Alexandria Engineering Journal (2021) 60, 1055–1065

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Alexandria Engineering Journal

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Traveling wave with beta derivative spatial-temporal evolution for describing the nonlinear directional couplers with metamaterials via two distinct methods

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Received 15 December 2019; revised 30 August 2020; accepted 7 October 2020
Available online 9 November 2020

KEYWORDS

Fractional Beta derivative evolution;
Nonlinear directional couplers with metamaterials;
Traveling wave solutions;
The auxiliary ordinary differential equation method;
The generalized Riccati method

Abstract This work is reported the analytical solutions for describing the nonlinear directional couplers with metamaterials by including spatial–temporal fractional beta derivative evolution. The auxiliary ordinary differential equation method and the generalized Riccati method with the properties of beta derivative are implemented to secure such solutions. The solutions are obtained in the new forms by involving of some useful mathematical functions. The constraint conditions among the traveling wave parameters are evaluated. Some of the obtained solutions are presented graphically to illustrate the effectiveness of beta derivative parameter and mathematical techniques. It is investigated that the amplitudes of soliton are increased with the increase of fractional beta derivative parameter. The obtained results would be very useful to examine and understand the physical issues in nonlinear optics, especially in twin-core couplers with optical metamaterials.

1. Introduction

It is well confirmed that optical nonlinear couplers are widely applicable devices that distribute light from a main fiber into one or more branch fibers. It may be used to multiplex two incoming bit streams onto a fiber and also to demultiplex a single-bit stream. Basically, such couplers can be used by making planar devices with the help of semiconductor material or as dual-core, single-mode fibers with solitons propagating in each core. Besides, nonlinear directional couplers (NLDCs) are attracted for a lot of successful applications in developing optical gates and signal processing units in optical networks. Indeed, the pragmatic design and fabrication of optical metamaterials (OMMs), so called the photonic metamaterials are produced from the synthetic materials. NLDCs along with

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☆ Peer review under responsibility of Faculty of Engineering, Alexandria University.

https://doi.org/10.1016/j.aej.2020.10.030
1110-0168

the above metamaterials are generated a new dimension in research because of their innovative and exhilarating prospects for practical industrial applications, especially in nonlinear optics. For instance, the exchange of energy is possible between the pulses propagating in two orthogonal modes when a soliton pulse is transmitted via a birefringent fiber. Two orthogonally polarized solitons may be trapped each other and propagate at a same group velocity. Such phenomenon is so called the soliton trapping, which is very essential for describing optical soliton switching. Such type of switching in fiber depends on the intensity and power as well as phase of the pulse. Due to above facts, the optical solitons are generated on the basic factors across the trans-continental and trans-oceanic distances in fiber optic communication [1–12]. In most of the studies, the importance of traveling wave propagation are already focused in describing various types pattern formation in crystal fibers, optical switching, optical metamaterials and metasurfaces, DWDM systems, magneto-optic wave guides, cascaded systems, etc. To do so, several kinds of theoretical and computational techniques have been explored to secure optical solitons by considering only the conserved as well as local physical models [14–19]. For instance, Ekici et. al. [20] have been studied the dynamics of optical solitons in birefringent fibers with Kerr law nonlinearity. Li et al. [21] have been investigated the oscillating collisions between the three solitons for a dual-mode fiber coupler system. Mirzazadeh et al. [22] and Banaza et al. [23] have been explained soliton solutions in optical couplers by implementing the $(G\prime/G)$-expansion scheme and the ansatz approach, respectively. Xiang et al. [24] have been explained controllable Raman soliton self-frequency shift in nonlinear MMs. Additionally, in Refs. [25–33] have been focused various kinds of bright, dark and singular soliton solutions in describing the OMMs. Very recently, the study of optical solitons in NLDC with OMMs have also been attracted much attention to many scholars where the conserved as well as locality arise in the physical model. Arnous et al. [34], Arsheda et al. [35] and Vega-Guzman et al. [36] have been studied such physical phenomena with the aid of trial function method, $\exp(-\Phi(\xi))$ expansion [37] and the principle of undetermined co-efficient, respectively. However, no work has been conveyed to understand the behaviors of coherent structures when the non-local and non-conservative physical issues arise in such models, because the classical models are not convenient to divulge the effect of physical issues due to a long time arise in the dynamical systems [38]. At these stages, the fractional derivatives (e.g. conformable derivative, Caputo derivative, Riemann–Liouville derivative, Hadamard derivative etc.) are an arena to overcome the difficulties arising in such models. But, most of these derivatives are not satisfied some fundamental theorem of calculus. To overcome this limitation of the aforementioned fractional derivatives, the newly derivative, so called the ‘‘beta-derivative (BD)” is very recently introduced in calculus by Atangana et al. [39], which is satisfied the entire fundamental properties of calculus. Such derivative can not only be employed as fractional derivative but also as a natural extension of the classical derivative. Hence, there are still now opportunities to further study on the nonlinear physical scenarios for NLDCs with OMMs by including the beta derivative evolution (BDE). Hence, this research work is first time reported the exact analytical solutions of Twin-Core Couplers (TCCs) having Kerr law nonlinearity with OMMs and BDE.

2. Twin-core couplers with beta derivative evolution

The twin-Core Couplers equations (TCCs) with spatial–temporal BD evolution can be written in the following form:

$$\begin{aligned}{}_0^{\ A}D_t^{\beta}Q+\alpha_1\ {}_0^{\ A}D_{xx}^{\beta}Q+F\left(|Q|^2\right)Q\\ =l_1\ {}_0^{\ A}D_{xx}^{\beta}\left(|Q|^2\right)Q+m_1\left(|Q|^2\right)\ {}_0^{\ A}D_{xx}^{\beta}Q\\ +g_1Q^2\ {}_0^{\ A}D_{xx}^{\beta}Q^*+k_1R\end{aligned}\tag{1}$$

$$\begin{aligned}{}_0^{\ A}D_t^{\beta}R+\alpha_2\ {}_0^{\ A}D_{xx}^{\beta}R+F\left(|R|^2\right)R\\ =l_2\ {}_0^{\ A}D_{xx}^{\beta}\left(|R|^2\right)R+m_2\left(|R|^2\right)\ {}_0^{\ A}D_{xx}^{\beta}R\\ +g_2R^2\ {}_0^{\ A}D_{xx}^{\beta}R^*+k_2Q\end{aligned}\tag{2}$$

where $0<\beta\leqslant 1$. Here, $Q(x,t)$ and $R(x,t)$ are represented the complex valued functions describing the optical issues in two core, respectively. ${}_0^{\ A}D_t^{\beta}$ and ${}_0^{\ A}D_x^{\beta}$ are denoted the beta derivative with regards to time and space, respectively. The constant coefficients of dispersion terms, that is, α_1,α_2,g_1 and g_2 are obtained by normalizing via the physical parameters related to TCCs. The constants l_1,l_2,m_1 and m_2 are obtained due to the trapping in the phase hole. The other constant k_1 and k_2 are denoted the coupling coefficient of TCCs. It is noted that the Eqs. (1) and (2) can be reduced to integer order derivative evolution by setting $\beta=1$, which is good agreement with the earlier studies. The useful definition of beta derivative has been defined in Ref. [39] as

$${}_0^{\ A}D_\chi^{\beta}F(\chi)=\lim_{\epsilon\to 0}\frac{F\left(\chi+\epsilon\left(\chi+\frac{1}{\Gamma(\beta)}\right)^{1-\beta}\right)-F(\chi)}{\epsilon}\tag{3}$$

Based on the definition, the derivative properties are obtained as follows:

$$(i).{}_0^{\ A}D_\chi^{\beta}\{mF(\chi)+nG(\chi)\}=m_0^AD_\chi^{\beta}\{F(\chi)\}+n_0^AD_\chi^{\beta}\{G(\chi)\}\tag{4}$$

$$(ii).{}_0^{\ A}D_\chi^{\beta}\{\mu\}=0\tag{5}$$

$$\begin{aligned}(iii).{}_0^{\ A}D_\chi^{\beta}\{F(\chi).G(\chi)\}=F(\chi)_0^AD_\chi^{\beta}\{G(\chi)\}\\ +G(\chi)_0^AD_\chi^{\beta}\{F(\chi)\}\end{aligned}\tag{6}$$

$$(iv).{}_0^{\ A}D_\chi^{\beta}\{F(\chi)/G(\chi)\}=\frac{G(\chi)_0^AD_\chi^{\beta}\{F(\chi)\}-F(\chi)_0^AD_\chi^{\beta}\{G(\chi)\}}{G^2(\chi)}\tag{7}$$

where, $m,n,\mu\in\Re$. $G\neq 0$ and F are two functions β-differentiable with $\beta\in(0,1]$. By introducing $\epsilon=\left(\chi+\frac{1}{\Gamma(\beta)}\right)^{\beta-1}h$, when $\epsilon\to 0, h\to 0$ in Eq. (3), one can derive another useful property as

$${}_0^{\ A}D_\chi^{\beta}F(\chi)=\left(\chi+\frac{1}{\Gamma(\beta)}\right)^{1-\beta}\frac{dF(\chi)}{d\chi}\tag{8}$$

To convert Eq. (1) and (2) into nonlinear ODE, one can assume the following useful traveling wave transform:

$$Q(x,t)=G_1(\eta)e^{i\mu(x,t)},\tag{9}$$

$$R(x,t)=G_2(\eta)e^{i\mu(x,t)},\tag{10}$$

where,

$$\eta = \frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^{\beta}, \tag{11}$$

and

$$\mu(x,t) = -\frac{\kappa}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0, \tag{12}$$

Here, $G_p(\eta), p = 1,2$ represents the amplitude component of the wave, c represents its velocity and l is any constant, while $\mu(x,t)$ is the phase component,e κ is the frequency, ω is the wave number and θ_0 is the phase constant, respectively. It is noted that the phases of TCCs are identical, which is so called as the phase-matching condition. For the integrability aspects of TCCs, such phase matching condition is very useful to divulge soliton solutions. Now, substituting Eq. (9) and (10) into Eq. (1) and (2), the imaginary part is obtained as

$$(c - 2\alpha_p\kappa l)G\prime_p + 2\kappa l(3l_p + m_p - g_p)G_p^2 G\prime_p = 0, p = 1,2 \tag{13}$$

Setting the coefficients of two linearly independent functions to zero, yields

$$c = 2\alpha_p\kappa l, \tag{14}$$

$$(3l_p + m_p - g_p) = 0. \tag{15}$$

Equating the two values of the soliton speed, one obtains

$$\alpha_1 = \alpha_2 = \alpha. \tag{16}$$

Eq. (14) can therefore be written as

$$c = 2\alpha\kappa l. \tag{17}$$

Besides, the real parts of the considered equations are obtained as

$$\alpha_p l^2 G_p'' - (\alpha_p\kappa^2 + \omega)G_p + F\left(G_p^2\right)G_p + (l_p + m_p + g_p)\kappa^2 G_p^3 - 6l_p l^2 G_p (G\prime_p)^2$$
$$-l^2(3l_p + m_p + g_p)G_p^2 G_p'' - k_p G_{p^*} = 0, \tag{18}$$

where, $\alpha_1 = \alpha_2 = \alpha, p = 1,2$ and $p^* = 3 - p$. According to balancing principle gives

$$G_p = G_{p^*} \tag{19}$$

Eq. (18) is therefore transformed to

$$\alpha l^2 G_p'' - (\alpha\kappa^2 + \omega + k_p)G_p + F\left(G_p^2\right)G_p + 2(g_p - l_p)\kappa^2 G_p^3$$
$$- 6l_p l^2 G_p (G\prime_p)^2 - 2l^2 g_p G_p^2 G_P'' = 0, \tag{20}$$

For the twin-core couplers with Kerr law nonlinearity, that is $F(h) = sh$, Eqs. (1) and (2) can be reduced to

$$ {}_0^A D_t^{\beta} Q + \alpha_1\, {}_0^A D_{xx}^{\beta} Q + s_1|Q|^2 Q$$
$$= l_1\, {}_0^A D_{xx}^{\beta}\left(|Q|^2\right)Q + m_1\left(|Q|^2\right){}_0^A D_{xx}^{\beta} Q$$
$$+ g_1 Q^2\, {}_0^A D_{xx}^{\beta} Q^* + k_1 R \tag{21}$$

$$ {}_0^A D_t^{\beta} R + \alpha_1\, {}_0^A D_{xx}^{\beta} R + s_2|R|^2 R$$
$$= l_1\, {}_0^A D_{xx}^{\beta}\left(|R|^2\right)R + m_1\left(|R|^2\right){}_0^A D_{xx}^{\beta} R$$
$$+ g_1 R^2\, {}_0^A D_{xx}^{\beta} R^* + k_2 Q \tag{22}$$

Based on the above two equations, Eq. (20) is converted to

$$\alpha l^2 G_p'' - (\alpha\kappa^2 + \omega + k_p)G_p + \left[s_p + 2(g_p - l_p)\kappa^2\right]G_p^3$$
$$- 6l_p l^2 G_p (G\prime_p)^2 - 2l^2 g_p G_p^2 G_P'' = 0, \tag{23}$$

To introduce the following transformation for obtaining the traveling wave solutions,

$$l_p = g_p = 0, \tag{24}$$

Eq. (23) can be rewritten as

$$\alpha l^2 G_p'' - (\alpha\kappa^2 + \omega + k_p)G_p + s_p G_p^3 = 0. \tag{25}$$

3. Traveling wave solutions of TCCs with beta derivative evolution via AODEM

First of all, the description of the AODEM is ignored for simplicity. Because the detailed description of this method is given in Ref. [40]. According to the AODEM, the analytical solutions can be written in the following form:

$$G_1(\eta) = \sum_{n=0}^{1} a_n F^n(\eta), \tag{26}$$

and

$$G_2(\eta) = \sum_{n=0}^{1} u_n F^n(\eta), \tag{27}$$

where, $a_n, u_n (n = 0,1)$ are real constants with $a_1 \neq 0$, and $u_1 \neq 0$ and $F(\eta)$ satisfies the following auxiliary ODE:

$$F\prime(\eta) = \sqrt{aF^2(\eta) + bF^4(\eta) + cF^6(\eta)}. \tag{28}$$

It is noted that Eq. (28) are provided several types of general solutions by demanding on the real parametric values of a, b and c, which are given in Ref. [34]. With the aid of Eqs. (26), (27) along with Eq. (28), one can form a polynomial in $F(\eta)$ from Eq. (25). Equating the coefficients of this polynomial, one can obtain the systems of algebraic equations from Eq. (25) as

For $p = 1$,

$$3\alpha l^2 a_1 c = 0,$$
$$2\alpha b l^2 a_1 + s a_1^3 = 0,$$
$$3 s a_0 a_1^2 = 0,$$
$$-(\alpha\kappa^2 + \omega + k_1)a_1 + a\alpha l^2 a_1 + 3 s a_0^2 a_1 = 0,$$
$$-(\alpha\kappa^2 + \omega + k_1)a_0 + s a_0^3 = 0.$$

For $p = 2$,

$$3\alpha l^2 u_1 c = 0,$$
$$2\alpha b l^2 u_1 + s u_1^3 = 0,$$
$$3 s u_0 u_1^2 = 0,$$
$$-(\alpha\kappa^2 + \omega + k_2)u_1 + a\alpha l^2 u_1 + 3 s u_0^2 u_1 = 0,$$
$$-(\alpha\kappa^2 + \omega + k_2)u_0 + s u_0^3 = 0.$$

By simplifying the above algebraic equations, one obtains

For $p = 1$,

$$a_0 = 0, a_1 = \sqrt{-\frac{2\alpha b l^2}{s}}, \omega = a\alpha l^2 - \alpha\kappa^2 - k_1, a = a, b = b, c = 0 \tag{29}$$

For $p = 2$,

$$u_0 = 0, u_1 = \sqrt{-\frac{2\alpha bl^2}{s}}, \omega = a\alpha l^2 - \alpha\kappa^2 - k_2, a = a, b = b, c = 0 \tag{30}$$

Based on the Eqs. (9), (10), (26), (27) and the solutions of Eq. (28), the following exact traveling wave solutions for understanding the nonlinear behavior of Q and R with BDE are obtained:

When $a > 0$,

$$Q_1(x,t) = \sqrt{\frac{2a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_1}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \operatorname{sec}h\left\{\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}, \tag{31}$$

$$R_1(x,t) = \sqrt{\frac{2a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_2}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \operatorname{sec}h\left\{\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}, \tag{32}$$

$$Q_2(x,t) = \sqrt{-\frac{2a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_1}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \operatorname{csc}h\left\{\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}, \tag{33}$$

$$R_2(x,t) = \sqrt{-\frac{2a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_2}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \operatorname{csc}h\left\{\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}, \tag{34}$$

$$Q_3(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_1}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\cosh\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{35}$$

$$R_3(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_2}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\cosh\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{36}$$

$$Q_4(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_1}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\pm\sinh\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{37}$$

$$R_4(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_2}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\pm\sinh\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{38}$$

$$Q_5(x,t) = 4\sqrt{-\frac{2a\alpha bl^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_1}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{\exp\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}}{\left(\exp\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-4b\right)^2}}, \tag{39}$$

$$R_5(x,t) = 4\sqrt{-\frac{2a\alpha bl^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_2}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{\exp\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}}{\left(\exp\left\{2\sqrt{a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-4b\right)^2}}, \tag{40}$$

When $a < 0$,

$$Q_6(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_1}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\cos\left\{2\sqrt{-a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{41}$$

$$R_6(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_2}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\cos\left\{2\sqrt{-a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{42}$$

$$Q_7(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_1}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\sin\left\{2\sqrt{-a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{43}$$

$$R_7(x,t) = 2\sqrt{-\frac{a\alpha l^2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{a\alpha l^2-\alpha\kappa^2-k_2}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \sqrt{\frac{1}{\sin\left\{2\sqrt{-a}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta\right]\right\}-1}}, \tag{44}$$

where, $c = 2\alpha\kappa l$. Here, the above traveling wave solutions are found by assuming $\varepsilon = 1$ from the solutions of Eq. (28) [see Ref. [40]]. One can easily achieve another set of solutions by considering $\varepsilon = -1$ in the solutions of Eq. (28), which are ignored for convenience. Some of the above analytical solutions are presented graphically (see Figs. 1–4) for illustrating the potentiality of the AODEM and fractional beta derivative parameter by considering the constant values of the remaining parameters.

4. Traveling wave solutions of TCCs with beta derivative evolution via GRM

This section start with the generalized Riccati method in a concise manner. According to the GRM, the analytical solutions can be written in the following form:

$$G_1(\eta) = a_0 + \sum_{n=1}^{N}\left(a_n F^n(\eta) + \frac{b_n}{F^n(\eta)}\right), \tag{45}$$

and

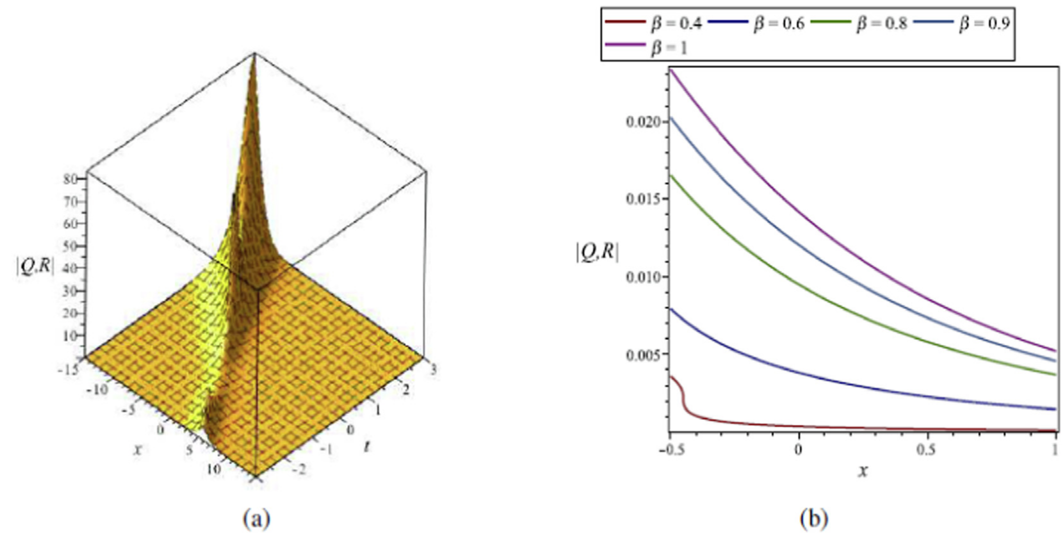


Fig. 1 Shape of $|Q,R|$ with regards to (a) x and t for $\beta = 0.9$ and (b) x and $t = 0.5$ for different values of β, respectively according to the analytical solutions as mentioned in Eqs. (18) and (18) of Eq. (1) and (2), respectively. The other parametric values are considered as $k_1 = k_2 = 0.5, s = 1, \kappa = 1, a = 1, l = 1$ and $b = 1$.

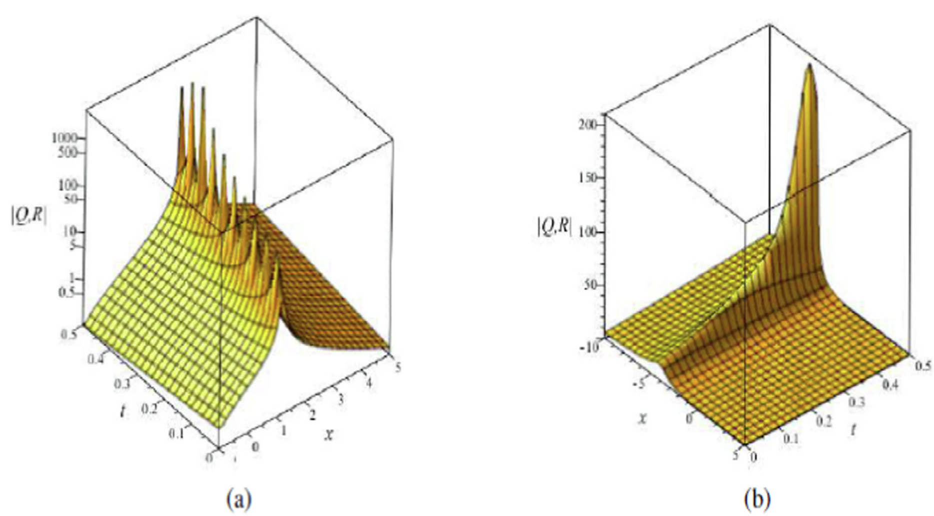


Fig. 2 Shape of $(a)|Q,R|$ with $k_1 = k_2 = 1, \alpha = -1, s = 1, \kappa = 1, a = 1, l = 1$ and $b = 1$ according to the analytical solutions as mentioned in Eqs. (18) and (36), and (b) $|Q,R|$ with $k_1 = k_2 = 1, s = 1$ $\kappa = 1, \alpha = 1, l = 1$ and $a = b = 1$ according to the analytical solutions as mentioned in Eqs. (18) and (38) of Eq. (1) and (2), respectively.

$$G_2(\eta) = u_0 + \sum_{n=1}^{N}\left(u_n F^n(\eta) + \frac{v_n}{F^n(\eta)}\right), \quad (46)$$

where, $a_0, a_n, b_n, u_0, u_n, v_n (n = 1, 2, 3 \ldots\ldots, N)$ are real constants with $a_N \neq 0$, or $b_N \neq 0$ and $u_N \neq 0$, or $v_N \neq 0$ to be determined later and $F(\eta)$ satisfies the following Riccati equation:

$$F\prime(\eta) = b + F^2(\eta). \quad (47)$$

It is noted that Eq. (47) are provided the following several types of general solutions by depending on the real parametric values of b.

If $b < 0$, then

$$(i).F(\eta) = -\sqrt{-b}\tanh\left(\sqrt{-b}\eta\right), \quad (48)$$

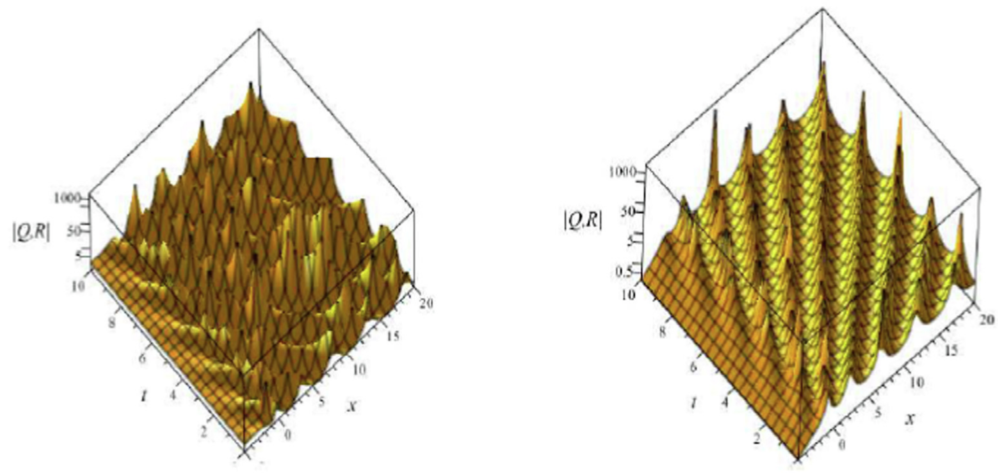
$$(ii).F(\eta) = -\sqrt{-b}\coth\left(\sqrt{-b}\eta\right), \quad (49)$$

If $b > 0$, then

$$(iii).F(\eta) = \sqrt{b}\tan\left(\sqrt{b}\eta\right), \quad (50)$$

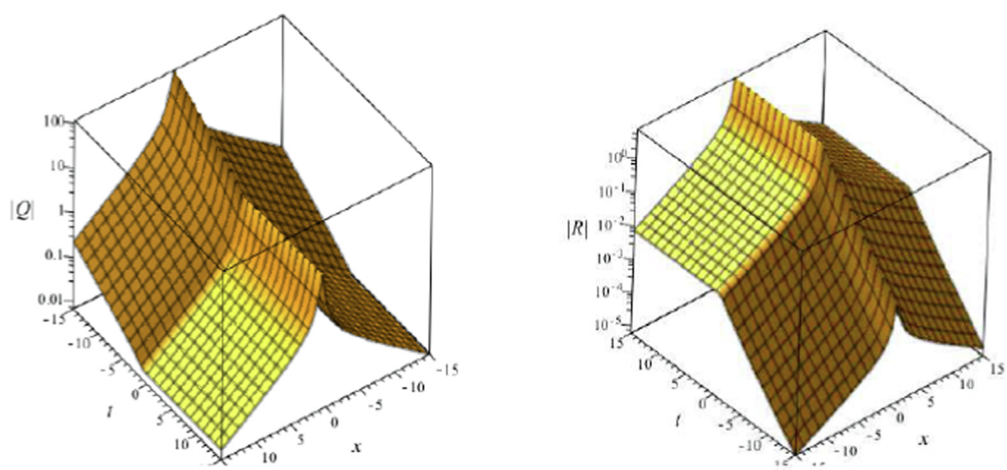
$$(iv).F(\eta) = -\sqrt{b}\cot\left(\sqrt{b}\eta\right), \quad (51)$$

If $b = 0$, then



(a) (b)

Fig. 3 Shape of $(a)|Q,R|$ with $k_1 = k_2 = 1, \alpha = 1, s = -1, \kappa = 1, a = -1, l = 1, \beta = 0.8$ and $b = 1$ according to the analytical solutions as mentioned in Eqs. (18) and (42), and (b) $|Q,R|$ with $k_1 = k_2 = 1, s = 1, \kappa = 1, \alpha = -1, \beta = 0.8, l = 1, a = -1$ and $b = 1$ according to the analytical solutions as mentioned in Eqs. (18) and (18) of Eq. (1) and (2), respectively.



(a) (b)

Fig. 4 Shape of (a) $|Q|$ and (b) $|R|$ with regards to x and t, respectively according to the analytical solutions as mentioned in Eqs. (18) and (18) of Eq. (1) and (2), respectively. The other parametric values are considered as $a = 0.5, b = 0.9, k_1 = 0.8, k_2 = -1.9, \beta = 0.85, s = 0.1, \kappa = 0.01, \alpha = 0.05$ and $l = 0.5$.

$$(v).F(\eta) = \frac{-1}{\eta}. \tag{52}$$

Aceording to the homogeneous balancing principle, one can obtain the value of N from Eq. (25) as $N = 1$. Therefore, the solutions of Eq. (15) in the following form:

$$G_1(\eta) = a_0 + a_1 F(\eta) + b_1 F^{-1}(\eta), \tag{53}$$

and

$$G_2(\eta) = u_0 + u_1 F(\eta) + v_1 F^{-1}(\eta), \tag{54}$$

With the assistance of Eqs. (53), (54) along with Eq. (47), one can get a polynomial in $F(\eta)$ from Eq. (25). Equating each coefficients of this polynomial to zero, one can obtain the systems of algebraic equations as

For $p = 1$,

$$2\alpha l^2 a_1 + s a_1^3 = 0,$$

$$3 s a_0 a_1^2 = 0,$$

$$2\alpha b l^2 a_1 - (\alpha\kappa^2 + \omega + k_1) a_1 + 3 s a_0^2 a_1 + 3 s a_1^2 b_1 = 0,$$

$$-(\alpha\kappa^2 + \omega + k_1) a_0 + s a_0^3 + 6 s a_0 a_1 b_1 = 0,$$

$$2\alpha b l^2 b_1 - (\alpha\kappa^2 + \omega + k_1) b_1 + 3 s a_0^2 b_1 + 3 s a_1 b_1^2 = 0,$$

$$3 s a_0 b_1^2 = 0,$$

$$2\alpha b^2 l^2 b_1 + s b_1^3 = 0.$$

For $p = 2$,

$$2\alpha l^2 u_1 + s u_1^3 = 0,$$
$$3 s u_0 u_1^2 = 0,$$
$$2\alpha b l^2 u_1 - (\alpha\kappa^2 + \omega + k_2)u_1 + 3 s u_0^2 u_1 + 3 s u_1^2 v_1 = 0,$$
$$-(\alpha\kappa^2 + \omega + k_2)u_0 + s u_0^3 + 6 s u_0 u_1 v_1 = 0,$$
$$2\alpha b l^2 v_1 - (\alpha\kappa^2 + \omega + k_2)v_1 + 3 s u_0^2 v_1 + 3 s u_1 v_1^2 = 0,$$
$$3 s u_0 v_1^2 = 0,$$
$$2\alpha b^2 l^2 v_1 + s v_1^3 = 0.$$

Simplifying the above algebraic equations, one obtains the following three cases:

Case-1:

For $p = 1$,

$$a_0 = a_1 = 0, b_1 = \pm\sqrt{-\frac{(\alpha\kappa^2+\omega+k_1)^2}{2\alpha s l^2}}, b = \frac{1}{2}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}. \tag{55}$$

For $p = 2$,

$$u_0 = u_1 = 0, v_1 = \pm\sqrt{-\frac{(\alpha\kappa^2+\omega+k_2)^2}{2\alpha s l^2}}, b = \frac{1}{2}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}. \tag{56}$$

Case-2:

For $p = 1$,

$$a_0 = 0, a_1 = \pm\sqrt{-\frac{2\alpha l^2}{s}}, b_1 = 0, b = \frac{1}{2}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}. \tag{57}$$

For $p = 2$,

$$u_0 = 0, u_1 = \pm\sqrt{-\frac{2\alpha l^2}{s}}, v_1 = 0, b = \frac{1}{2}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}. \tag{58}$$

For above both cases, the analytical solutions of Eqs. (1) and (2) are archived as below:

When $b < 0$,

$$Q_1(x,t) = \mp\sqrt{\frac{\alpha\kappa^2+\omega+k_1}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\coth\left(\sqrt{-\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{59}$$

$$R_1(x,t) = \mp\sqrt{\frac{\alpha\kappa^2+\omega+k_2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\coth\left(\sqrt{-\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{60}$$

$$Q_2(x,t) = \mp\sqrt{\frac{\alpha\kappa^2+\omega+k_1}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\tanh\left(\sqrt{-\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{61}$$

$$R_2(x,t) = \mp\sqrt{\frac{\alpha\kappa^2+\omega+k_2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\tanh\left(\sqrt{-\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{62}$$

When $b > 0$,

$$Q_3(x,t) = \pm\sqrt{-\frac{\alpha\kappa^2+\omega+k_1}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\cot\left(\sqrt{\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{63}$$

$$R_3(x,t) = \pm\sqrt{-\frac{\alpha\kappa^2+\omega+k_2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\cot\left(\sqrt{\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{64}$$

$$Q_4(x,t) = \mp\sqrt{-\frac{\alpha\kappa^2+\omega+k_1}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\tan\left(\sqrt{\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{65}$$

$$R_4(x,t) = \mp\sqrt{-\frac{\alpha\kappa^2+\omega+k_2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\tan\left(\sqrt{\frac{1}{2}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{66}$$

Case-3(i):

$$a_0 = 0, a_1 = \pm 2\alpha l\sqrt{\frac{-1}{2\alpha s}}, b_1 = \mp\frac{\alpha\kappa^2+\omega+k_1}{2l}\sqrt{\frac{-1}{8\alpha s}}, b = \frac{1}{8}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}$$
$$u_0 = 0, u_1 = \pm 2\alpha l\sqrt{\frac{-1}{2\alpha s}}, v_1 = \mp\frac{\alpha\kappa^2+\omega+k_2}{2l}\sqrt{\frac{-1}{8\alpha s}}, b = \frac{1}{8}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}$$
.

For $b < 0$, the analytical solutions of Eqs. (1) and (2) are archived as below:

$$Q_5(x,t) = \frac{1}{2}\sqrt{\frac{\alpha\kappa^2+\omega+k_1}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{-\tanh\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right) - \coth\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{67}$$

$$R_5(x,t) = \frac{1}{2}\sqrt{\frac{\alpha\kappa^2+\omega+k_2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{-\tanh\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right) - \coth\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{68}$$

$$Q_6(x,t) = \frac{1}{2}\sqrt{\frac{\alpha\kappa^2+\omega+k_1}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times\left\{\tanh\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right) + \coth\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{69}$$

$$R_6(x,t) = \frac{1}{2}\sqrt{\frac{\alpha\kappa^2+\omega+k_2}{s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \theta_0\right]\right) \times \left\{\tanh\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right) + \coth\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} + \frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\}, \tag{70}$$

where $K_1 = -(\alpha\kappa^2 + \omega + k_1)/8\alpha l^2$ and $K_2 = -(\alpha\kappa^2 + \omega + k_2)/8\alpha l^2$.

For $b > 0$, the analytical solutions of Eqs. (1) and (2) are archived as below:

$$Q_7(x,t)=\frac{1}{2}\sqrt{-\frac{\alpha\kappa^2+\omega+k_1}{s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{\tan\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.-\cot\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{71}$$

$$R_7(x,t)=\frac{1}{2}\sqrt{-\frac{\alpha\kappa^2+\omega+k_2}{s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{\tan\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.-\cot\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{72}$$

$$Q_8(x,t)=\frac{1}{2}\sqrt{-\frac{\alpha\kappa^2+\omega+k_1}{s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{-\tan\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.+\cot\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{73}$$

$$R_8(x,t)=\frac{1}{2}\sqrt{-\frac{\alpha\kappa^2+\omega+k_2}{s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{-\tan\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.+\cot\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{74}$$

where $K_3=(\alpha\kappa^2+\omega+k_1)/8\alpha l^2$ and $K_4=(\alpha\kappa^2+\omega+k_2)/8\alpha l^2$.

Case-3(ii):

$$a_0=0, a_1=\pm 2\alpha l\sqrt{\frac{-1}{2\alpha s}}, b_1=\mp\frac{\alpha\kappa^2+\omega+k_1}{2l}\sqrt{\frac{-1}{2\alpha s}}, b=-\frac{1}{4}\frac{\alpha\kappa^2+\omega+k_1}{\alpha l^2}$$
$$u_0=0, u_1=\pm 2\alpha l\sqrt{\frac{-1}{2\alpha s}}, v_1=\mp\frac{\alpha\kappa^2+\omega+k_2}{2l}\sqrt{\frac{-1}{2\alpha s}}, b=-\frac{1}{4}\frac{\alpha\kappa^2+\omega+k_2}{\alpha l^2}$$
.

Hence, for $b<0$, the analytical solutions of Eqs. (1) and (2) are archived as below:

$$Q_9(x,t)=\sqrt{-\frac{\alpha\kappa^2+\omega+k_1}{2s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{\tanh\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.-\coth\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{75}$$

$$R_9(x,t)=\sqrt{-\frac{\alpha\kappa^2+\omega+k_2}{2s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{\tanh\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.-\coth\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{76}$$

$$Q_{10}(x,t)=\sqrt{-\frac{\alpha\kappa^2+\omega+k_1}{2s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{\coth\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.-\tanh\left(\sqrt{K_1}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{77}$$

$$R_{10}(x,t)=\sqrt{-\frac{\alpha\kappa^2+\omega+k_2}{2s}}\times\exp\left(i\left[-\frac{\kappa}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{\omega}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\theta_0\right]\right)\times$$
$$\left\{\coth\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right.$$
$$\left.-\tanh\left(\sqrt{K_2}\left[\frac{l}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}+\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^{\beta}\right]\right)\right\},\tag{78}$$

where $K_1=(\alpha\kappa^2+\omega+k_1)/4\alpha l^2$ and $K_2=(\alpha\kappa^2+\omega+k_2)/4\alpha l^2$.

Also, for $b>0$, the analytical solutions of Eqs. (1) and (2) are archived as below:

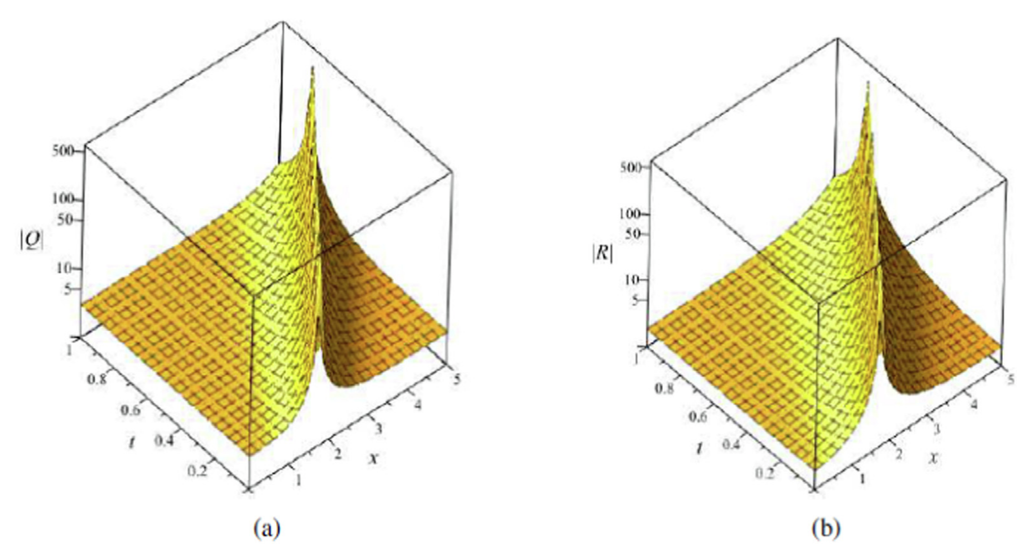


Fig. 5 Shape of (a) $|Q|$ and (b) $|R|$ with regards to x and t, respectively according to the analytical solutions as mentioned in Eqs. (18) and (18) of Eq. (1) and (2), respectively. The other parametric values are considered as $k_1=-0.000001, k_2=-5.5, \beta=0.5, s=1, \kappa=1, \alpha=-1, \omega=10$ and $l=1$.

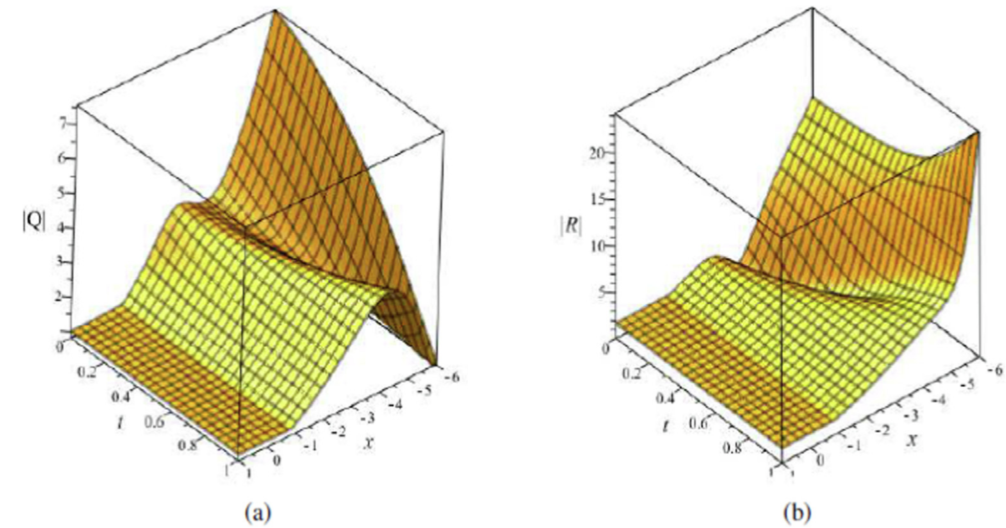


Fig. 6 Shape of $(a)|Q|$ and $(b)|R|$ with regards to x and t, respectively according to the analytical solutions as mentioned in Eqs. (18) and (18) of Eq. (1) and (2), respectively. The other parametric values are considered as $k_1 = -3, k_2 = -5, \beta = 0.85, s = 1, \kappa = 1, \alpha = 1$ and $l = 1$.

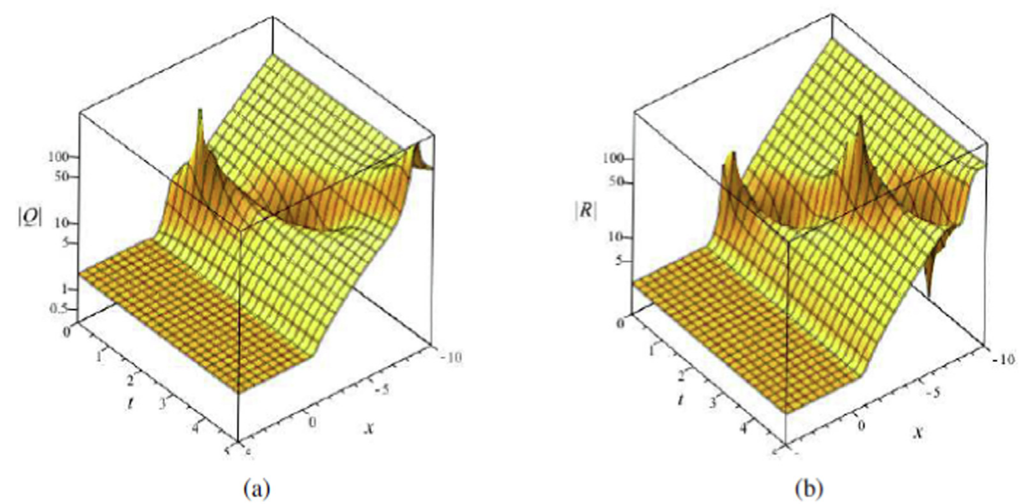


Fig. 7 Shape of $(a)|Q|$ and $(b)|R|$ with regards to x and t, respectively according to the analytical solutions as mentioned in Eqs. (18) and (18) of Eq. (1) and (2), respectively. The other parametric values are considered as $k_1 = -3, k_2 = -5, \beta = 0.8, s = 0.5, \kappa = 1, \alpha = 0.5$ and $l = 0.4$.

$$Q_{11}(x,t) = \sqrt{\frac{\alpha\kappa^2 + \omega + k_1}{2s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{\omega}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \left\{\tan\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right) + \cot\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right)\right\}, \tag{79}$$

$$R_{11}(x,t) = \sqrt{\frac{\alpha\kappa^2 + \omega + k_2}{2s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{\omega}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \left\{\tan\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right) + \cot\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right)\right\}, \tag{80}$$

$$Q_{12}(x,t) = \sqrt{\frac{\alpha\kappa^2 + \omega + k_1}{2s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{\omega}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \left\{-\tan\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right) - \cot\left(\sqrt{K_3}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right)\right\}, \tag{81}$$

$$R_{12}(x,t) = \sqrt{\frac{\alpha\kappa^2 + \omega + k_2}{2s}} \times \exp\left(i\left[-\frac{\kappa}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{\omega}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta + \theta_0\right]\right) \times \left\{-\tan\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right) - \cot\left(\sqrt{K_4}\left[\frac{l}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta + \frac{c}{\beta}\left(t + \frac{1}{\Gamma(\beta)}\right)^\beta\right]\right)\right\}, \tag{82}$$

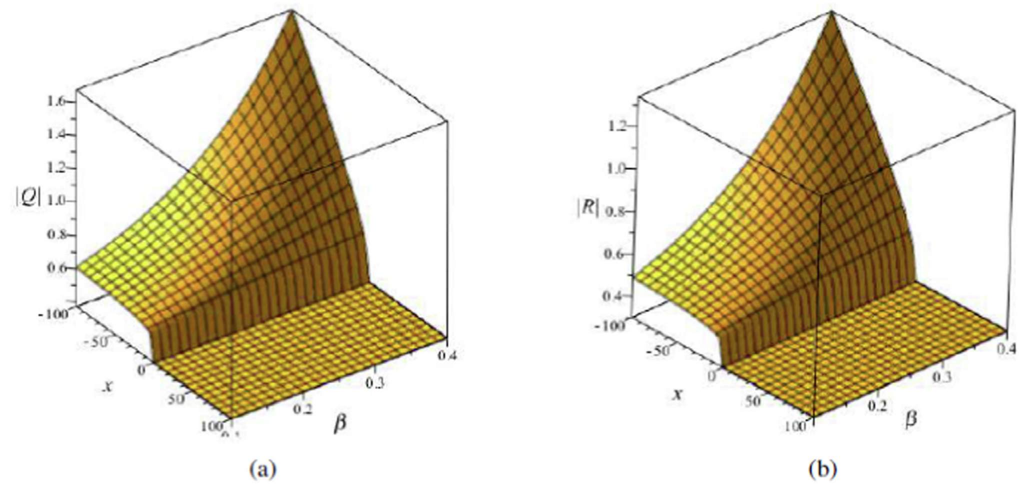


Fig. 8 Shape of $(a)|Q|$ and $(b)|R|$ with regards to x and t, respectively according to the analytical solutions as mentioned in Eqs. (18) and (18) of Eq. (1) and (2), respectively. The other parametric values are considered as $k_1 = -0.15, k_2 = -0.1, t = 10, s = 1, \kappa = 0.4, \alpha = 0.01, \omega = 0.01$ and $l = 0.4$.

where, $K_3 = -(\alpha\kappa^2 + \omega + k_1)/4\alpha l^2$ and $K_4 = -(\alpha\kappa^2 + \omega + k_2)/4\alpha l^2$. It is noted that some of the above analytical solutions are presented graphically (see Figs. 4–8) for illustrating the effectiveness of the GRM and fractional beta derivative parameter by considering the constant values of the remaining parameters.

5. Results and discussions

The dimensionless forms of the optical fields in the respective cores of the optical fibers, that is Q and R are determined in terms of various mathematical functions and BDE for understanding the physical issues in nonlinear optics, such as intensity-dependent switches and devices for separating a compressed soliton from its broad pedestal. It is noted that Eqs. (1) and (2) are not actually integrable due to the presence of the arbitrary functional form and the coupling terms. Hence, the traveling wave solutions are constructed by employing two very useful mathematical techniques with some assumptions, that is,$\alpha_1 = \alpha_2 = \alpha$ and Eq. (24). Some of the obtained outcomes are presented graphically in Figs. 1 to 8. It is seen form these figures that serval types of traveling waves, like soliton, periodic and kink shaped structures as well as rouge waves are obtained from the obtained analytical solutions of the considered equations. It is also found that the beta fractional parameter are significantly modified the wave dynamics when β lies between 0 to 1. On the other hand, Eq. (25) can reduce in the integral form by multiplying $G\prime$ as $(1/2)(G\prime)^2 + V(G) = 0$, where $V(G) = -((\alpha k^2 + \omega + k_p)/2\alpha l^2)G^2 + (s/4\alpha l^2)G^3$ $(s_p = s)$. By setting $V\prime(G) = 0$, one obtains $G = 0$ and $G = \pm\sqrt{(\alpha k^2 + \omega + k_p)/s}$. The existence condition of traveling wave solutions based on the considered equations is obtained as $V''(G) = 0|_{G=0} = -((\alpha k^2 + \omega + k_p)/\alpha l^2) < 0$. It is provided that traveling waves are strongly depended on the physical parameter related to TCCs and existed whenever the condition $(\alpha k^2 + \omega + k_p) < 0$ is satisfied. It is also predicted that one can be applied the obtained results in this manuscript for better understanding the physical phenomena of wave propagation at relatively high field intensities in two cores by determining energy $(E = \int_{-\infty}^{\infty}(|Q|^2 + |R|^2)d\eta)$, linear momentum, and the Hamiltonian etc.

6. Conclusion

In this paper, the twin-core NLDCs with OMMs with spatial–temporal BDE and kerr law nonlinearity have considered. The analytical solutions of this equations have constituted by implementing the auxiliary ordinary differential equation method and the generalized Riccati method. The obtained solutions have represented in terms of several kinds of mathematical functions with beta derivative and other related parameters. It is noted that the solutions of the considered equations are obtained in new forms because of BDE, which is indicated that no comparison are needed with the previous studies. This work is not only focused on BDE but also the effectiveness of applicable methods to obtain the traveling wave solutions of model equations. The outcomes of this study would be beneficial to understand not only the behaviors of wave propagation in nonlinear sciences, especially in nonlinear optics but also further studies in laboratory, where the considered model equation is applicable. It is also noted that one can employ the AODEM and GRM for finding the analytical solutions of any other nonlinear evolution equation with BDE, but beyond the scope of this study.

Declaration of Competing Interest

All authors declare no conflicts of interest in this paper.

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Further reading

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