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Research article

The (2+1)-dimensional hyperbolic nonlinear Schrödinger equation and its optical solitons

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Abstract: A comprehensive study on the (2+1)-dimensional hyperbolic nonlinear Schrödinger (2D-HNLS) equation describing the propagation of electromagnetic fields in self-focusing and normally dispersive planar wave guides in optics is conducted in the current paper. To this end, after reducing the 2D-HNLS equation to a one-dimensional nonlinear ordinary differential (1D-NLOD) equation in the real regime using a traveling wave transformation, its optical solitons are formally obtained through a group of well-established methods such as the exponential and Kudryashov methods. Some graphical representations regarding optical solitons that are categorized as bright and dark solitons are considered to clarify the dynamics of the obtained solutions. It is noted that some of optical solitons retrieved in the current study are new and have been not retrieved previously.

Keywords: (2+1)-dimensional hyperbolic nonlinear Schrödinger equation; electromagnetic fields; traveling wave transformation; exponential and Kudryashov methods; bright and dark solitons
Mathematics Subject Classification: 35-XX, 35C08

1. Introduction

There are many nonlinear phenomena in nonlinear optics and other areas of scientific disciplines that are modeled by nonlinear Schrödinger equations. The Sasa–Satsuma equation [1–3], the complex Ginzburg–Landau equation [4–6], the Fokas–Lenells equation [7–9], the perturbed Gerdjikov–Ivanov equation [10–12], the Biswas–Arshed equation [13–15], the Kudryashov equation [16–18], and the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation [19–27] are a family of nonlinear Schrödinger equations that model nonlinear phenomena related to themselves. For example, the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation [19–27]

$$i\frac{\partial u}{\partial y}+\frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial t^2}\right)+|u|^2u=0, \qquad (1)$$

models the propagation of electromagnetic fields in self-focusing and normally dispersive planar wave guides in optics. In Eq (1), $u=u(x,y,t)$ is a dependent variable and x, y, and t represent spatial and temporal variables. The importance of exploring the 2D-HNLS equation has tempted a lot of researchers to consider Eq (1) as a canonical model in their studies. In this regard, Ai-Lin and Ji [22] adopted the Lie group symmetry method to find Lie point symmetries and exact traveling solutions of the 2D-HNLS equation. Aliyu et al. [23] investigated optical solitary waves of the 2D-HNLS equation using the solitary wave ansatz. Apeanti et al. [24] applied a generalized elliptic expansion method to look for optical solitons of the 2D-HNLS equation. Durur et al. [25] obtained periodic and singular wave solutions of the 2D-HNLS equation via the projected method. Tala-Tebue and his collaborators [26] considered the 2D-HNLS equation in their article and found its optical solitons through the modified Jacobi elliptic method. Very recently, exact solutions of the 2D-HNLS equation were constructed by Ur Rehman [27] using a group of well-organized methods.

Today, due to the development of computer algebra systems like MAPLE and MATHEMATICA, handling symbolic computations has become easier and more convenient than the past. Such an evolution led to the establishment of a series of effective methods to construct soliton solutions of nonlinear partial differential equations (NLPDEs). Two useful techniques that profit from the existence of symbolic computation packages in extracting soliton solutions of NLPDEs are the exponential and Kudryashov methods [28–40]. The exponential and Kudryashov methods are two easy-to-use techniques that have demonstrated their performance in dealing with NLPDEs. To address a series of recent applications of these useful methods, Zafar et al. [35] applied the exponential method as a newly well-designed method to seek new exact solutions of the conformable time-fractional Cahn–Allen equation. Hosseini et al. [40] derived optical solitons of an integrable (2+1)-dimensional nonlinear Schrödinger system using the Kudryashov methods. The need for further studies on the existence of other optical solitons of the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation cheered the authors to apply the exponential and Kudryashov methods for performing such a key goal. More works can be found in [41–51].

The organization of this paper is as follows: In Section 2, a detailed review regarding the exponential and Kudryashov methods is presented. In Section 3, after reducing the 2D-HNLS equation to a 1D-NLOD equation in the real regime using a traveling wave transformation, its optical solitons are formally obtained through the exponential and Kudryashov methods. Additionally, some graphical representations regarding bright and dark solitons are considered to clarify their dynamics. The paper concludes with a short review of the outcomes.

2. Methods and their review

This section presents a detailed review regarding the exponential and Kudryashov methods. For this aim, let's consider the following NLOD equation

$$P(U,U',U'',\dots)=0,\qquad {}'=\frac{d}{d\epsilon}, \tag{2}$$

where P is a polynomial in terms of U and its derivatives.

2.1. Exponential method

The exponential method profits from applying a solution for Eq (2) as follows

$$U(\epsilon)=\frac{a_0+a_1a^{\epsilon}+\dots+a_Na^{N\epsilon}}{b_0+b_1a^{\epsilon}+\dots+b_Na^{N\epsilon}},\quad a_N\neq 0,\quad b_N\neq 0, \tag{3}$$

where $a_i, i=0,1,\dots,N$ and $b_i, i=0,1,\dots,N$ are computed later and $N\in\mathbb{Z}^+$. By substituting Eq (3) into the NLOD Eq (2) and using a number of operations, we attain a set of nonlinear algebraic equations whose solution yields soliton solutions of Eq (2).

2.2. Kudryashov methods

The Kudryashov method adopts a solution for Eq (2) as follows

$$U(\epsilon)=a_0+a_1K(\epsilon)+\dots+a_NK^N(\epsilon),\quad a_N\neq 0, \tag{4}$$

where $a_i, i=0,1,\dots,N$ are evaluated later, N is obtained by the balance principle, and $K(\epsilon)$ is the following function

$$K(\epsilon)=\frac{4A}{(4A^2-\eta)\sinh(\epsilon)+(4A^2+\eta)\cosh(\epsilon)},\quad \eta=4AB,$$

satisfying a nonlinear equation as

$$\left(K'(\epsilon)\right)^2=K^2(\epsilon)\left(1-\eta K^2(\epsilon)\right).$$

By inserting Eq (4) into the NLOD Eq (2) and using a number of operations, we reach a set of nonlinear algebraic equations whose solution gives soliton solutions of Eq (2).

It is noteworthy that instead of Eq (4), the solution of Eq (2) can be considered as the following form [40–42]

$$U(\epsilon)=a_0+\sum_{i=1}^{N}\left(\frac{K(\epsilon)}{1+K^2(\epsilon)}\right)^{i-1}\left(a_i\frac{K(\epsilon)}{1+K^2(\epsilon)}+b_i\frac{1-K^2(\epsilon)}{1+K^2(\epsilon)}\right),\quad a_N \text{ or } b_N\neq 0, \tag{5}$$

where a_0, a_i $(i=1,2,\dots,N)$, and b_i $(i=1,2,\dots,N)$ are found later, N is derived by the balance technique, and $K(\epsilon)$ is a function in the form

$$K(\epsilon) = \frac{4A}{(4A^2 - \eta)\sinh(\epsilon) + (4A^2 + \eta)\cosh(\epsilon)}, \quad \eta = 4AB,$$

satisfying

$$\left(K'(\epsilon)\right)^2 = K^2(\epsilon)\left(1 - \eta K^2(\epsilon)\right).$$

By setting Eq (5) in the NLOD Eq (2) and using a number of operations, we arrive at a set of nonlinear algebraic equations whose solution results in soliton solutions of Eq (2).

3. 2D-HNLS equation and its optical solitons

The current section presents optical solitons of the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation that are formally derived through adopting a series of effective methods such as the exponential and Kudryashov methods. To begin, let's employ a traveling wave transformation as follows

$$u(x, y, t) = U(\epsilon)e^{i(x+\alpha_2 y+\beta_2 t)}, \quad \epsilon = x + \alpha_1 y + \beta_1 t, \tag{6}$$

where speed and frequency are represented by β_1 and β_2, respectively. After substituting Eq (6) into Eq (1) and distinguishing the real and imaginary expressions, one obtains the following second-order NLOD equation

$$\left(1 - {\beta_1}^2\right)\frac{d^2U(\epsilon)}{d\epsilon^2} + \left({\beta_2}^2 - 2\alpha_2 - 1\right)U(\epsilon) + 2U^3(\epsilon) = 0, \tag{7}$$

where

$$\alpha_1 = \beta_1\beta_2 - 1.$$

3.1. Exponential method and its application

Because $N \in \mathbb{Z}^+$, consequently, the solution of Eq (7) can be expressed as follows

$$U(\epsilon) = \frac{a_0 + a_1 a^{\epsilon} + a_2 a^{2\epsilon}}{b_0 + b_1 a^{\epsilon} + b_2 a^{2\epsilon}}, \quad a_2 \neq 0, \quad b_2 \neq 0, \tag{8}$$

where $a_i, i = 0,1,2$ and $b_i, i = 0,1,2$ are computed later. By substituting Eq (8) into the NLOD Eq (7) and using a number of operations, we attain a set of nonlinear algebraic equations as

$$a_0{b_0}^2{\beta_2}^2 - 2a_0\alpha_2{b_0}^2 + 2{a_0}^3 - a_0{b_0}^2 = 0,$$

$$(\ln(a))^2a_0b_0b_1{\beta_1}^2 - (\ln(a))^2a_1{b_0}^2{\beta_1}^2 - (\ln(a))^2a_0b_0b_1 + (\ln(a))^2a_1{b_0}^2 + 2a_0b_0b_1{\beta_2}^2 + a_1{b_0}^2{\beta_2}^2 - 4a_0\alpha_2b_0b_1 - 2a_1\alpha_2{b_0}^2 + 6{a_0}^2a_1 - 2a_0b_0b_1 - a_1{b_0}^2 = 0,$$

$$4(\ln(a))^2a_0b_0b_2{\beta_1}^2 - (\ln(a))^2a_0{b_1}^2{\beta_1}^2 + (\ln(a))^2a_1b_0b_1{\beta_1}^2 - 4(\ln(a))^2a_2{b_0}^2{\beta_1}^2 - 4(\ln(a))^2a_0b_0b_2 + (\ln(a))^2a_0{b_1}^2 - (\ln(a))^2a_1b_0b_1 + 4(\ln(a))^2a_2{b_0}^2 + 2a_0b_0b_2{\beta_2}^2 + a_0{b_1}^2{\beta_2}^2 + 2a_1b_0b_1{\beta_2}^2 + a_2{b_0}^2{\beta_2}^2 - 4a_0\alpha_2b_0b_2 - 2a_0\alpha_2{b_1}^2 - 4a_1\alpha_2b_0b_1 - 2a_2\alpha_2{b_0}^2 + 6{a_0}^2a_2 + 6a_0{a_1}^2 - 2a_0b_0b_2 - a_0{b_1}^2 - 2a_1b_0b_1 - a_2{b_0}^2 = 0,$$

$$-3(\ln(a))^2 a_0 b_1 b_2 {\beta_1}^2 + 6(\ln(a))^2 a_1 b_0 b_2 {\beta_1}^2 - 3(\ln(a))^2 a_2 b_0 b_1 {\beta_1}^2 + 3(\ln(a))^2 a_0 b_1 b_2 - 6 (\ln(a))^2 a_1 b_0 b_2 + 3(\ln(a))^2 a_2 b_0 b_1 + 2a_0 b_1 b_2 {\beta_2}^2 + 2a_1 b_0 b_2 {\beta_2}^2 + a_1 {b_1}^2 {\beta_2}^2 + 2a_2 b_0 b_1 {\beta_2}^2 - 4 a_0 \alpha_2 b_1 b_2 - 4a_1 \alpha_2 b_0 b_2 - 2a_1 \alpha_2 {b_1}^2 - 4a_2 \alpha_2 b_0 b_1 + 12a_0 a_1 a_2 - 2a_0 b_1 b_2 + 2{a_1}^3 - 2a_1 b_0 b_2 - a_1 {b_1}^2 - 2a_2 b_0 b_1 = 0,$$

$$-4(\ln(a))^2 a_0 {b_2}^2 {\beta_1}^2 + (\ln(a))^2 a_1 b_1 b_2 {\beta_1}^2 + 4(\ln(a))^2 a_2 b_0 b_2 {\beta_1}^2 - (\ln(a))^2 a_2 {b_1}^2 {\beta_1}^2 + 4 (\ln(a))^2 a_0 {b_2}^2 - (\ln(a))^2 a_1 b_1 b_2 - 4(\ln(a))^2 a_2 b_0 b_2 + (\ln(a))^2 a_2 {b_1}^2 + a_0 {b_2}^2 {\beta_2}^2 + 2 a_1 b_1 b_2 {\beta_2}^2 + 2a_2 b_0 b_2 {\beta_2}^2 + a_2 {b_1}^2 {\beta_2}^2 - 2a_0 \alpha_2 {b_2}^2 - 4a_1 \alpha_2 b_1 b_2 - 4a_2 \alpha_2 b_0 b_2 - 2a_2 \alpha_2 {b_1}^2 + 6 a_0 {a_2}^2 - a_0 {b_2}^2 + 6{a_1}^2 a_2 - 2a_1 b_1 b_2 - 2a_2 b_0 b_2 - a_2 {b_1}^2 = 0,$$

$$-(\ln(a))^2 a_1 {b_2}^2 {\beta_1}^2 + (\ln(a))^2 a_2 b_1 b_2 {\beta_1}^2 + (\ln(a))^2 a_1 {b_2}^2 - (\ln(a))^2 a_2 b_1 b_2 + a_1 {b_2}^2 {\beta_2}^2 + 2 a_2 b_1 b_2 {\beta_2}^2 - 2a_1 \alpha_2 {b_2}^2 - 4a_2 \alpha_2 b_1 b_2 + 6a_1 {a_2}^2 - a_1 {b_2}^2 - 2a_2 b_1 b_2 = 0,$$

$$a_2 {b_2}^2 {\beta_2}^2 - 2a_2 \alpha_2 {b_2}^2 + 2{a_2}^3 - a_2 {b_2}^2 = 0.$$

By utilizing a computer algebra system like MAPLE, we find:

Case 1.

$$a_1 = \frac{b_1 \ln(a)}{2\sqrt{\left({\beta_1}^2-1\right)^{-1}}}, \quad b_0 = \frac{2\sqrt{\left({\beta_1}^2-1\right)^{-1}}\, a_0}{\ln(a)},$$

$$b_2 = \frac{2a_2}{\ln(a)\left({\beta_1}^2-1\right)\sqrt{\left({\beta_1}^2-1\right)^{-1}}}, \quad \beta_2 = \pm\frac{1}{2}\sqrt{-2(\ln(a))^2 {\beta_1}^2 + 2(\ln(a))^2 + 8\alpha_2 + 4}.$$

Therefore, the following solitons to the 2D-HNLS equation are acquired

$$u_{1,2}(x,y,t) = \frac{a_0 + \frac{b_1 \ln(a)}{2\sqrt{\left({\beta_1}^2-1\right)^{-1}}} a^{x+\alpha_1 y+\beta_1 t} + a_2 a^{2(x+\alpha_1 y+\beta_1 t)}}{\frac{2\sqrt{\left({\beta_1}^2-1\right)^{-1}}\, a_0}{\ln(a)} + b_1 a^{x+\alpha_1 y+\beta_1 t} + \frac{2a_2}{\ln(a)\left({\beta_1}^2-1\right)\sqrt{\left({\beta_1}^2-1\right)^{-1}}} a^{2(x+\alpha_1 y+\beta_1 t)}}$$

$$\times e^{i\left(x+\alpha_2 y \pm \left(\frac{1}{2}\sqrt{-2(\ln(a))^2 {\beta_1}^2 + 2(\ln(a))^2 + 8\alpha_2 + 4}\right) t\right)},$$

$$\alpha_1 = \beta_1 \beta_2 - 1, \quad \beta_2 = \pm\frac{1}{2}\sqrt{-2(\ln(a))^2 {\beta_1}^2 + 2(\ln(a))^2 + 8\alpha_2 + 4}.$$

Case 2.

$$a_1 = -\frac{b_1 \ln(a)}{2\sqrt{\left({\beta_1}^2-1\right)^{-1}}}, \quad b_0 = -\frac{2\sqrt{\left({\beta_1}^2-1\right)^{-1}}\, a_0}{\ln(a)},$$

$$b_2 = -\frac{2a_2}{\ln(a)\left({\beta_1}^2-1\right)\sqrt{\left({\beta_1}^2-1\right)^{-1}}}, \quad \beta_2 = \pm\frac{1}{2}\sqrt{-2(\ln(a))^2 {\beta_1}^2 + 2(\ln(a))^2 + 8\alpha_2 + 4}.$$

Thus, the following solitons to the 2D-HNLS equation are gained

$$u_{3,4}(x,y,t)=\frac{a_0-\frac{b_1\ln(a)}{2\sqrt{(\beta_1{}^2-1)}^{-1}}a^{x+\alpha_1y+\beta_1t}+a_2a^{2(x+\alpha_1y+\beta_1t)}}{-\frac{2\sqrt{(\beta_1{}^2-1)}^{-1}a_0}{\ln(a)}+b_1a^{x+\alpha_1y+\beta_1t}-\frac{2a_2}{\ln(a)(\beta_1{}^2-1)\sqrt{(\beta_1{}^2-1)}^{-1}}a^{2(x+\alpha_1y+\beta_1t)}}$$

$$\times e^{i\left(x+\alpha_2y\pm\left(\frac{1}{2}\sqrt{-2(\ln(a))^2\beta_1{}^2+2(\ln(a))^2+8\alpha_2+4}\right)t\right)},$$

$$\alpha_1=\beta_1\beta_2-1,\ \ \beta_2=\pm\frac{1}{2}\sqrt{-2(\ln(a))^2\beta_1{}^2+2(\ln(a))^2+8\alpha_2+4}.$$

The graphical representations of $|u_1(x,y,t)|$ demonstrating the dark solitons have been considered in Figure 1. The appropriate values that have been utilized to portray Figure 1 are $a_0=1$, $a_2=1$, $b_1=1$, $\alpha_2=0.9$, $\beta_1=1.5$, $a=2.7$, and (a) $t=-1$ (b) $t=1$.

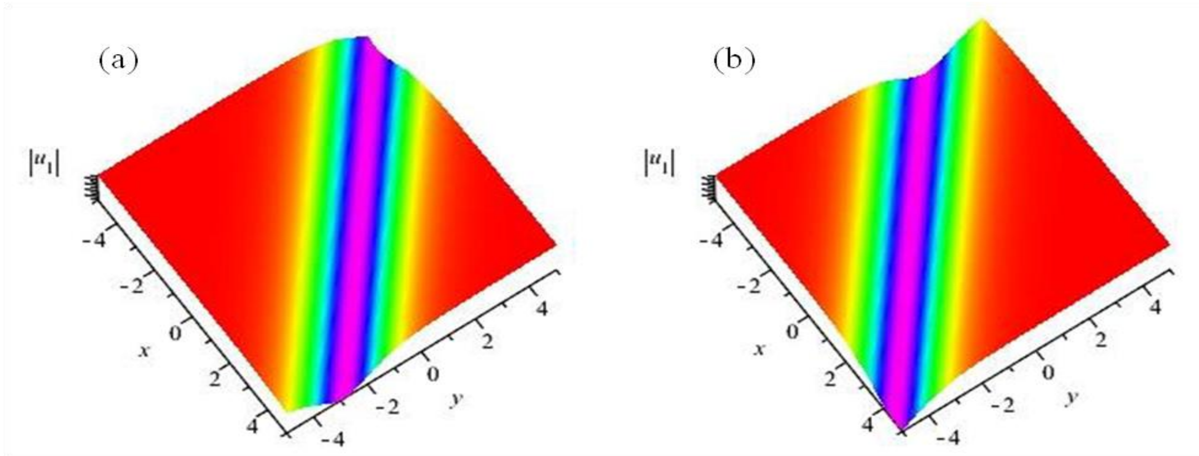

Figure 1. The graphical representations of $|u_1(x,y,t)|$ for $a_0=1$, $a_2=1$, $b_1=1$, $\alpha_2=0.9$, $\beta_1=1.5$, $a=2.7$, and (a) $t=-1$ (b) $t=1$.

3.2. Kudryashov methods and their applications

Based on Eq (7), the balance principle, and the Kudryashov method, a solution for Eq (7) is considered as follows

$$U(\epsilon)=a_0+a_1K(\epsilon),\quad a_1\neq 0, \tag{9}$$

where a_0 and a_1 are evaluated later. By inserting Eq (9) into Eq (7) and using a number of operations, we reach the following set of nonlinear algebraic equations

$$2\eta a_1\beta_1{}^2+2a_1{}^3-2\eta a_1=0,$$

$$6a_0a_1{}^2=0,$$

$$6a_0{}^2a_1-a_1\beta_1{}^2+a_1\beta_2{}^2-2a_1\alpha_2=0,$$

$$2a_0{}^3+a_0\beta_2{}^2-2a_0\alpha_2-a_0=0.$$

By adopting a computer algebra system like MAPLE, we find:

Case 1.

$$a_0 = 0,\ \ a_1 = \sqrt{-\eta{\beta_1}^2 + \eta},\ \ \beta_2 = \pm\sqrt{{\beta_1}^2 + 2\alpha_2}.$$

Therefore, the following solitons to the 2D-HNLS equation are acquired

$$u_{1,2}(x,y,t) = \frac{4A\sqrt{-\eta{\beta_1}^2+\eta}}{(4A^2-\eta)\sinh(x+\alpha_1 y+\beta_1 t)+(4A^2+\eta)\cosh(x+\alpha_1 y+\beta_1 t)} e^{i\left(x+\alpha_2 y\pm\sqrt{{\beta_1}^2+2\alpha_2 t}\right)},$$

$$\eta = 4AB,\ \ \alpha_1 = \beta_1\beta_2 - 1,\ \ \beta_2 = \pm\sqrt{{\beta_1}^2 + 2\alpha_2}.$$

Case 2.

$$a_0 = 0,\ \ a_1 = -\sqrt{-\eta{\beta_1}^2 + \eta},\ \ \beta_2 = \pm\sqrt{{\beta_1}^2 + 2\alpha_2}.$$

Thus, the following solitons to the 2D-HNLS equation are gained

$$u_{3,4}(x,y,t) = -\frac{4A\sqrt{-\eta{\beta_1}^2+\eta}}{(4A^2-\eta)\sinh(x+\alpha_1 y+\beta_1 t)+(4A^2+\eta)\cosh(x+\alpha_1 y+\beta_1 t)} e^{i\left(x+\alpha_2 y\pm\sqrt{{\beta_1}^2+2\alpha_2 t}\right)},$$

$$\eta = 4AB,\ \ \alpha_1 = \beta_1\beta_2 - 1,\ \ \beta_2 = \pm\sqrt{{\beta_1}^2 + 2\alpha_2}.$$

Figure 2 represents the graphical representations of $|u_1(x,y,t)|$ that signify the bright solitons. The appropriate values that have been used to plot Figure 2 are $A = 2,\ B = 1,\ \alpha_2 = 0.3,\ \beta_1 = 1.5$, and (a) $t = -1.5$ (b) $t = 1.5$.

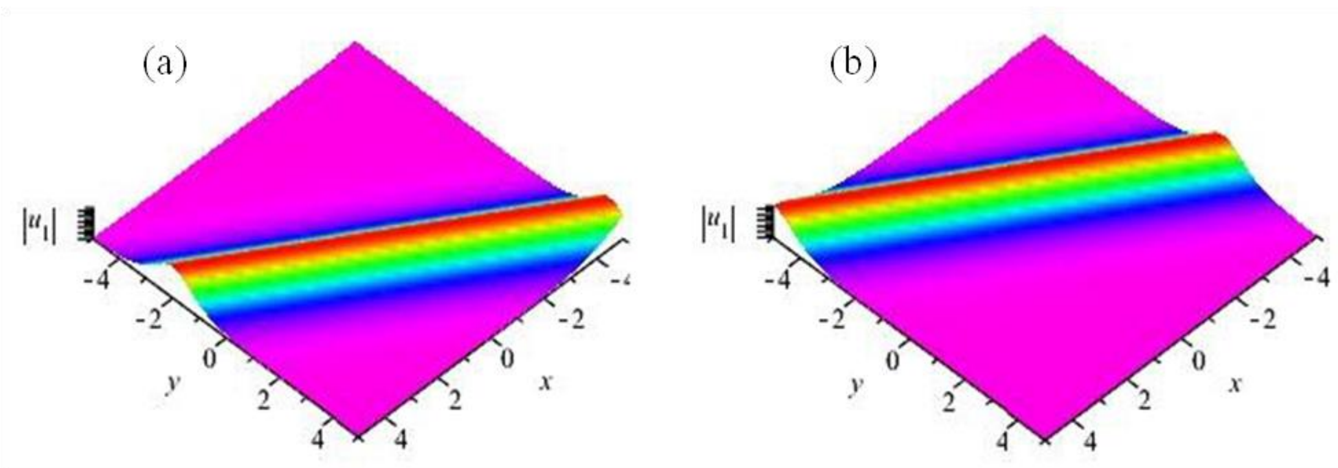

Figure 2. The graphical representations of $|u_1(x,y,t)|$ for $A = 2,\ B = 1,\ \alpha_2 = 0.3$, $\beta_1 = 1.5$, and (a) $t = -1.5$ (b) $t = 1.5$.

It is noteworthy that instead of Eq (9), the solution of Eq (7) can be considered as

$$U(\epsilon)=a_0+a_1\frac{K(\epsilon)}{1+K^2(\epsilon)}+a_2\frac{1-K^2(\epsilon)}{1+K^2(\epsilon)},\quad a_1 \text{ or } a_2\neq 0, \tag{10}$$

where a_0, a_1, and a_2 are found later. By setting Eq (10) in Eq (7) and using a number of operations, we arrive at a set of nonlinear algebraic equations as

$$16ABa_2{\beta_1}^2-16ABa_2+2{a_0}^3-6{a_0}^2a_2+6a_0{a_2}^2+a_0{\beta_2}^2-2{a_2}^3-a_2{\beta_2}^2-2a_0\alpha_2+2a_2\alpha_2-a_0+a_2=0,$$

$$-24ABa_1{\beta_1}^2+24ABa_1+6{a_0}^2a_1-12a_0a_1a_2+6a_1{a_2}^2-a_1{\beta_1}^2+a_1{\beta_2}^2-2a_1\alpha_2=0,$$

$$-48ABa_2{\beta_1}^2+48ABa_2+6{a_0}^3-6{a_0}^2a_2+6a_0{a_1}^2-6a_0{a_2}^2+3a_0{\beta_2}^2-6{a_1}^2a_2+6{a_2}^3-8a_2{\beta_1}^2-a_2{\beta_2}^2-6a_0\alpha_2+2a_2\alpha_2-3a_0+9a_2=0,$$

$$8ABa_1{\beta_1}^2-8ABa_1+12{a_0}^2a_1+2{a_1}^3-12a_1{a_2}^2+6a_1{\beta_1}^2+2a_1{\beta_2}^2-4a_1\alpha_2-8a_1=0,$$

$$6{a_0}^3+6{a_0}^2a_2+6a_0{a_1}^2-6a_0{a_2}^2+3a_0{\beta_2}^2+6{a_1}^2a_2-6{a_2}^3+8a_2{\beta_1}^2+a_2{\beta_2}^2-6a_0\alpha_2-2a_2\alpha_2-3a_0-9a_2=0,$$

$$6{a_0}^2a_1+12a_0a_1a_2+6a_1{a_2}^2-a_1{\beta_1}^2+a_1{\beta_2}^2-2a_1\alpha_2=0,$$

$$2{a_0}^3+6{a_0}^2a_2+6a_0{a_2}^2+a_0{\beta_2}^2+2{a_2}^3+a_2{\beta_2}^2-2a_0\alpha_2-2a_2\alpha_2-a_0-a_2=0.$$

By utilizing a computer algebra system like MAPLE, we find:

Case 1.

$$B=0,\ a_0=0,\ a_1=2\sqrt{-{\beta_2}^2+2\alpha_2+1},\ a_2=0,\ \beta_1=\pm\sqrt{{\beta_2}^2-2\alpha_2}.$$

Therefore, the following solitons to the 2D-HNLS equation are acquired

$$u_{1,2}(x,y,t)=2A\sqrt{-{\beta_2}^2+2\alpha_2+1}\,\frac{\sinh(x+\alpha_1y+\beta_1t)+\cosh(x+\alpha_1y+\beta_1t)}{(A\sinh(x+\alpha_1y+\beta_1t)+A\cosh(x+\alpha_1y+\beta_1t))^2+1}\,e^{i(x+\alpha_2y+\beta_2t)},$$

$$\alpha_1=\beta_1\beta_2-1,\ \beta_1=\pm\sqrt{{\beta_2}^2-2\alpha_2}.$$

Case 2.

$$B=0,\ a_0=0,\ a_1=-2\sqrt{-{\beta_2}^2+2\alpha_2+1},\ a_2=0,\ \beta_1=\pm\sqrt{{\beta_2}^2-2\alpha_2}.$$

Thus, the following solitons to the 2D-HNLS equation are derived

$$u_{3,4}(x,y,t)=-2A\sqrt{-{\beta_2}^2+2\alpha_2+1}\,\frac{\sinh(x+\alpha_1y+\beta_1t)+\cosh(x+\alpha_1y+\beta_1t)}{(A\sinh(x+\alpha_1y+\beta_1t)+A\cosh(x+\alpha_1y+\beta_1t))^2+1}\,e^{i(x+\alpha_2y+\beta_2t)},$$

$$\alpha_1=\beta_1\beta_2-1,\ \beta_1=\pm\sqrt{{\beta_2}^2-2\alpha_2}.$$

Case 3.

$$B=-\frac{1}{2A},\ \ a_0=-a_2,\ \ a_1=0,\ \ \beta_1=\sqrt{-a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-4a_2{}^2+2\alpha_2+1}.$$

Consequently, the following solitons to the 2D-HNLS equation are obtained

$$u_{5,6}(x,y,t)=-\frac{8a_2A^2}{2\sinh(x+\alpha_1y+\beta_1t)\cosh(x+\alpha_1y+\beta_1t)(4A^4-1)+2(\cosh(x+\alpha_1y+\beta_1t))^2(4A^4+1)-4A^4-1}$$

$$\times e^{i(x+\alpha_2y+\beta_2t)},$$

$$\alpha_1=\beta_1\beta_2-1,\ \ \beta_1=\sqrt{-a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-4a_2{}^2+2\alpha_2+1}.$$

Case 4.

$$B=-\frac{1}{2A},\ \ a_0=-a_2,\ \ a_1=0,\ \ \beta_1=-\sqrt{-a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-4a_2{}^2+2\alpha_2+1}.$$

Accordingly, the following solitons to the 2D-HNLS equation are gained

$$u_{7,8}(x,y,t)=-\frac{8a_2A^2}{2\sinh(x+\alpha_1y+\beta_1t)\cosh(x+\alpha_1y+\beta_1t)(4A^4-1)+2(\cosh(x+\alpha_1y+\beta_1t))^2(4A^4+1)-4A^4-1}$$

$$\times e^{i(x+\alpha_2y+\beta_2t)},$$

$$\alpha_1=\beta_1\beta_2-1,\ \ \beta_1=-\sqrt{-a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-4a_2{}^2+2\alpha_2+1}.$$

Case 5.

$$B=0,\ \ a_0=0,\ \ a_1=2ia_2,\ \ \beta_1=\sqrt{4a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-2a_2{}^2+2\alpha_2+1}.$$

So, the following exact solutions to the 2D-HNLS equation are acquired

$$u_{9,10}(x,y,t)=\frac{a_2\big((A^2+1)\sinh(x+\alpha_1y+\beta_1t)+(A^2-1)\cosh(x+\alpha_1y+\beta_1t)+2iA\big)}{(A^2-1)\sinh(x+\alpha_1y+\beta_1t)+(A^2+1)\cosh(x+\alpha_1y+\beta_1t)}e^{i(x+\alpha_2y+\beta_2t)},$$

$$\alpha_1=\beta_1\beta_2-1,\ \ \beta_1=\sqrt{4a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-2a_2{}^2+2\alpha_2+1}.$$

Case 6.

$$B=0,\ \ a_0=0,\ \ a_1=2ia_2,\ \ \beta_1=-\sqrt{4a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-2a_2{}^2+2\alpha_2+1}.$$

Therefore, the following exact solutions to the 2D-HNLS equation are derived

$$u_{11,12}(x,y,t)=\frac{a_2\big((A^2+1)\sinh(x+\alpha_1y+\beta_1t)+(A^2-1)\cosh(x+\alpha_1y+\beta_1t)+2iA\big)}{(A^2-1)\sinh(x+\alpha_1y+\beta_1t)+(A^2+1)\cosh(x+\alpha_1y+\beta_1t)}e^{i(x+\alpha_2y+\beta_2t)},$$

$$\alpha_1=\beta_1\beta_2-1,\ \ \beta_1=-\sqrt{4a_2{}^2+1},\ \ \beta_2=\pm\sqrt{-2a_2{}^2+2\alpha_2+1}.$$

Case 7.

$$B = 0,\ a_0 = 0,\ a_1 = -2ia_2,\ \beta_1 = \sqrt{4a_2{}^2 + 1},\ \beta_2 = \pm\sqrt{-2a_2{}^2 + 2\alpha_2 + 1}.$$

Thus, the following exact solutions to the 2D-HNLS equation are obtained

$$u_{13,14}(x, y, t) = \frac{a_2\big((A^2+1)\sinh(x+\alpha_1 y+\beta_1 t)+(A^2-1)\cosh(x+\alpha_1 y+\beta_1 t)-2iA\big)}{(A^2-1)\sinh(x+\alpha_1 y+\beta_1 t)+(A^2+1)\cosh(x+\alpha_1 y+\beta_1 t)} e^{i(x+\alpha_2 y+\beta_2 t)},$$

$$\alpha_1 = \beta_1\beta_2 - 1, \qquad \beta_1 = \sqrt{4a_2{}^2 + 1},\ \beta_2 = \pm\sqrt{-2a_2{}^2 + 2\alpha_2 + 1}.$$

Case 8.

$$B = 0,\ a_0 = 0,\ a_1 = -2ia_2,\ \beta_1 = -\sqrt{4a_2{}^2 + 1},\ \beta_2 = \pm\sqrt{-2a_2{}^2 + 2\alpha_2 + 1}.$$

Consequently, the following exact solutions to the 2D-HNLS equation are gained

$$u_{15,16}(x, y, t) = \frac{a_2\big((A^2+1)\sinh(x+\alpha_1 y+\beta_1 t)+(A^2-1)\cosh(x+\alpha_1 y+\beta_1 t)-2iA\big)}{(A^2-1)\sinh(x+\alpha_1 y+\beta_1 t)+(A^2+1)\cosh(x+\alpha_1 y+\beta_1 t)} e^{i(x+\alpha_2 y+\beta_2 t)},$$

$$\alpha_1 = \beta_1\beta_2 - 1, \qquad \beta_1 = -\sqrt{4a_2{}^2 + 1},\ \beta_2 = \pm\sqrt{-2a_2{}^2 + 2\alpha_2 + 1}.$$

Note 1: It should be stated that our results were examined by MAPLE, confirming their correctness.

Note 2: It is noteworthy that optical solitons generated by the first and third methods are new and have been not retrieved previously.

The graphical representations of $|u_5(x, y, t)|$ demonstrating the bright solitons have been considered in Figure 3. The appropriate values that have been utilized to portray Figure 3 are $A = 1$, $a_2 = 0.5$, $\alpha_2 = 1$, and (a) $t = -1.5$ (b) $t = 1.5$.

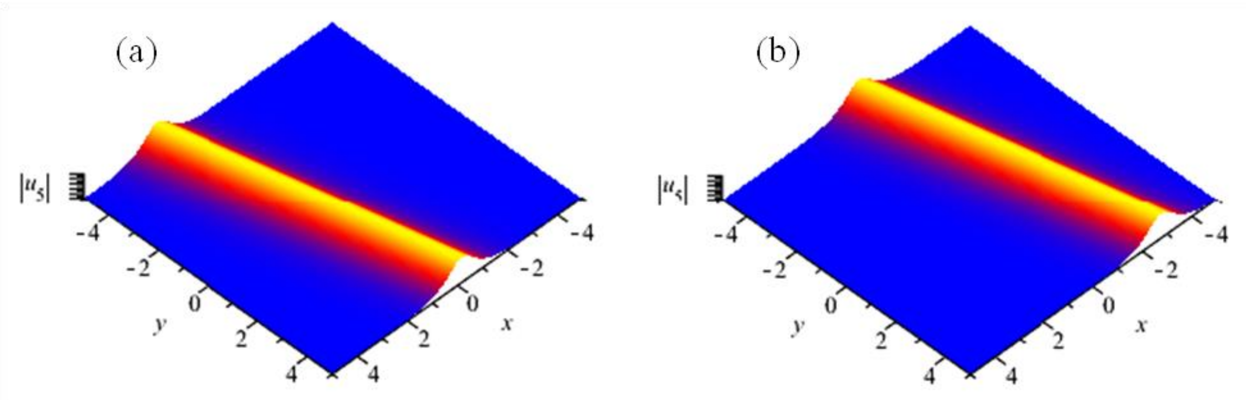

Figure 3. The graphical representations of $|u_5(x, y, t)|$ for $A = 1$, $a_2 = 0.5$, $\alpha_2 = 1$, and (a) $t = -1.5$ (b) $t = 1.5$.

4. Conclusions

The present paper studied comprehensively the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation describing the propagation of electromagnetic fields in self-focusing and normally dispersive planar wave guides in optics. The intended purpose was accomplished by reducing the 2D-HNLS equation to a one-dimensional nonlinear ordinary differential equation in the real regime using a traveling wave transformation and solving the resulting 1D-NLOD equation through the exponential and Kudryashov methods. As a result, several new optical solitons to the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation were formally obtained that are categorized as bright and dark solitons. Several graphical representations regarding the bright and dark solitons were represented to clarify the dynamics of the obtained solutions.

Conflict of interest

The authors declare no conflict of interest.

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