

## EDITORIAL

### SPECIAL ISSUE SECTION ON FRACTAL AI-BASED ANALYSES AND APPLICATIONS TO COMPLEX SYSTEMS: PART I

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The universe, with its fractal structure patterns, includes infinite array of elements interacting in complex systems, while manifesting adaptability, self-organization and sensitivity to the external environment. A fractal system is also a nonlinear, complex and interactive system capable of adapting to a vague environment where Fractional Calculus (FC) reflects the actual state properties of the related systems which have latent and unforeseeable variations, so it enables integration and differentiation. Correspondingly, Fractal Artificial Intelligence (Fractal AI) allows the derivation of novel computational methods related to complex data and the respective advancements in computation, employed to make critical decisions, extract hidden information from the dynamics of complex system, quantizing processes and accelerating experiments by performing big data analyses for exploration in line with the modern science paradigm. Within this framework, fractal, fractional calculus, quantum, wavelet and entropy-based applications are some of the most widely used methods to solve the related multifaceted problems; and fractal-based analysis relies on self-similarity and contractivity with iterative function systems.

The key objective of this Special Issue is directed towards the new developments based on fractal, fractional calculus and multivariate fractional methods in conjunction with their applications in complex systems, including the theoretical and numerical dimensions. Robust solutions for problems of complex systems, thus, tend to have dynamic, nonlinear, nondifferentiable, transient and chaotic properties. In view

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of that, various papers on fractal, fractional calculus, quantum, wavelet, entropy-based as well as fractal AI-based analyses and applications have been incorporated into this Special Issue following the completion of detailed peer-review processes and related procedures.

The key points and contributions of each accepted paper of the Part I of our Special Issue are outlined in the following:

The paper entitled “On the Approximate Solutions for a System of Coupled Korteweg–de Vries Equations with Local Fractional Derivative” addresses the utilization of local fractional reduced differential transform method (LFRDTM) and local fractional Laplace variational iteration method (LFLVIM) for the purpose of obtaining the approximate solutions for coupled KdV equations. The results of the study following comparative analysis reveal that the algorithms proposed prove to be effective in handling linear and nonlinear problems in the fields of engineering and sciences while showing the techniques’ powerful aspect to obtain approximate or analytical solutions of other problems related to Local Fractional Derivative Operators (LFDOs).

In the work named “Complex Fractional-order HIV Diffusion Model Based on Amplitude Equations with Turing Patterns and Turing Instability”, the Turing instability due to fractional diffusion in Human Immunodeficiency Virus (HIV) model is dealt with through the application of the Turing bifurcation point and weakly nonlinear analysis within the complex fractional order dynamics. The highly relevant experimental results point that the related effects can be explored and implemented for different mathematical, physical, engineering and biological models. The study also underlines the importance of Fractional Calculus due to the fact that it enables the revealing of the underlying processes and dynamic phenomena that occur in biological tissues at multiscales.

The paper entitled “Solution of the Local Fractional Generalized KdV Equation Using Homotopy Analysis Method”, is concerned with solution of the  $n$ -Generalized KdV equation by local fractional homotopy analysis method (LFHAM), which is followed by the analysis of the approximate solution in the form of nondifferentiable generalized functions defined on Cantor sets along with the discussion of examples and special cases based on the main results. The contribution is the proving of two theorems and the presentation of the illustrative examples, which corroborate the method’s efficiency.

In another paper, “A Fractal Fractional Model for Cervical Cancer due to *Human Papillomavirus Infection*”, the authors investigate cervical cancer based on the construction of the compartmental model, studying the disease dynamics by the extended fractal–fractional model. The future direction of the scheme proposed in the study is that the methods employed can be applied efficiently on extended fractal-fractional predator–prey models and chemical reaction models, with the idea proposed having the potential of being extended to the reaction–diffusion models.

The existence theory of fractional differential equations that involve Atangana–Baleanu derivative of order  $1 < \alpha \leq 2$ , with nonseparated and integral-type boundary conditions, is addressed in the work entitled “Existence Results for ABC-Fractional Differential Equations with Non-separated and Integral Type of Boundary Conditions”. Using the Krasnoselskii’s fixed-point theorem, an existence result for the solutions of given AB-fractional differential equation has been proven through the related examples. The future direction is pointed towards the assumption of the problem with general sort of boundary conditions such as integral-type nonseparated boundary conditions and multistrips.

In the following work entitled “Fractional Mayer Neuro-swarm Heuristic Solver for Multi-Fractional Order Doubly Singular Model based on Lane–Emden Equation”, the authors present a novel fractional Mayer neuro-swarming intelligent solver to treat the multifractional order doubly singular Lane–Emden equation in a numerical way through the utilization of combination of Mayer Wavelet (MW) neural networks (NNs)

optimized by the global search effectiveness of particle swarm optimization (PSO) and interior-point (IP) method. Based on the convergence, robustness, stability and accuracy, comparative investigations are made relying on the reference results on different measures while statistical interpretations demonstrate the performance of the fractional Mayer neuro-swarming intelligent solvers proposed by the authors who also point towards future studies which can explore the MW-NN-PSOIP method's differential equations containing integer order as well as fractional order representations of the variables.

In another work named “New Newton's Type Estimates Pertaining to Local Fractional Integral via Generalized  $p$ -Convexity with Applications”, the notion of  $p$ -convex functions on fractal sets  $\mathbb{R}^{\hat{\alpha}}$ , ( $0 < \hat{\alpha} \leq 1$ ) is investigated, having derived an auxiliary result depending on a three-step quadratic kernel through the employment of generalized  $p$ -convexity. The authors established novel Newton's type of variants for the local differentiable functions and the application is done in cumulative distribution function as well as generalized special weighted means for the confirmation of the considered method's computational effectiveness in pure and applied science domains when approximation schemes are involved.

Several novel classes of convex functions involving arbitrary nonnegative functions are elaborated in Hilbert space in the study named “New Multi-Functional Approach for  $K$ th-Order Differentiability Governed by Fractional Calculus via Approximately Generalized  $(\psi, \hbar)$ -Convex Functions in Hilbert Space”. Based on the HOS-generalized  $(\psi, \hbar)$ -convex function representation, several theorems have been established and related novel consequences are presented, which all demonstrate better performance for the HOS-generalized  $\psi$ -quasiconvex functions. As future direction, the advanced convexity properties in the prein-  
vexity context can be extended to inequality theory, quantum calculus, artificial intelligence, robotics and forecasting applications in different promising areas.

The paper “Dynamical Analysis of the Meminductor-based Chaotic System with Hidden Attractor”, provides the design of a simple meminductor-based chaotic system proposing the scheme as a mathematical model for application in nonlinear circuits. The study provides the analysis of the system's dynamic characteristics through theoretical analysis as well as numerical simulation, revealing novel nonlinear phenomena, such as transient chaos and state transitions. As future work, the signals generated by the related complex chaotic system can be applied to information security and communication encryption in a well-suited way.

The abstract version of the Moore–Gibson–Thompson equation:  $au_{ttt} + \beta u_{tt} + c^2 \Delta u + b \Delta u_t + \int_0^t h(t-s) \Delta u(s) ds = 0$  depending on the parameters  $\alpha, \beta, b > 0$  is considered in the study entitled “Solvability of the Moore–Gibson–Thompson Equation with Viscoelastic Memory Term and Integral Condition via Galerkin Method”. The aim of the study is to investigate the solvability of the Moore–Gibson–Thompson equation with viscoelastic memory term and integral condition via the Galerkin method.

In another work entitled “Design of Neuro-swarming Heuristic Solver for Multi-pantograph Singular Delay Differential Equation”, the authors focus on the design of a neural-swarming heuristic procedure for numerical investigations of Singular Multi-Pantograph Delay Differential (SMP-DD) equation through the application of the function approximation aptitude of Artificial Neural Networks (ANNs)-optimized efficient swarming mechanism based on Particle Swarm Optimization (PSO) integrated with the convex optimization with Active Set (AS) algorithm for rapid refinements (ANN-PSO-AS). The future direction is pointed towards the use of accurate stochastic numerical procedure based on the ANN-PSO-AS algorithm for higher-order functional differential model, dynamical analysis of computational fluid mechanics problems and so forth.

In the following paper “A Powerful Iterative Approach for Quintic Complex Ginzburg–Landau Equation within the Frame of Fractional Operator”, which is directed towards the analysis of the behavior of various models, the main aim is to find the iterative solution for generalized quintic complex Ginzburg–Landau (GCGL) equation through the employment of fractional natural decomposition method (FNDM) in Fractional Calculus framework. The plots with different fractional orders demonstrate the significance

of integrating fractional concept into systems that manifest nonlinear complex phenomena in science and technology.

The study entitled “Qualitative Properties of Solutions of Fractional Differential and Difference Equations Arising in Physical Models” seeks to formulate some of the discrete fractional nonlinear inequalities with  $\xi$  fractional sum operators employed with certain conventional and forthright inequalities. The explicit bounds of Gronwall type inequalities are recreated through the observation of the principle of discrete fractional calculus for unknown functions. The main results may be proven by the use of the analysis process and the methodology of the mean value theorem, which can be employed as a beneficial device in subjective examination solutions of discrete fractional difference equations.

The study entitled “Stability Result and Well Posedness for Timoshenko’s Beam Laminated with Thermoelectric and Past History” proves the stability combined well-posedness results of solutions for a linear Timoshenko’s beam laminated with thermoplastic and past history along with distributed delay terms for the both cases non-equal and equal speeds of wave propagation. By considering two dissipations, which are the infinite memory and distributed delay, on the laminated Timoshenko beam system, it is revealed that that sort of damping mechanism is effective in other systems as well due to obtaining a well-posedness and general decay results with minimal conditions, which is important as future work considering nonlocal conditions such as integral boundary conditions.

The work “New Computations of Ostrowski Type Inequality Pertaining to Fractal Style with Applications” is a study providing novel estimates of Ostrowski-type inequalities in a simpler manner of some recent significant results in the context of a fractal set  $\mathbb{R}^\alpha$ . An auxiliary result which correlates with generalized convex (GC) and concave functions for absolutely continuous functions with second-order local differentiable mappings has been established by the new approach presented in the study whose contribution is to provide a better understanding of fractal analysis in the exploration of real-world phenomena.

The following paper, “Achieving More Precise Bounds Based on Double and Triple Integral as Proposed by Generalized Proportional Fractional Operators in the Hilfer Sense”, presents a convenient and user-friendly approach that depends on nonlocal kernel. The study provides some novel cases with special conditions, which are also analyzed comparatively. Accordingly, the outcomes demonstrate that the plans proposed are significant computationally so that comparable sorts of differential equations can be addressed in a robust way to investigate specific classes of integro-differential equations. As future research, the new techniques obtained can be extended further to achieve analytical solutions of the fractional Schrödinger equations besides high-dimensional fractional equations concerning image processing.

The last work in Part I of our Special Issue is named “Monotonicity and Asymptotic Properties of Solutions for Parabolic Equations via a Given Initial Value Condition on Graphs”, which establishes several results that involve the minimum and maximum principles and the comparison principles for elliptic equations and parabolic equations on finite graphs. An illustration with numerical experiments has been presented to demonstrate the main results in the study which can show a future direction considering time-fractional equations and parabolic equations.

We are of the opinion and anticipation that our Special Issue will open up a new frontier so that novel and robust solutions can be provided by scientists and researchers working on applied sciences, engineering, medicine, biology, finance and so forth in order to address challenges concerning complex systems. Our Special Issue has its subsequent continuation as Part II, Part III and Part IV for further investigation.

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