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MASTER THESIS

FUZZY RULE-BASED MODELING IN QUEUING SYSTEMS

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ABSTRACT

FUZZY RULE-BASED MODELING
IN QUEUING SYSTEMS

Farzaneh GHOLAMI ZANJANBAR

M.S, Social Sciences

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In this thesis, a new hard clustering method is proposed to provide objective knowledge for the fuzzy queuing systems. In this method, locally linear controllers are extracted and translated into the first-order Takagi-Sugeno rule based fuzzy model. In this extraction process, the region of fuzzy subspaces of available inputs corresponding to different implications is used to obtain the clusters of outputs of the queuing system. Then, the multiple regression functions associated with these separate clusters are used to interpret the performance of queuing systems. Some applications of the proposed method including calculations of performances and cost analysis with some comparisons are presented and the results are discussed.

Keywords: queuing system, fuzzy rule base, clustering, linear controller, performance.

ÖZ

KUYRUK SİSTEMLERİNDE BULANIK KURAL TABANLI MODELLEME

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Sistemlerin kontrol uygulamarında bulanık mantığın kullanıldığı tipdekı problemlerde büyük başarı ve beğeni kazanılmıştır. Dolayısıyla, gerçek yaşamlarda, kuyruk sistemlerinin bulanık kavramı ile tasarımı ve kontrolü daha gerçekçi ve uygulanabilir olacaktır.

Bu tez çalışmasında, bulanık kuyruk sistemleri için nesnel bilgi sağlamaya yönelik yeni bir sabit kümeleme yöntemi önerilmiştir. Bu yöntemde, yerel doğrusal denetleyicileri ayıklanıp birinci dereceden bulanık kural tabanlı Takagi-Sugeno modeline dönüştürülmektedir. Bu ayıklama işleminde, kuyruk sisteminin çıktı kümelerini elde etmek için farklı yansımalara karşılık gelen varolan girdilerin bulanık altuzayları bölgesi kullanılır. Daha sonra, bu ayrı kümeler ile ilişkili çoklu regresyon fonksiyonları kuyruk sistemlerinin performansını değerlendirmek için kullanılmıştır. Önerilen yöntem ile performanslar ve maliyet analizi hesaplamalarını içine alan bazı uygulamalar sunulmuştur ve sonuçlar tartışılmıştır.

Anahtar Kelimeler: kuyruk sistemi, bulanık kural tabanı, kümeleme, lineer kontrolör, performans.

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INTRODUCTION

In this thesis, a new hard clustering method is presented to provide objective knowledge for the fuzzy queuing systems. In many literatures, probability distributions estimate the arrival times and service times. On the other hand, in many real-world applications there are the linguistic terms, such as “Crowded” arrivals, “Fast” or “Slow” services that describe the arrival and service patterns instead of the probability distributions. As you know in most of practical applications, both arrival times and service times are possibilistic. Therefore, design and control of the queuing system with fuzzy concept is more realistic and applicable. Controlling the queues occupy is an important place in our lives where control applications in decision making and management based on fuzzy logic has had the highest success.

With using the Zadeh’s extension principle (Zadeh, 1978), the possibility concept, and fuzzy Markov chain (Stanford, 1982), the problem of fuzzy queues has been inquired by Li & Lee (1989), Buckley (1990), Negi & Le (1992) and so on. Aydın & Apaydin (2008) the fuzzy queuing control parameters with different membership functions are considered. Wang & Yang and Li. (1999) the fuzzy queues are transformed to a group of crisp queues by using the $\alpha - cut$ method and Zadeh’s extension principle.

Three key features of control systems are: inputs, outputs, and control parameters or, control actions (Timothy, 2004). For instance, priority dicipline machine for entering customers to different queues in the banks is a control mechanism where inputs are arrival and service time rates of customers, outputs are the length of the queues, and the control actions are the altering the queue disipline, capacity and etc.

Consider to a wide-range of real-world practical applications, description of dynamic systems with available input-output data is a critical moot point of the scientific research. Usually the input and output relationship of a process in fuzzy logic controller is expressed by “if-then rules”, as:

If *the interarrival is crowded* ***then*** *the length of queue is long.*

Since many real systems are innately nonlinear, the conventional systems can not identify these systems by linear models (Ljung, 1987). Currently, there are many capable studies to improve the nonlinear system identification methods using available data. The TSK (Takagi, Sugeno, & Kang) method was proposed in generating fuzzy rules using available input-output data set (Takagi and Sugeno, 1985; Sugeno and Kang, 1988). In the TSK rule based fuzzy model, a linear membership function in each implication is formed to describe the real relation of input-output in the system. Comparisons of clustering algorithms in the identification of Takagi-Sugeno model (Fazel Zarandi, 2012; Abonyi, 2000; Johansen, 2000) are presented by Vernieuwe (2006).

In recent researches, clustering technique is being utilized for extracting fuzzy rule consequences which requires the user to identify structure of the knowledge or the rule base. Clustering is a method of classification of patterns or data item or observations into clusters or groups and is helpful in constructing fuzzy rules from data (Timothy, 2004). The clustering algorithm needs the user to define the initial location of the cluster. Every cluster represents a set of typical data points covering the range of data behavior. There are various clustering algorithms using optimization techniques to identify the antecedents of a system in some literatures such as Gath-Geva clustering algorithm (1989), modified Gath–Geva fuzzy clustering algorithm (Abonyi, 2002), the Gustafson-Kessel clustering algorithm (1979), the subtractive clustering algorithm (Chiu, 1994).

This thesis shows a new hard clustering method in identification and simulation of fuzzy queuing systems just only using available input data set (Şentarlı and G. Zanjanbar, 2013). We proposed estimated an output data set and develop a mathematical approach in generating a rule based fuzzy model using a given input and the virtual output data sets. A crisp output data set is produced, using arrival and service rate data sets and the queuing system performance expression function. The calculated output data set is separated into few clusters due to the region of fuzzy subspaces of available inputs. Each cluster generates approximate linear membership function for related implication. In this thesis, a computer source code is provided for the new mathematical approach which derives the linear membership functions to explain the real input and calculated output relation of the queuing system performances based on human interpretable information.

This thesis consists of 3 chapters. In the next section, we have a glance on the basic knowledge about fuzzy theory and the classical and fuzzy queuing systems with infinite capacity. Chapter 2 will be devoted to define the proposed new method of clustering to develop the first-order Takagi-Sugeno rule based fuzzy model on field of multi server queuing systems, using input data. In chapter 3, the realistic examples are illustrated the applicability of proposed approach. In this chapter, a cost analysis of a queuing system via new method and the comparisons of the proposed method with the conventional method are presented. The comparisons between new proposed model and the fuzzy N-Policy queuing system based upon α-cut method results on predicting performances in a queuing system are presented, too. Conclusions are provided at the end of this study.

CHAPTER I

FUZZY LOGIC ESSENTIALS

1.1. BENEFITS OF FUZZY LOGIC

Fuzzy logic is a technique that systematically and mathematically attempts to analyze human reasoning and decision making. Fuzzy logic provides engineers with a clear and intuitive way to implement control systems, decision-making and diagnostic systems in various branches of industry (Babuska, 1997). So that, fuzzy logic allows exploiting engineers' empirical knowledge represented in the "if/then" rules and transfer it to a function. Fuzzy logic algorithms can be used for advanced applications in industrial automation such as:

- ***Intelligent control systems*****:** Fuzzy control solutions are especially useful for complex systems where standard control fails. Fuzzy logic is an advantage in conventional analytical-process models which are too involved. Fuzzy logic can be easily combined with conventional controllers and mainly improved their functionality, that, it is another advantage of fuzzy logic. For instance, fuzzy rules interpolate between a sets of locally linear controllers and plan improvements of a system controller based on changing operating conditions. So fuzzy rules do not necessarily have to displace conventional control methods, but rather develop their potentialities.
- ***Process diagnostics, fault detection*****:** If an analytical process model is not accessible or is too compound to be run in real-time, empirical knowledge can be used to classify process conditions and early detect faults.
- ***Decision-making and expert systems*****:** Fuzzy rules can analyze an experienced human operator in real time, e.g. select appropriate ingredients, components or machines according to specific situations in the manufacturing process.

1.2. FUZZY SETS

Prof. Lotfi Zadeh introduced the concept of fuzzy sets in 1965. Since then, the theory has been developed by many researchers and application engineers.

In classical set theory, a membership function define a set that assigns each element a degree of membership (0 or 1) so that, 0 means the element is not member of the set and 1 means the element is member of the set. The classical (crisp) Fuzzy sets include the degree of memberships in which are any values in the real unit interval [0, 1].

Let's assume that we have defined three classical sets "Not Crowded", "Moderate" and "Crowded" for variable arrival rate (see Figure. 1.1). If we want to classify (evaluate degree of membership) for example for value 23 to these sets, we get value 1 for set "Moderate" and 0 for sets "Not Crowded" and "Crowded". Vague classification will be more realistic and thus closer to human reasoning, because no sharp distinction usually exists between moderate and crowded arrival rates, as one arrival rate reading can be moderate to some extent (0.8) and crowded to another one (0.2), see Figure. 1.2.

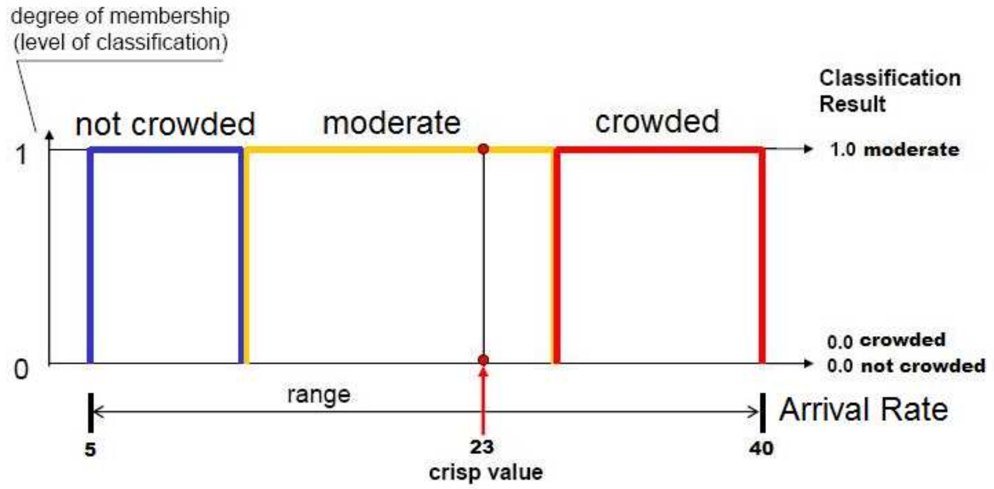


Figure 1.1 Classical sets.

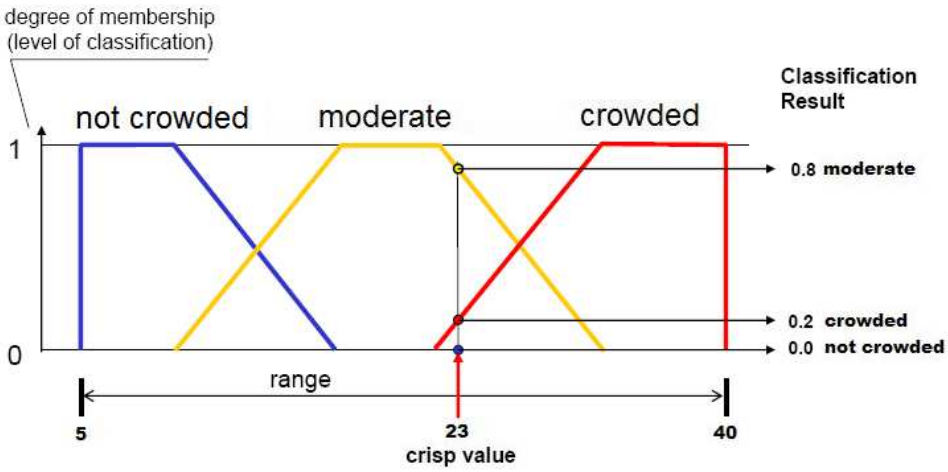


Figure 1.2 Fuzzy sets.

1.3. MEMBERSHIP FUNCTIONS

A function-theoretic form maps fuzzy set elements to a universe of membership values. In this text, a set symbol with a tilde above strike, denote a fuzzy set; for example, Ã is the fuzzy set A. each element of a fuzzy set Ã is assigned to a real number value on the interval 0 to 1 with a membership function.

If x is a member of fuzzy set Ã, then this mapping is extracted by $\eta_{\tilde{A}}(x) \in [0,1]$.

In our particular example, the variable x is the arrival rate, X is the range [5, 40], Ã is e.g. “moderate” and for x=23 we get $\eta_{\tilde{A}}(23) = 0.8$.

The variable x is called the *linguistic variable* and corresponding fuzzy sets defined on the range are called *linguistic terms* described by membership function. For example, the linguistic variable arrival rate has terms “Not Crowded”, “Moderate” and “Crowded”. The process of classification of a particular value of the variable x to corresponding fuzzy sets is called *fuzzification.*

The most commonly used membership functions are in Figure 1.3. Singleton, whose degree of membership is 1 just for a single value c and 0, otherwise, is used just for output linguistic variables.

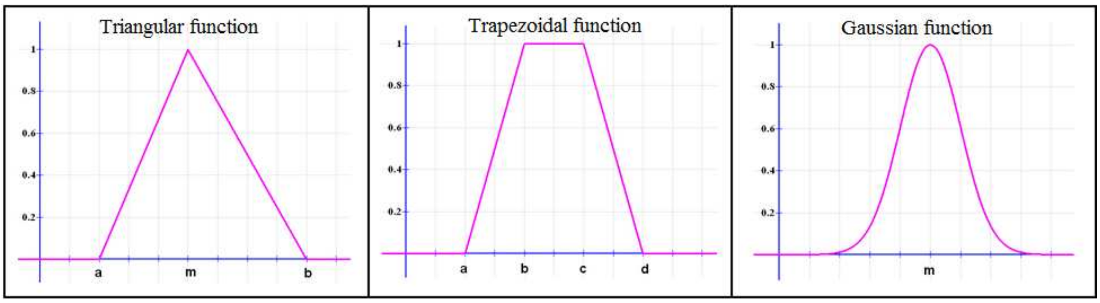


Figure 1.3 Types of membership functions.

1.4. FUZZY SET OPERATIONS

Suppose two fuzzy sets $\tilde{A}$ and $\tilde{B}$ on the universe X. For $\forall x \in X$, the following union, intersection, and complement function operations are denoted for $\tilde{A}$ and $\tilde{B}$ on X:

Union $\quad \eta_{(\tilde{A} \cup \tilde{B})}(x) = \eta_{\tilde{A}}(x) \vee \eta_{\tilde{B}}(x)$

Intersection $\quad \eta_{(\tilde{A} \cap \tilde{B})}(x) = \eta_{\tilde{A}}(x) \wedge \eta_{\tilde{B}}(x)$

Complement $\quad \eta_{\bar{\tilde{A}}}(x) = 1 - \eta_{\tilde{A}}(x)$

1.5. FUZZY SYSTEMS

A fuzzy system is a static or dynamic system which utilizes fuzzy sets or fuzzy logic. To involve the fuzzy sets in a system, there are some ways (Babuska, 1997), such as:

- ***In the explanation of the system*:** A number of if-then rules or fuzzy relations with fuzzy means can explain a system. For instance, the relationship between the arrival rate and the length of queue in a queuing system is described by a fuzzy rule as :

If *the arrival rate is crowded* ***then*** *the expected length of queue will be long*

- ***In the system's parameters specification*:** An algebraic or differential equation can explain a fuzzy system, where the parameters are fuzzy numbers, not real numbers.

- ***The fuzzy input sets and system state variables*****:** Vague information related to human linguistic terms can extract fuzzy inputs, such as fast, slow, etc. Fuzzy systems can process the information of this type of data, which is not usable with conventional (crisp) systems.

Some of the above properties can be included in a fuzzy system. Table 1.1 presents the relationships in fuzzy and crisp system descriptions (Babuska, 1999).

Table 1.1 Crisp and fuzzy data in some system descriptions.

System description	**Input data**	**Conclusion**	**Mathematical method**
Crisp	Crisp	Crisp	conventional analysis
Crisp	Fuzzy	Fuzzy	Zadeh's and Mamdani extension principle
Fuzzy	Crisp/Fuzzy	Fuzzy	fuzzy analysis

1.5.1. Fuzzy Inference

A fuzzy inference is a mechanism for evaluation of the fuzzy system, i.e. computing output values from input values. The fuzzy analysis consists of the following steps:

1. ***Fuzzification:*** Inputs are classified to corresponding linguistic terms to get premises membership functions.
2. ***Fuzzy rules evaluation*****:** The membership function term of conclusions are calculated from premise membership function terms and logical operations.
3. ***Defuzzification*****:** Output linguistic terms are converted to a real crisp value according to their membership functions.

1.5.1.1.Defuzzification

In the most of processes, to analyze the output result of a fuzzy inference, we need a single scalar amount instead of a fuzzy set. Just as fuzzification is the alteration of an exact amount to a fuzzy amount, defuzzification is the alteration of a fuzzy amount to an exact amount. Recently, in many literatures, *weighted average method* is proposed for defuzzifying fuzzy output membership functions.

- ***Weighted average method***: The output is computed with weighted average of the each output of the set of rules, based on knowledge of the system. This type of defuzzification method is defined as:

$$Y = \frac{\sum_{i=1}^{n} w_i y^i}{\sum_{i=1}^{n} w_i}$$

where Y is the defuzzified output, y^i is the membership function of the output of each rule, and w_i is the weight of each rule. This fast and easy computable method gives adequately precise conclusion. In this thesis, this method has been first summarized and then illustrated in examples.

- ***α – cut method:*** The study begin by considering a fuzzy set $\tilde{A}$, then define an *α* -cut set, A_α, where 0 ≤ *α*≤ 1.The set A_α is a crisp set called the (*α*)-cut set of the fuzzy set $\tilde{A}$,where $A_\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\}$. Note that the *α* -cut set $\tilde{A}$ does not have a tilde above score; it is a crisp set derived from its parent fuzzy set, $\tilde{A}$. Whereas an infinite number of values *α* in the interval [0, 1], every fuzzy set $\tilde{A}$ can be translated into an infinite number of *α* -cut sets. Any element $x \in A_\alpha$ associates to $\tilde{A}$ with a grade of membership that is greater than or equal to the value *α*.

1.5.2. Some Practical Relevancy of Fuzzy Modeling

- ***Lacking or Ambiguous knowledge about systems.*** The system behaviors can be described by conventional system theory just using crisp mathematical methods. For instance, mathematical models of queuing systems can be achieved with algebraic equations. In the most of systems, the comprehensions' of the fundamental phenomenon's is incompletely and crisp conventional methods can not be analyzed or too complex. The biotechnology, finance, chemical, sociology, ecology are examples of such systems. The knowledge of human expert's is available as a useful part of information about these systems. Explanation of this ambiguous and uncertain knowledge may be too difficult for crisp conventional methods. On the other hand, it seems often possible to define the behavior of systems in the form of if-then rules, by terms of natural language. "Fuzzy rule-based systems can be used as knowledge-based models constructed by using knowledge of experts in the given field of interest "(Pedrycz, 1990; Yager and Filev, 1994). Hence, fuzzy systems are very much alike to intelligence systems researched widely in the "symbolic" artificial intelligence (Buchanan and Shortliffe, 1984; Patterson, 1990).

- ***Imprecise information processing*.** Crisp exact numerical results with conventional mathematical methods only can be extracted with the correctly known parameters and input data. A modeling framework is required the processable data and associated uncertainty. Dealing with uncertainty is a usual way in stochastic approach. However, the stochastic framework cannot deal all types of uncertainty. Fuzzy logic and set theory is one of various alternative approaches which have been proposed (Smets, 1988).
- ***Fuzzy modeling and identification*.** Today's, in scientific researches, identification of dynamic systems from input data are an important matter. Linear models used in conventional system identification, cannot extract many nonlinear real systems (Ljung, 1987). Newly, the nonlinear system methods are developed successfully from available data. Mathematical approaches in fuzzy systems can approximate other approaches or functions flexibility with a wanted precision. This effect is called "general function approximation" (Kosko, 1994; Wang, 1994; Zeng and Singh, 1995). In comparison to other well-known techniques like artificial neural networks, fuzzy systems provide a more clear description of the system according to the possible linguistic reasoning by the structure of implications. The logical structure of the rules makes the analysis of the model easier and close to human linguistics.

1.6. RULE-BASED FUZZY MODELS

The relationships between input-output data in rule-based fuzzy models are described by means of fuzzy if–then rules as form of:

If antecedent proposition **then** consequent proposition.

The type "x is Ã" is always a fuzzy antecedent proposition where x is a linguistic variable and Ã is a linguistic term. The degree of x in fuzzy set Ã is a real number between zero and one. Two main types of rule-based fuzzy models are specified based on the form of the consequent:

- ***Linguistic fuzzy model*:** the fuzzy propositions are composed of the antecedent as well as the consequent
- ***Takagi–Sugeno (TS) fuzzy model*:** The only fuzzy proposition is antecedent and the consequent is in the form of a crisp function.

Notice to the explanation of these two different fuzzy models in the following subsections in detailed meaning.

1.6.1. Linguistic Fuzzy Model

Available qualitative knowledge in this model (Zadeh, 1973; Mamdani, 1977), translates into the form of if–then rules:

R_i : **If** x **is** $\tilde{A}_i$ **then** y **is** $\tilde{B}_i$ i=1, 2, ..., n.

In this expression, x denotes the antecedent input variable, and $\tilde{A}_i$ resembles the antecedent linguistic terms. Alike, y denotes the consequent output variable and $\tilde{B}_i$ resembles the consequent linguistic terms. The crisp x or y are defined in the regions of their universe base variable sets: $x \in X$ and $y \in Y$. The membership functions of the antecedent and consequent fuzzy sets are: $\eta(x): X \to [0,1]$, $\eta(y): Y \to [0,1]$. Fuzzy domains are defined by fuzzy sets $\tilde{A}^i$ in the antecedent space, depending on the related consequent space. The linguistic terms $\tilde{A}^i$ and $\tilde{B}_i$ are usually chosen from sets of terms, such as "Slow", "Fast", etc. By meaning of these sets by $\tilde{A}$ and $\tilde{B}$, we have $\tilde{A}^i \subset \tilde{A}$ and $\tilde{B}^i \subset \tilde{B}$, respectively. The rule set

$R = \{R^i | i = 1,2,...,n\}$ and the constitute of the sets $\tilde{A}$ and $\tilde{B}$, make available the basic knowledge consider to the linguistic model.

1.6.2. Takagi–Sugeno (TS) Fuzzy Model

The linguistic model explains a given system behaviors by means of linguistic if-then rules with the fuzzy antecedents and fuzzy consequents using Zadeh's extension principle. On the contrary, instead of fuzzy consequent, the crisp consequent functions are used in the TS fuzzy rule based model. Therefore, there is a relation between linguistic and mathematical regression equation where the fuzzy domains are defined by antecedents according to the input space in which consequent equations are valid. The TS rules are produced as:

R^i : If x is $\tilde{A}^i$ then $y^i = f^i(x)$, i=1, 2…, n (1.1)

In most cases, a vector-valued function f^i is extracted from a nonlinear function. The functions f^i have the same structure with the different parameters in each rule. Simple and practically useful linear parameterizations in the rules have the form:

R^i : If x is $\tilde{A}^i$ then $y^i = a^{i^T} x + b^i$, i=1, 2… n (1.2)

where a^i is a parameter vector and b^i is a scalar offset.

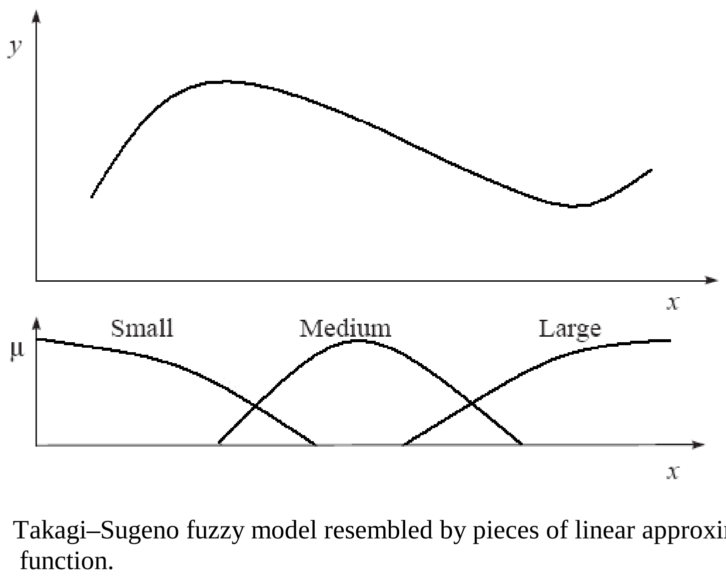


Figure 1.4 Takagi–Sugeno fuzzy model resembled by pieces of linear approximations of a nonlinear function.

1.7. QUEUING SYSTEM WITH INFINITE CAPACITY (M/M/S)

In this thesis, a multi servers M/M/s queuing system with infinite capacity is considered. In this system, following assumptions are supposed:

- There is a single server in each waiting line.
- The frequencies of arrival rate are resembled by a Poisson distribution (λ).
- The service times obey an exponential probability distribution.
- There is first-come, first-served (FCFS) discipline in the queue.
- There is no balking or reneging.

Various arrival and service time stochastic processes within the system are assumed to be independent of each other. Here, L_q and W_q resemble the expected number of customers in the queue and the expected waiting time in the queue, respectively. Consider the two system performances L_q and W_q can easily be derived by a Markov chain approach. It follows that

$$L_q = \frac{(\lambda/\mu)^S (\lambda/s\mu)}{S!(1-\lambda/s\mu)^2 \Pr_0} \quad (1.3)$$

where $$\Pr_0 = \frac{1}{\left[\sum_{n=0}^{S-1} 1/n!(\lambda/\mu)^n\right] + 1/S!(\lambda/\mu)^S (\frac{s\mu}{s\mu-\lambda})}$$

$$W_q = \frac{L_q}{\lambda} \quad (1.4)$$

For the steady-state conditions, we have $0 < \lambda/\mu < 1$ and $s\mu > \lambda$.

1.8. FUZZY QUEUING SYSTEM WITH INFINITE CAPACITY FM/FM/S

A fuzzy multi server queuing system with infinite capacity is considered with following assumptions:

- The frequencies of fuzzy arrival rate are resembled by a Poisson distribution $\tilde{\lambda}$.

- The fuzzy service times obey an exponential probability distribution..
- $\tilde{\lambda}$ and $\tilde{\mu}$ are uncertain.
- $\tilde{\lambda}$ and $\tilde{\mu}$ by the convex fuzzy sets are represented.
- $\eta_{\tilde{\lambda}}(\lambda)$ and $\eta_{\tilde{\mu}}(\mu)$ define the trapezoidal membership functions of $\tilde{\lambda}$ and $\tilde{\mu}$, respectively.

For this FM/FM/s queue model we have:

$$\tilde{\lambda} = \{(x, \eta_{\tilde{\lambda}}(x)) | x \in \lambda\}$$

$$\tilde{\mu} = \{(x, \eta_{\tilde{\mu}}(x)) | x \in \mu\}$$

where λ and μ consists of the crisp sets of the arrival and service rates, respectively as:

$$\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}, \mu = \{\mu_1, \mu_2, \ldots, \mu_n\}$$

1.9. FUZZY N-POLICY QUEUING SYSTEM BASED UPON α-CUT MODEL

In this part, we consider to present an FM/FM/1 N-policy fuzzy queuing model with infinite capacity based upon α-cut model. This fuzzy queuing model is developed by the N-policy M/M/1 queuing system utilizing the fuzzy set theory. The fuzzy arrival rate and the fuzzy service rate values are used to develop the membership function using a mathematical approach of the system performance. The fuzzy queues are translated to a group of crisp queues based on the α-cut model and Zadeh's extension principle.

Let W_q define the expected waiting time in the queue. W_q can be simply derived by a Markov chain approach. It is:

$$W_q = \frac{N-1}{2\lambda} + \frac{\lambda}{\mu(\mu - \lambda)} \qquad (1.5)$$

where $0 < \lambda / \mu < 1$ since we have the steady-state condition.

Suppose $f(x, y)$ define the system performances. Since $\tilde{\lambda}$ and $\tilde{\mu}$ are fuzzy numbers, $f(\tilde{\lambda}, \tilde{\mu})$ is also a fuzzy number. With Zadeh's extension principle (Zadeh, 1978 and Zimmermann, 2001), the membership function of the system performance $f(\tilde{\lambda}, \tilde{\mu})$ is obtained as:

$$\eta_{f(\tilde{\lambda},\tilde{\mu})}(z) = \sup_{\Omega} \min\{\eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y) \mid z = f(x,y)\} \quad (1.6)$$

where $\Omega = \{x \in X, y \in Y \mid x \rangle 0, y \rangle 0\}$

In the N-policy M/M/1 queuing system, when the arrival rate x and the service rate y are crisp values, the crisp expected waiting time in the queue is such as:

$$W_q = \frac{N-1}{2x} + \frac{x}{y(y-x)} \quad (1.7)$$

The membership functions of $\tilde{W}_q$ as a result, become:

$$\eta_{\tilde{W}_q}(z) = \sup_{\Omega} \min\left\{\eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y) \middle| \frac{N-1}{2x} + \frac{x}{y(y-x)}\right\} \quad (1.8)$$

For practical use, the membership function in equation (1.8) is not explained in the typical forms and inferring the shapes of the related membership function with $\tilde{W}_q$ is too difficult. Therefore, the α-cuts of $\tilde{W}_q$ can be derived by applying the Zadeh's extension principle to solve this problem. Consider to above explains, let us define the α-cuts of $\tilde{\lambda}$ and $\tilde{\mu}$ as follows:

$$\lambda(\alpha) = \{x \in X \mid \eta_{\tilde{\lambda}}(x) \geq \alpha\} \quad (1.9)$$

$$\mu(\alpha) = \{y \in Y \mid \eta_{\tilde{\mu}}(y) \geq \alpha\} \quad (1.10)$$

$\tilde{\lambda}$ resembles the fuzzy arrival rate and $\tilde{\mu}$ resembles the fuzzy service rate of the N-policy FM/FM/1 system. Hence, consider to equations (1.9) and (1.10), the crisp α-cut sets of $\tilde{\lambda}$ and $\tilde{\mu}$ can be redefined in such as:

$$\lambda(\alpha) = [x_\alpha^L, x_\alpha^U] = \left[\min_{x \in X}\{x \in X \mid \eta_{\tilde{\lambda}}(x) \geq \alpha\}, \max_{x \in X}\{x \in X \mid \eta_{\tilde{\lambda}}(x) \geq \alpha\}\right] \quad (1.11)$$

$$\mu(\alpha) = [y_\alpha^L, y_\alpha^U] = \left[\min_{y \in Y}\{y \in Y \mid \eta_{\tilde{\mu}}(y) \geq \alpha\}, \max_{y \in Y}\{y \in Y \mid \eta_{\tilde{\mu}}(y) \geq \alpha\}\right] \quad (1.12)$$

"From equations (1.11) and (1.12), it indicates that $\tilde{\lambda}$ and $\tilde{\mu}$ are lying the range of $[x_\alpha^L, x_\alpha^U]$ and $[y_\alpha^L, y_\alpha^U]$, respectively, at a possible level α" (Yin Wang, 2010). As you see, the N-policy FM/FM/1 queue decreases to a group of crisp N-policy D/D/1 queue with different α-level sets $\{\lambda(\alpha) \mid 0 < \alpha < 1\}$ and $\{\mu(\alpha) \mid 0 < \alpha < 1\}$. The upper and lower bounds of $\tilde{\lambda}$ and $\tilde{\mu}$ can be presented as functions of α as $x_\alpha^U = \max \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $x_\alpha^L = \min \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $y_\alpha^U = \max \eta_{\tilde{\mu}}^{-1}(\alpha)$ and $y_\alpha^L = \min \eta_{\tilde{\mu}}^{-1}(\alpha)$ by the principal property of convex fuzzy numbers (Zimmermann, 2001). Since, α is the parameter of the membership function $\tilde{W}_q$ therefore, the α-cuts approaches are used to develop the membership function $\tilde{W}_q$.

The membership function of the expected waiting time in the queue $\eta_{\tilde{W}_q}(z)$ is the minimum of $\eta_{\tilde{\lambda}}(x)$ and $\eta_{\tilde{\mu}}(y)$ that is derived based on the Zadeh's extension principle.

By solving the corresponding parametric nonlinear program the lower $(W_q)_\alpha^L$ and $(W_q)_\alpha^U$ upper limiting values of α-cuts of $\tilde{W}_q$ can be found as:

$$(W_q)_\alpha^L = \min_{\Omega}\left(\frac{N-1}{2x} + \frac{x}{y(y-x)}\right), \qquad (1.13)$$

$$\text{s.t.} \quad x_\alpha^L \le x \le x_\alpha^U \quad \text{and} \quad y_\alpha^L \le y \le y_\alpha^U$$

and

$$(W_q)_\alpha^U = \max_{\Omega}\left(\frac{N-1}{2x} + \frac{x}{y(y-x)}\right) \qquad (1.14)$$

$$\text{s.t.} \quad x_\alpha^L \le x \le x_\alpha^U \quad \text{and} \quad y_\alpha^L \le y \le y_\alpha^U .$$

The optimal solution changing's are characterized by mathematical approaches in this model with x_α^L, x_α^U, y_α^L and y_α^U when value of α modifies between 0 and 1 . This model is a part of parametric NLP (Gal, 1979).

According to equations (1.10) and (1.11), we can replace $x \in \lambda(\alpha)$ and $y \in \mu(\alpha)$ by $x \in [x_\alpha^L, x_\alpha^U]$ and $y \in [y_\alpha^L, y_\alpha^U]$, respectively. It is worthy of mention that α-cuts of x and y form an embedded structure based upon α (Zimmermann, 2001). Considering the two possibility levels α_1 and α_2 , we have

$$[x_{\alpha_1}^L, x_{\alpha_1}^U] \in [x_{\alpha_2}^L, x_{\alpha_2}^U] \text{ and } [y_{\alpha_1}^L, y_{\alpha_1}^U] \in [y_{\alpha_2}^L, y_{\alpha_2}^U]$$

where 0 < α_2 < α_1 ≤ 1. Thus, as α increase for $(W_q)^L_{\alpha_1} \geq (W_q)^L_{\alpha_2}$, $(W_q)^L_{\alpha}$ increases and for $(W_q)^U_{\alpha_1} \leq (W_q)^U_{\alpha_2}$, $(W_q)^U_{\alpha}$ decreases. Therefore, the membership function $\eta_{\tilde{w}_q}(z)$ will be computable. Defining an ascending function $(W_q)^L_{\alpha} : \alpha \to (W_q)^L_{\alpha}$ and a descending function $(W_q)^U_{\alpha} : \alpha \to (W_q)^U_{\alpha}$ help us to explain the membership function $\eta_{\tilde{w}_q}(z)$, with invertible the both $(W_q)^L$ and $(W_q)^U$, based upon α such as:

$$\eta_{\tilde{w}_q}(z) = \begin{cases} L(z), & (W_q)^L_{\alpha=0} \leq z \leq (W_q)^L_{\alpha=1} \\ 1, & (W_q)^L_{\alpha=1} \leq z \leq (W_q)^U_{\alpha=1} \\ R(z), & (W_q)^U_{\alpha=1} \leq z \leq (W_q)^U_{\alpha=0} \end{cases} \qquad (1.15)$$

where the left shape function $L(z)$ is $\left[(W_q)^L\right]^{-1}$ and the right shape function $R(z)$ is $\left[(W_q)^U\right]^{-1}$. In section 3.2.1, an example shows the alteration of the membership grade of the system performances at a different α level.

CHAPTER II

PROPOSED RULE BASED FUZZY MODEL IN QUEUING SYSTEMS

In this chapter, the new rule based fuzzy model in the field of queuing system is investigated making use of the Takagi-Sugeno-Kang (TSK) first-order rule based fuzzy model.

2.1 KNOWLEDGE-BASED DESIGN FOR BUILDING PROPOSED FUZZY MODEL

Generally, a fuzzy model according to available expert knowledge is designed as the following steps:

1) Choose variables resembling the input and output, the form of the rules, and the deduction, and methods to defuzzify the solutions.
2) Determine the required amount of linguistic terms for each rule and determine the related functions resembling the membership.
3) Translate the knowledge possessed using the fuzzy if-then rules.
4) Approve the model. If the expected performance is not visible, repeat the above steps.

2.2 FIRST-ORDER TSK RULE BASED FUZZY MODEL

The TSK first-order rule based fuzzy model a given system is described by means of linguistic if-then rules with two inputs and an output. A fuzzy system is represented by first-order TSK model as form of the following implications:

$$R^1: \text{ If } x_1 \text{ is } \tilde{A}^1_{x_1} \text{ and } x_2 \text{ is } \tilde{A}^1_{x_2} \text{ then } y^1$$

$$\vdots \qquad \vdots$$

$$R^n: \text{ If } x_1 \text{ is } \tilde{A}^n_{x_1} \text{ and } x_2 \text{ is } \tilde{A}^n_{x_2} \text{ then } y^n \qquad (2.1)$$

where

y^i Output linear membership functions in the consequences.

x_1, x_2 Crisp variables in the premises.

A_{x_1}, A_{x_2} Membership functions of the fuzzy sets in the premises.

P_0, P_1, P_2 Parameters of linear membership functions in the consequences.

n The number of implications (rules).

The crisp final output of the system is obtained via a weighted average defuzzification, as shown in Figure 2.1.

Final output $Y = \frac{\sum_{i=1}^{n} w_i y^i}{\sum_{i=1}^{n} w_i}$ (2.2)

where $w_i = Min(\tilde{A}^i_{x_1}, \tilde{A}^i_{x_2})$ (2.3)

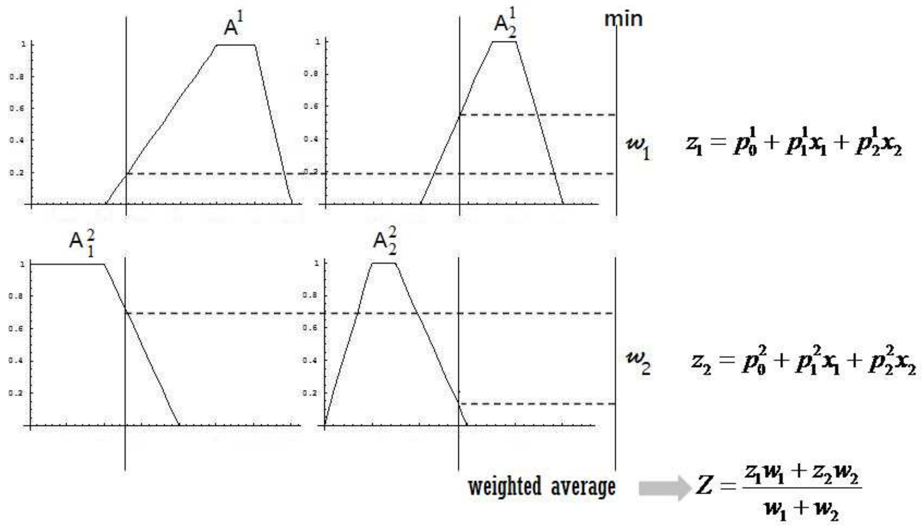


Figure 2.1 The first order Takagi-Sugeno-Kang fuzzy rule based model.

After above explanations, the proposed fuzzy model is denoted by a queuing system in detail in the next sections.

2.2.1 The Linguistic Terms in Premises:

A fuzzy queuing system process involving two fuzzy input sets is analyzed; arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$ to predict crisp performances as expected length of queue, L_q, and expected waiting time in queue, W_q.

In the form of TSK fuzzy rule based model in a queuing system, in order to model (2.1), we have:

R^i: If λ is $\tilde{\lambda}^i$ and μ is $\tilde{\mu}^i$ then y^i, i=1, 2…, n (2.4)

Where $\tilde{\lambda}^i$ and $\tilde{\mu}^i$ are the premise linguistic terms. The linguistic terms $\tilde{\lambda}^i$ and $\tilde{\mu}^i$ are selected from sets of human linguistic terms, such as crowded, slow, fast and etc. The fuzzy sets $\tilde{\lambda}^i$ and $\tilde{\mu}^i$ are defined in their regions over their respective crisp interval variables λ and μ. Fuzzy sets $\tilde{\lambda}^i$ and $\tilde{\mu}^i$ define fuzzy regions in the antecedent space, according to the respective consequent propositions. By denoting these sets by $\tilde{\lambda}$ and $\tilde{\mu}$ respectively, we have $\tilde{\lambda}^i \subset \tilde{\lambda}$ and $\tilde{\mu}^i \subset \tilde{\mu}$.

Example 3.1: Suppose that the crisp interval arrival rate set is λ= [1, 2, 3, 4] per hour and the interval service rate set is μ= [11, 12, 13, 14] per hour as premise values.

The linguistic terms "Crowded" and "Not Crowded" fuzzy subsets of arrival rate and "Fast" and "Slow" fuzzy subsets of service rate set show in Table 2.1 and Table 2.2.

Table 2.1 Linguistic terms, interval arrival rates set and its fuzzy subsets.

	Domain element			
linguistic term	1	2	3	4
Crowded	0	0	0.67	1
NotCrowded	1	1	0	0

Table 2.2 Linguistic terms, interval service time rates set and its fuzzy subsets.

	Domain element			
linguistic term	11	12	13	14
Fast	0	0	0.67	1
Slow	1	0.67	0	0

Trapezoidal membership functions define the meaning of the linguistic terms, as you see in Figure 2.2 and Figure 2.3.

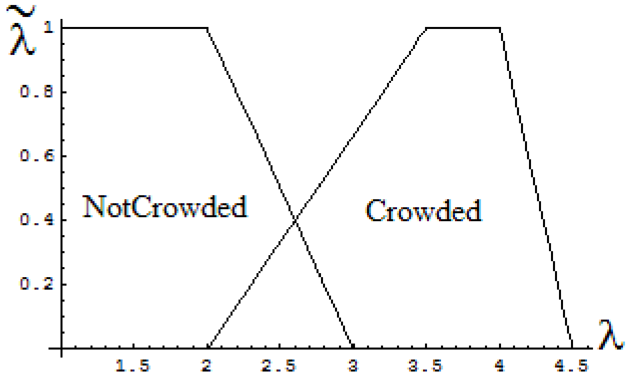


Figure 2.2 ''Crowded'' and ''NotCrowded'' arrival rates per hour.

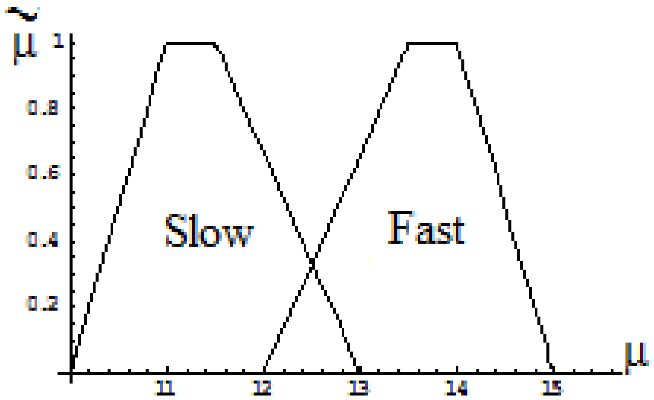


Figure 2.3 ''Fast'' and ''Slow'' service time rates per hour.

Remember that there is not universal way to define the linguistic terms. The numerical variables are selected arbitrarily.

As you see in Figure 2.4 when we use a "Crowded $\tilde{\lambda}_2$" arrival rate and "slow $\tilde{\mu}_1$" service rate, we get out of the process large values of performances in queue; when we input "Not Crowded $\tilde{\lambda}_1$"and " Fast $\tilde{\mu}_2$" into the system, the performances reach to small values; when we input "Not Crowded $\tilde{\lambda}_1$" and " Slow $\tilde{\mu}_1$" or "Crowded $\tilde{\lambda}_2$" and " Fast $\tilde{\mu}_2$" into the system, the performances reach to moderate values.

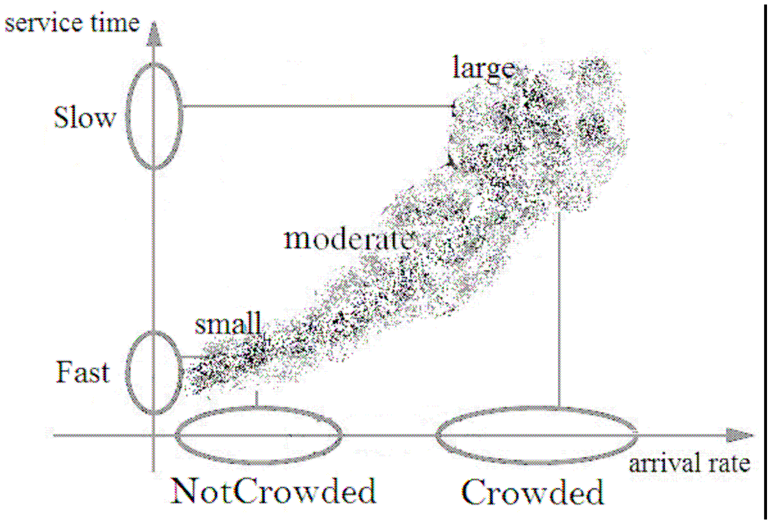


Figure 2.4 The little inputs produce small values and big inputs produce large values on the fuzzy queuing system performances.

2.2.2 Implication Numbers Identification:

In the rule based fuzzy method, the behavior of system translates to some implications. The fuzzy sets $\tilde{\lambda}^i$ and $\tilde{\mu}^i$ are defined in their regions over their respective crisp interval variables λ and μ .If the number of fuzzy subspaces $\tilde{\lambda}^i$ is "A" and the number of fuzzy subspaces $\tilde{\mu}^i$ is "B", (Sugeno, 1985) each model consists of $(A \times B)$ implications. Hence, a queuing system represented by some implications as:

$$\begin{cases} R^1: & \text{if } \lambda_k \text{ is } \tilde{\lambda}^1 \text{ and } \mu_k \text{ is } \tilde{\mu}^1 \text{ then } y^1 \\ \vdots & \vdots \\ R^{(A\times B)}: & \text{if } \lambda_k \text{ is } \tilde{\lambda}^{(A\times B)} \text{ and } \mu_k \text{ is } \tilde{\mu}^{(A\times B)} \text{ then } y^{(A\times B)} \end{cases} \qquad (2.5)$$

The rule set $R = \{R^i \mid i = 1,2,\ldots,(A \times B)\}$ and the knowledge base of the linguistic model is constituted with the sets λ and μ .

Example 3.2:

As you see in Example 3.1, $\tilde{\lambda}$ is divided into two fuzzy subspaces "Crowded" and "Not Crowded", and $\tilde{\mu}$ is divided into two fuzzy subspaces, "Fast" and "Slow".

Therefore, this model consists of (2×2) implications. If the y^i is the expected length of queue, implications having the form such as:

$$\begin{cases} R^1: & \text{if } \lambda \text{ is NotCrowded}(\tilde{\lambda}_1) \text{ and } \mu \text{ is fast}(\tilde{\mu}_2) \text{ then } L_q \text{ is short} \\ R^2: & \text{if } \lambda \text{ is NotCrowded}(\tilde{\lambda}_1) \text{ and } \mu \text{ is slow}(\tilde{\mu}_1) \text{ then } L_q \text{ is low moderate} \\ R^3: & \text{if } \lambda \text{ is crowded}(\tilde{\lambda}_2) \text{ and } \mu \text{ is fast}(\tilde{\mu}_2) \text{ then } L_q \text{ is high moderate} \\ R^4: & \text{if } \lambda \text{ is crowded}(\tilde{\lambda}_2) \text{ and } \mu \text{ is slow}(\tilde{\mu}_1) \text{ then } L_q \text{ is long} \end{cases}$$

2.2.3 Consequence Parameters Identification:

Suppose $f(x,y)$ define the system performance and let redefine an FM/FM/s queue system in order to model (2.4) by following rules as:

$$R^i: \text{If } \lambda \text{ is } \tilde{\lambda}^i \text{ and } \mu \text{ is } \tilde{\mu}^i \text{ then } y^i = f^i(\lambda,\mu), \quad i=1, 2\dots, n \quad (2.6)$$

where the functions of $f^i(\lambda,\mu)$ is a clarified piece by piece linearization of a nonlinear function with different parameters in each rule consequence. Commonly, the fuzzy clustering techniques can estimate these parameters from the available data.

In this thesis, for the purpose of to be able to use the linguistic terms to estimate vector-valued function $f^i(\lambda,\mu)$, an output data is produced with available input and expression performance system function. Using the new *hard clustering method*, we divide the computed output to some clusters and derive the $f^i(\lambda,\mu)$ of the queuing system performance for every implication that related to each cluster.

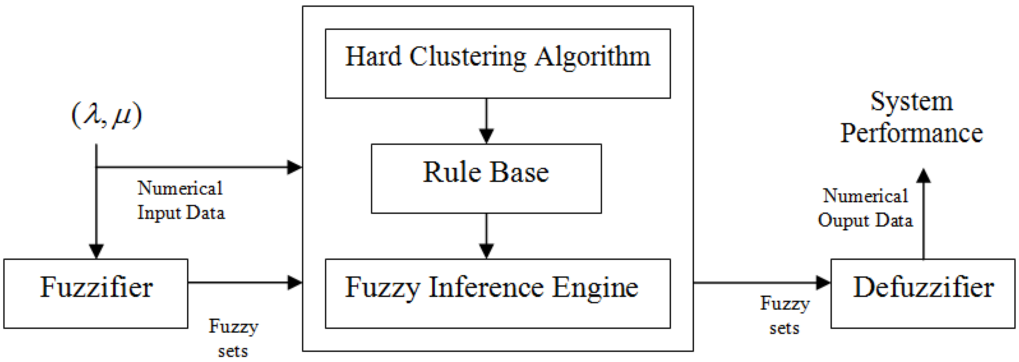


Figure 2.5 A fuzzy system includes the hard clustering method.

In the rule based model (2.6), $f^i(\lambda,\mu)$ is a vector-valued function with parameterization as form $f^i(\lambda,\mu) = p_1^{iT}.\lambda + p_2^{iT}.\mu + p_0^i$

where p_1^i, p_2^i are parameters of vector and p_0^i is a scalar offset.

The inference formula of the TSK model is a straightforward extension of the (2.2):

$$Y = \frac{\sum_{i=1}^{n} w_i (p_1^{iT}.\lambda + p_2^{iT}.\mu + p_0^{iT})}{\sum_{i=1}^{n} w_i}$$

The proposed rule based fuzzy model in a queuing system can be observed as a clarified piece-wise approximation of the interval performance function, depicted in Figure 2.6.

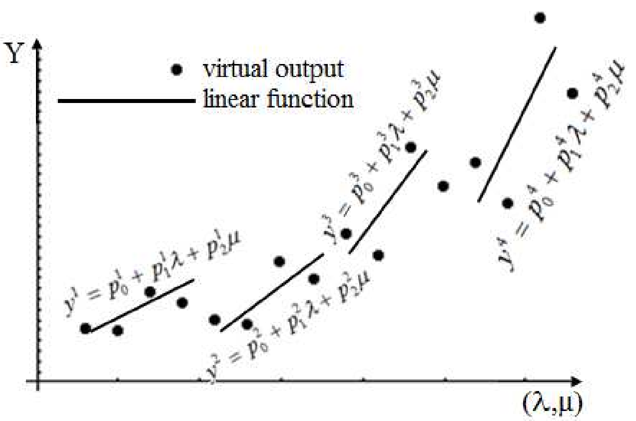


Figure 2.6 The proposed fuzzy model as a clarified piece-by-piece linearization of a nonlinear performance function.

The line y^i displays the linear function in the consequence of implication R^i. The equation in a consequence is extracted to define a law based on the fuzzy subspace definition in premise and its calculated output in a consequence.

The utilization of hard clustering method to extract the linear consequences is presented in the next subsection completely.

2.3 HARD CLUSTERING METHOD

Suppose a data universe **Y** with **n** data members. Clustering identifies the number of c clusters in **Y** where (**2 ≤ c < n**). For algebraic data, the member values of each cluster are more mathematically similar to each other than to members of other clusters. To clustering identification we must determine the similarity between observations and classify the partitions.

2.3.1 Application of Interval Function to Evaluation of Consequence Parameters

Fuzzy systems are inferred from descriptions of crisp interval-valued systems, which are crisp systems in general. Figure 2.7 illustrated this with an example of a function and its interval and fuzzy forms. In this figure, the evaluation of the function for crisp, interval and fuzzy data is depicted too. Remember that a function $f: X \rightarrow Y$ is a relation that can be observed as a subset of the Cartesian product $X \times Y$. As you see in the Figure 2.7, the vertical dashed lines show the extension of the given input into the product space $X \times Y$. After finding the intersection of this extension with the relation, this intersection is projected onto Y that is shown with horizontal dashed lines.

Since the function and the data (crisp, interval, fuzzy) are independent, this function evaluation makes clear up the use of fuzzy relations for inference in fuzzy modeling.

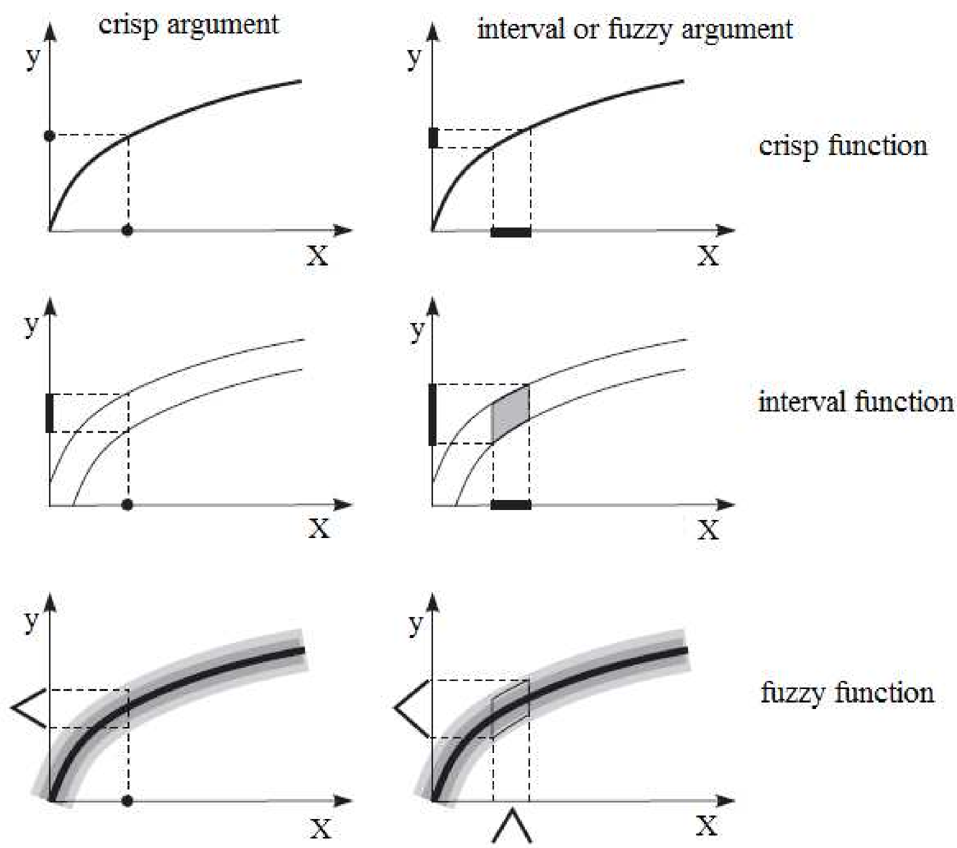


Figure 2.7 Crisp, interval and fuzzy arguments for evaluation of a crisp, interval and fuzzy function.

Fuzzy systems can supply various aims, such as data analysis, modeling, forecasting or decision making. In this research, a new fuzzy rule-based system is studied, for simplicity, attentive to its realized eventual purpose in field of queuing systems.

To introduce the evaluation interval function, we define a sample set of $(m \times n)$ data that we wish to classify:

$$Y = \begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \vdots & \vdots \\ y_{1m} & \cdots & y_{mn} \end{bmatrix} \qquad (2.7)$$

defined by $f : \lambda \times \mu \rightarrow Y$ where f is a binary function which takes two inputs λ with m factors and μ with n factors, where $\lambda = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_m]^T$, $\mu = [\mu_1 \quad \mu_2 \quad \cdots \quad \mu_n]$ and Y is the calculated output set.

It is noted that $\lambda_1 < \lambda_2 < \cdots < \lambda_m$ and $\mu_1 < \mu_2 < \cdots < \mu_n$.

The evaluated membership functions $f^i(\lambda, \mu)$ for a given input (arrival and service rates) of each implication are depicted in Figure 2.8.

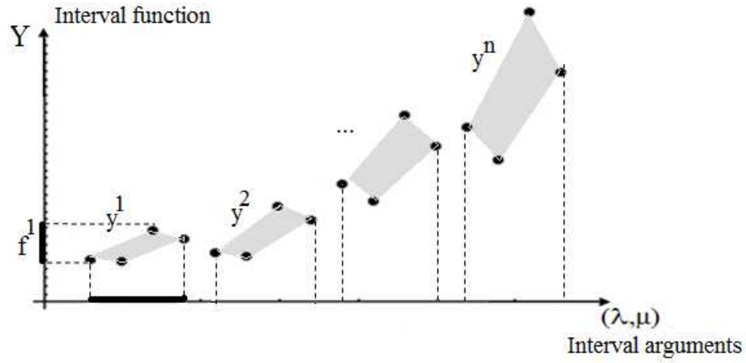


Figure 2.8 Evaluation of interval performance function with interval arrival and service time rates in the M/M/s queuing system.

To evaluate the interval performance function in each implication, we have to determine the related region (partition) in the calculated output Y.

When a hard clustering approach is used, it is required to divide the data into different and unique clusters, where each data element exists exactly in one cluster. Based on the similarity between members within a cluster, we consider the similarity in value components of every member of each cluster. This means, every member in each cluster is calculated with same pair (λ_i, μ_j) of same sub interval values of λ and μ. Hence, interval inputs λ and μ are divided to A subsets λ^i and B subsets μ^i in order to $\tilde{\lambda}^i$ and $\tilde{\mu}^i$ fuzzy subset numbers (2.2.2 section definition). We have $\lambda^i \subset \lambda$ and $\mu^i \subset \mu$.

In general, there seems no theoretical approach available for the number of divisions.

Let L and K are vectors of orders $(A-1)$ and $(B-1)$ respectively:

$$L = [l_1, l_2, ..., l_{A-1}] \text{ and } K = [k_1, k_2, ..., k_{B-1}] \qquad (2.8)$$

where l_a and k_b represent the division space points ($y_{l_a k_b}$) in matrix Y.

Therefore, the $Y^{(m \times n)}$ matrix is divided into $(A \times B)$ clusters so that the crisp data of every cluster are related to an implication as you see in Figure 2.9.

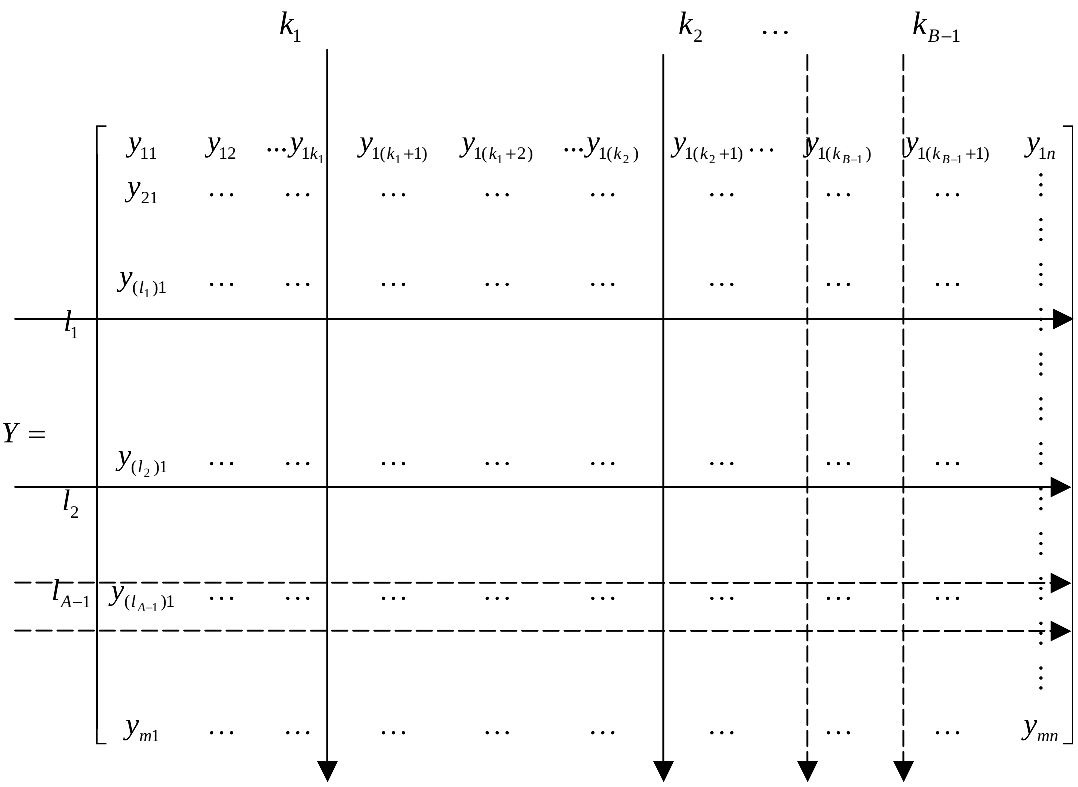


Figure 2.9 Division of matrix $Y^{(m \times n)}$ into $(A \times B)$ clusters.

Define a family of clusters { $C^{l_a k_b}$, a = 1, 2, . . . , A-1 , b=1,2,...,B-1 } as a hard c-partition of Y, where $c = (A \times B)$ is the number of clusters($2 \le c < (m \times n)$). The following set-theoretic forms apply to these partitions:

$$\bigcup_{a=1}^{A} \bigcup_{b=1}^{B} C^{l_a k_b} = Y \qquad \text{for all } l_a, k_b \qquad (2.9)$$

$$C^{l_a k_b} \bigcap C^{l_a k_b} = \phi \qquad \text{for all } (l_a, k_b) \neq (l_a, k_b) \qquad (2.10)$$

$$\phi \subset C^{l_a k_b} \subset Y \qquad \text{for all } l_a, k_b \qquad (2.11)$$

Equation (2.9) expresses the fact that the set of all clusters exhausts the universe of data sample. Equation (2.10) indicates that none of the clusters overlap in the sense that a data sample can belong to more than one cluster. Equation (2.11) expresses that a class cannot be empty and it cannot contain all the data samples.

Consequently, we define a *hard cluster* for Y as the following matrix set:

$$C_r^{\ ij} = \begin{cases} (\{\lambda_1, \lambda_2, ..., \lambda_{l_1}\}^T) \times (\{\mu_1, \mu_2, ..., \mu_{k_1}\}, & i = 1, \quad j = 1, \quad r = 1 \\ (\{\lambda_{(l_{i-1}+1)}, \lambda_{(l_{i-1}+2)}, ..., \lambda_{l_i}\}^T) \times (\{\mu_{(k_{j-1}+1)}, \mu_{(k_{j-1}+2)}, ..., \mu_{k_j}\}), & 1 \le i < A, \quad 1 \le j < B, \quad 2 \le r < c-1 \\ (\{\lambda_{l_{A-1}+1}, \lambda_{l_{A-1}+2}, ..., \lambda_m\}^T) \times (\{\mu_{k_{B-1}+1}, \mu_{k_{B-1}+2}, ..., \mu_n\}), & i = A, \quad j = B, \quad r = c \end{cases}$$

(2.12)

$\chi_{C^{l_a k_b}}(y_{ij})$ is defined as the hard membership function-theoretic expression which resembles the regression of all data y_{ij} in $C^{l_a k_b}$ cluster.

For simplicity in notation, our membership assignment of the $(i \times j)$ th data point in the rth $C^{l_a k_b}$ cluster of data universe Y, is defined to be

$$\chi_{C_r^{\ l_a k_b}}(y_{ij}) \equiv \chi_{C_r}(y_{ij}) = p_0^r + p_1^r.\lambda_i + p_2^{\ r}.\mu_j \qquad (2.13)$$

where P_0, P_1, P_2 are the linear function constant parameters.

In this sense, our system is inferred from descriptions of crisp interval-valued systems.

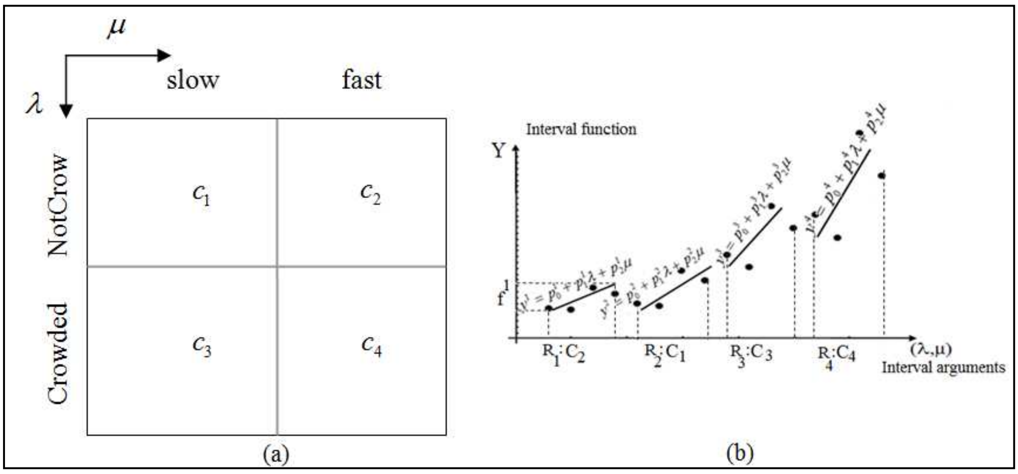


Figure 2.10 (a) The hard clustering model, (b) the function valued data points and their interpolation linear functions.

As you see in Figure 2.10, our model can be observed as a relation among the input data in premise and the parameters of a linear system as a consequence.

Therefore, using (2.5) and (2.13), the implications of queuing system have the form:

$$\begin{cases} R^1: \quad \text{if } \lambda_i \text{ is } \tilde{\lambda}^1 \text{ and } \mu_j \text{ is } \tilde{\mu}^1 \quad \text{then} \quad \mathrm{f}^1(\lambda,\mu) = P_0^1 + \mathrm{P}_1^1.\lambda_i + \mathrm{P}_2^1.\mu_j \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ R^{(A\times B)}: \text{ if } \lambda_k \text{ is } \tilde{\lambda}^{(A\times B)} \text{ and } \mu_k \text{ is } \tilde{\mu}^{(A\times B)} \quad \text{then} \quad \mathrm{f}^{(A\times B)}(\lambda,\mu) = P_0^{(A\times B)} + \mathrm{P}_1^{(A\times B)}.\lambda_i + \mathrm{P}_2^{(A\times B)}.\mu_j \end{cases}$$

The step-by-step procedures in this hard clustering method are summarized into six steps:

1. Calculate matrix Y with crisp interval input data sets λ and μ .
2. Divide λ and μ crisp sets into desired parts and initialize A and B values: $c = (A \times B)$

Then, do r = 1, 2...

3. Initialize $L = [l_1, l_2, ..., l_{A-1}]$ and $K = [k_1, k_2, ..., k_{B-1}]$
4. Determine $C_r^{l_a k_b}(y_{ij})$.
5. Obtain $\chi_{C_r}(y_{ij}) = p_0^r + p_1^r.\lambda_i + p_2{}^r.\mu_j$
6. If r=c,

   Stop; otherwise set $r = r + 1$ and return to step 4.

A good illustration of our clustering method is visible in the next chapter.

CHAPTER III

APPLICATION OF THE PROPOSED MODEL IN FUZZY CONTROL AND DISCUSSION OF THE RESULTS

In order to demonstrate applying the proposed fuzzy rule based model to the fuzzy queuing system, we considered some case studies in this chapter.

3.1 APPLICATION IN PERFORMANCE CALCULATIONS USING THE PROPOSED FUZZY MODEL

In this section, let us consider a case study on measuring performances in a fuzzy queuing system.

3.1.1. Computations of the Expected Length of Queue in the FM/FM/s Queuing System

To illustrate above examples, we have λ= [1, 2, 3, 4] and μ= [11, 12, 13, 14] per hour, and each set is divided into two parts (A=2, $\lambda^1 = \{1,2\}, \lambda^2 = \{3,4\}$ B=2, $\mu^1 = \{11,12\}, \mu^2 = \{13,14\}$ $L = [2]$ and $K = [2]$). The number of partitions is $c = (A \times B) = 4$. If the number of server s=2, and the expected length of queue, L_q, is a binary function, using (1.3) and (2.7) we have:

$$L_q = \begin{bmatrix} 0.00019 & 0.00014 & 0.00011 & 0.00009 \\ 0.00152 & 0.00117 & 0.00092 & 0.00073 \\ 0.0052 & 0.00397 & 0.00311 & 0.00249 \\ 0.01243 & 0.00952 & 0.00746 & 0.00595 \end{bmatrix}$$

According to fifth step of clustering algorithm, equivalent linear functions related to each cluster or implication in the fuzzy queuing system for expected length of queue, are:

$$L_q(\lambda,\mu)=\begin{cases} R^1: short & 0.001+0.0007\lambda-0.0001\mu \\ R^2: lowModerate & 0.0032+0.001\lambda+0.0003\mu \\ R^3: highModerate & -0.0004+0.0039\lambda-0.0006\mu \\ R^4: long & 0.009+0.0064\lambda-0.0021\mu \end{cases}$$

Clusters of the function valued crisp data points and their interpolated linear functions data of the expected length of queue are depicted in Figure 3.1.

$$\begin{array}{cc} \text{C1} & \text{C2} \end{array}$$

$$L_q=\left[\begin{array}{cc|cc} 0.00019 & 0.00014 & 0.00011 & 0.00009 \\ 0.00152 & 0.00117 & 0.00092 & 0.00073 \\ \hline 0.0052 & 0.00397 & 0.00311 & 0.00249 \\ 0.01243 & 0.00952 & 0.00746 & 0.00595 \end{array}\right]$$

$$\begin{array}{cc} \text{C3} & \text{C4} \end{array}$$

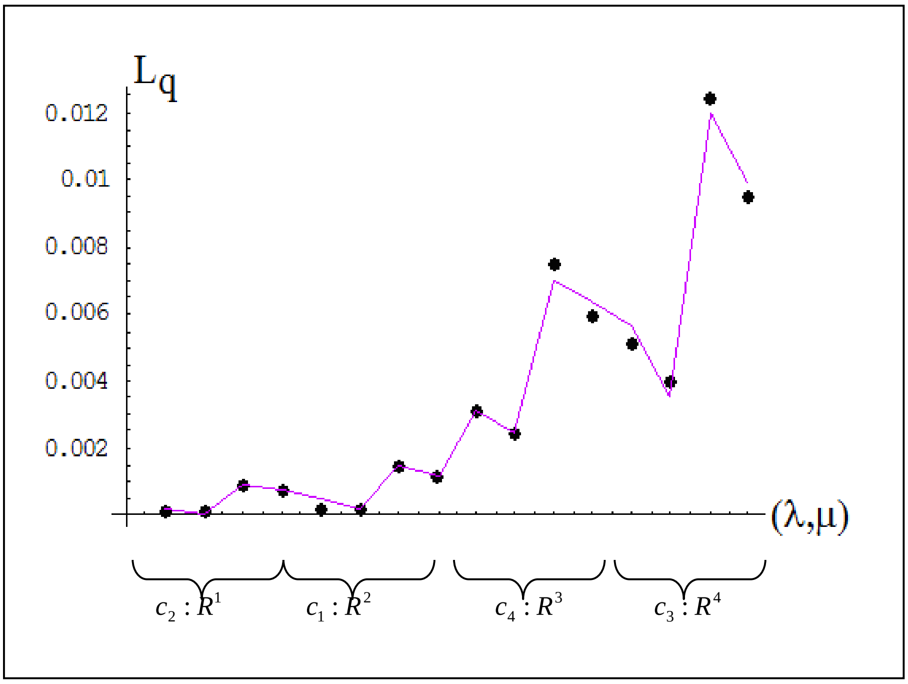


Figure 3.1 Clusters of the function-valued results and the corresponding linear interpolation for the expected length of queue as crisp data points in the M/M/s queuing system.

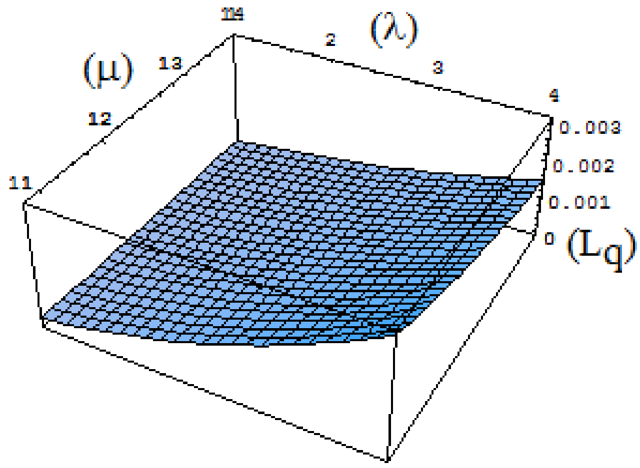


Figure 3.2 Input-output data of the length of queue in the crisp M/M/s queuing system.

The expected length of queue for all pairs (λ_i, μ_i) regarding to their position in a particular cluster and implication are calculated in related membership function y^i as Table 3.1:

Table 3.1 The expected lengths of queue for all pairs (λ_i, μ_i), according to the related linear membership function. (Shown in bold)

(λ_i, μ_i)	y^1	y^2	y^3	y^4
(1,11)	0.00039	**0.00048**	-0.00341	-0.0072
(1,12)	0.00028	**0.00014**	-0.00404	-0.0093
(1,13)	**0.00016**	-0.00019	-0.00466	-0.0113
(1,14)	**0.00004**	-0.00053	-0.00529	-0.0134
(2,11)	0.00112	**0.00151**	0.00047	-0.0008
(2,12)	0.00099	**0.00117**	-0.00015	-0.0029
(2,13)	**0.00088**	0.00083	-0.00077	-0.0049
(2,14)	**0.00077**	0.00049	-0.00140	-0.0070
(3,11)	0.00184	0.00254	0.00436	**0.0060**
(3,12)	0.00172	0.00220	0.00374	**0.0035**
(3,13)	0.00160	0.00186	**0.00311**	0.0015
(3,14)	0.00149	0.00152	**0.00249**	-0.0006
(4,11)	0.00256	0.00356	0.00825	**0.0120**
(4,12)	0.00244	0.00322	0.00763	**0.0099**
(4,13)	0.00232	0.00289	**0.00700**	0.0079
(4,14)	0.00221	0.00255	**0.00638**	0.0058

Table 3.2 Calculation of the expected waiting time in a fuzzy/FM/s queuing system.

(λ_i,μ_i)	w_1	w_2	w_3	w_4	$Y_{L_q}=\sum_{i=1}^{4}w_i.y^i/\sum_{i=1}^{4}w_i$
(1,11)	0	0	0	1	0.000394
(1,12)	0	0	0	0.67	0.000185
(1,13)	0	0	0.67	0	-0.000129
(1,14)	0	0	1	0	-0.0005314
(2,11)	0	0	0	1	0.001115
(2,12)	0	0	0	0.67	0.000666
(2,13)	0	0	0.67	0	0.000555
(2,14)	0	0	1	0	0.0004947
(3,11)	0	0.67	0	0	0.0029097
(3,12)	0	0.67	0	0	0.00249
(3,13)	0.67	0	0	0	0.00149
(3,14)	0.67	0	0	0	-0.000566
(4,11)	0	0.67	0	0	0.00825
(4,12)	0	1	0	0	0.00508
(4,13)	0.67	0	0	0	0.007897
(4,14)	0.67	0	0	0	0.00584

Table 3.2 presents the w_1, w_2, w_3 and w_4 for every implication and the final weighted defuzzification crisp output of the expected length of queue.

(λ_i, μ_i)	L_q	Y_{L_q}	%deviation
(1,11)	0.00019	0.000394	107.4
(1,12)	0.00014	0.000185	32.1
(1,13)	0.00011	-0.000129	217.3
(1,14)	0.00009	-0.000531	690.4
(2,11)	0.00152	0.001115	26.6
(2,12)	0.00117	0.000666	43.1
(2,13)	0.00092	0.000555	39.7
(2,14)	0.00073	0.000495	32.2
(3,11)	0.0052	0.002910	44.0
(3,12)	0.00397	0.00249	37.3
(3,13)	0.00311	0.00149	52.1
(3,14)	0.00249	-0.000566	122.7
(4,11)	0.01243	0.00825	33.6
(4,12)	0.00952	0.00508	46.6
(4,13)	0.00746	0.0079	5.9
(4,14)	0.00595	0.00584	1.8

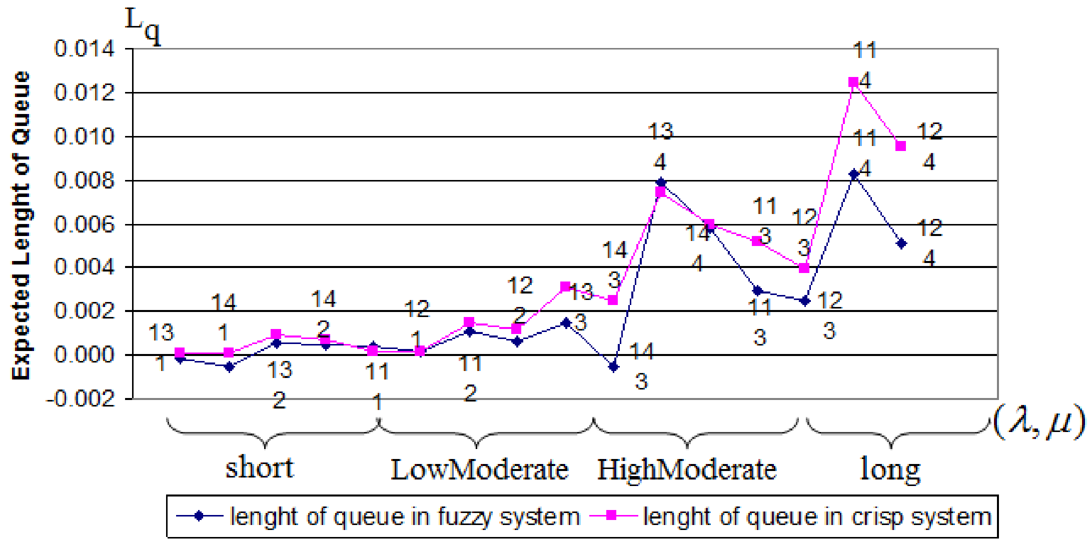


Figure 3.3 The comparison of the final output results of the proposed fuzzy model and the conventional approach for the expected length of queue.

Referring to Figure 3.3, the results of the expected length of queue in the proposed fuzzy model and crisp inferences are close approximately, expect of (3,13)and(3,14) points in “High Moderate” region and (3,11), (3,12), (4,11), and

(4,12) points in “long” region. In these points, the expected length of queue in fuzzy inference is estimated shorter.

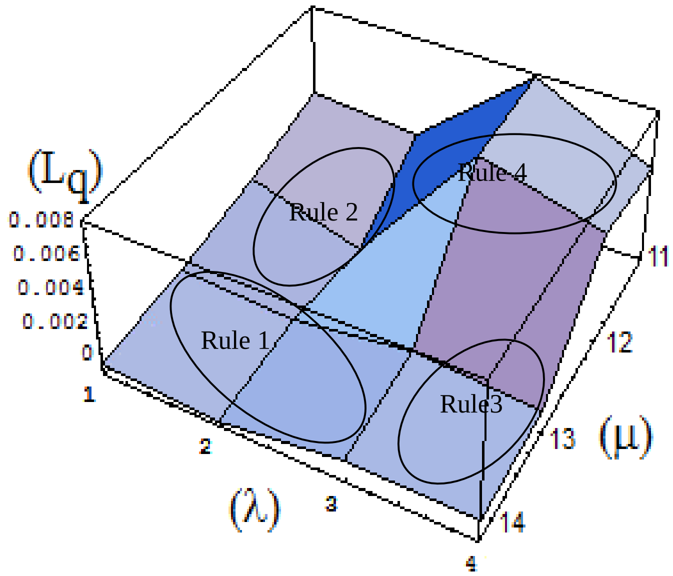


Figure 3.4 The input-output relation found using the proposed fuzzy model to interpret the expected length of queue.

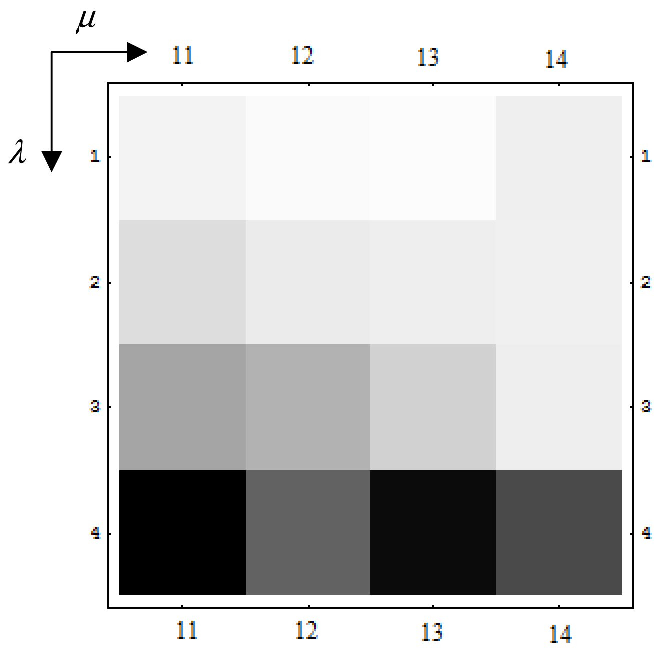


Figure3.5 Darker squares correspond to bigger values for the expected length of queue in the fuzzy system.

3.1.2. Computations of the Expected Waiting Time in Queue in the FM/FM/s Queuing System

According to the expected waiting time equation (1.3) and matrix (2.7) for W_q as matrix Y we have:

$$W_q = \begin{bmatrix} 0.00019 & 0.00014 & 0.00011 & 0.00009 \\ 0.00076 & 0.00058 & 0.00046 & 0.00037 \\ 0.00172 & 0.00132 & 0.00104 & 0.00083 \\ 0.00311 & 0.00238 & 0.00186 & 0.00148 \end{bmatrix}$$

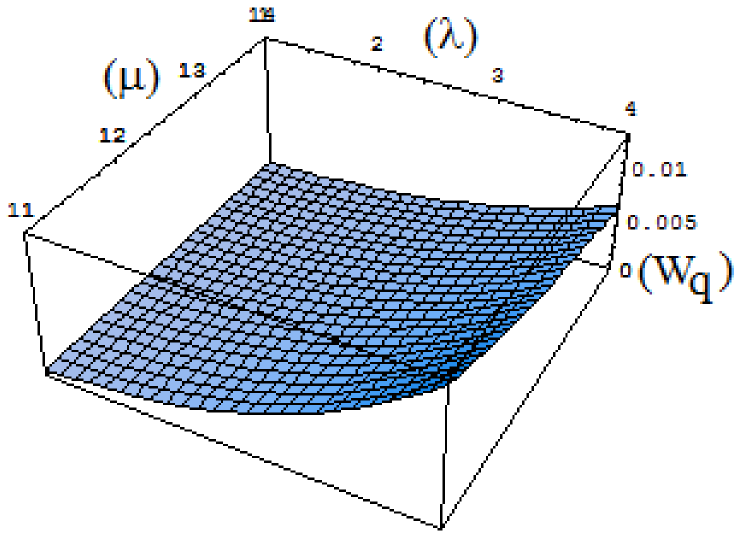


Figure 3.6 Input-output data of the expected waiting time in queue in the crisp M/M/s queuing system.

According to fifth step of clustering algorithm, equivalent linear functions related to each cluster or implication in the fuzzy queue system for expected time in queue are:

$$W_q(\lambda,\mu) = \begin{cases} R^1: fast & 0.0006 + 0.00028\lambda - 0.00006\mu \\ R^2: lowModerate & 0.000096 + 0.00057\lambda - 0.0004\mu \\ R^3: highModerate & 0.00287 + 0.00074\lambda - 0.000306\mu \\ R^4: slow & 0.00448 + 0.00122\lambda - 0.00057\mu \end{cases}$$

Clusters of the function-valued crisp data points and their interpolation linear functions of expected waiting time in queue are depicted in Figure 3.7.

C1 C2

$$W_q = \left[\begin{array}{cc|cc} 0.00019 & 0.00014 & 0.00011 & 0.00009 \\ 0.00076 & 0.00058 & 0.00046 & 0.00037 \\ \hline 0.00172 & 0.00132 & 0.00104 & 0.00083 \\ 0.00311 & 0.00238 & 0.00186 & 0.00148 \end{array}\right]$$

C3 C4

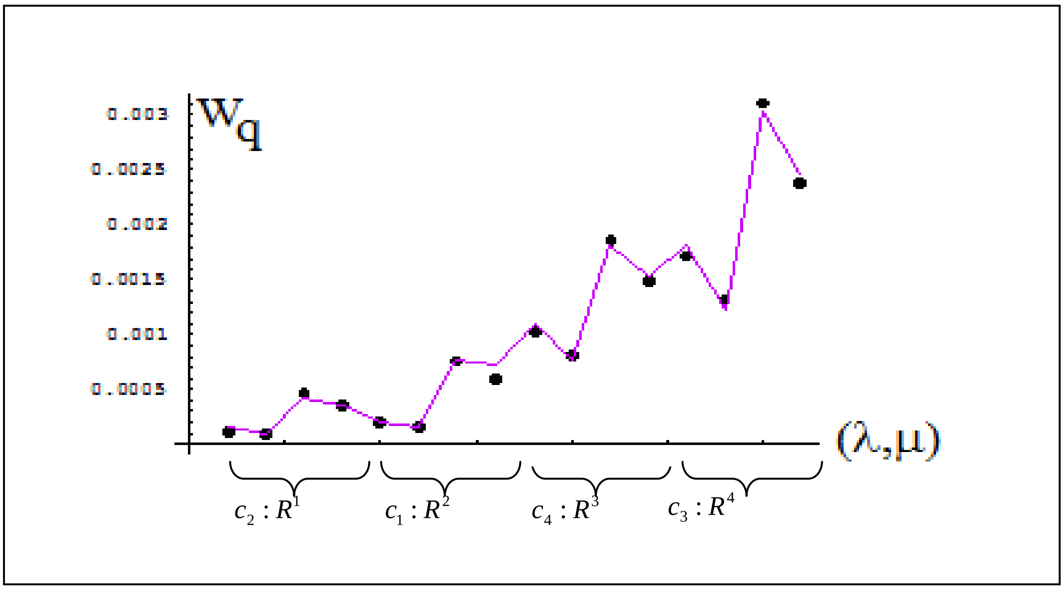


Figure 3.7 Clusters of the function-valued and their interpolation linear functions crisp data points of expected waiting time in the M/M/s queuing system.

The expected waiting times in queue for all pairs (λ_i, μ_i) regarding to their position in a particular cluster and implication are calculated in related membership function y^i as Table 3.3:

Table 3.3 The expected waiting time in queue for all pairs (λ_i, μ_i), regarding to the related linear membership functions. (Shown in bold)

(λ_i, μ_i)	y^1	y^2	y^3	y^4
(1,11)	0.00026	**0.00019**	0.00022	-0.00063
(1,12)	0.00021	**0.00015**	-0.00086	-0.00120
(1,13)	**0.00015**	0.00010	-0.00039	-0.00178
(1,14)	**0.00009**	0.00006	-0.00070	-0.00235
(2,11)	0.00054	**0.00076**	0.00096	0.00059
(2,12)	0.00048	**0.00072**	0.00065	0.00002
(2,13)	**0.00042**	0.00067	0.00035	-0.00056
(2,14)	**0.00037**	0.00063	0.00004	-0.00113
(3,11)	0.00081	0.00133	0.00170	**0.00181**
(3,12)	0.00076	0.00128	0.00139	**0.00124**
(3,13)	0.00070	0.00124	**0.00109**	0.00066
(3,14)	0.00064	0.00120	**0.00078**	0.00009
(4,11)	0.00109	0.00190	0.00244	**0.00303**
(4,12)	0.00103	0.00185	0.00213	**0.00246**
(4,13)	0.00097	0.00181	**0.00183**	0.00188
(4,14)	0.00092	0.00177	**0.00152**	0.00131

Table 3.4 Calculations of the expected waiting time in queue in a fuzzy FM/FM/s queuing system.

(λ_i, μ_i)	(w_1)	(w_2)	(w_3)	(w_4)	$Y_{W_q} = \sum_{i=1}^{4} w_i . y^i / \sum_{i=1}^{4} w_i$
(1,11)	0	0	0	1	0.000263
(1,12)	0	0	0	0.67	0.000137
(1,13)	0	0	0.67	0	0.000068
(1,14)	0	0	1	0	0.000059
(2,11)	0	0	0	1	0.000539
(2,12)	0	0	0	0.67	0.000320
(2,13)	0	0	0.67	0	0.000448
(2,14)	0	0	1	0	0.000629
(3,11)	0	0.67	0	0	0.001134
(3,12)	0	0.67	0	0	0.000930
(3,13)	0.67	0	0	0	0.000662
(3,14)	0.67	0	0	0	0.000087
(4,11)	0	0.67	0	0	0.002441
(4,12)	0	1	0	0	0.001423
(4,13)	0.67	0	0	0	0.001881
(4,14)	0.67	0	0	0	0.001306

Table 3.4 presents the w_1, w_2, w_3 and w_4 for every implication and the final weighted defuzzification crisp output of the expected waiting time, Y_{Wq}.

(λ_i, μ_i)	W_q	Y_{W_q}	% Deviation
(1,11)	0.00019	0.000263	38.4
(1,12)	0.00014	0.000137	2.1
(1,13)	0.00011	0.000068	38.2
(1,14)	0.00009	0.000059	34.4
(2,11)	0.00075	0.000539	28.1
(2,12)	0.00058	0.000320	44.8
(2,13)	0.00046	0.000448	2.6
(2,14)	0.00037	0.000629	70.0
(3,11)	0.00172	0.001134	34.1
(3,12)	0.00132	0.000930	29.5
(3,13)	0.00103	0.000662	35.7
(3,14)	0.00083	0.000087	89.5
(4,11)	0.00311	0.002441	21.5
(4,12)	0.00238	0.001423	40.2
(4,13)	0.00186	0.001881	1.1
(4,14)	0.00149	0.001306	12.3

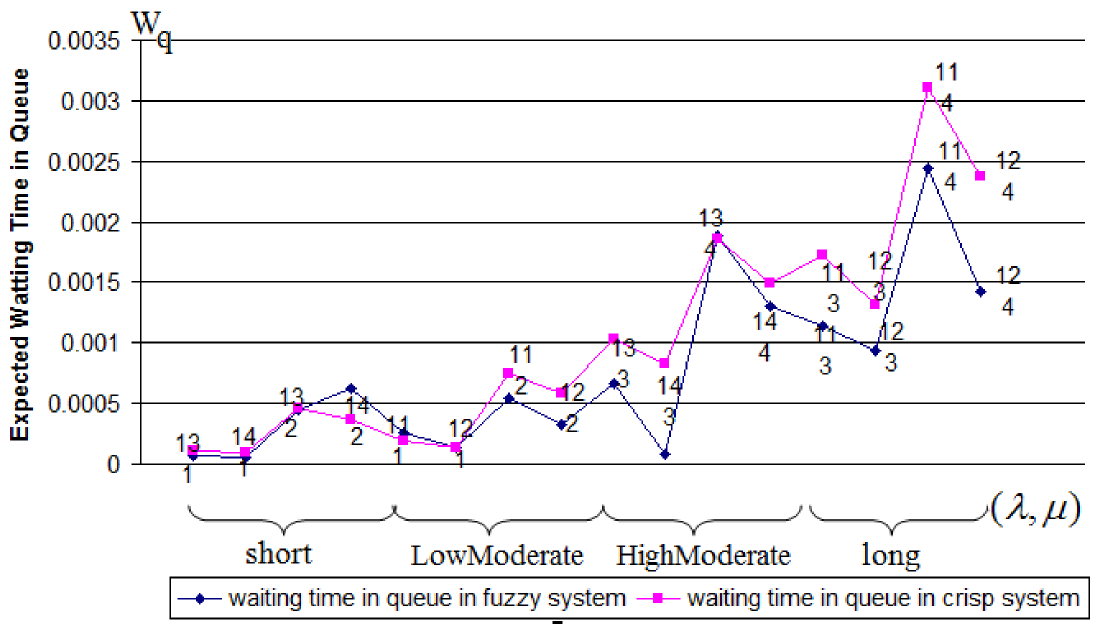


Figure 3.8 The comparison of the final output results of the proposed fuzzy model and the conventional approach for the expected waiting time.

Referring to Figure 3.8, the results of the expected waiting time in queue in the proposed fuzzy model and conventional inferences are close, except points (3,13) and (3,14) and points in “High Moderate” region and points (3,11), (3,12), (4,11), and (4,12) in “long” region. In these points the expected waiting time in queue in the fuzzy inference is estimated shorter.

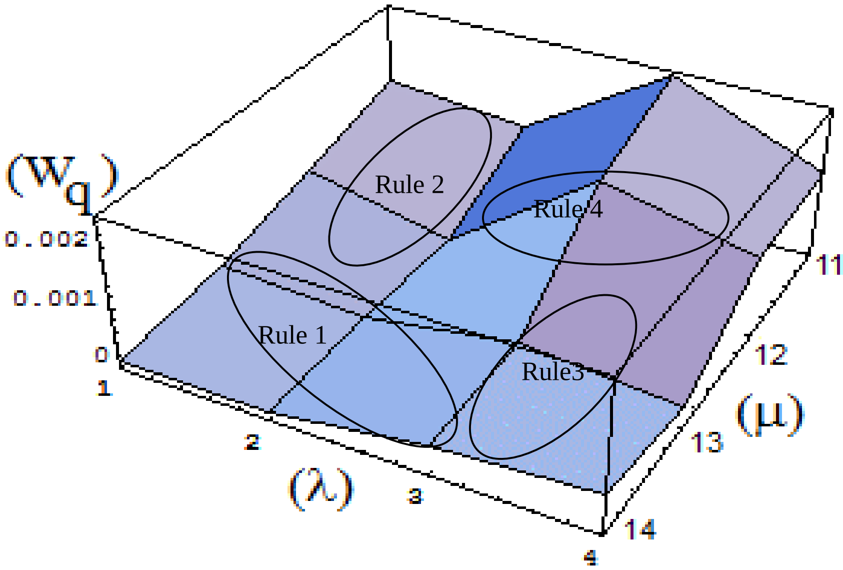


Figure 3.9 The input-output data of the proposed fuzzy model to interpret the expected waiting time in queue.

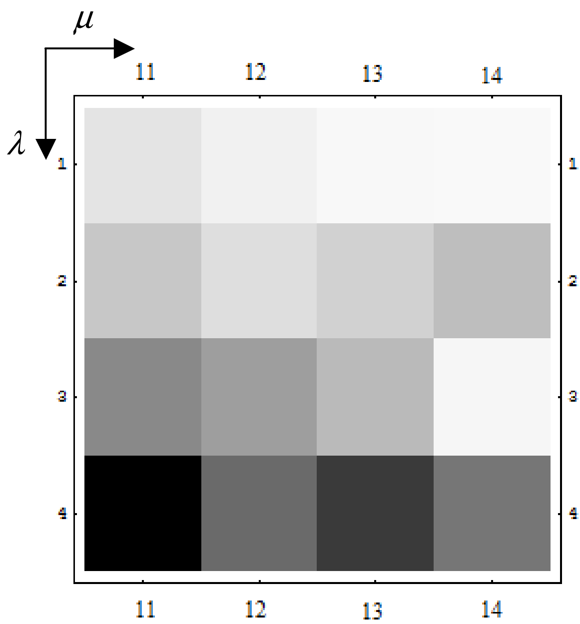


Figure 3.10 Darker squares correspond to bigger values for the expected waiting time in queue in the fuzzy system.

According to Figure 3.3 and Figure 3.8, in spite of the fact that new hard clustering method are evaluated with linguistic terms, the inferring results have the same rank in both of the conventional and the proposed fuzzy model.

These observations provide engineers with a clear and intuitive way to implement control systems and decision-making in various conditions of queuing system.

3.2. COMPARISON OF THE PROPOSED MODEL AND THE FUZZY N-POLICY QUEUING SYSTEM BASED UPON α-CUT MODEL

In this section, we supposed a different case problem to compare the fuzzy N-Policy queuing system based upon α-cut model (Wang& Yang and Li. 2010) and our new fuzzy rule based model. In section 3.2.1, the sample problem is solved using the fuzzy model upon α-cut method. In section 3.2.2, the sample problem is solved using the new fuzzy rule based model and in section 3.2.3, the results of the two methods are compared.

3.2.1. Expected Waiting Time in the Queue using the Fuzzy N-Policy Queue Model Based upon α-Cut

In the Example 3.1, the crisp interval arrival rate set is λ= [1, 2, 3, 4] per hour and the interval service rate set is μ= [11, 12, 13, 14] per hour as knowledge based (Yin Wang, 2010). It is simple to determine

$$\left[x_\alpha^L, x_\alpha^U\right] = \left[\min \mu_{\tilde{\lambda}}^{-1}(\alpha), \max \mu_{\tilde{\lambda}}^{-1}(\alpha)\right] = \left[1+\alpha, 4-\alpha\right]$$

$$\left[y_\alpha^L, y_\alpha^U\right] = \left[\min \mu_{\tilde{\mu}}^{-1}(\alpha), \max \mu_{\tilde{\mu}}^{-1}(\alpha)\right] = \left[11+\alpha, 14-\alpha\right]$$

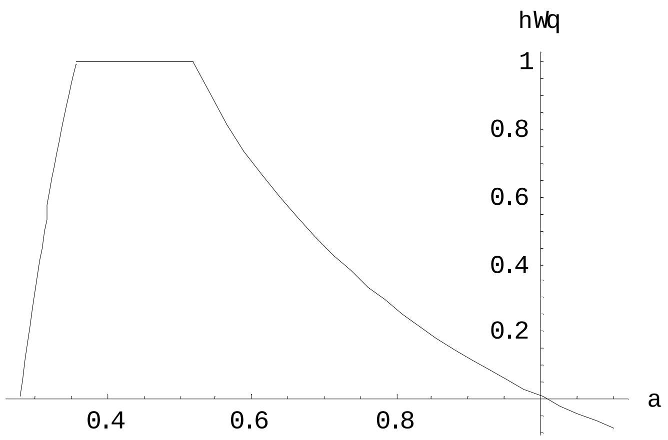


Figure 3.11 The trapezoidal membership function of the expected waiting time in queue using the fuzzy N-policy queue model based upon α-cut.

Clearly, the minimum value of expected waiting time in the queue is achieved when $x = x_\alpha^U$ and $y = y_\alpha^U$ and the maximum value is achieved when $x = x_\alpha^L$ and $y = y_\alpha^L$. According to Equations (2.13) and (2.14), the lower and upper bounds of the α-cut of $\tilde{W}_q$, respectively, are given by

$$(W_q)_\alpha^L = \frac{N-1}{4-\alpha} + \frac{(4-\alpha)}{10(14-\alpha)}$$

$$(W_q)_\alpha^U = \frac{N-1}{2(1+\alpha)} + \frac{(1+\alpha)}{10(11+\alpha)}$$

The inverse functions of $(W_q)^L$ and $(W_q)^U$ are exists. We have

$$\eta_{\tilde{W}_q}(z) = \begin{cases} \dfrac{9-90\alpha+5\sqrt{-3+20\alpha+100\alpha^2}}{1-10\alpha}, & \dfrac{39}{140} \le z \le \dfrac{139}{390} \\ 1, & \dfrac{139}{390} \le z \le \dfrac{31}{60} \\ \dfrac{6-60\alpha+5\sqrt{-3+20\alpha+100\alpha^2}}{-1+10\alpha} & \dfrac{31}{60} \le z \le \dfrac{111}{110} \end{cases}$$

Table 3.5 The membership function of the expected waiting time in queue for the fuzzy N-policy queue model based upon **α** -cut.

α	x_α^L	x_α^U	y_α^L	y_α^U	$(W_q)^L$	$(W_q)^U$
0.00	1	4	11	14	0.279	1.009
0.10	1.1	3.9	11.1	13.9	0.284	0.919
0.20	1.2	3.8	11.2	13.8	0.291	0.844
0.30	1.3	3.7	11.3	13.7	0.279	0.781
0.40	1.4	3.6	11.4	13.6	0.304	0.727
0.50	1.5	3.5	11.5	13.5	0.312	0.680
0.60	1.6	3.4	11.6	13.4	0.319	0.639
0.70	1.7	3.3	11.7	13.3	0.328	0.603
0.80	1.8	3.2	11.8	13.2	0.337	0.571
0.90	1.9	3.1	11.9	13.1	0.346	0.542
1.00	2	3	12	13	0.356	0.517

The expected waiting time in the queue for eleven different α levels are presented in Table 3.5. As you see, the results appear between 0.356 and 0.517 when α = 1. When α =0, at the maximum value, the fuzzy expected waiting time in the queue appears impossibly below 0.279 or exceed 1.009.

3.2.2.Expected Waiting Time in Queue using the Proposed Model

Since our method is flexible to accept any function, interval values and fuzzy terms, according to matrix (2.7), the expected waiting time in queue function (1.5) is replaced as binary function which takes two interval inputs λ=[1, 2, 3, 4] and μ= [11, 12, 13, 14] as inter-arrival and service times, respectively. We have

$$W_q = \begin{bmatrix} 1.00909 & 1.00758 & 1.00641 & 1.00549 \\ 0.520202 & 0.516667 & 0.513986 & 0.511905 \\ 0.367424 & 0.361111 & 0.35641 & 0.352814 \\ 0.301948 & 0.291667 & 0.284188 & 0.278571 \end{bmatrix}$$

Two fuzzy terms “slow” and “fast” for service time rates and “Crowded” and “Not Crowded” for arrival rates are considered. In result, the calculated output will divided to four partitions. If implications over the system have the form as implication of example 3.2, regarding to fifth step of clustering algorithm, equivalent linear functions related to each cluster or implication in the fuzzy queuing system for expected waiting time in queue, are:

$$W_q(\lambda,\mu) = \begin{cases} R^1: fast & 1.51 - 0.49\lambda - 0.0009\mu \\ R^2: lowModerate & 1.52 - 0.49\lambda + 0.0015\mu \\ R^3: highModerate & 0.615 - 0.073\lambda - 0.003\mu \\ R^4: slow & 0.65 - 0.069\lambda - 0.0063\mu \end{cases}$$

If use of adjusting techniques for designing fuzzy models is considered, the proposed rule based fuzzy model using new clustering method is a more condensed and calculable than α-cut method. These adjusting techniques are used to extract the membership functions from available data to construct best fuzzy model systems.

(λ_i,μ_i)	W_q	Y_{W_q}	% Deviation
(1,11)	1.009090	1.00825	0.1
(1,12)	1.007580	0.671555	33.3
(1,13)	1.006410	0.670713	33.4
(1,14)	1.005490	1.00456	0.1
(2,11)	0.520202	0.514668	1.1
(2,12)	0.516667	0.342498	33.7
(2,13)	0.513986	0.343437	33.2
(2,14)	0.511905	0.513645	0.3
(3,11)	0.367424	0.24155	34.3
(3,12)	0.361111	0.239437	33.7
(3,13)	0.356410	0.354797	0.5
(3,14)	0.352814	0.348483	1.2
(4,11)	0.301948	0.289465	4.1
(4,12)	0.291667	0.190863	34.6
(4,13)	0.284188	0.285353	0.4
(4,14)	0.278571	0.279039	0.2

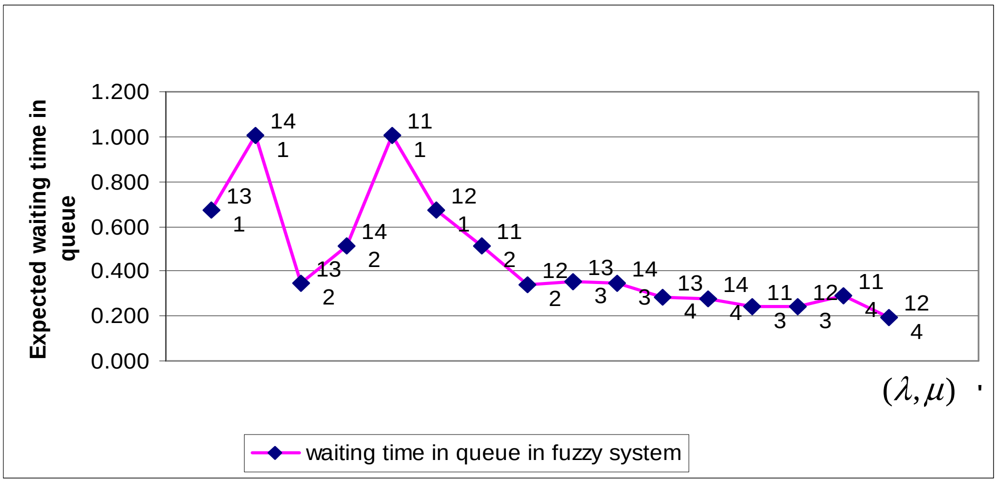


Figure 3.12 Prediction of the expected waiting time in a queuing system using the proposed model.

3.2.3. Comparison of New Proposed Model and Fuzzy Model Based upon α-cut Method

Some final results about the two different methods are concluded as:

1. The two different methods are computationally efficient.
2. Both of the approach works well with optimization and flexible techniques.
3. The two different methods are well matched to mathematical analysis.
4. They are flexible to use human input.
5. The proposed model is intuitive and has widespread acceptance to give any function to extract the liner membership functions, but there is not any guaranty to produce an invertible membership function with the fuzzy model based upon α-cut method.
6. The new proposed model works with linear techniques smoothly, but the other method does not.
7. In the new method, the acceptable and possible results are produced but there are always some impossible results in the other one method.

3.3. APPLICATION IN FUZZY COST ANALYSIS IN QUEUING SYSTEMS

The feature of cash-flow modeling is often uncertainty and is involved with re-processing of used products in ***cost analysis*** of data. Since the data in quality, supply, and disassembly times is uncertainty, therefore the data is not objective.

Hence, decision-makers have to rely on fuzzy data for analysis to make the more really results. Notice that, the data of both new products and used products are taken into account with cost analysis. In this section, an economic fuzzy cost analysis in a queuing system is presented using the new proposed rule based fuzzy model and then is compared with conventional method.

3.3.1. The Cost Relationships in Queuing Systems Analysis

As you see in the cost curves in Figure 3.13, there is generally a converse relationship between service cost and the cost of waiting in queue. According to the level of service, by rises the number of servers, the cost of service increases, and waiting cost decreases. Commonly in system analysis, the minimum point on the total cost curve should be aligned with the level of service. The cost of providing the service is usually related to cost of the servers, such as the cost of the staffs at the bank. When the number of servers increases to reduce the waiting time, service cost will increase. We can specify easily the service cost to compute. Contrary, the waiting cost is not clear to determine. The loss of business is occurred when customers get satisfied of waiting for long time and leave the system. This business loss may be occurred once or more time. As cost of waiting, the cost of business losses is especially difficult to determine, although such data, is provided by some organizations for businesses and industries seldom. The loss of salary for staffs and production time, load or unload transportations, waiting to use equipment, can be mentioned for some types of waiting costs.

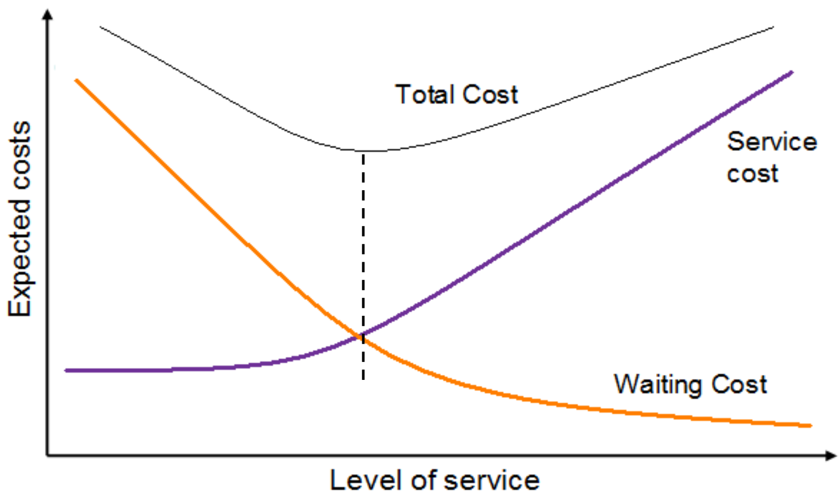


Figure 3.13 The cost relationship in queuing system analysis.

3.3.1.1.Queuing System Costs and Quality Service

The modern approach to quality management is to believe that the relationship between quality and cost is a short-run view that underestimates the potential long-term loss of business from poor quality. In the long-run, a higher level of quality will gain market share and increase business and thus is more cost-effective. Further, as the company focuses on improving quality service, the cost of accomplish good quality will be less because of the novelties in processes and work design that will result. This level of better-quality, that is, quicker, service will, in the long-run, growth business and be more cost-effective than the traditional view implies.

3.3.2.Conventional Economic Analysis in Queuing Systems

As this text has shown, we can use fuzzy queuing models to estimate performance measures of a queuing system. To analyze a queuing system economically, the information afforded by the linguistic terms is surveyed to extract a cost model for the waiting line system. So, this model can help us to balance the cost of waiting customers and the cost of providing the service. This balancing procedure is an important issue in the area of operations management.

In developing a cost model for the input problem in both conventional and fuzzy models, we will consider only the waiting time and servicing time variables, and the cost of the queuing system. Here, C_W the hourly waiting cost of each customer and C_S , the cost of each server per hour. Since, the cost of waiting customer per unit time cannot be correctly estimated; managers have to predict an acceptable value to reflect the loss probability of future revenue if an unsatisfied customer passes to another competitor company. In conventional method, suppose C_W is estimated to be \$50 per hour. The cost of operating each service facility as the wages of any server or the cost of equipment, including maintenance is more easily determined.
Let us assume that C_S =\$100 per hour. Therefore, the total cost per minute is $T_{\cos t} = C_W L_q + C_S S$, where L_q, is the average number of customers in the queuing system and S is number of servers. Table 3.6 summarizes the computed results as the cost for the two-three- and four server scenarios. We can obvious clearly that the economic advantages of a three-server system in conventional calculation.

Table 3.6 Results of the economic cost analysis of a queuing system design using the conventional method.

System	S	System Cost	L_q	Customer Cost	Total cost
Two -server	2	200	5	250.00	450.00
Three-server	3	300	0.36	18.00	318.00
Four-server	4	400	0.21	10.50	410.50

3.3.3. Fuzzy Economic Cost Analysis of Queuing Systems Using the Proposed Model

To control the truth of the result for the case given above, an interval data is supposed for expected length of queue with 2 servers as:

$L_{q2} = [0.02, 3, 7, 10]$

Where L_{q2}, the average of this interval data is 5, approximately.

This range is divided to two subsets according to the linguistic terms “short” and “long”.

In this case study, an interval data is supposed for the waiting cost per hour per customer as:

$C_W = [0, 35, 70, 100]$

The linguistic terms “low cost”, ($\tilde{C}_W^{\,1}$) and “high cost”, ($\tilde{C}_W^{\,2}$) fuzzy subsets of waiting cost per customer are supposed as given in Table 3.7.

Table 3.7 Linguistic terms and interval cost of waiting time set and its fuzzy subsets.

	Domain element			
linguistic term	0	35	70	100
low cost	0	0.9	0.2	0
high cost	0.0011	0.41	0.97	0.11

$\tilde{L}_q^{\,i}$ and $\tilde{C}_W^{\,i}$ are fuzzy sets explained in the domains of their corresponding base crisp interval variables L_q and C_W. Fuzzy sets $\tilde{L}_q^{\,i}$ and $\tilde{C}_W^{\,i}$ define fuzzy domains in the antecedent space, according to the respective consequent propositions. By denoting these sets by $\tilde{L}_q^{\,i}$ and $\tilde{C}_W^{\,i}$ respectively, we have $\tilde{L}_q^{\,i} \subset L_q$ and $\tilde{C}_W^{\,i} \subset \tilde{C}_W$.

The meaning of the linguistic terms is defined by “Pimf” membership functions, illustrated in Figure 3.14.

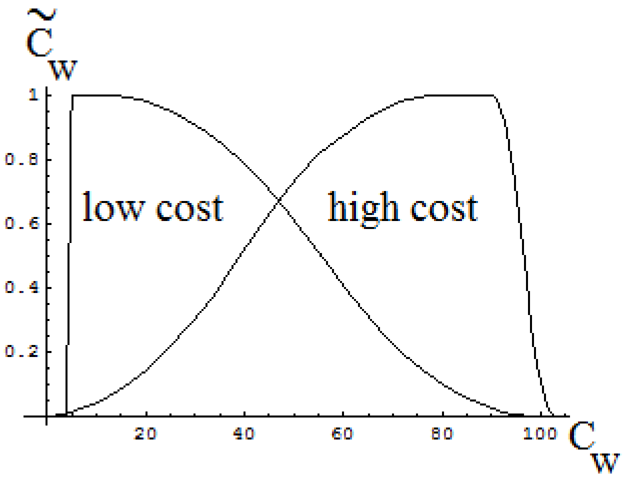


Figure 3.14 ''low cost'' and ''high cost'' waiting cost per customer.

Since the sets of expected length of queue and waiting cost are divided to two sub fuzzy sets, the four implications of the fuzzy cost model have the form of:

$$
\begin{cases}
R^1: & \text{if } L_q \text{ is } short \text{ and } C_W \text{ is } low \text{ then } T_{cost} \text{ is small} \\
R^2: & \text{if } L_q \text{ is } short \text{ and } C_W \text{ is } high \text{ then } T_{cost} \text{ is low moderate} \\
R^3: & \text{if } L_q \text{ is } long \text{ and } C_W \text{ is } low \text{ then } T_{cost} \text{ is high moderate} \\
R^4: & \text{if } L_q \text{ is } long \text{ and } C_W \text{ is } high \text{ then } T_{cost} \text{ is big}
\end{cases}
$$

As we have $T_{\cos t} = C_W L_q + C_S S$, therefore the matrix value of total cost is:

$$C_1 = \{0,35\} \times \{0.4,4.5\} \qquad C_2 = \{0,35\} \times \{11,15\}$$

$$
T_{\cos t} = \begin{bmatrix} 800 & 814 & 828 & 840 \\ 800 & 957.5 & 1115 & 1250 \\ 800 & 1185 & 1570 & 1900 \\ 800 & 1325 & 1850 & 2300 \end{bmatrix}
$$

$$C_3 = \{70,100\} \times \{0.4,4.5\} \qquad C_4 = \{70,100\} \times \{11,15\}$$

According to fifth step of clustering algorithm, equivalent linear functions related to each cluster or implication in the fuzzy queuing system for expected length of queue, are:

$$T_{\cos t}(C_W, L_q) = \begin{cases} R^1 : small & 2260.0187L_q - 24.8826C_W + 823.2925 \\ R^2 : lowModerate & 8.1091C_W + 40.8241L_q + 377.5043 \\ R^3 : highModerate & 8.1091C_W + 40.8241L_q + 377.5043 \\ R^4 : big & 201.061C_W - 194.4854L_q \end{cases}$$

All pairs (L_q, C_W) according to their position in which clusters and implications are calculated in related membership function y^i. The final weighted defuzzification crisp output total cost is presented in $T_{\cos t}$ column in Table 3.8 and depicted in Figure3.15:

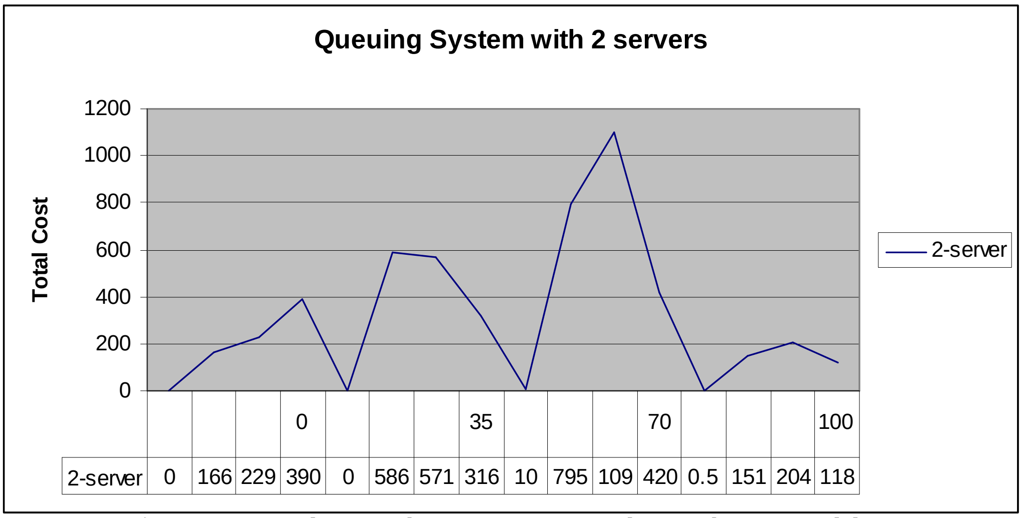


Figure 3.15 Total cost with 2 servers computed using the new model.

	w1	w2	w3	w4	y1	y2	y3	y4	Total Cost
1	0.0000222222	0	0	0.00118977	-0.00641333	0	0	-0.283375	-233.816
2	0.0000222222	0.854564	0.0000222222	0.407198	-0.000968889	171.511	-0.000469855	-7.48768	165.575
3	0.0000222222	0.227244	0.0000222222	0.970256	0.00447556	66.0598	0.00349267	195.41	229.243
4	0.0000222222	0.	0.	0.106509	0.00914222	0.	0.	41.5161	389.718
5	0.00118977	0	0	0.00118977	-0.0951814	0	0	-0.152251	-64.0785
6	0.407198	0.854564	0.5	0.407198	67.1877	427.282	74.6992	37.3896	586.407
7	0.5	0.227244	0.227244	0.96875	205.	134.074	74.4706	301.872	570.505
8	0.106509	0.	0.	0.106509	66.0355	0.	0.	53.2544	316.036
9	0.00118977	0	0	0.00118977	0.237954	0	0	0.0237553	10.2211
10	0.407198	0.28125	0.854564	0.28125	181.203	253.616	323.293	67.431	795.075
11	0.970256	0.227244	0.227244	0.28125	669.477	225.368	126.49	129.246	1097.1
12	0.106509	0.	0.	0.106509	95.858	0.	0.	69.0106	419.824
13	0.00118977	0.	0	0.	0.487805	0.	0	0.	0.487805
14	0.125	0.	0.125	0.	81.875	0.	68.75	0.	150.625
15	0.125	0.	0.125	0.	112.5	0.	91.0392	0.	203.539
16	0.106509	0.	0.	0.	118.225	0.	0.	0.	118.225

	w1	w2	w3	w4	y1	y2	y3	y4	Total Cost
1	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.277216	0	0	-0.0750571	-31.2655
2	0.00973499	$\frac{6769}{7921}$	0.00973499	$\frac{1369}{3362}$	2.50676	257.266	2.72571	26.486	283.171
3	0.00973499	$\frac{1800}{7921}$	0.00973499	$\frac{1631}{1681}$	2.74527	70.5593	2.87222	187.429	230.189
4	0.00973499	0	0	$\frac{18}{169}$	2.9497	0	0	32.2722	280.575
5	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.30577	0	0	-0.0465027	-19.237
6	$\frac{1369}{3362}$	$\frac{6769}{7921}$	0.723094	$\frac{1369}{3362}$	114.626	264.445	217.67	36.2587	611.899
7	0.723094	$\frac{1800}{7921}$	$\frac{1800}{7921}$	$\frac{1631}{1681}$	221.267	72.4681	71.8265	210.715	463.667
8	$\frac{18}{169}$	0	0	$\frac{18}{169}$	34.8284	0	0	34.8284	198.328
9	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.324807	0	0	-0.0274664	-11.2179
10	$\frac{1369}{3362}$	0.817729	$\frac{6769}{7921}$	$\frac{1369}{3362}$	121.141	257.625	269.23	42.7739	665.198
11	$\frac{1631}{1681}$	$\frac{1800}{7921}$	$\frac{1800}{7921}$	0.817729	312.422	73.7407	75.0133	190.673	546.204
12	$\frac{18}{169}$	0	0	$\frac{18}{169}$	36.5325	0	0	36.5325	208.033
13	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.35693	0	0	0.00465734	2.31418
14	$\frac{1369}{3362}$	0.0711997	$\frac{6769}{7921}$	0.0711997	132.136	23.1043	289.453	9.40153	451.389
15	$\frac{1631}{1681}$	0.0711997	$\frac{1800}{7921}$	0.0711997	338.619	23.7771	80.3909	18.5243	456.613
16	$\frac{18}{169}$	0	0	0.0711997	39.4083	0	0	26.3439	187.65

	w1	w2	w3	w4	y1	y2	y3	y4	Total Cost
1	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.422368	0	0	-0.315866	-132.32
2	0.005	0.0246914	0.005	0.0246914	1.8555	9.88519	1.88329	0.34788	19.4822
3	0.005	0.0246914	0.005	0.0246914	1.936	9.98889	1.96379	7.25095	135.994
4	0.005	0	0	0.0246914	2.005	0	0	13.1679	445.497
5	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.437835	0	0	-0.17769	-74.2366
6	$\frac{1369}{3362}$	$\frac{6769}{7921}$	0.82	$\frac{1369}{3362}$	156.405	353.789	318.203	53.0276	849.703
7	0.82	$\frac{1800}{7921}$	$\frac{1800}{7921}$	$\frac{1631}{1681}$	328.164	95.0335	91.8413	397.611	692.169
8	$\frac{18}{169}$	0	0	$\frac{18}{169}$	44.0947	0	0	69.1706	368.812
9	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.447353	0	0	-0.0926594	-38.4927
10	$\frac{1369}{3362}$	$\frac{6769}{7921}$	$\frac{6769}{7921}$	$\frac{1369}{3362}$	159.662	360.968	337.608	82.1295	890.784
11	$\frac{1631}{1681}$	$\frac{1800}{7921}$	$\frac{1800}{7921}$	0.954086	396.058	96.9423	93.4347	459.172	779.46
12	$\frac{18}{169}$	0	0	$\frac{18}{169}$	44.9467	0	0	76.7827	405.399
13	$\frac{2}{1681}$	0	0	$\frac{2}{1681}$	0.475907	0	0	0.162434	68.7387
14	0.163265	0.0293848	0.163265	0.0293848	67.9347	13.1526	67.9347	12.227	180.756
15	0.163265	0.0293848	0.163265	0.0293848	70.5633	13.276	70.5633	20.4422	207.458
16	$\frac{18}{169}$	0	0	0.0293848	47.503	0	0	27.4838	249.748

In continuous, two interval data for expected length of queue with 3 and 4 servers as, $L_{q3} = [0.03, 0.27, 0.43, 0.7]$ and $L_{q4} = [0.01, 0.14, 0.22, 0.46]$, are supposed respectively, where the average of the interval data L_{q3} is 0.36 and L_{q4} is 0.21, approximately.

The results of calculations are presented in Table 3.9 and Table 3.10.

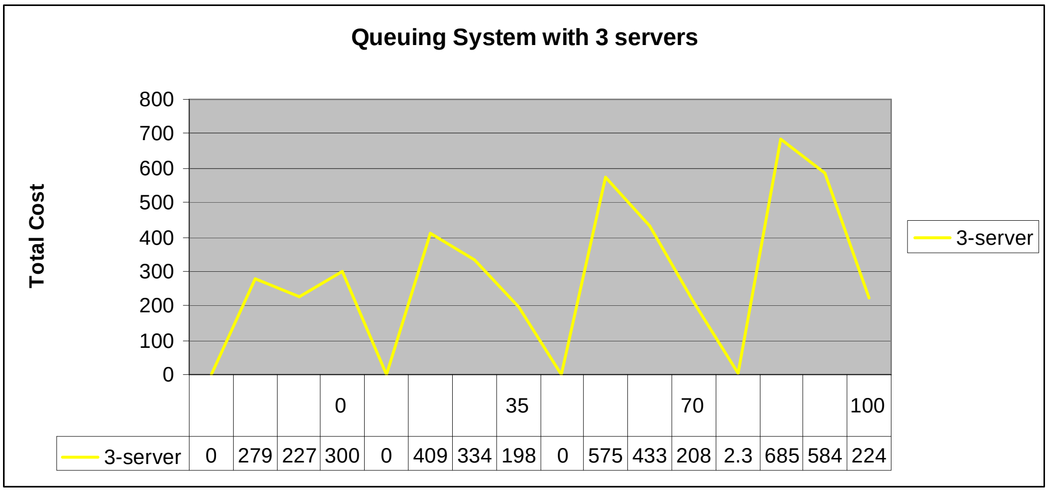


Figure 3.16 Total cost with 3 servers computed using the new model.

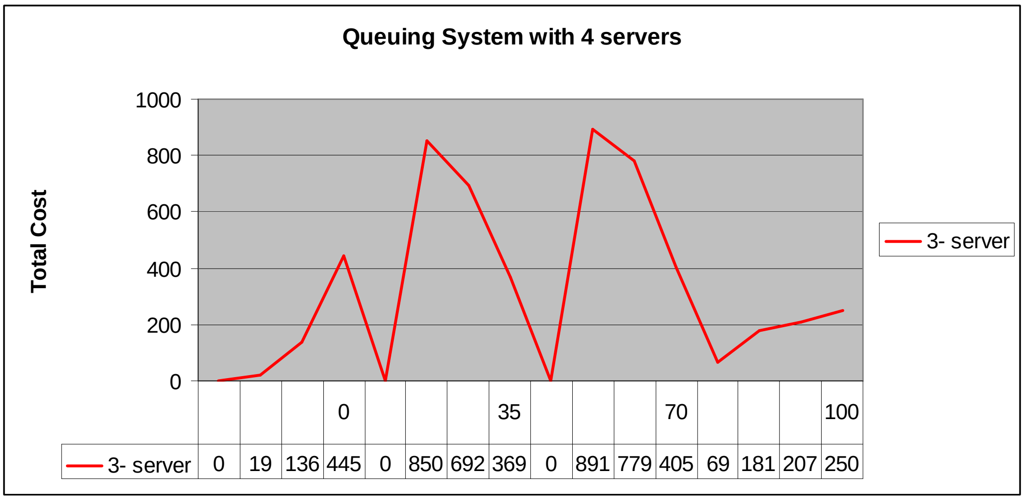
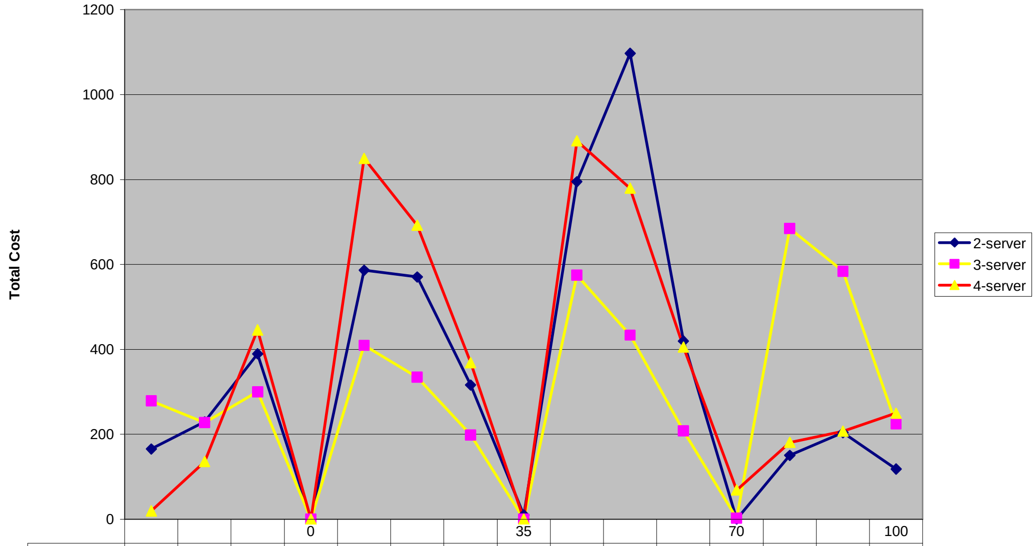


Figure 3.17 Total cost with 4 servers computed using the new model.

These results can be graphed to look for patterns and trends.



				0				35				70			100
2-server	165.58	229.24	389.72	0	586.41	570.51	316.04	10.221	795.08	1097.1	419.82	0.4878	150.63	203.54	118.23
3-server	278.8	227.42	300.28	0	409.45	334.3	198.33	0	574.65	433.31	208.03	2.3142	684.98	584.07	224.41
4-server	19.482	135.99	445.5	0	849.7	692.17	368.81	0	890.78	779.46	405.4	68.739	180.76	207.46	249.75

Figure 3.18 Total cost results in the queuing system with 2, 3 and 4 servers computed using the new model.

You can see in Figure 3.18, the economic advantages of a two servers system for range of $C_w = [70 - 100]$\$, and a three servers for range of $C_w = [0 - 70]$\$, in fuzzy analysis using the proposed model.

CONCLUSIONS

In this thesis a new hard interval clustering method is presented to describe consequences in developed TSK first order rule based fuzzy model in field of queuing system. This method is proposed based on interval input data sets(λ and μ) and linguistic fuzzy terms such as "Crowded"," Slow"," Fast" and so on. An output data set is calculated by a binary system performance function, on pairs of available data (λ_i, μ_i). The output data is divided to some clusters so that each cluster is distinguished by input subsets based on fuzzy term approximations. The interpolated linear membership function of every cluster describes the relation between premise and consequence of the related implication on system.

The new hard clustering method has been applied to a conventional queuing system. In the example problems, the coupling affects of $\tilde{\lambda}$ and $\tilde{\mu}$ on fuzzy system performances $\tilde{L}_q$ and $\tilde{W}_q$ has been revealed. These observations may provide managers with a clear and intuitive way to implement control systems in various conditions of queuing system.

This thesis provides distinctive practical results for designing queuing systems with fuzzy control and examines its benefits compared to other control methods. Using this method, the queuing system can be controlled more smooth and flexible in fuzzy mode in the real life.

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