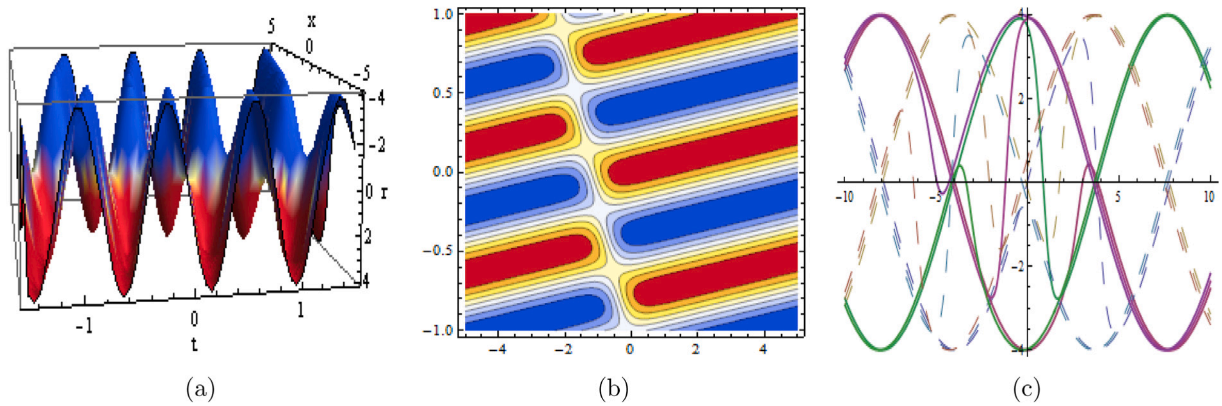


Fig. 5. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (17) at $b=0.4, c=0.2, a_4=-0.5, a_5=2, a_6=4, m_0=2.3, m_1=2.9, m_2=2, y=0.4$ respectively.

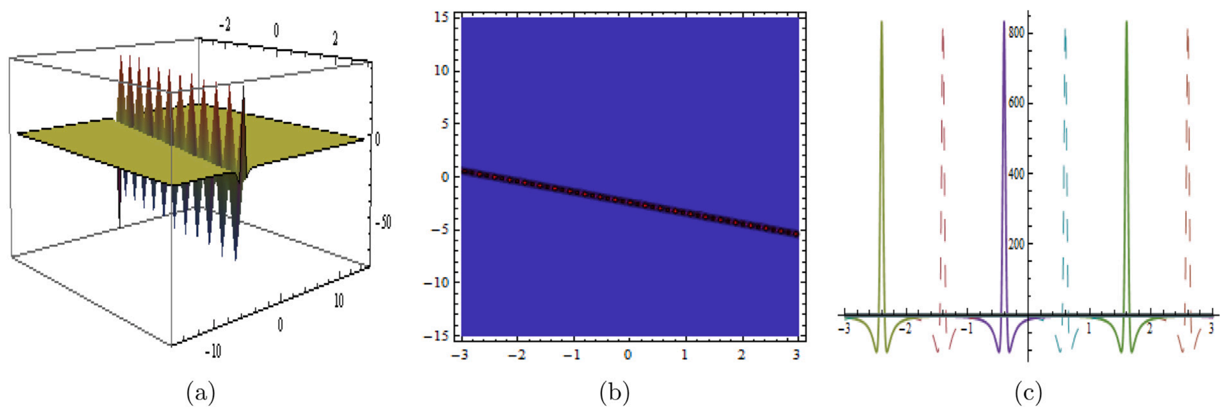


Fig. 6. 3D, 2D and contour plot of $s(x,y,t)$ in Eq. (17) at $a_4=3, a_5=-1, a_6=-5, m_0=-1.15, m_1=2.8, m_2=1, y=0.6$ respectively.

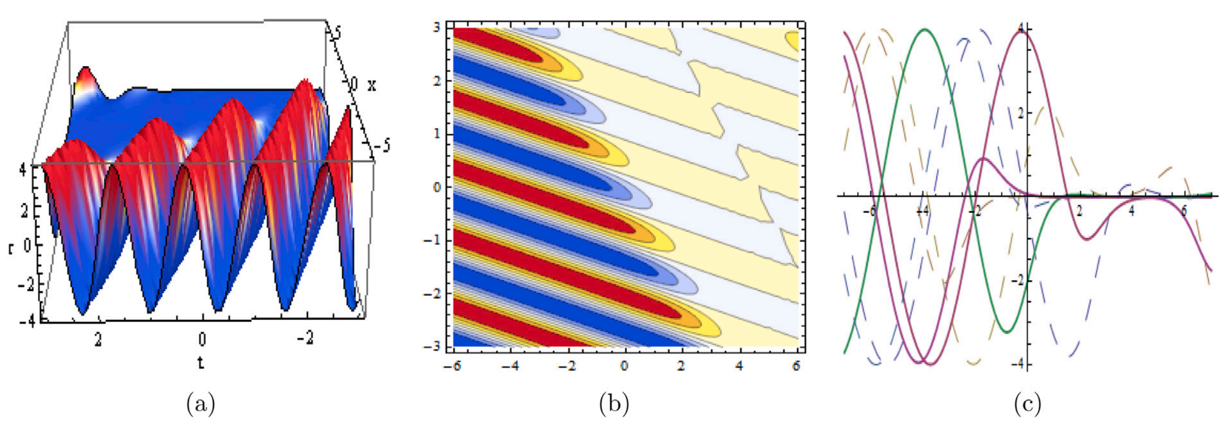


Fig. 7. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (20) at $b=0.9, c=0.8, e=1, a_1=2, a_2=-10, a_4=-1, a_6=9, m_0=2.3, m_1=2.9, m_2=2, y=0.5$ respectively.

By using these Parameters into Eq. (28), to get the required sol. for Eq. (12) by using $R=2(\ln p)_\eta$,

$$R(\eta)=\frac{4a_3^2\eta}{a_2^2+a_3^2\eta^2}. \tag{30}$$

By using Eqs. (30), (9) and (11) to get required M-shaped solution for Eq. (1),

$$r(x,y,t)=\frac{4a3^2e^{i\left(\frac{(-6a_3^2+a_2^2b^2)t}{a_2^2}+bx+cy\right)}\left(\frac{t}{7}+x+y\right)}{a_2^2+a_3^2\left(\frac{t}{7}+x+y\right)^2}, \tag{31}$$

$$s(x,y,t)=-\frac{14a_3^2\left(\frac{t}{7}+x+y\right)^2}{\left(a_2^2+a_3^2\left(\frac{t}{7}+x+y\right)^2\right)^2}.$$

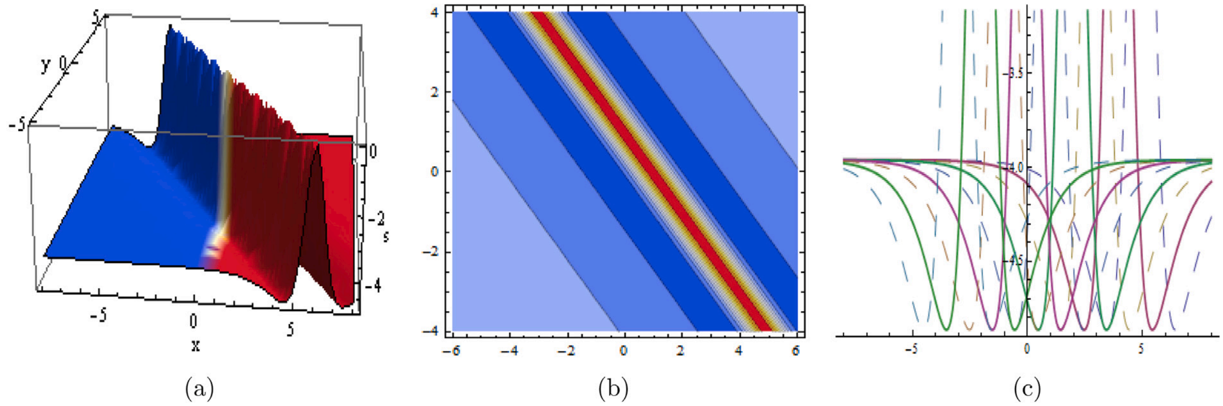


Fig. 8. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (20) at $e = 0.01, a_1 = -1, a_2 = 1, a_4 = -5, a_6 = 1, m_0 = -3, m_1 = 10, m_2 = -1, t = 5$ respectively.

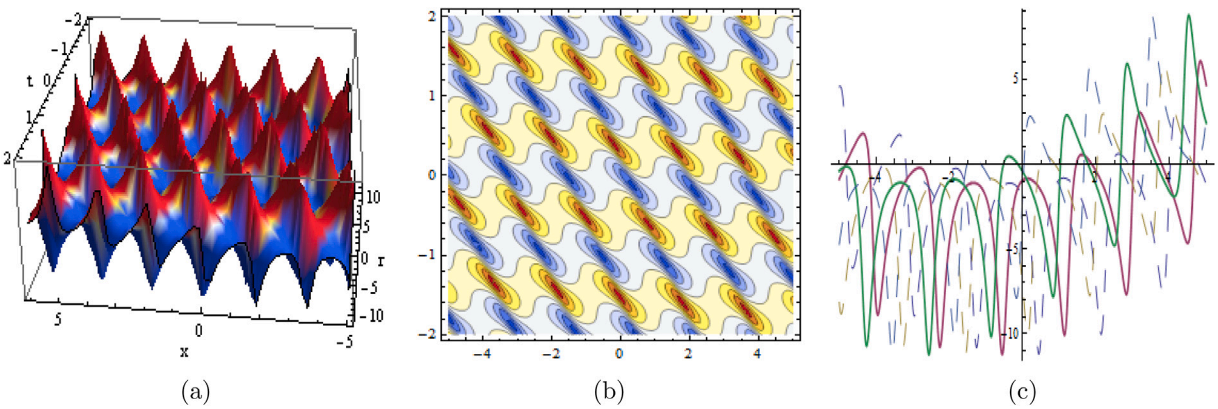


Fig. 9. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (24) at $b = 0.3, c = -1, a_4 = -2, a_5 = 1, a_6 = 3, m_1 = 0.6, m_2 = 1.5, q = 0.9, q_1 = 1.8, y = 1$ successively.

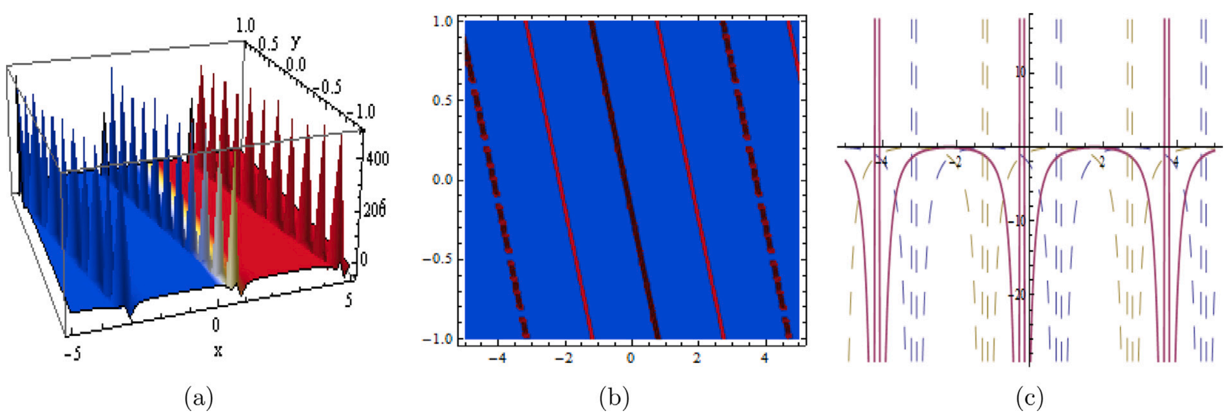


Fig. 10. 3D, 2D and contour plot of $s(x,y,t)$ in Eq. (24) at $a_4 = -2, a_5 = -1, a_6 = 5, m_1 = 0.6, m_2 = 1.5, q = 0.9, q_1 = 0.8, t = 0.3$ successively.

Set II. For $a_4 = 0$,

$$b = b, c = c, d = b^2, e = 7, a_1 = a_1, a_2 = a_2, a_3 = a_3, a_5 = -\frac{a_2^2 a_3^2}{a_1^2 + a_3^2}. \tag{32}$$

By using Eq. (32) into Eq. (28), to get the required rational sol. for Eq. (12) by using $R = 2(\ln p)_\eta$,

$$R(\eta) = \frac{4(a_1^2 + a_3^2)}{a_1 a_2 + a_1^2 \eta + a_3^2 \eta}. \tag{33}$$

By using Eqs. (33), (9) and (11) to get required M-shaped solution for Eq. (1),

$$r(x,y,t) = \frac{4(a_1^2 + a_3^2) e^{i(b^2 t + bx + cy)}}{a_1 a_2 + a_1^2(7t + x + y) + a_3^2(7t + x + y)}, \tag{34}$$

$$s(x,y,t) = -\frac{2(a_1^2 + a_3^2)^2}{(a_1 a_2 + a_1^2(7t + x + y) + a_3^2(7t + x + y))^2}.$$

Periodic cross-kink solutions

For this purpose, we use the following form [47],

$$p = e^{-(a_1\eta + a_2)} + m_1 e^{a_1\eta + a_2} + m_2 \cos\left(a_3\eta + a_4\right) + m_3 \cosh\left(a_5\eta + a_6\right) + a_7. \tag{35}$$

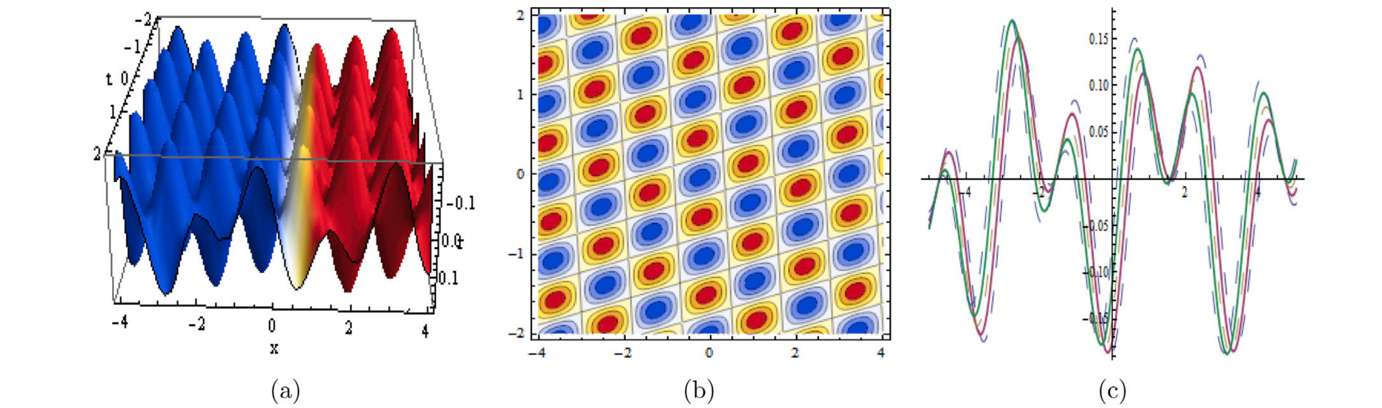


Fig. 11. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (27) at $b=1, c=0.6, e=0.1, a_4=3, a_5=-3, a_6=1, m_1=-2, m_2=2.5, q=1.2, q_1=0.9, y=0.02$ successively.

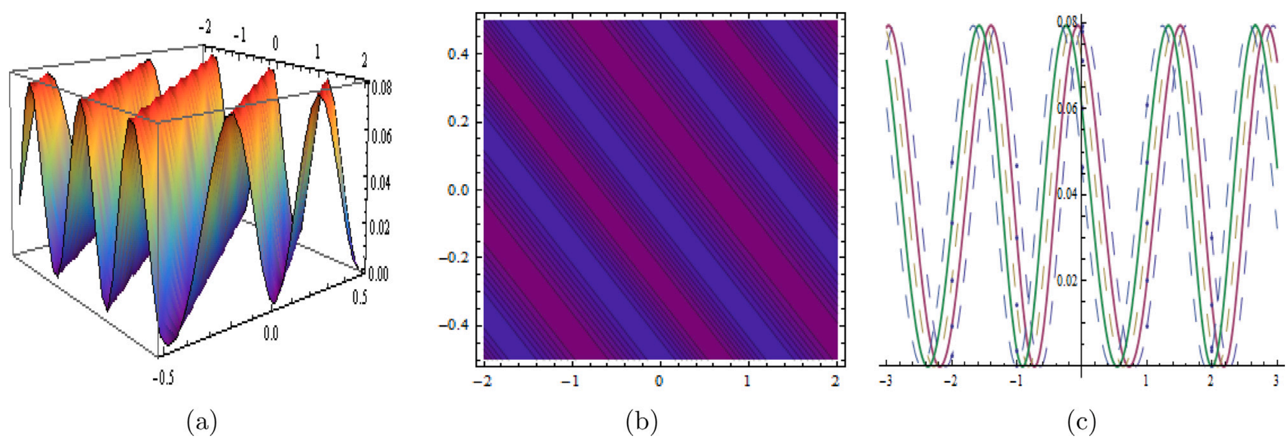


Fig. 12. 3D, 2D and contour plot of $s(x,y,t)$ in Eq. (27) at $e=3, a_4=0.5, a_5=1.2, a_6=-1, m_1=0.8, m_2=0.3, q=1, q_1=-1.8, x=0.2$ successively.

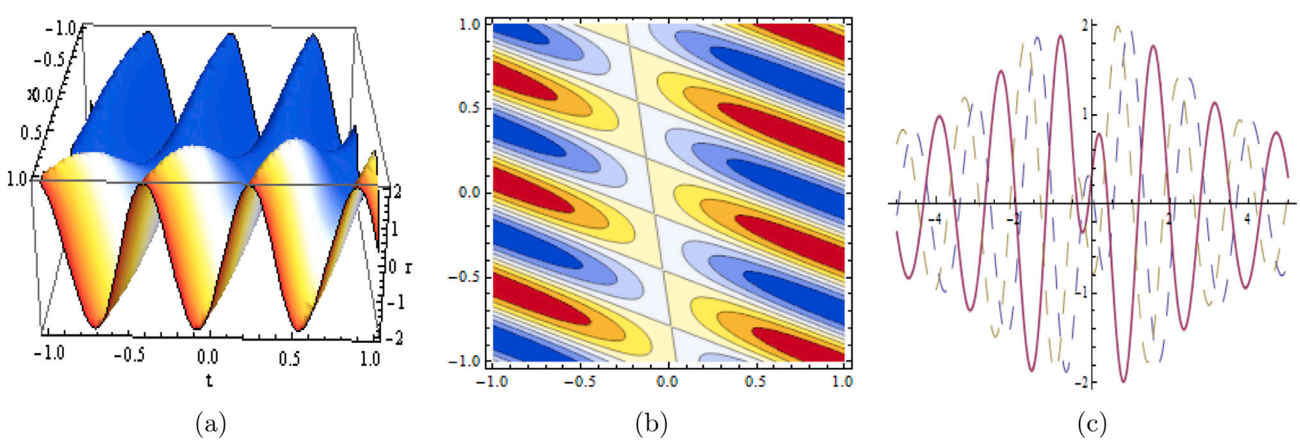


Fig. 13. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (31) at $b=4, c=0.01, a_2=-0.1, a_3=0.1, y=0.1$ respectively.

Here $a_i (1 \le i \le 7)$, m_1, m_2 and m_3, all are real constants to be investigate. Inserting p into Eq. (13) via computational Mathematica and computing the coefficients of $e^{(a_1\eta+a_2)}$, $e^{-(a_1\eta+a_2)}$, $e^{a_1\eta+a_2+2(a_1\eta+a_2)}$, $e^{-a_1\eta-a_2+2(a_1\eta+a_2)}$, $\cos(a_3\eta+a_4)$, $\sin(a_3\eta+a_4)\sinh(a_5\eta+a_6)$, $\cos(a_3\eta+a_4)\cosh(a_5\eta+a_6)$, $e^{-a_1\eta-a_2+2(a_1\eta+a_2)}\cos(a_3\eta+a_4)$, $e^{-a_1\eta-a_2+2(a_1\eta+a_2)}\sin(a_3\eta+a_4)$, $e^{-a_1\eta-a_2+2(a_1\eta+a_2)}\cos(a_3\eta+a_4)\cosh(a_5\eta+a_6)$, $\cos(a_3\eta+a_4)\cosh(a_5\eta+a_6)\sinh(a_5\eta+a_6)$, and $e^{-a_1\eta-a_2+2(a_1\eta+a_2)}\sin(a_3\eta+a_4)\sinh(a_5\eta+a_6)$, then we get a system of equations. After solving the system of equations, we attain some parametric values:

Set I.

$$
\begin{aligned}
&b=b, c=c, d=-2a_5^2+b^2, e=1, a_1=a_5, a_2=a_2, a_3=ia_5,\\
&a_4=a_4, a_5=a_5, a_6=a_6,\\
&a_7=0, m_1=m_1, m_2=m_2, m_3=m_3.
\end{aligned}
\tag{36}
$$

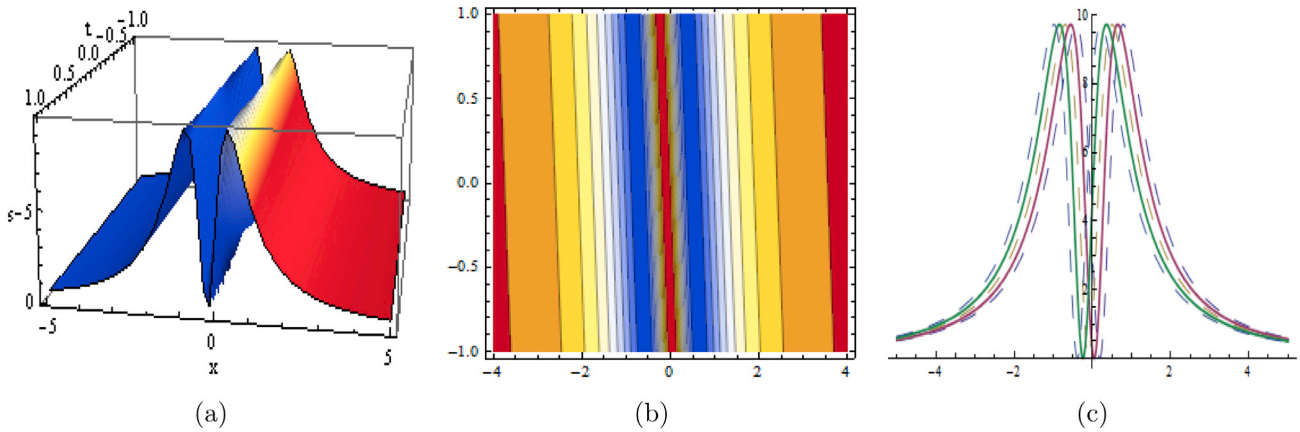


Fig. 14. 3D, 2D and contour plot of $s(x,y,t)$ in Eq. (31) at $a_2=-0.09, a_3=0.15, y=0.1$ respectively.

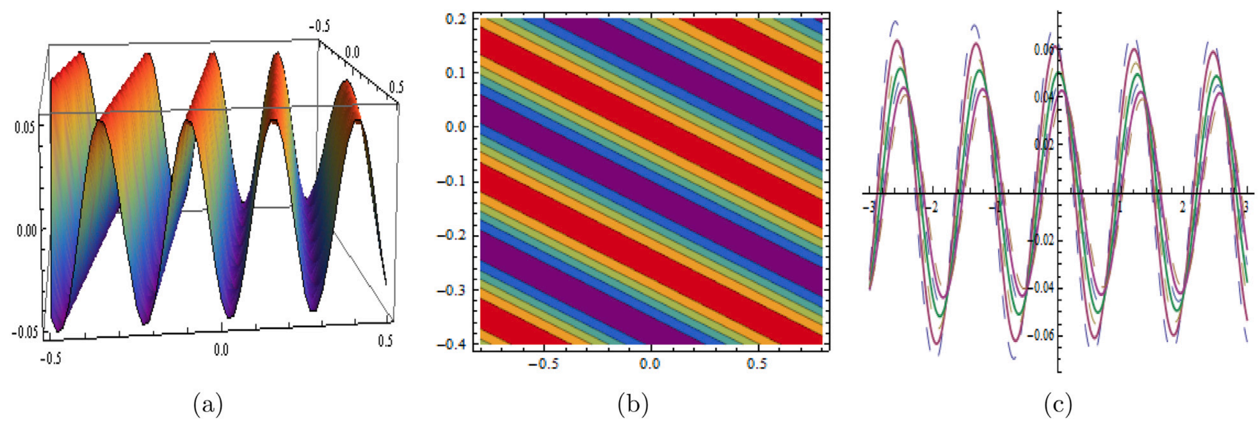


Fig. 15. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (34) at $b=5, c=0.01, a_1=-0.1, a_2=-8, a_3=0.01, y=0.09$ respectively.

By using these Parameters into Eq. (35), to get the required sol. for Eq. (12) by using $R=2(\ln p)_\eta$,

$$R(\eta) = \frac{2\left(-a_5 e^{-a_2-a_5\eta}+a_5 e^{a_2+a_5\eta}m_1-ia_5 m_2\sin(a_4+ia_5\eta)+a_5 m_3\sinh(a_6+a_5\eta)\right)}{e^{-a_2-a_5\eta}+e^{a_2+a_5\eta}m_1+m_2\cos(a_4+ia_5\eta)+m_3\cosh(a_6+a_5\eta)}. \quad (37)$$

By using Eqs. (37), (9) and (11) to get required breather solution for Eq. (1) Eq. (38) is given in Box IV, where $\Omega=a_6+a_5(t+x+y)$.

Set II.

$$b=b, c=c, d=a_1^2-3a_5^2+b^2, e=-\frac{5a_1^2-3a_5^2}{a_1^2-3a_5^2}, a_1=a_1, a_2=a_2,$$
$$a_3=a_3, a_4=a_4, a_5=a_5, \quad (39)$$
$$a_6=a_6, a_7=a_7, m_1=\frac{1}{4}\frac{a_7^2a_5^2}{(a_1-a_5)(a_1+a_5)}, m_2=0, m_3=m_3.$$

By using these Parameters into Eq. (35), to get the required sol. for Eq. (12) by using $R=2(\ln p)_\eta$,

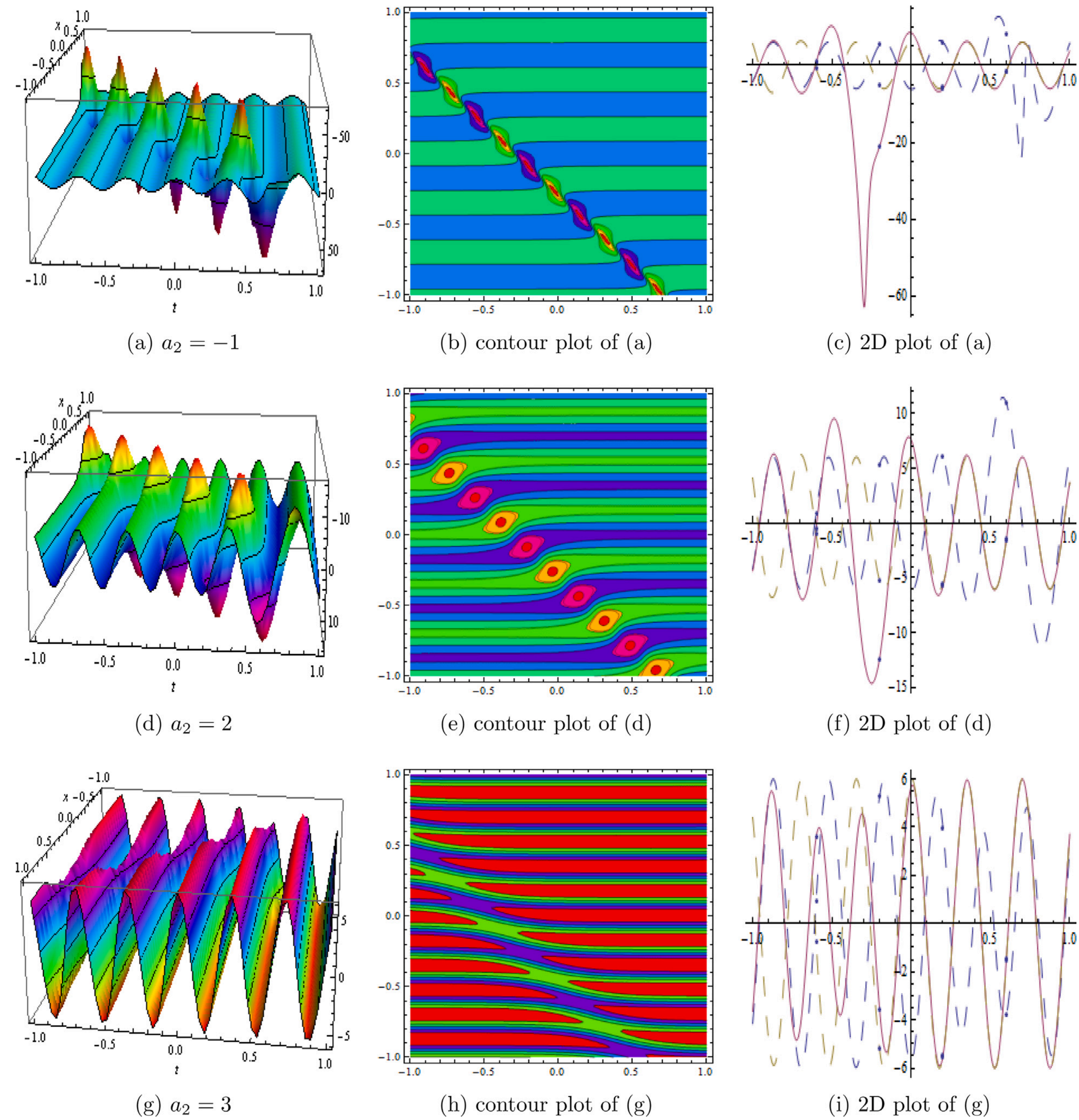
$$R(\eta)=\frac{2\left(-a_1e^{-a_2-a_1\eta}+\frac{a_1a_5^2a_7^2e^{a_2+a_1\eta}}{4(a_1-a_5)(a_1+a_5)}+a_5m_3\sinh(a_6+a_5\eta)\right)}{a_7+e^{-a_2-a_1\eta}+\frac{a_5^2a_7^2e^{a_2+a_1\eta}}{4(a_1-a_5)(a_1+a_5)}+m_3\cosh(a_6+a_5\eta)}. \quad (40)$$

By using Eqs. (40), (9) and (11) to get required breather solution for Eq. (1) Eq. (41) is given in Box V, where $\Omega=a_6+a_5\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)$.

Result and discussions

Here, we make a detailed comparison of our attained results with the earlier work for Maccari-system. Baskonus et al. studied complex hyperbolic-function solutions for Maccari-system [36]. Demiray et al. investigated rational, traveling wave, Jacobi elliptic function, periodic and hyperbolic solutions for Maccari-system. [37]. Hafez et al. found traveling wave solutions for Maccari-system with $exp(-\phi(\xi))$ - expansion scheme [38]. Xu et al. studied N-dark soliton solutions for coupled Maccari-system [39]. Jiang et al. studied rogue-wave and two families of homoclinic breather for Maccari-system [40], Maccari attained rogue-wave solutions for Maccari-system [41]. And we investigated rogue, multi-wave, homoclinic breather, M-shaped rational, and periodic-kink solutions for Maccari-system.

We can see from Eqs. (6) and (8) when Λ & $\Delta\to\infty$ then both r and s approaches to zero. Now from Figs. 1 and 3, we can see the behavior of traveling wave solutions. In Fig. 1(a) we can see a periodic wave, in 1(b) we see the dark and bright surfaces, in 1(c) we attained one bright and two dark faces, in 1(d) dark and bright surfaces appear and, in 1(e) we obtained periodic wave same as of 1(a). Fig. 2 represents the contour structures of Fig. 1 respectively. In Fig. 3(a) we can see some waves appearing, in 3(b) some non-periodic waves appear, in 3(c) we have seen a large size bright lump wave appears, and in 3(d) again show non-periodic waves like as of 3(b), and in 3(e) waves attains their original form as of 3(a). Fig. 4 represents the contour structures of Fig. 3 respectively. In Fig. 5, we seen M-shaped periodic wave and multiple dark and bright solutions. In Fig. 6, we seen multiple dark and bright solutions of equal sizes. In Fig. 7, we obtain



(a) $a_2 = -1$ (b) contour plot of (a) (c) 2D plot of (a)

(d) $a_2 = 2$ (e) contour plot of (d) (f) 2D plot of (d)

(g) $a_2 = 3$ (h) contour plot of (g) (i) 2D plot of (g)

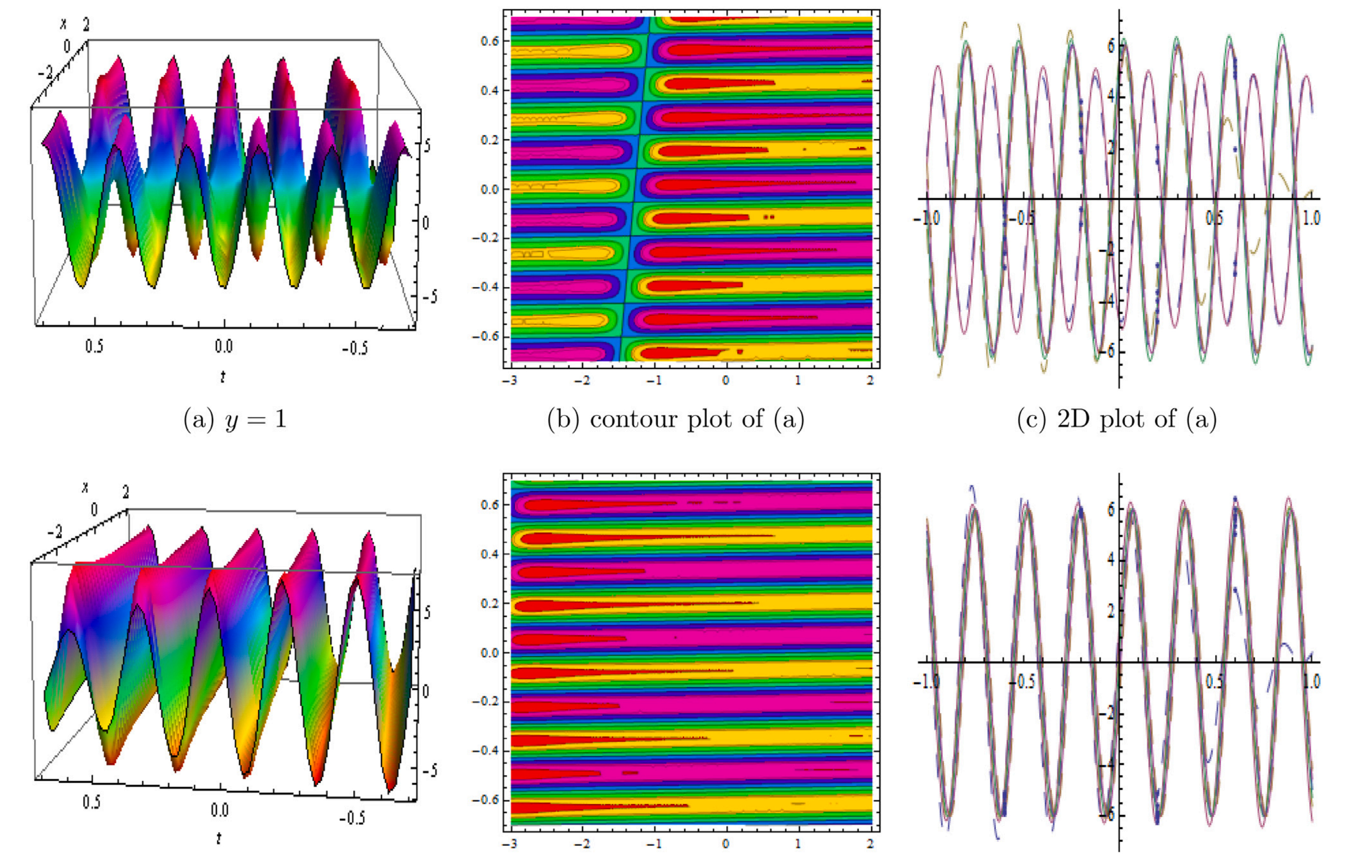
Fig. 16. 3D, 2D and contour plot of $r(x, y, t)$ in Eq. (38) at $b = 0.01, c = 0.5, a_4 = -0.9, a_5 = 3, a_6 = 2, m_1 = 1, m_2 = 2, m_3 = -3, y = 0.1$ successively.

$$r(x,y,t) = \frac{2a_5 e^{i(-2a_5^2 t + b^2 t + bx + cy)} \left(-e^{-a_2 - a_5(t+x+y)} + e^{a_2 + a_5(t+x+y)} m_1 - i m_2 \sin(a_4 + i a_5(t+x+y)) + m_3 \sinh(\Omega)\right)}{e^{-a_2 - a_5(t+x+y)} + e^{a_2 + a_5(t+x+y)} m_1 + m_2 \cos(a_4 + i a_5(t+x+y)) + m_3 \cosh(a_6 + a_5(t+x+y))}, \tag{38}$$

$$s(x,y,t) = -\frac{2a_5^2 \left(e^{-a_2 - a_5(t+x+y)} - e^{a_2 + a_5(t+x+y)} m_1 + i m_2 \sin(a_4 + i a_5(t+x+y)) - m_3 \sinh(a_6 + a_5(t+x+y))\right)^2}{\left(e^{-a_2 - a_5(t+x+y)} + e^{a_2 + a_5(t+x+y)} m_1 + m_2 \cos(a_4 + i a_5(t+x+y)) + m_3 \cosh(a_6 + a_5(t+x+y))\right)^2},$$

Box IV.

multi M-shaped periodic waves. In Fig. 8, we have seen large multi-peak waves. In Fig. 9, we obtained a large number of multiple dark and bright solutions. In Fig. 10, we seen a large number of bright lump type multiple parallel waves. In Fig. 11, we have seen multiple dark and bright solution of equal sizes. In Fig. 12, we have seen M-shaped



(a) $y = 1$ (b) contour plot of (a) (c) 2D plot of (a)

(d) $y = 3$ (e) contour plot of (d) (f) 2D plot of (d)

Fig. 17. 3D, 2D and contour plot of $r(x,y,t)$ in Eq. (41) at $b = 0.1, c = 0.5, a_1 = -2, a_2 = -1, a_5 = 3, a_6 = 2, a_7 = 4, m_3 = -3$ successively.

$$r(x,y,t) = \frac{2e^{i((a_1^2-3a_5^2+b^2)t+bx+cy)}\left(-a_1 e^{-a_2-a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)} + \frac{a_1a_5^2a_7^2 e^{a_2+a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)}}{4(a_1-a_5)(a_1+a_5)} + a_5m_3\sinh(\Omega)\right)}{a_7 + e^{-a_2-a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)} + \frac{a_5^2a_7^2 e^{a_2+a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)}}{4(a_1-a_5)(a_1+a_5)} + m_3\cosh(\Omega)},$$

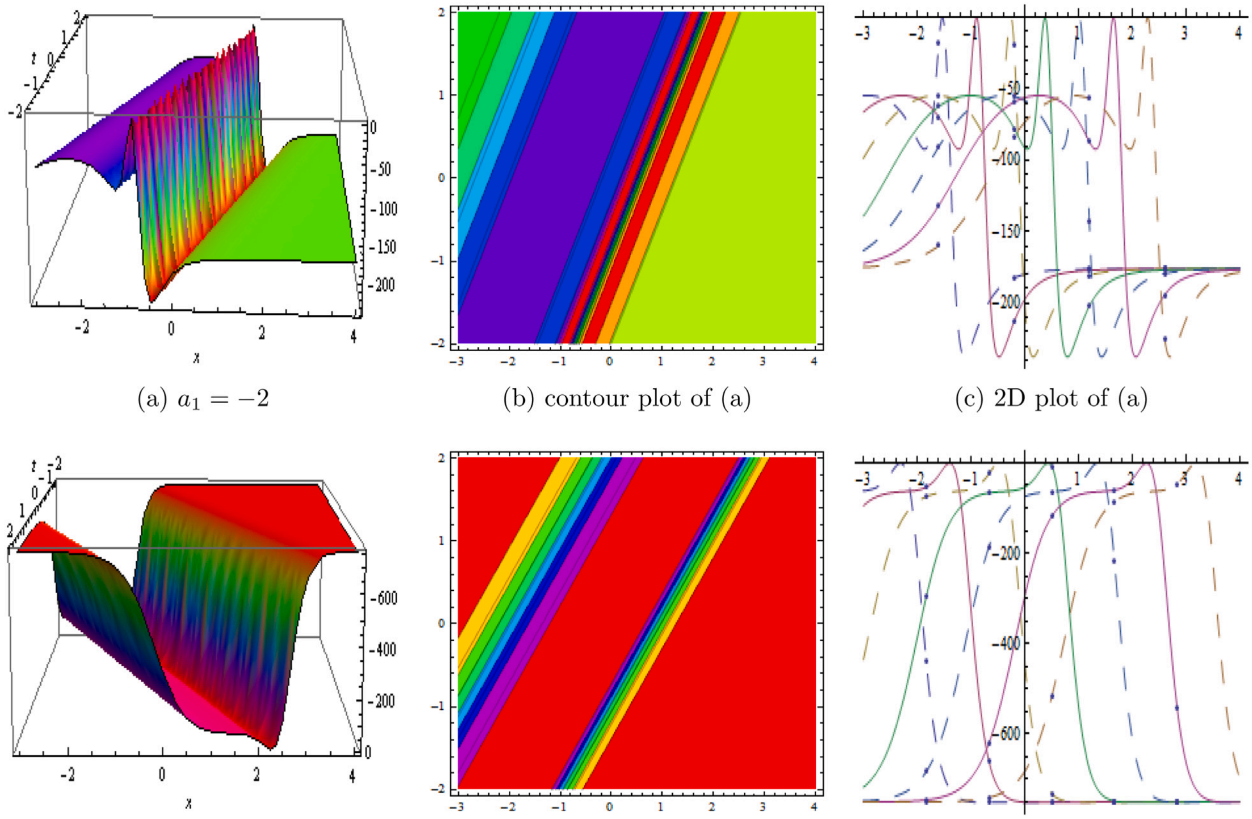
$$s(x,y,t) = \frac{(a_1^2-3a_5^2)\left(-a_1 e^{-a_2-a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)} + \frac{a_1a_5^2a_7^2 e^{a_2+a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)}}{4(a_1-a_5)(a_1+a_5)} + a_5m_3\sinh(\Omega)\right)^2}{a_1^2\left(a_7 + e^{-a_2-a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)} + \frac{a_5^2a_7^2 e^{a_2+a_1\left(\frac{-5a_1^2t+3a_5^2t}{a_1^2-3a_5^2}+x+y\right)}}{4(a_1-a_5)(a_1+a_5)} + m_3\cosh(\Omega)\right)^2}, \tag{41}$$

Box V.

periodic having large amplitude waves. In Fig. 13, we have seen M-shaped periodic waves. In Fig. 14, we seen M-shape solution. In Fig. 15, we get an M-shaped large-amplitude periodic waves. In Figs. 16–18, we represents the evolution of kink and periodic wave, and also draw their contour and 2D plots. In Fig. 16, we obtained multiple bright and dark solitons with periodic wave for disjoint values of a_2. We can see at $a_2 = 3$ the structure completely turned into a periodic wave. In Fig. 17, a strong periodic wave and multiple bright and dark solitons appear for solution set r. Fig. 18 represents the solution set s at different values of a_1 respectively.

Concluding remarks

In this article, we studied some earlier researcher's work and investigates some new results. Our aim for this paper was to obtained rogue, multi-waves, breathers, M -shaped rational solitons and periodic



(a) $a_1 = -2$ (b) contour plot of (a) (c) 2D plot of (a)

(d) $a_1 = -1$ (e) contour plot of (d) (f) 2D plot of (d)

Fig. 18. 3D, 2D and contour plot of $s(x, y, t)$ in Eq. (41) at $a_2 = 2, a_5 = 4, a_6 = -3, a_7 = 8, m_3 = -1, y = 1$ successively.

cross kink solutions for Maccari-system. For this we used rational and traveling wave transformations and distinct bilinear forms and also set up their 3D, 2D and contour structures. These forms provide us better and powerful scientific tools for solving nonlinear systems.

CRediT authorship contribution statement

Syed T.R. Rizvi: Visualization, Investigation. **Aly R. Seadawy:** Conceptualization, Methodology. **M. Aamir Ashraf:** Data curation, Writing – original draft. **Muhammad Younis:** Software, Validation, Supervision. **Abdul Khaliq:** Reviewing and Editing. **Dumitru Baleanu:** Reviewing and Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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