



**A MILK RUN DISTRIBUTION SYSTEM DESIGN FOR HYBRID  
USAGE OF DRONES**

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A MILK RUN DISTRIBUTION SYSTEM DESIGN FOR HYBRID USAGE OF  
DRONES

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## **ABSTRACT**

### **A MILK RUN DISTRIBUTION SYSTEM DESIGN FOR HYBRID USAGE OF DRONES**

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The importance and unstoppable progress of the logistics industry in the world is increasing rapidly. The advancement of technology, the production of new vehicles, ideas and devices greatly affect the activities in the logistics sector. Drones are one of these vehicles and it seems that they are one of the main actors of the breakthrough developments in the logistics industry.

Milk run distribution is one of the most important and well-known logistic systems. There are studies in the literature that include delivery models with drones in the milk run distribution system.

The aim of this study is to reveal the benefit analysis of the proposed hybrid milk run model (HMR) against the classical milk run. While doing this study, a nonlinear mixed integer mathematical model was developed and converted to a quadratic model to reduce the difficulties of nonlinearity. Some of the problem-specific parameter values are taken from Özbilge (2021) but some of them are defined by us.

The proposed HMR model is run on a numerical setting consisting of two truck operating and fuel costs, two opportunity costs, and two operating and battery usage costs for drone parameters and 10 customer locations for each. While taking these runs, the GAMS optimization program was used within the scope of the study.

Our main finding is that delivery using drones with truck in hybrid milk run (HMR) distribution is 35.68% on average more beneficial than classical milk run (CMR) distribution in terms of costs in all problem setting.

**Keywords:** Logistics, Drone, Hybrid Milk Run Distribution, Mathematical Modeling, Nonlinear Mixed Integer Programming Model



## ÖZ

### İNSANSIZ HAVA ARAÇLARININ MELEZ KULLANIMI İÇİN BİR SÜREKLİ SEVKİYAT DAĞITIM SİSTEMİ TASARIMI

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Lojistik sektörünün dünyadaki önemi ve durdurulamaz gelişimi hızla artmaktadır. Teknolojinin ilerlemesi, yeni araçların, cihazların üretilmesi, yeni fikirlerin ortaya çıkması lojistik sektöründeki faaliyetleri büyük ölçüde etkilemektedir. İnsansız hava aracı (İHA) da bu araçlardan biri ve lojistik sektöründe çığır açan gelişmelerin baş aktörlerinden biri olacak gibi görünüyor.

Sürekli sevkiyat, en önemli ve en iyi bilinen lojistik sistemlerden biridir. Literatürde sürekli sevkiyat sisteminde İHA ile teslimat modellerini içeren çalışmalar bulunmaktadır.

Bu çalışmanın amacı, önerilen melez sürekli sevkiyat modelinin klasik sürekli sevkiyat modeline karşı fayda analizini ortaya çıkarmaktır. Bu çalışma yapılırken, doğrusal olmayan bir karışık tamsayı matematiksel modeli geliştirilmiş ve doğrusal olmamanın getirdiği zorlukları azaltmak için model, ikinci dereceden bir modele dönüştürülmüştür. Probleme özel parametre değerlerinin bir kısmı Özbilge (2021)'den alınmıştır, ancak bazıları tarafımızdan tanımlanmıştır.

Önerilen HMR modeli, iki kamyon işletim ve yakıt maliyeti, iki fırsat maliyeti ve iki İHA işletim ve pil kullanım maliyeti parametresinden ve her biri için 10 müşteri konumundan oluşan sayısal bir ayar üzerinde çalıştırılmıştır. Bu çalıştırmalar yapılırken çalışma kapsamında GAMS optimizasyon programı kullanılmıştır.

Bu alıřmadaki ana bulgumuz, melez srekli sevkiyat dađıtımında kamyonla beraber İHA kullanımının, tm parametre ayarlarında klasik srekli sevkiyat (CMR) dađıtımına gre maliyet bakımından ortalama 35.68% daha faydalı olmasındır.

**Anahtar Kelimeler:** Lojistik, İnsansız Hava Aracı (İHA), Melez Srekli Sevkiyat Dađıtımı, Matematiksel Modelleme, Dođrusal Olmayan Karıřık Tamsayı Programlama



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## NOMENCLATURE

Use	Synonym	Abbr.	Turkish	Synonym
Milk run			Sürekli Sevkiyat	Döngüsel Hareket
Vehicle	Truck	V	Araç	Kamyon
Drone	UAV (unmanned aerial vehicles)	D	İHA (İnsansız Hava Aracı)	
Item	Parts/materials	I	Ürün	
Location	Customer	L	Müşteri	
Depot	Receiving plant		Depo	
Arrival		arr	Varış	İniş
Departure		dep	Kalkış	
Limit			Limit	Üst sınır
Setup			Kurulum	
Classical Milk Run	CMR			
Drone Milk Run	DMR			
Hybrid Milk Run	HMR			
Launch	Dispatch, Take-off, Release, Send-off		Kalkış	
Land	Reconvene		İniş	
Departure			Çıkış	
Arrive			Varış	
Loaded			Yüklü	Dolu
Empty			Boş	

# CHAPTER 1

## INTRODUCTION

The world logistics market has an important place today and the market is expanding every year. It is around \$ 5.2 trillion (IMARC, 2020). The technology is the main factor at the concept of logistics. New developments in logistics are seen in the world when the technology is improved. Logistics never gets old, and it needs improvements continuously according to the conditions of today.

Distribution systems are composed of flow of goods and services from a source to the customer. The distribution system is the necessary part of the businesses to stay in competitive level. If a company's distribution system is on not adequate level, it may affect the company badly. The distribution systems may have many goals. Availability of product in the market, providing convenience to customers, reducing the cost, distribution of goods, maximizing sales, enhancing competitiveness, controlling inventory and promotion of products can be given as examples to the objectives of the distribution systems (Bhasin, 2020). There are different types of distribution systems today and it is vital to choose right system for business owner. State of the art technology for delivery is Yandex.Rover which is autonomous delivery robot. Yandex plans to use the technology to deliver food through Yandex.Eats, small packages to and from offices, and potentially products to individuals affected by COVID-19. In this example, small packages, types of foods and beverages are suitable to deliver for Yandex.Rover but when we think medical stuff or heavy packages, this system would not be right model for the business (Petrella et al., 2021).

In this thesis, we consider a system in which we aim to minimize the cost in an environment where customers and their demands are known, and deliveries might be done with truck and/or drones in a milk run distribution system.

Milk run is one of the oldest distribution systems, which is still one of the most important and well-known logistic systems. The name of this system comes from the system developed by the people who distribute milk in the 19th century at United States of America (Sadjadi et al., 2009). The concept of milk run logistics originates from the dairy industry. According to Kovács (2011), milk run method is a delivery system where vehicles are rotating based on a schedule defined before, stopping at every department to collect products that have been ordered. Kong, Jia, and Gan (2013) also defined the milk run system as milkman walks to the doors of the consumers' houses with vehicle in a defined route and delivers the milk bottles to consumers and after that takes back the empty bottles. Milk run is a delivery model that a truck exits from the distribution center, along the designed route, in the fixed time window to the specified vendors to pick up parts and then returns to the distribution center. The Figure 1 shows the milk run operations in the below.

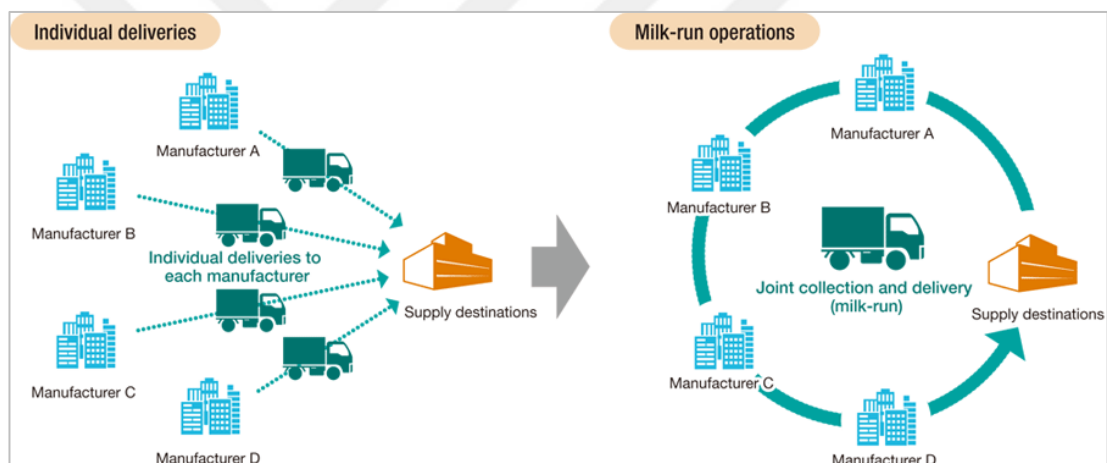


Figure 1. The Difference between Individual Deliveries and Milk Run Operations. (Source: Retrieved from <https://lojistikbilimi.com/tedarik-zincirinde-dagitim-turleri-nelerdir/>)

According to You and Jiao (2014), to implement the milk run concept in the express delivery industry results in improved load factors, more efficient use of time, shorter shipment distances, lower logistics transportation costs, and better transport demand fulfillment.

The advantages of milk run system can be listed as follows:

- Lowers inventories while increasing material transfer between facilities.
- Improves freight loading performance.
- More efficient use of vehicles (Kong, Jia, and Gan, 2013)

- Safety stock levels decrease, and control of stock levels increases.
- With the decrease in stock levels, the pallet and the areas used are also reduced.
- A more active supplier-customer communication is carried out.
- The estimates on delivery times are very close to reality.
- Different materials can be transported at the same time.
- Transportation costs are reduced thanks to consolidated transportation, which offsets small lot transportation.
- Synchronization improves the assembly line of manufacturer and increases the precision of JIT products delivery. Milk run may help strengthen logistics procurement processes by providing a consolidated collection of goods.
  - Increase the rate of vehicle loading and reduce the overall distance traveled. It can improve the teamwork of different suppliers and manufacturers, improve supply agility and flexibility, as well as the manufacturer's ability to respond and system performance.
  - It lowers the probability of poor product quality in the event of a crisis. To mitigate the effect on sales, manufacturers should easily identify and notify corresponding suppliers.
  - It alters logistics techniques, reducing in-process inventory, increasing capital flows, and lowering investment costs by using third-party logistics. (Brar and Saini, 2011)
  - The disadvantages of milk run system can be listed as follows:
  - It may not be an applicable method for every supplier in the logistics industry.
  - When routing errors are made or delays caused by suppliers, trips created to compensate will be costly.
  - Applicable for small size and less cost goods.
  - The roads and the decided route on milk run system determine the suppliers. This situation causes a loss of flexibility in supplier selection.

Drones are flying robots that can be remotely controlled or fly autonomously through software-controlled flight plans in their embedded systems, working together with sensors and GPS. Also, they are known as unmanned aerial vehicles (UAVs) (Earls, 2019). Using drones in distribution systems is getting more popular day by day. The aim is to transport packages with a higher speed but also less cost. The applications



of drone deliveries are seen mainly in healthcare delivery, food delivery, postal delivery, and ship resupply. The concept of drone delivery was introduced by Amazon founder Jeff Bezos in December 2013. He announced that Amazon was planning the delivery system which is composed of UAVs. Deutsche Post DHL which is a German logistics company started a project and they transported medicine with drone in 2013. In 2014, Google introduced the Wing drones which can deliver larger items than Amazon's Prime Air. Also, the United Arab Emirates explained their project to use drones in order to distribute official government documents in 2014 (Dorling et al., 2016). FedEx, Matternet, UPS and Alibaba also have shown interest and used drones in the areas which can be defined as last-mile delivery (Chung et al., 2020).

The drone deliveries are one of the results of these new concepts in logistics business. Jeff Bezos, who is Amazon's CEO, supports this idea and adds that drone deliveries will be ordinary in the near future (Kitjacharoenchai et al., 2019). Currently, trend of using drones in last mile delivery has increased (Amazon, UPS, Wing etc.). Drones can be an effective way of delivery if used with trucks. Some published articles support this idea (Wang and Sheu, 2019). This reason encourages our study, and this is one of our motivations.

As indicated before, the milk run logistics system is an effective way for processes of distribution and collection. Therefore, we choose to use this distribution method in our problem, and this is the second motivation for our study.

Finally, this thesis is also within the scope of Scientific Research Projects (BAP). Özbilge (2021) evaluated the classical milk run and only drone system separately within the scope of the Scientific Research Projects (BAP). We, on the other hand, continue this study with the hybrid milk run (HMR) model and investigate a benefit analysis of the use of HMR compared to the classical milk run.

Hybrid milk run (HMR) problem solution is expected to reduce the cost of delivering all packages (and returning back to the original location) compared to drone-only and truck-only milk run distributions of Özbilge (2021). We will analyze the benefits of using hybrid milk run structure.

The research questions we focus on in this study is given as follows:

- What are the benefits of using hybrid milk run (HMR) over classical milk run (CMR)?
- How do benefits vary with problem characteristics?

## **CHAPTER 2**

### **LITERATURE REVIEW**

Today, online purchases have become even more important with the impact of COVID-19. People now prioritize the preference of remote shopping. For the goods sold over the internet, companies have started to look for more effective and cheaper ways with the development of technology in the delivery process. Therefore, the concept of drone delivery has been one of the most discussed studies recently. The idea of preferring drones over any other vehicle is to be faster, to avoid possible traffic situations, to have the minimum possible waiting time, and to have the ability to move in all directions. In addition, the lack of a human pilot is also a factor. Based on these reasons, a significant reduction in cost is expected.

First, big companies such as Amazon, Google, DHL, and Walmart made this breakthrough by introducing drone delivery projects in their package delivery processes, and the idea of drone delivery gained popularity (Yoo et al., 2018).

Poikonen et al. (2017) develop a model with objective of minimum total time. In this model, there are trucks and drones. These drones are carried by trucks and both vehicles can deliver packages to the customers. When launching or landing operation of drone, truck must be stationary at customer locations or depot. When the drone completes its service, it returns back to the top of the truck. This study shows that using delivery drone in tandem and launched from the truck can reduce the total delivery time.

Moshref-Javadi et al. (2020) proposed an efficient truck and drone routing algorithm and model to solve real world problem. In this study, solution comparison of proposed model and algorithm with truck only model is made. The objective of the study is to reduce the waiting times of the customers in the system. The model is applied for e-commerce delivery in Sao Paulo and their numerical results show significant reductions in customer waiting time to be gained from the proposed multi-modal delivery model.

Agatz et al. (2018) proposes a system where the delivery truck and drone serve all customers together. The delivery truck makes delivery to the customer locations and drone serves to another customer points. So, their customer sets are different from each other. After the drone completes its delivery, it goes back to truck to take another package to serve. In this study, they introduce the traveling salesman problem with drones, the TSP-D as they call it. They model a new integer program formulation for this study. Besides, fast route-first and cluster second heuristics are developed based on local search and dynamic programming. They prove worst case approximation ratios for the heuristics and test their performance by comparing the solutions to the optimal solutions for small instances.

Ferrandez et al. (2016) investigate the effectiveness of implementing unmanned aerial delivery vehicles in delivery networks. In this study, their aim is to investigate the time and energy associated to a truck-drone delivery network compared to only truck or drone. Also, proposing an optimization algorithm that determines the optimal number of launch sites and locations and developing mathematical formulations for closed form estimations for the optimal number of launch locations and the optimal total time of delivery are the other objectives of them. They want to know how much time, money, and energy a hub configuration (star-distance) takes to compare it to truck-only delivery on a TSP route. To meet all deliveries, TSP truck route is computed via genetic algorithm. As a result of this study, multiple drones per truck are found more optimal and contribute to savings in both energy and time.

Pugliese et al. (2020) address the routing problem including fleet of trucks equipped with drones to perform deliveries in last mile delivery process. They formulate mixed integer linear program and develop heuristics to serve all customers at minimum cost, under time window, capacity, and flying endurance constraints. The computational results are carried out on instances generated by starting from vehicle routing problem with time windows benchmarks. The formulation they create can solve instances with up to 15 customers. The solutions of the MILP are used as benchmark to evaluate the effectiveness of the proposed heuristic procedure.

Wang et al. (2020) address the bi-objective problem which includes both truck and drone delivery within the system of TSP with drones (TSP-D). The objectives of the problem are introduced as combine of minimizing cost and minimizing total time. An improved non-dominated sorting genetic algorithm (INSGA-II) is proposed to

solve the problem. The proposed algorithm is validated as competitive performance according to the computational results.

Salama and Srinivas (2020) address delivering orders to a set of customer locations using several drones that operate in conjunction with a single truck. In this study, delivery missions are paralleled by simultaneous launch of drones from a truck parked at a focal point to nearby customer locations. Therefore, the key decisions to optimize are dividing the delivery locations into small clusters, setting one focal point per cluster, and routing the truck to all focal points such that customer orders in each cluster are fulfilled by a drone or truck. They present a mathematical programming models to jointly optimize all the decisions involved. While choosing a cluster focal point, they consider two policies: restricting it to one of the customer locations or allowing it to be anywhere in the delivery area. At the end of the research, it is found that the proposed joint optimization approach outperforms the sequential heuristic method for all the cases. Also, allowing the focal points to be anywhere in the delivery area provides better savings in terms of cost and time than restricting focal points to one of the customer locations.

Baloch & Gzara (2020) investigate the economic evaluation of drone delivery in terms of its impact on distribution network of an e-retailer. They consider customer preferences, regulatory and technological limitations, and decisions of location. They consider an e-retailer which offers several same day deliveries with drones. They develop distribution network design formulation to compete in industry. A nonlinear mathematical formulation is designed, and a novel logic-based Benders decomposition approach is developed to solve it. Also, numerical testing based on New York City is carried out and performed sensitivity analyses over delivery charge, delivery time, government regulations, technological limitations, customer behavior, and market size. The results of this study show that government regulations, technological limitations, and service charge decisions play a vital role in the future of UAV delivery.

In our study, we will use a similar mathematical programming model to Moshref-Javadi et.al. (2020) under the numerical setting of Özbilge (2021) and extend those two works by using both drones and the truck to model hybrid milk run (HMR) delivery to minimize total cost. Considering multiple drones, hybrid milk run distribution is a complicated problem. Considering multi-usage of drones on a tour, synchronization of truck and drone routes, etc. make the hybrid milk run (HMR)

problem further complicated. In this study, we will optimize the HMR problem under the numerical settings used by Özbilge (2021).

The comparison of our study which is hybrid milk run with Poikonen et al. (2017) and Moshref-Javadi et al. (2020) is given in the Table 1 below.

*Table 1. Comparison of Other Studies' Milk Run Distribution Structures with HMR*

<b>Structure of milk run distibution</b>	<b>Poikonen et al. (2017)</b>	<b>Moshref-Javadi et al. (2020)</b>	<b>Hybrid milk run</b>
Objective	<b>Minimize the completion time</b>	<b>Minimize the total customer waiting times</b>	<b>Minimize total cost</b>
Does truck make delivery?	<b>YES</b>	<b>YES</b>	<b>YES</b>
Is truck speed less than drone speed?	<b>N.C.</b>	<b>N.C.</b>	<b>YES</b>
Does truck stop at customer points?	<b>YES</b>	<b>YES</b>	<b>YES</b>
Does truck stop at specific points?	<b>YES</b>	<b>NO</b>	<b>NO</b>
Does drone make delivery?	<b>YES</b>	<b>YES</b>	<b>YES</b>
Does the truck have to stop at customer point or depot to launch the drone?	<b>YES</b>	<b>YES</b>	<b>NO</b>

(N.C. = Not Considered)

## **CHAPTER 3**

### **PROBLEM DEFINITION**

We explain problem environment in Section 3.1. Then we describe an abstraction of the problem in Section 3.2. In the abstraction of the problem section, we show the concept of the structure for both classical milk run, and hybrid milk run deliveries. After that, we give samples to two types of milk run systems we mentioned in the abstraction, show how we did their calculations in Section 3.3. In the calculation part, we explain the calculation methodology that we created using data such as the speed of the truck, the loaded and empty speeds of the drones, and the distances between locations. We compare the results we have obtained according to the calculation methodology we have described.

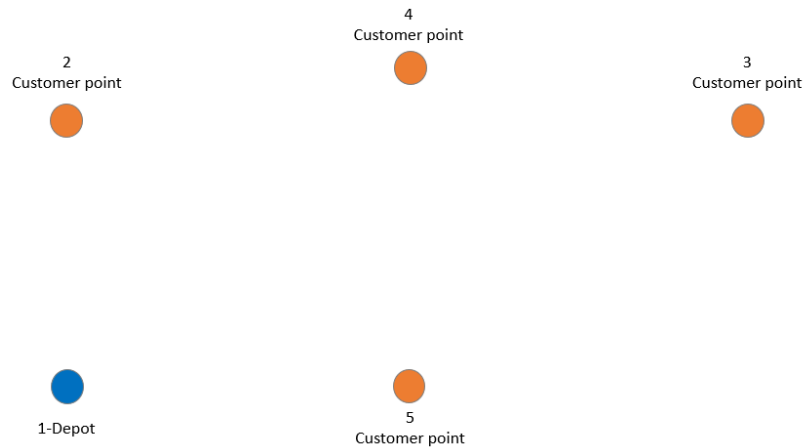
#### **3.1. PROBLEM ENVIRONMENT**

This problem has a theoretical setting environment including depot and customer locations. These customer locations have demands and these demands are known before the delivery operation starts. Delivery operations may include both truck's and drones' deliveries simultaneously. In this problem, the drones are kept in the truck. As the truck follows its route, it may launch the drone(s) to the customer locations where the drone(s) reach. The launch operation of drones can happen when the truck stops or while the truck is moving along the route. On the other hand, landing operations of drones are performed only when truck stops at customer or depot point. The capacity of a drone is assumed as one unit demand in this problem. Each customer location is served by truck or drone. It is not possible to send both truck and drone to same customer location. In this problem, multiple drones can be launched or landed simultaneously, and the speeds of empty and loaded drones are accepted as different. Also, the flight ranges of empty and loaded drones are different. When the drones

return to the truck after delivery, drones can be charged in the truck if necessary. After all demands are served, truck and drones return to depot together or separately.

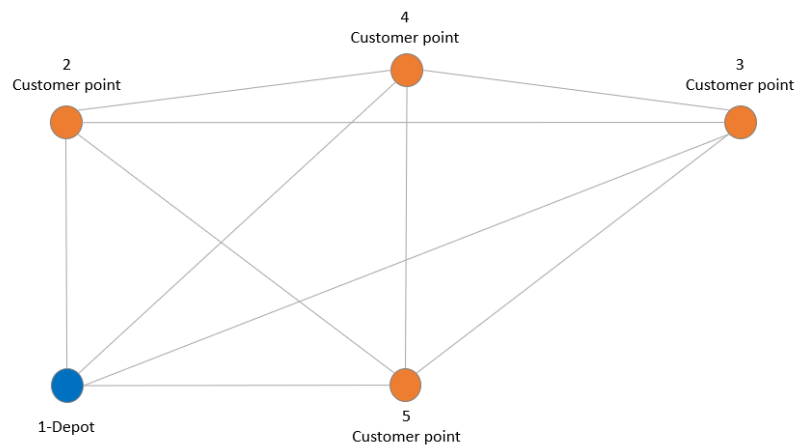
### 3.2. AN ABSTRACTION OF THE PROBLEM

Before we explain the hybrid milk run model illustration, we will show the classical milk run model with only truck. In this case, there are total of 5 nodes. Node 1 illustrates the depot, and the others represent the customer points. Location illustration of the nodes is shown in Figure 2.



*Figure 2. Location Illustration of the Nodes*

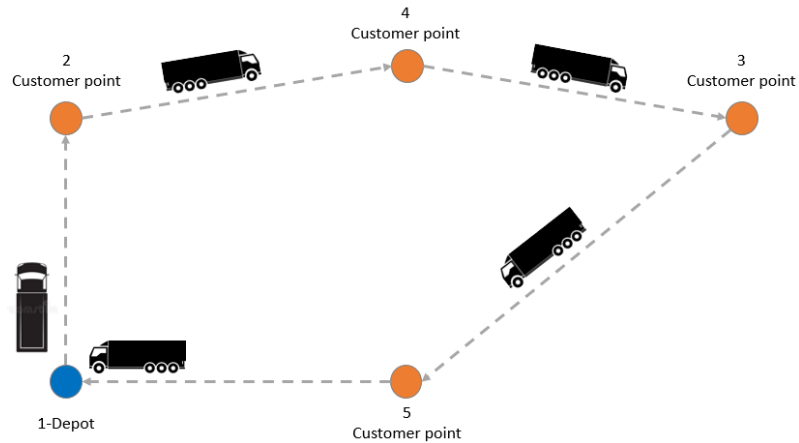
These nodes are complete graph for truck route. As shown in Figure 3, the connections between nodes, it is possible to reach all points with truck.



*Figure 3. Complete Graph of the Customer Points*

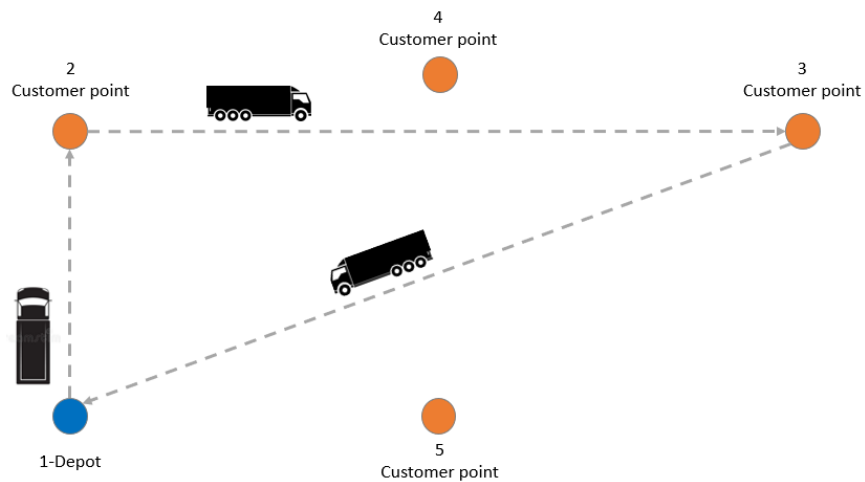
In the classical milk run, the truck starts its predetermined route from the depot, then delivers customer demands and returns to the depot. In a real delivery system, some of the problems that the truck may encounter are heavy traffic, bad weather, illness, etc. However, in the classical milk run model that we will illustrate below in Figure 4, it is assumed that the truck will deliver at an average speed in the city. To

summarize the Figure 4; after the truck picks up the packages to be delivered from the depot, it starts to move towards the customer located at the 2nd point on its specified route. After making its delivery, it proceeds to the customer at the 4th point and makes the delivery. It then proceeds to the customer at the 3rd point and delivers the customer's demand. The truck goes to point 5, the final customer point, and makes its final delivery. Having completed the entire delivery, the empty truck returns to the depot.



*Figure 4. Classical Milk Run Delivery Illustration*

Now we will explain the general features of the hybrid milk run model which we study in our thesis. As seen in the Figure 5, there are again 5 locations. This time truck goes to customer points of 2 and 3. The demands of the customer points 4 and 5 will be satisfied by drones.



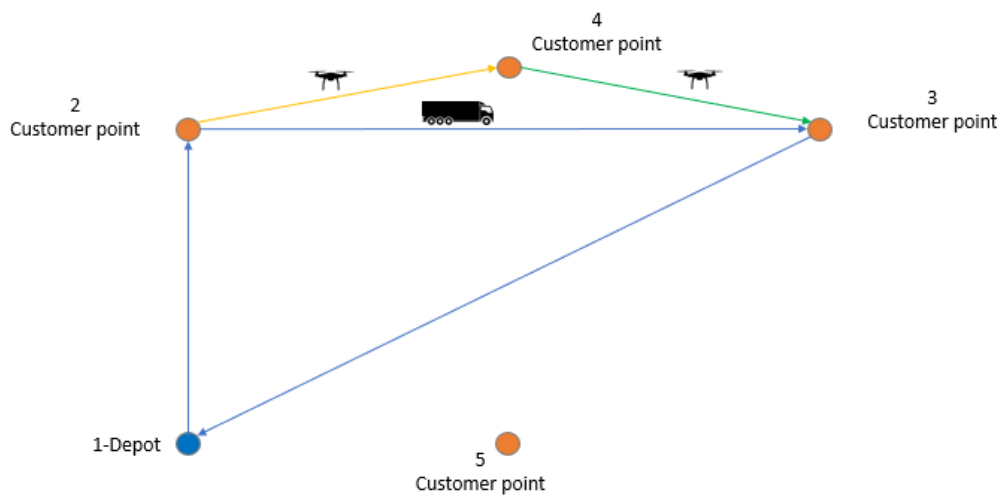
*Figure 5. An Illustration to the Delivery Route of Truck in HMR*

The truck exits from depot to start serving customer demands and goes to customer point 2. After that, it continues its route and goes to customer 3. After the service is done, truck returns to the depot. In this scenario, demands of customer points



4 and 5 are not satisfied. The Figure 5 represents only truck movement, but my study includes both drone and truck delivery at the same time.

In the hybrid milk run (HMR) model, while the truck follows the route and goes to the customer locations, the drones go to the locations where the truck does not go and serve these locations. Figure 6 shows the working principle of truck and drone together. After the truck delivers to the 2nd customer point, it continues its way without stopping by the 4th customer and reaches the 3rd customer point. When the truck is at the 2nd customer point, it launches the drone to serve the 4th customer point, because truck does not go to this point. After completing the delivery of the drone, it meets with truck at the 3rd customer point.



*Figure 6. An Illustration to the Drone Delivery of 4th Customer Location*

Figure 7 shows the continuation of the route. At the 3rd customer location, after the truck and drone meet, the truck continues its route and goes towards the depot. The drone is launched to serve 5th customer location as it waits for its demand to be met. It is an important point to launch the drone over the road while the truck goes to the depot. After the drone delivery is completed, the drone returns to the depot with truck.

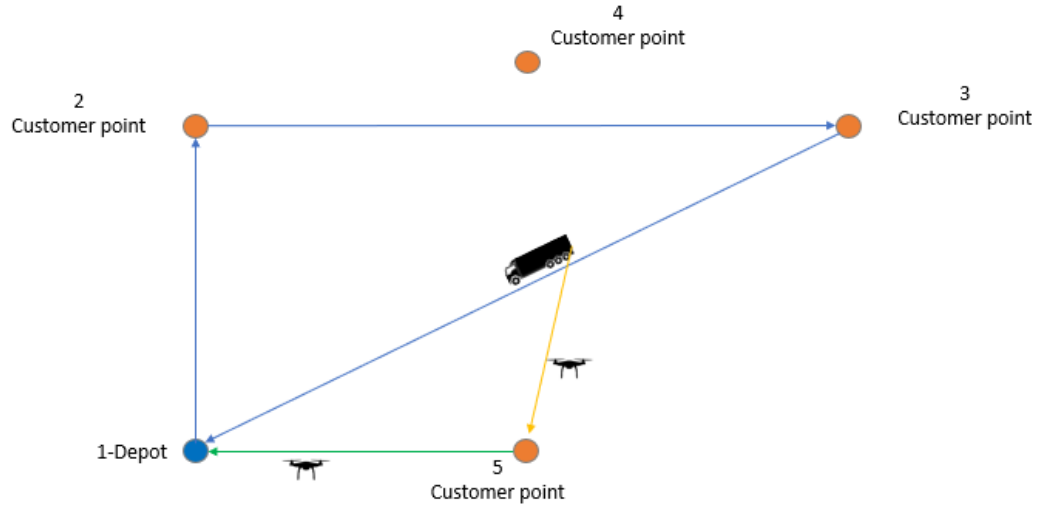









Figure 7. An Illustration to the Drone Delivery of 5th Customer Location

The figure representations of the model are given in Table 2. These figures we show will help us explain our model.

Table 2. Figure Representations of the Model

Figure Representation	Definition
	Depot
	Customer Points
	Truck
	Drone
	Truck Movement
	Loaded Drone Movement
	Empty Drone Movement

### 3.3. SAMPLE PROBLEM AND CALCULATION OF FLIGHT TIMES

In this section, we will show a sample problem based on the example we used in the abstraction of the problem. The sample problem will be applied to both classical milk run, and hybrid milk run models. First, we will show a sample problem for the

classical milk run and calculate the total delivery time. Next, we will show a sample for the hybrid milk run and calculate the total delivery time of this system for trucks and drones.

We will start with a sample of classical milk run problem. As you can see in Figure 8, the route followed by the truck is starting from the depot which is the 1st location, the 2nd customer location, the 4th customer location, the 3rd customer location, the 5th customer location, and the starting point is the depot. The coordinates of the locations are indicated in the Figure 9.

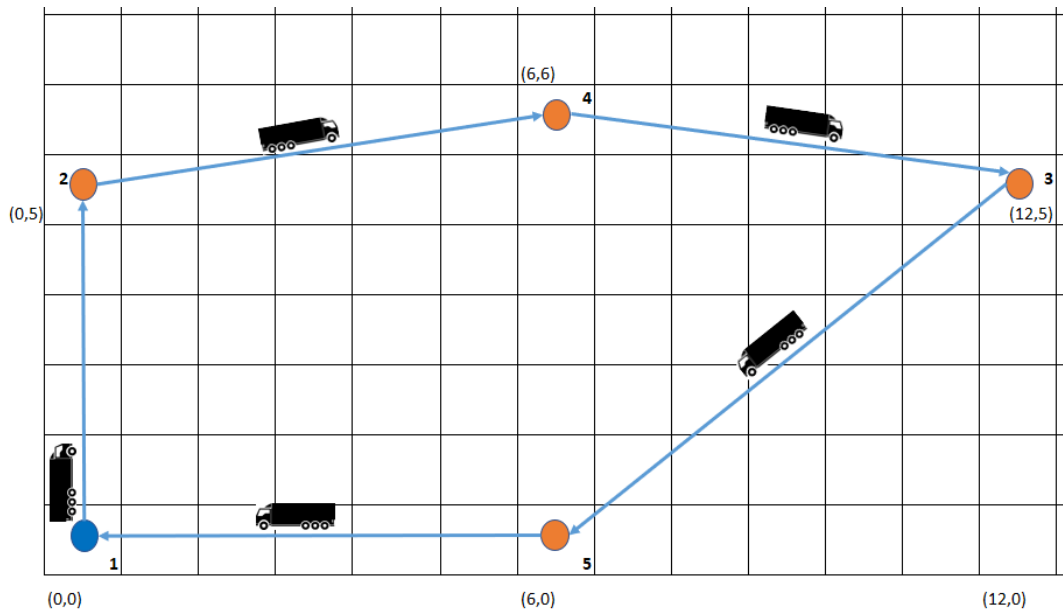


Figure 8. Sample Problem for Classical Milk Run Model

The distances between the locations are found with the Euclidean distance formula. In this example problem, the distances between locations are calculated as follows:

$$\text{Depot 1 - Customer 2: } \sqrt{(0 - 0)^2 + (0 - 5)^2} = 5$$

$$\text{Customer 2 - Customer 4: } \sqrt{(0 - 6)^2 + (5 - 6)^2} = 6.08$$

$$\text{Customer 4 - Customer 3: } \sqrt{(6 - 12)^2 + (6 - 5)^2} = 6.08$$

$$\text{Customer 3 - Customer 5: } \sqrt{(12 - 6)^2 + (5 - 0)^2} = 7.81$$

$$\text{Customer 5 - Depot 1: } \sqrt{(6 - 0)^2 + (0 - 0)^2} = 6$$

The speed of truck is accepted as 35 km/h. We will calculate the total delivery time of the truck based on this accepted speed and the distances we calculated above. These calculations were made according to the formula  $t = x/v$  (*time = distance/velocity*).

Depot 1 - Customer 2:  $(5/35).60 = 8.57$  minutes

Customer 2 - Customer 4:  $(6.08/35).60 = 10.42$  minutes

Customer 4 - Customer 3:  $(6.08/35).60 = 10.42$  minutes

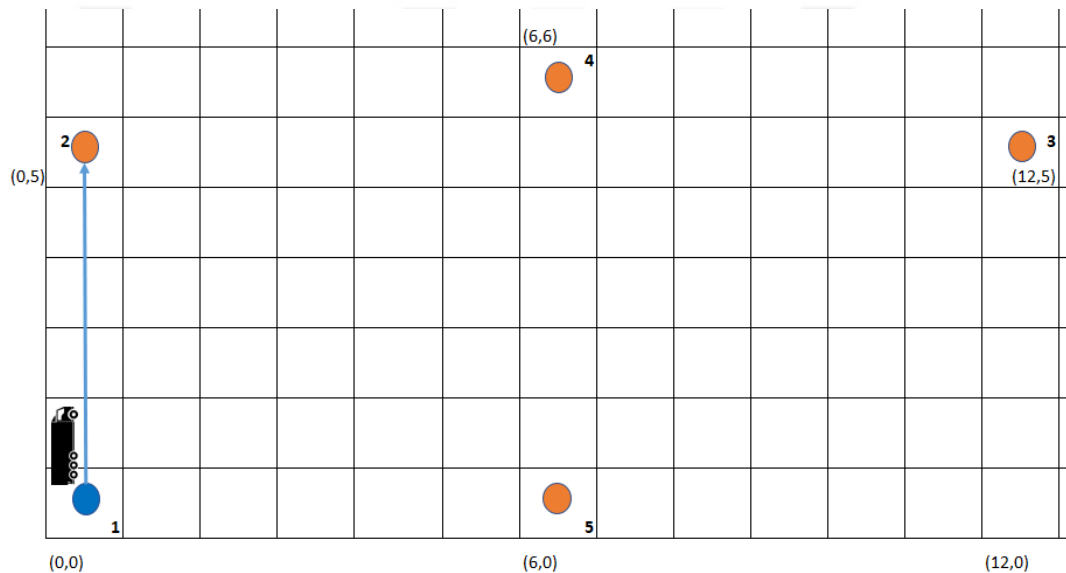
Customer 3 - Customer 5:  $(7.81/35).60 = 13.38$  minutes

Customer 5 – Depot 1:  $(6/35).60 = 10.28$  minutes

Total delivery time =  $8.57+10.42+10.42+13.38+10.28 = 53.07$  minutes

In the example we showed for the classical milk run, the total delivery time was found to be 53.07 minutes when customer locations were served only by truck.

We will now create a sample problem for the hybrid milk run model. As you can see in the Figure 9 below, the truck is on its way from the depot to the 2nd customer point with the drone inside.



*Figure 9. Sample Problem for Hybrid Milk Run Model (Step 1)*

As seen in the Figure 10 below, after the truck served the 2nd customer, the truck will continue its route with the 3rd customer. However, customer 4 also needs to be delivered, so the drone will serve customer 4. According to the calculations we will show below, the drone will be launched from the truck at customer point 2 to customer point 4.

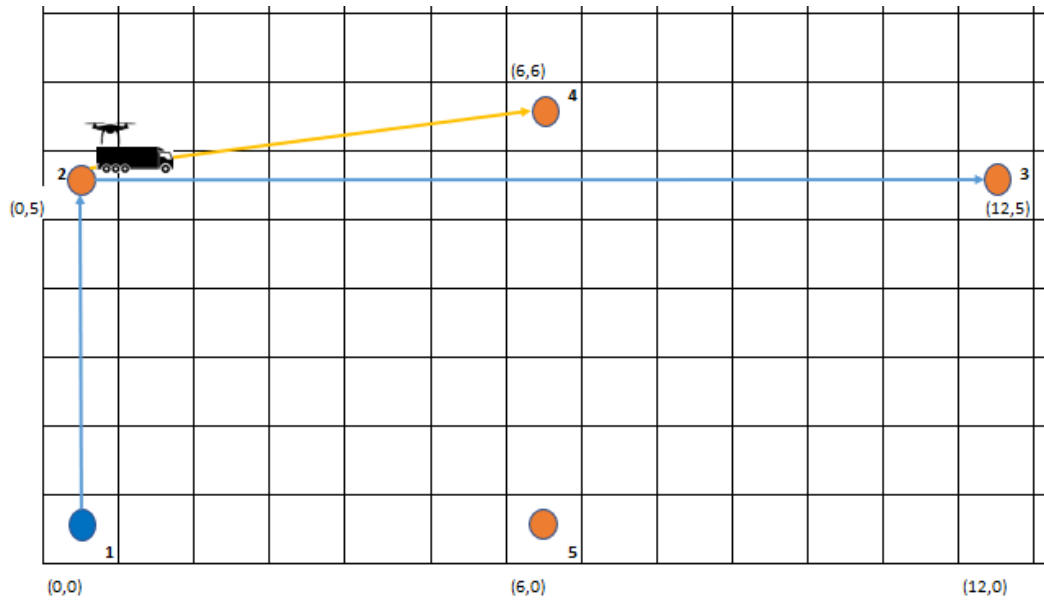


Figure 10. Sample Problem for Hybrid Milk Run Model (Step 2)

As seen in the Figure 11 below, while the drone delivers at the 4th customer point, the truck continues its way to the 3rd customer point.

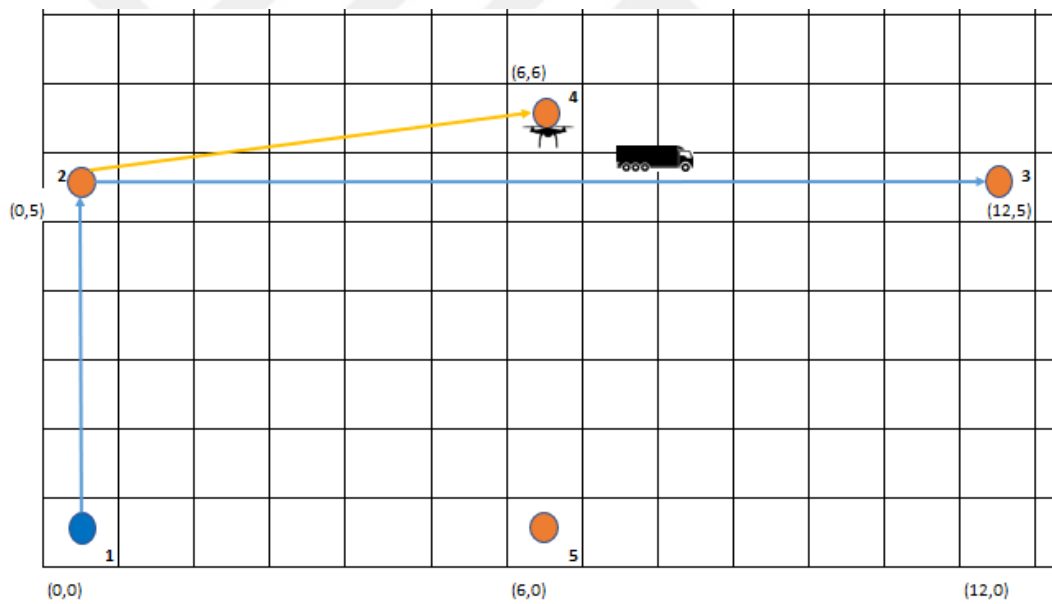


Figure 11. Sample Problem for Hybrid Milk Run Model (Step 3)

As seen in the Figure 12 below, after the drone delivers to the 4th customer point, it will return to the truck waiting for it at the 3rd customer point.

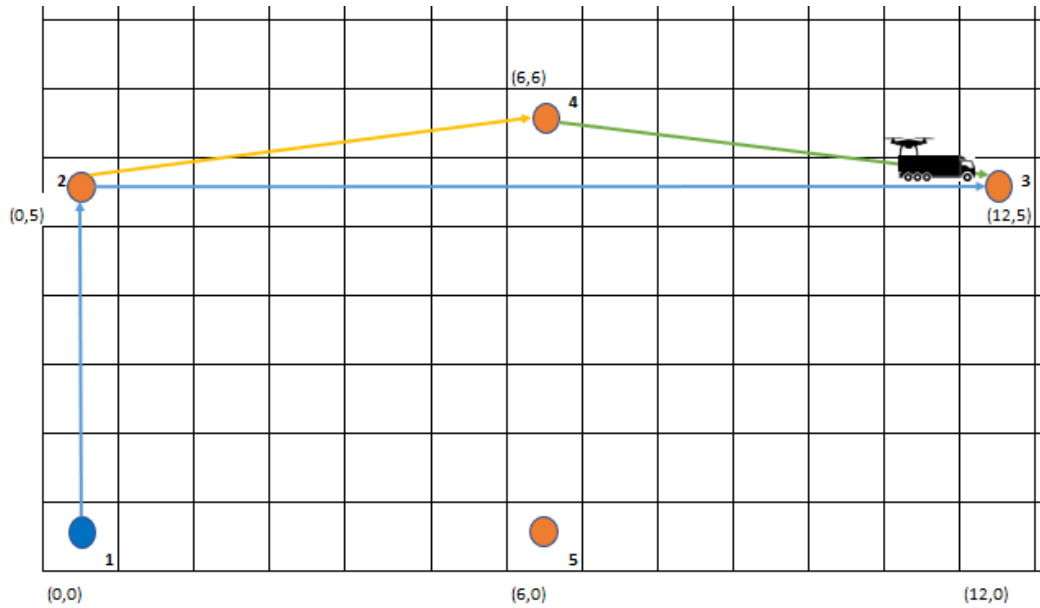


Figure 12. Sample Problem for Hybrid Milk Run Model (Step 4)

As seen in the Figure 13 below, here is the calculation methodology we used to determine where the drone will launch from the truck:

First of all, returning the empty drone to the truck after delivery forms the basis of this methodology, and we start the calculation from this part. We know that after the drone delivers to the 4th customer point, it will definitely meet the truck at the 3rd customer point. The important point here is whether the drone will be launched from the previous customer point where the truck was located, or from a point on the road as the truck continues its way.

The circle represents the drone range of serving customer point 4. The radius of circle is determined and drawn according to the remaining range based on loaded drone.

We will calculate and explain where the drone will be launched from, and how the circle will be drawn, using numerical data.

In addition, all numerical settings about drone ranges and drone speeds are taken from Özbilge's (2021) study which is A Milk-Run Distribution System Design for Integrating Drones.

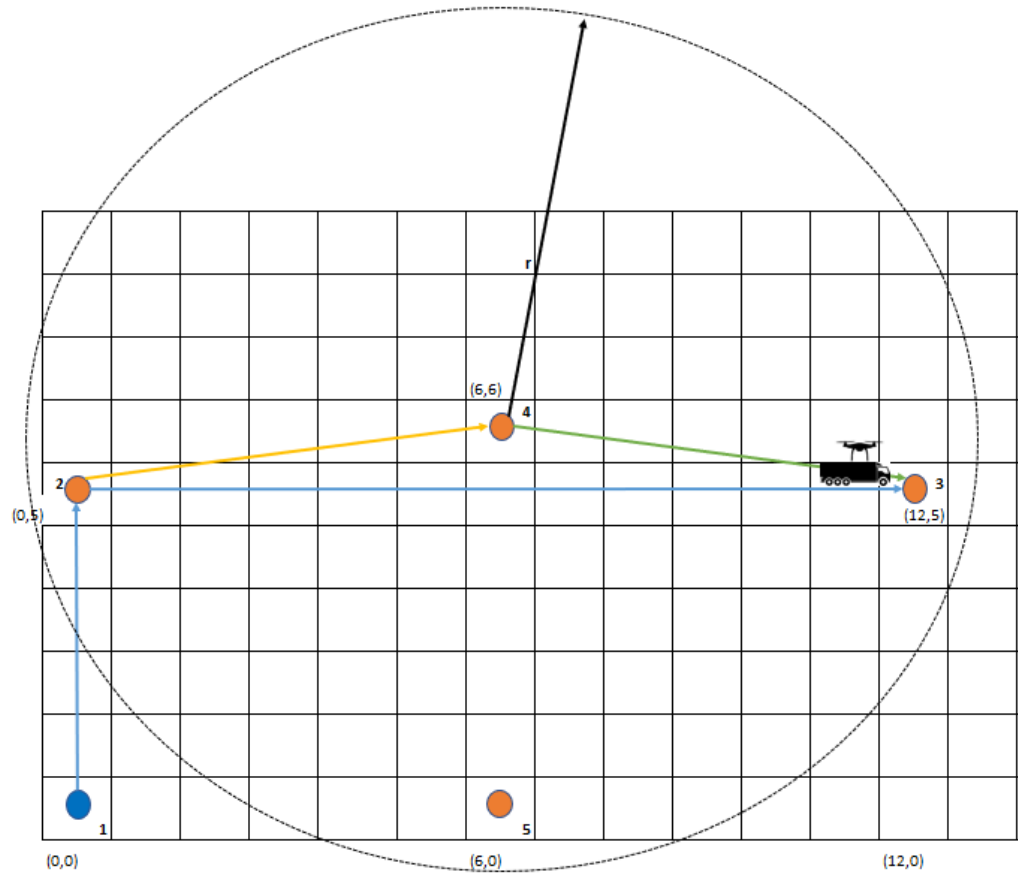


Figure 13. Calculation of Drone Flight Times for 4<sup>th</sup> Customer Point

### Calculation of Drone Flight Times for 4<sup>th</sup> Customer Point

Empty drone speed, loaded drone speed and beta values are as follows:

- Empty drone speed = 60 km/h
- Loaded drone speed = 45 km/h
- Beta = Loaded drone speed / Empty drone speed = 45 km/h / 60 km/h = 0.7

RETURN (  )

- Empty drone range = 15.79
- Returning distance of drone (Customer 4 - Customer 3) =  $\sqrt{6^2 + 1^2} = 6.08$
- Possible range of empty drone = 15.79 – 6.08 = 9.70

LEAVE (  )

- Remaining range based on loaded (r) = 9.703 \* 0.7 = 6.79
- Leaving distance of drone (Customer 2 - Customer 4) =  $\sqrt{6^2 + 1^2} = 6.08$

FLIGHT TIMES

- Flight time for empty drone = (6.08 / 60) \* 60 = 6.08 minutes
- Flight time for loaded drone = (6.08 / 45) \* 60 = 8.11 minutes
- Total flight time = 6.08 + 8.11 = 14.19 minutes

As seen in the Figure 14 below, the truck that picks up the drone at the 3rd customer point continues its way to the depot. According to the calculations we will show below, while the truck is moving on the road, it launches the drone towards the 5th customer point.

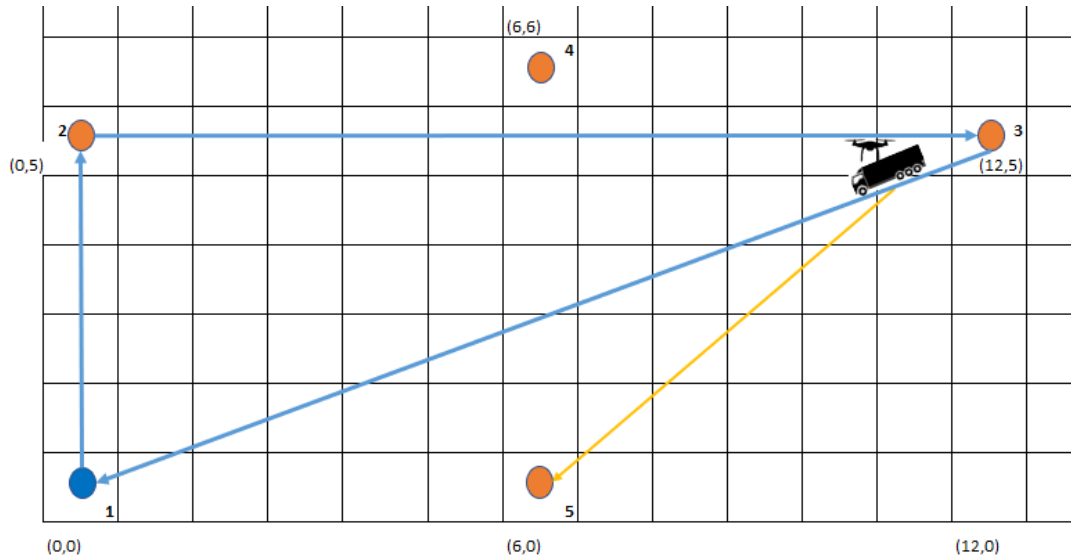


Figure 14. Sample Problem for Hybrid Milk Run Model (Step 5)

As seen in the Figure 15 below, while the drone delivers at the 5th customer point, the truck continues its way to the depot.

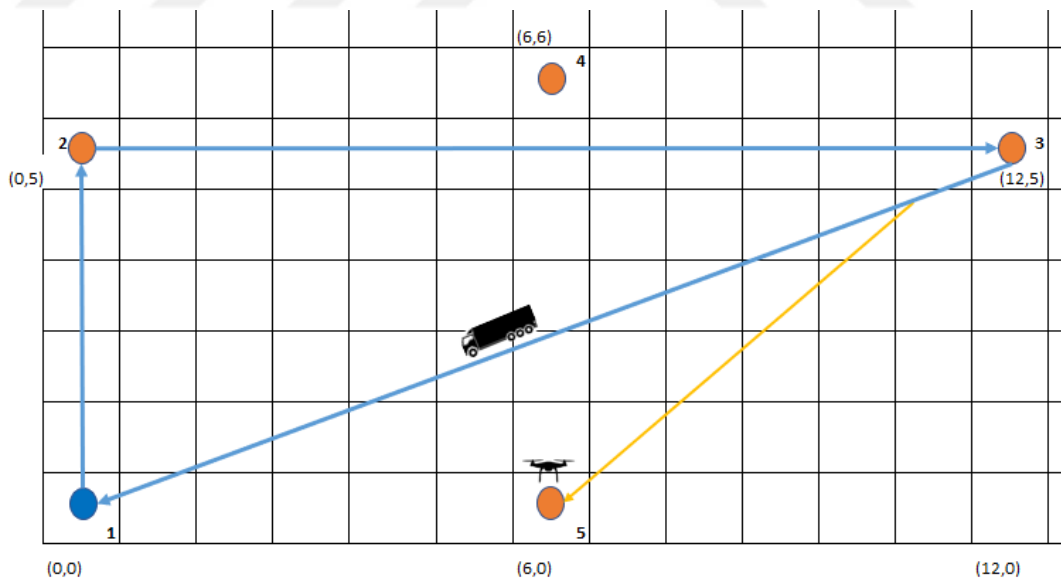


Figure 15. Sample Problem for Hybrid Milk Run Model (Step 6)

As seen in the Figure 16 below, after the drone delivers to the 5th customer point, it will return to the truck waiting for it at the depot.



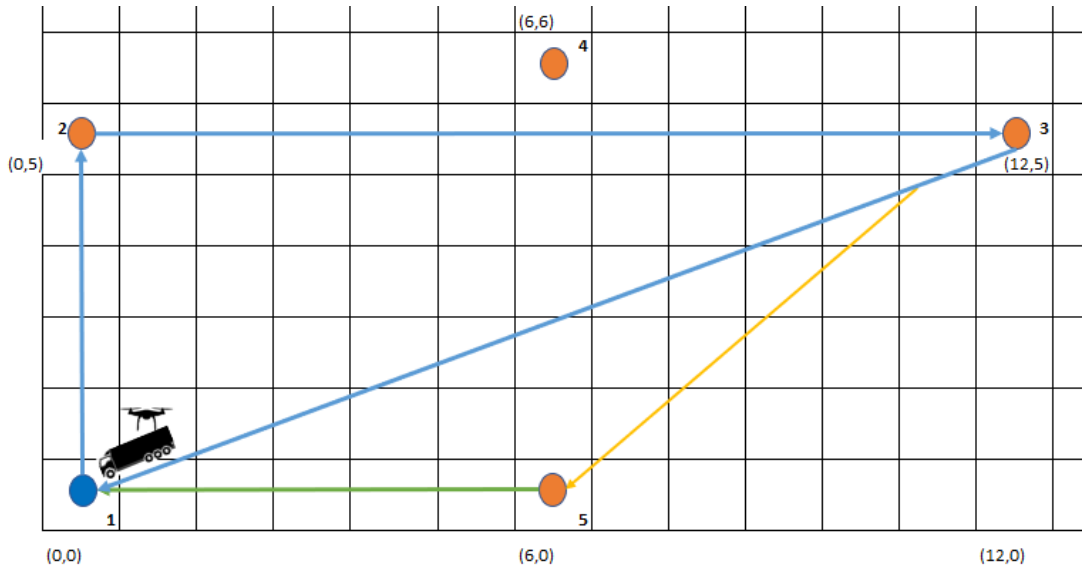


Figure 16. Sample Problem for Hybrid Milk Run Model (Step 7)

As seen in the Figure 17 below, this time we will look at the calculations for drone delivery to the 5th customer point. The circle represents the drone range of serving customer point 5. The radius of circle is determined and drawn according to the remaining range based on loaded drone.

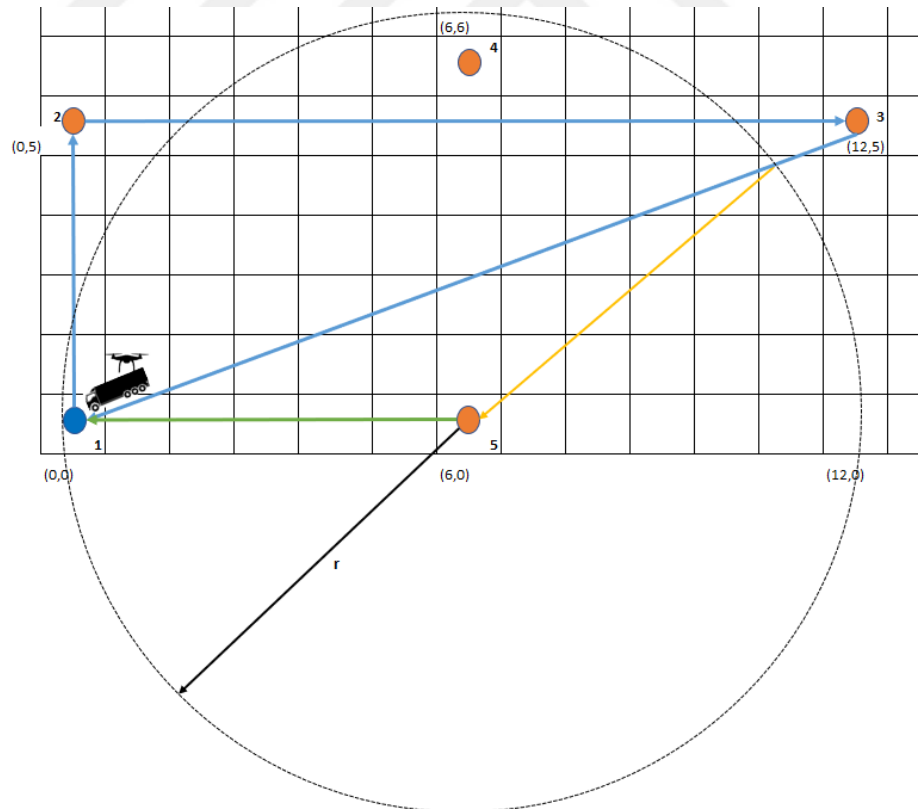


Figure 17. Calculation of Drone Flight Times for 5th Customer Point

### Calculation of Drone Flight Times for 5th Customer Point

Empty drone speed, loaded drone speed and beta values are as follows:

- Empty drone speed = 60 km/h
- Loaded drone speed = 45 km/h
- Beta = Loaded drone speed / Empty drone speed = 45 km/h / 60 km/h = 0.7

RETURN ( → )

- Empty drone range = 15.79
- Returning distance of drone (Customer 5 – Depot 1) =  $\sqrt{6^2 + 0^2} = 6$
- Possible range of empty drone = 15.79 – 6 = 9.79

LEAVE ( → )

- Remaining range based on loaded (r) = 9.79 \* 0.7 = 6.85
- Leaving distance of drone (A Point Between Customer 3 & Depot 1 - Depot 1)  
=  $\sqrt{(4.65)^2 + 5^2} = 6.83$

FLIGHT TIMES

- Flight time for empty drone = (6 / 60) \* 60 = 6 minutes
- Flight time for loaded drone = (6.83 / 45) \* 60 = 9.10 minutes
- Total flight time = 6 + 9.10 = 15.10 minutes

The total delivery distances and times for the truck also can be calculated as follows:

$$\text{Depot 1 - Customer 2: } \sqrt{(0 - 0)^2 + (0 - 5)^2} = 5$$

$$\text{Customer 2 - Customer 3: } \sqrt{(0 - 12)^2 + (5 - 5)^2} = 12$$

$$\text{Customer 3 - Depot 1: } \sqrt{(12 - 0)^2 + (5 - 0)^2} = 13$$

$$\text{Depot 1 - Customer 2: } (5/35).60 = 8.57 \text{ minutes}$$

$$\text{Customer 2 - Customer 3: } (12/35).60 = 20.57 \text{ minutes}$$

$$\text{Customer 3 - Depot 1: } (13/35).60 = 22.28 \text{ minutes}$$

## CHAPTER 4

### MODELING APPROACH

In this chapter, we will show the model structure for proposed model which is hybrid milk run model (HMR). Before explaining this model, we will first describe the classical milk run model (CMR). This stage is important for making comparisons in the next sections. After that, the proposed HMR model will be introduced. The objective function, decision variables, parameters, sets and constraints will be shown for both models and will be explained in detail. The problem is modelled and run in GAMS. The GAMS models for the CMR and HMR are given in Appendix A and Appendix B, respectively.

#### 4.1. CLASSICAL MILK RUN MODEL (CMR)

Before moving on to the mathematical model of the hybrid milk run (HMR) model we propose, we will explain the classical milk run to compare (Winston, 2004).

To explain how the classical milk run works in practice; in the classical milk run, the route is not optimized and is determined before the operation. The truck leaves the depot, goes to the locations on its route, and finally returns to the depot. If a location on the route does not need service, the truck does not stop there and continues from the next location.

In a nutshell, all customer demands are known before the start of operations and all customers must be served. Also, the truck stops at all customer locations for delivery from the truck during its tour and speed of truck is fixed through its whole travel.

##### 4.1.1. Sets

The set of customer locations for truck is the only set of the classical milk run model and it is given as follows:

$$i \quad : \quad S^l = \{i: i = 1, 2, \dots, i, \dots, I\} \text{ where } 1 \text{ corresponds to the depot}$$

#### 4.1.2. Parameters

The parameters of the classical milk run model are given as follows:

$\zeta$  : Effective speed of truck in city (km/h)

$D_i$  : Demand at customer location  $i$  (in unit size)

$L_{ij}$  : The distance between node  $i$  and  $j$  (km)

$T_{ij}^{trk}$  : Travel time between the node  $i$  and node  $j$  for the truck (in minutes)

$T^{trk}$  : Service time of the truck to serve a customer (in minutes)

$C^{trk}$  : Operating and fuel cost of truck ( $\phi$ /km)

$C^{opt}$  : Opportunity cost of time ( $\phi$ /minutes)

#### 4.1.3. Decision Variables

The decision variables of the classical milk run model are given as follows:

$x_{ij}$  :  $\begin{cases} 1 & \text{if truck moves from location } i \text{ to location } j \\ 0 & \text{otherwise} \end{cases}$

$t_i$  : The departure time of truck from location  $i$

$\tau$  : The completion time of the tour

$z^{trk}$  : Total truck cost

$z^{opt}$  : Total opportunity cost

#### 4.1.4. Objective Function

The objective function of the classical milk run model is minimizing the summation of total truck operation cost and the opportunity cost. Knowing that (1) defines the objective function to minimize. The opportunity cost is the expense incurred because of the overall time difference between the classical milk run tour and the proposed hybrid milk run tour. The notion is that if the tour were conducted according to the model described in this thesis, the total time spent on the tour would be less than that of a classical milk run. The company has the chance to serve more customers and earn more money thanks to this time difference.

$$\text{Min } z^{trk} + z^{opt} \quad (1)$$

$$z^{trk} = C^{trk} \sum_{\forall i} \sum_{\forall j} L_{ij} x_{ij} \quad (2)$$

$$z^{opt} = C^{opt} \tau \quad (3)$$

#### 4.1.5. Model Constraints

The constraints of the classical milk run model are given as follows:

$$\sum_{\forall i} x_{ij} = 1 \quad \forall j \quad (4)$$

$$\sum_{\forall k} x_{jk} = 1 \quad \forall j \quad (5)$$

$$t_1 = 0 \quad (6)$$

$$u_i - u_j + I x_{ij} \leq I - 1, \quad \text{for } i \neq j, \quad i, j = 2, 3, \dots, I \quad (7)$$

$$t_j \geq (t_i + T_{ij}^{trk} + T^{trk}) x_{ij} \quad j = 2, 3, \dots, I ; \forall i \quad (8)$$

$$x_{i1}(t_i + T_{i1}^{trk}) \leq \tau \quad \forall i \quad (9)$$

$$x_{ii} = 0 \quad (10)$$

The constraints can be explained as follows: Constraints (4) and (5) ensure that each customer location is arrived once, and each customer location is left once. The constraint (6) ensures that truck always starts delivery from location 1. The equation (7) ensures that any set of  $x_{ij}$ 's containing a subtour will be infeasible and any set of  $x_{ij}$ 's that forms a tour will be feasible. The constraint (8) guarantees that if truck moves from location  $i$  to location  $j$ , the departure time of truck from location  $j$  should be greater than or equal to the summation of the departure time of truck from location  $i$ , travel time between the  $i$  and  $j$ , and service time of the truck. The constraint (9) confirms that departure time of truck from location  $i$  plus travel time between the node  $i$  and depot for the truck must be less than or equal to the completion time of the tour. Finally, the constraint (10) ensures the loop between locations.

## **4.2. THE HYBRID MILK RUN MODEL (HMR)**

In this section, the assumptions, sets, parameters, decision variables, objective functions and constraints of the proposed model which is hybrid milk run will be explained.

### ***4.2.1. Assumptions***

In the hybrid milk run model to be used in this thesis, the settings for the milk run distribution system are as follows: The truck visits several locations at a given time using a predefined route. It collects the parts/materials and delivers them to the origin. The truck can take multiple drones within the tour and it does not have a fixed capacity for maximum number of drones. After the parts/materials are delivered to customers, the truck returns back to the depot. The milk run system is a loop that begins and ends at the same location. The system allows the exchange of loaded and empty drones.

On the other hand, the settings of the drones can be explained as follows: Each drone can only carry one item at a time; therefore, its capacity is one unit, and drones have a limited range of flight. Drone speed is faster compared to average speed of vehicle moving on route in city. Also, drones land on the truck when truck makes a stop. Drones launch off the truck when either truck makes a stop or during movement on path. The synchronization of truck and drone in delivery is ensured. In addition, after drone returns back to the truck, drone's battery can be replaced instantaneously.

In this study, we assumed the average speed of the truck to be 35 km/h. We call this assumed speed as effective speed, which corresponds to the speed that Euclidean distance between two points is traveled. For the sake of simplicity, we assume that the travel time of the truck and drones between any pair of locations is proportional to the Euclidean distance. Boysen et al. (2018) stated that they determine the drone and truck distances according to the Euclidean distance. Also, Agatz et al. (2018) stated that the travel time of the truck between any locations is considered according to the Euclidean distance.

Finally, item variety is not assumed in this study. We only consider one unit load. Even if it is, it can be applied in a technology where several drones will work together, but we did not include this in the model.

The assumptions of the hybrid milk run model can be summarized as follows: (A-1) All customer demands are known before the start of operations and all customers must be served.

(A-2) The truck stops at some customer location for delivery from the truck during its tour. Launching operation of drones are performed when either truck stops or during the movement of truck on path. Landing operations of drones are performed as the truck stops. Drones deliver to customers which are not visited by the truck.

(A-3) If the drone is launched off the truck during the movement on path  $(i,j)$ , drone returns to the  $j$  location (always next location).

(A-4) A specific drone can make single movement through a single tour.

(A-5) The capacity of a drone is one unit demand. Item variety is not considered.

(A-6) Demand of each customer can be fulfilled by truck and/or drones.

(A-7) Multiple drones can be launched / landed simultaneously.

(A-8) Speed of truck and drones are fixed through their whole travel; drone speed is different if it is loaded or unloaded.

(A-9) Flight range of drones is fixed and different if they are loaded or unloaded.

(A-10) Travel times and flight times depend on distance between locations. Congestion effect is not considered.

(A-11) We use effective average speed for truck between Euclidean distances.

#### **4.2.2. Sets**

The nonlinear mixed integer programming for the proposed model (HMR) is developed with the following steps.

Before moving on to mathematical modeling, I will describe the sets of the model. The set you see in the first line is the customer locations for trucks and drones. Here, the 1st location represents the depot. To express better the locations in the model constraints, alias was created for  $i, j$  and  $k$ . The other set  $d$  represents the number of drones.

$I$  :  $S^I = \{i: i = 1, 2, \dots, i, \dots, I\}$  where 1 corresponds to the depot  
Alias  $(i, j, k)$

$D$  :  $S^D = \{d: d = 1, 2, \dots, d, \dots, D\}$ , number of drones

#### **4.2.3. Parameters**

The parameters of the model are as follows:

$\zeta$  : Effective speed of truck in city, (km/h)

$D_i$  : Demand at customer location  $i$ , (in unit size)

$D_i^B$  :  $\begin{cases} 1 & \text{If demand at customer location } i \text{ is positive} \\ 0 & \text{otherwise} \end{cases}$

$L_{ij}$  : The distance between node  $i$  and  $j$  (km)

$T_{ij}^{trk}$  : Travel time between the node  $i$  and node  $j$  for the truck (in minutes)

$T_{ijk}^{drn}$  : The flight time of drone if the truck is on path  $(i,j)$  and the drone is launched to serve customer location  $k$  (in minutes)

$P_{ijk}$  :  $\begin{cases} 2 & \text{If drone launches from node } i \text{ to serve customer location } k \text{ over } p \\ 1 & \text{If drone launches from a point on path } (i,j) \text{ to serve customer loc} \\ 0 & \text{Not possible} \end{cases}$

$S_{ijk}^{trk}$  : The travel time of truck up to launch of drone to serve customer location  $k$  over path  $(i,j)$  when  $P_{ijk}^B = 1$  (in minutes)

$T^{max}$  : Upper bound on total operation time (in minutes)

$T^{trk}$  : Service time of the truck to serve a customer (in minutes)

$C^{trk}$  : Operating and fuel cost of truck ( $\text{¢/km}$ )

$C^{opt}$  : Opportunity cost of time ( $\text{¢/min}$ )

$C^{drn}$  : Operating and battery usage cost of drone ( $\text{¢/min}$ )

#### 4.2.4. Decision Variables

In this section, the decision variables of the hybrid milk run model are given. The decision variable  $x_{ij}$  is the binary variable that takes the value of one if truck moves from location  $i$  to location  $j$ . Next binary decision variable is  $w_i$  and it takes one if customer location  $i$  is served by truck.  $y_i$  is another binary decision variable and it takes one if location  $i$ 's demand is delivered by drone(s).  $v_{ijk}$  is a binary decision variable and it takes one if flight(s) of drone(s) for location  $k$  is (are) associated with truck movement from location  $i$  to  $j$ .  $r_{dk}$  is a binary decision variable and it takes one if customer location  $k$  is served by drone  $d$ .  $t_i$  is the departure time of truck from location  $i$ .  $f_{dk}$  is the departure time of drone  $d$  to serve location  $k$ .  $\tau$  is the completion time of the tour.  $z^{trk}$  is the total truck



operation cost, and  $z^{drn}$  is the total drone operation cost. Also,  $z^{opt}$  is the total opportunity cost. The decision variables of the model are given as follows:

$$x_{ij} : \begin{cases} 1 & \text{If truck moves from location } i \text{ to location } j \\ 0 & \text{otherwise} \end{cases}$$

$$w_i : \begin{cases} 1 & \text{If customer location } i \text{ is served by truck} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i : \begin{cases} 1 & \text{If location } i \text{'s demand is delivered by drone(s)} \\ 0 & \text{otherwise} \end{cases}$$

$$v_{ijk} : \begin{cases} 1 & \text{If flight of drone for location } k \text{ is associated with truck movement from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

$$r_{dk} : \begin{cases} 1 & \text{If customer location } k \text{ is served by drone } d \\ 0 & \text{otherwise} \end{cases}$$

$$t_i : \text{The departure time of truck from location } i$$

$$f_{dk} : \text{The departure time of drone } d \text{ to serve location } k$$

$$\tau : \text{The completion time of the tour}$$

$$z^{trk} : \text{Total truck cost}$$

$$z^{drn} : \text{Total drone cost}$$

$$z^{opt} : \text{Total opportunity cost}$$

#### 4.2.5. Objective Function

The objective of this model is to minimize the total cost. This cost consists of three components. The first of these is the fuel cost of the truck moving. The second is the cost due to drone operation. Finally, another cost is the opportunity cost. The cost information about the truck, drone and opportunity used in this model are taken from Özbilge's (2021) study which is A Milk-Run Distribution System Design for Integrating Drones. The objective function of the model is given as follows:

$$\text{Min } z^{trk} + z^{drn} + z^{opt} \quad (11)$$

$$z^{trk} = C^{trk} \sum_{\forall i} \sum_{\forall j} L_{ij} x_{ij} \quad (12)$$

$$z^{opt} = C^{opt} \tau \quad (13)$$

$$z^{drn} = C^{drn} \cdot \left[ \sum_{\forall i} \sum_{\forall j} \sum_{\forall k} (v_{ijk} T_{ijk}^{drn} \sum_{\forall d} r_{dk}) \right] \quad (14)$$

#### 4.2.6. Model Constraints

The constraints of the model are given in the below.

$$\mu = \sum_{\forall i} w_i \quad i = 2,3,4,\dots,I \quad (15)$$

$$w_i + y_i = D_i^B \quad i = 2,3,4,\dots,I \quad (16)$$

$$v_{ijk} \leq x_{ij} \quad \forall i, j, k \quad (17)$$

$$y_k = \sum_{\forall i} \sum_{\forall j} v_{ijk} \quad \forall k \quad (18)$$

$$w_j = \sum_{\forall i} x_{ij} \quad j = 2,3,\dots,I \quad (19)$$

$$w_j = \sum_{\forall k} x_{jk} \quad j = 2,3,\dots,I \quad (20)$$

$$\sum_{\forall i} x_{i1} = 1 \quad (21)$$

$$\sum_{\forall k} x_{1k} = 1 \quad (22)$$

$$u_i - u_j + (\mu + 1)x_{ij} \leq \mu, \quad \text{for } i \neq j, \quad i, j = 2,3,\dots,I \quad (23)$$

$$\sum_{\forall d} r_{dk} = D_k y_k \quad \forall k \quad (24)$$

$$f_{dk} \leq T^{\max} r_{dk} \quad \forall k, d \quad (25)$$

$$t_j \geq (t_i + T_{ij}^{trk} + T^{trk}) x_{ij} \quad j = 2,3,\dots,I; \quad \forall i \quad (26)$$

$$(f_{dk} + T_{ijk}^{drn}) v_{ijk} r_{dk} \leq t_j \quad \forall d, j, k, i \quad (27)$$

$$f_{dk} \geq (t_i - T^{trk}) v_{ijk} P_{ijk} r_{dk} \quad \forall d, k, i, j \quad (28)$$

$$f_{dk} \geq (t_i + S_{ijk}^{trk}) v_{ijk} (2 - P_{ijk}) r_{dk} \quad \forall d, k, i, j \quad (29)$$

$$t_i + S_{ijk}^{trk} v_{ijk} \leq \tau \quad \forall k, i, j \quad (30)$$

$$x_{i1}(t_i + T_{i1}^{trk}) \leq \tau \quad \forall i \quad (31)$$

$$w_1 = 1 \quad i = 2, 3, 4, \dots, I \quad (32)$$

$$t_1 = 0 \quad (33)$$

$$x_{ii} = 0 \quad \forall i \quad (34)$$

Constraint (15) calculates how many nodes the truck has visited. Constraint (16) ensures that if customer location  $i$  has positive demand it should be satisfied via truck or drone. Constraint (17) ensures that if customer location  $k$  is served by drone(s), where flight(s) is(are) associated with truck movement from location  $i$  to  $j$ , then truck should move from location  $i$  to  $j$ . Constraint (18) guarantees that the location  $k$ 's demand is delivered by drone if and only if flight(s) of drone(s) for location  $k$  is(are) associated with a truck movement. Constraint (19) ensures that if the truck moves from location  $i$  to location  $j$  then location  $j$ 's demand satisfied by truck and constraint (20) ensures that if the truck moves from location  $j$  to location  $k$  then location  $j$ 's demand satisfied by truck. Constraints (21) and (22) ensures that truck moves from any customer location to depot once and, truck moves from depot to any customer location once. Constraint (23) ensures that any set of  $x_{ij}$ 's containing a subtour will be infeasible and any set of  $x_{ij}$ 's that forms a tour will be feasible (subtour elimination). Constraint (24) ensures that if the demand of location  $k$  is satisfied with drone(s) then there should be same number of launched drones with demand. Constraint (25) guarantees that if customer location  $k$  is served by drone, then its maximum total operation time should be greater than or equal to the departure time of drone  $d$  to serve

location  $k$ . Constraint (26) guarantees that if truck moves from location  $i$  to location  $j$ , the departure time of truck from location  $j$  should be greater than or equal to the summation of the departure time of truck from location  $i$ , travel time between the  $i$  and  $j$ , and service time of the truck. Constraint (27) ensures that the truck can depart from location  $j$  if flight(s) of drone(s) for location  $k$  is (are) associated with truck movement from location  $i$  to  $j$ , customer location  $k$  is served by drone  $d$ , time at summation of departure time of the drone  $d$  and its flight time. Constraint (28) guarantees that the departure time of drone  $d$  to serve location  $k$  should be greater than or equal to if customer location  $k$  is served by drone  $d$ , flight(s) of drone(s) for location  $k$  is (are) associated with truck movement from location  $i$  to  $j$ , drone launches from node  $i$  or point on path  $(i,j)$  to serve customer location  $k$  and multiplication of difference between the departure time of truck from location  $i$  and service time of the truck to serve a customer. Constraint (29) ensures that the departure time of drone  $d$  to serve location  $k$  should be greater than or equal to if customer location  $k$  is served by drone  $d$ , flight(s) of drone(s) for location  $k$  is (are) associated with truck movement from location  $i$  to  $j$ , drone launches from node  $i$  or point on path  $(i,j)$  to serve customer location  $k$  and summation of the departure time of truck from location  $i$  and the travel time of truck up to launch of drone to serve customer location  $k$  over path  $(i,j)$  when  $P_{ijk}^B = 1$ . Constraint (30) guarantees that the departure time of truck from location  $i$  plus travel time of truck up to launch of drone to serve customer location  $k$  over path  $(i,j)$  when  $P_{ijk} = 1$  if flight(s) of drone(s) for location  $k$  is (are) associated with truck movement from location  $i$  to  $j$  must be less than or equal to the completion time of the tour. Constraint (31) guarantees that the departure time of truck from location  $i$  plus travel time between the node  $i$  and depot for the truck must be less than or equal to the completion time of the tour. Constraint (32) ensures that truck always goes to first location which is depot. Constraint (33) enforces that truck always starts delivery from first location which is depot. Constraint (34) ensures the loop between locations.

### **4.3. REDUCTION OF NONLINEARITIES OF THE HMR MODEL TO QUADRATIC FORM**

This model formulation was created using the GAMS program. For this reason, Çankaya University purchased a GAMS license to enable us to carry out our work and provided us with this opportunity.

Since the formulation we wrote has a nonlinear structure, we encountered difficulties in the solution phase of the model. Three constraints of the model contain nonlinearity. For example, in constraint (27), decision variables which are  $f_{dk}$ ,  $v_{ijk}$  and  $r_{dk}$  are multiplied with each other and this complicates our work. As a result of our correspondence with GAMS company to overcome this problem, we learned that the solver named Gurobi is suitable for this model and Gurobi license was obtained.

Gurobi makes the model quadratic. This actually is ensured with a very simple mathematical trick. Instead of multiplying these three variables, another variable ( $fr_{dk}$ ) is created by multiplying two variables, and the remaining variable ( $v_{ijk}$ ) is multiplied with it.

Accordingly, constraint (27) is rewritten in quadratic form as follows:

$$(fr_{dk}v_{ijk} + T_{ijk}^{drn}v_{ijk}r_{dk}) \leq t_j \quad \forall d, j, k, i \quad (35)$$

We convert the constraint (28) and constraint (29) into quadratic form with the same approach in constraint (35).

## CHAPTER 5

### NUMERICAL STUDY

#### 5.1. DATA AND PARAMETERS

We explain how the parameters used in this study were determined as follows:

As customer locations, 10 locations were selected randomly in each instance from the data of 96 locations prepared by Özbilge (2021) and worked on with this location data. So, our problem consists of 10 customer locations in each instance and there are total of four instances. We perform our numerical study on these locations. The names of these locations and their coordinates for first instance are given in the Table 3 below. Location 1 represents the depot and others are customer locations. The locations of other instances are given in Appendix C.

*Table 3. Center Locations of Customers for Instance-1*

Locations	Xc	Yc
1	14.32	0
21	5	9
28	5	7
33	-9	5
52	-9	-1
54	11	-1
58	-9	-3
67	11	-5
70	-7	-7
85	9	-9
86	-7	-11

In his study, Özbilge (2021) specified the range and speed values for the drone, according to two different drone models, and included these values in his numerical study. We will continue our study using Özbilge's findings. Accordingly, we accept

the empty drone speed as 60 km/h and the loaded drone speed as 45 km/h. Alpha ( $\alpha$ ) is the speed ratio of loaded drone to empty drone which we use in this study. This value is 0.8 which is come from ratio of truck speed to empty drone speed which is 45/60. However, the ranges of loaded and empty drones are 11.05 km and 15.79 km, respectively. To show ratio of loaded drone range to empty drone range, we use beta ( $\beta$ ). On the other hand, while Özbilge accepted the speed of the truck as 90 km/h because he used the highway on his study, we determine this value as 35 km/h because we use the truck in the city. We explain to how we decided on this value when describing the distances between locations and truck speed.

Specific ratio parameters that will be used to determine the database to be used in the numerical study part of our thesis are given in Table 4 below.

*Table 4. Ratio Parameters*

<b>Ratio Parameter</b>	<b>Explanation</b>	<b>Value Range</b>
$\alpha$	Speed ratio of loaded drone to empty drone	$0 < \alpha < 1$
$\beta$	Range ratio of loaded drone to empty drone	$0 < \beta < 1$
$\xi$	Speed ratio of truck in city to empty drone	$0 < \xi < 1$

$D_i$  represents the demand at customer location  $i$  and the demand values used in this study are taken from Özbilge (2021). The number of demands for the selected customer locations for the first instance are given in the Table 5 below. The demands of the locations for other instances are given in Appendix D.

Table 5. Demands of the Locations for Instance-1

Locations	# of Demands
1	0
21	2
28	2
33	3
52	3
54	1
58	5
67	3
70	2
85	4
86	3

$L_{ij}$  represents the distances between customer locations  $i$  and  $j$ . The distances are calculated from the Euclidean distance formula. According to the randomly selected customer locations and the calculation methodology, these parameter values for the first instance are given in the Table 6 below. For other instances, distance matrices are given in the Appendix E.

Table 6. Distance Matrix for the Instance-1

EUCLIDEAN DISTANCES		x	14.32	5.00	5.00	-9.00	-9.00	11.00	-9.00	11.00	-7.00	9.00	-7.00
		y	0.00	9.00	7.00	5.00	-1.00	-1.00	-3.00	-5.00	-7.00	-9.00	-11.00
x	y	Locations	1	21	28	33	52	54	58	67	70	85	86
14.32	0.00	1	0.00	12.96	11.66	23.85	23.35	3.47	23.52	6.00	22.44	10.46	23.99
5.00	9.00	21	12.96	0.00	2.00	14.56	17.20	11.66	18.44	15.23	20.00	18.44	23.32
5.00	7.00	28	11.66	2.00	0.00	14.14	16.12	10.00	17.20	13.42	18.44	16.49	21.63
-9.00	5.00	33	23.85	14.56	14.14	0.00	6.00	20.88	8.00	22.36	12.17	22.80	16.12
-9.00	-1.00	52	23.35	17.20	16.12	6.00	0.00	20.00	2.00	20.40	6.32	19.70	10.20
11.00	-1.00	54	3.47	11.66	10.00	20.88	20.00	0.00	20.10	4.00	18.97	8.25	20.59
-9.00	-3.00	58	23.52	18.44	17.20	8.00	2.00	20.10	0.00	20.10	4.47	18.97	8.25
11.00	-5.00	67	6.00	15.23	13.42	22.36	20.40	4.00	20.10	0.00	18.11	4.47	18.97
-7.00	-7.00	70	22.44	20.00	18.44	12.17	6.32	18.97	4.47	18.11	0.00	16.12	4.00
9.00	-9.00	85	10.46	18.44	16.49	22.80	19.70	8.25	18.97	4.47	16.12	0.00	16.12
-7.00	-11.00	86	23.99	23.32	21.63	16.12	10.20	20.59	8.25	18.97	4.00	16.12	0.00



Zeta ( $\zeta$ ) is the effective speed of the truck in city. We take this value as 35 km/h, considering that the speed of the truck in the city is slower than the speed of the drone. In the process of finding this value, we created 7 separate routes in certain regions in Ankara and examined these distances on Google maps. The important point here is that the distances are created from the Euclidean distance formula, not the actual road distance. For this reason, the effective speed determined for the truck is not a real-life speed, but a speed based on the location distances we have created, and this value is 35 km/h. In addition, all of the routes used in the phase of finding the effective speed have been prepared with the thought that the truck will take off at 10:00.

As you can see in the Figure 18, the route between the two locations (ACity and Panora AVM) is shown. According to this route, a car is expected to reach its destination between 16 and 28 minutes. Since the truck is in question in this parameter, it is obvious that the car and the truck cannot move at the same speed both in terms of engine power and according to traffic laws.

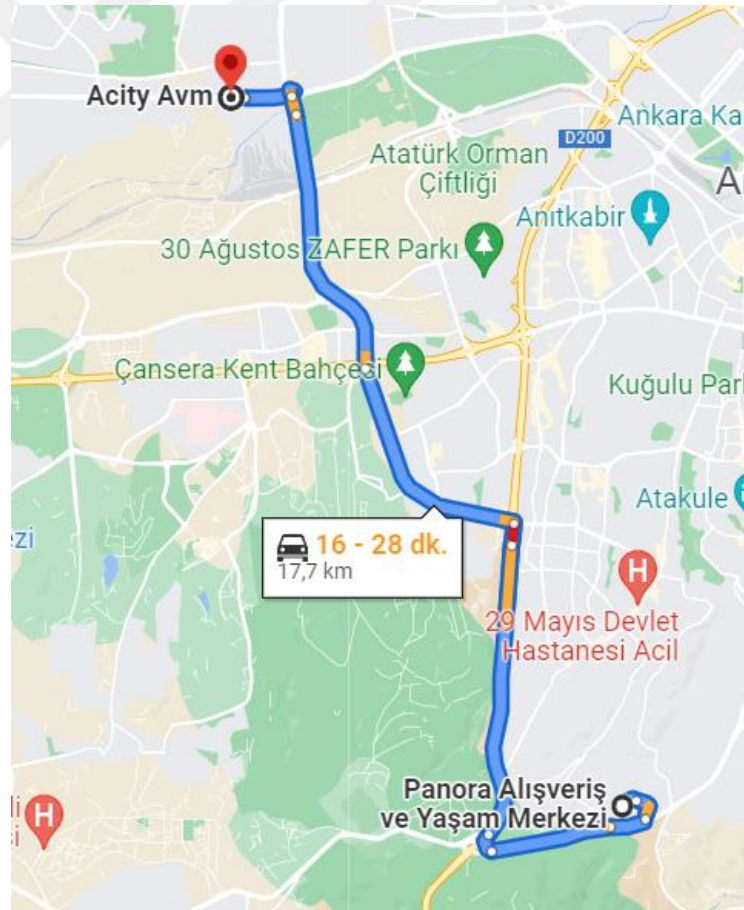


Figure 18. An Example Route Based on Real Road Distance

We chose the highest value for the time required to reach the target point. According to the results we found for 7 different routes, it was determined that the

speed of the car was 34 km/h in its slowest state. This value is found for a car assuming it is going really slow enough. Accordingly, we thought that 35 km/h would be an ideal average speed for the truck. The factor in our decision that this value was ideal was that we have a problem setting with calculated Euclidean distances, as you can see in the Figure 19. The actual distance value between these locations is not covered in our problem environment. We focus only on Euclidean distances. So, the route, which was originally 17.7 km, was reduced to 12.23 km with the Euclidean distance approach.

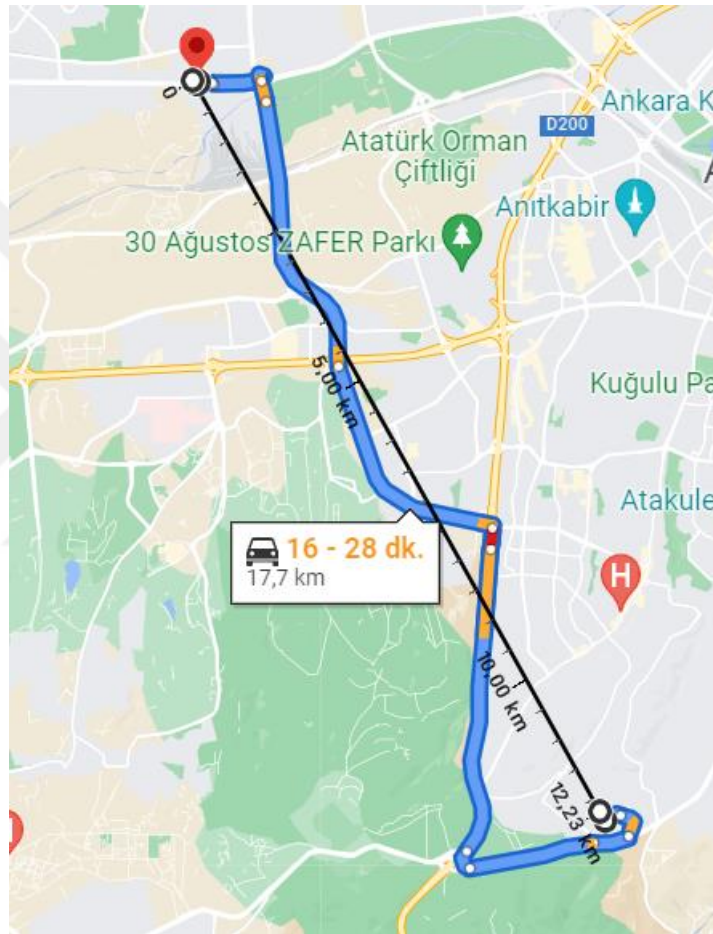


Figure 19. An Example Route Based on Euclidean Distance

The analysis study, where we determined the effective speed of the truck as 35 km/h, is given in the Appendix J.

$\xi$  is also ratio parameter which means speed ratio of truck in city to empty drone. This value is 0.6 which is come from ratio of truck speed to empty drone speed which is 35/60.

$T_{ij}^{trk}$  is the total travel time between location  $i$  and location  $j$  for the truck. The formula we use when calculating these travel times between customer locations is

(distance between locations/effective speed of truck) \* 60. The travel times of the truck in terms of minutes, which we determined according to this formula, are given in Table 7 below. For other instances, travel times of truck are given in the Appendix F.

Table 7. Total Travel Times of Truck for Instance-1

Locations	1	21	28	33	52	54	58	67	70	85	86
1	0.00	22.22	19.99	40.89	40.02	5.95	40.31	10.29	38.47	17.93	41.13
21	22.22	0.00	3.43	24.96	29.49	19.99	31.61	26.11	34.29	31.61	39.98
28	19.99	3.43	0.00	24.24	27.64	17.14	29.49	23.00	31.61	28.27	37.09
33	40.89	24.96	24.24	0.00	10.29	35.80	13.71	38.33	20.86	39.09	27.64
52	40.02	29.49	27.64	10.29	0.00	34.29	3.43	34.96	10.84	33.77	17.48
54	5.95	19.99	17.14	35.80	34.29	0.00	34.46	6.86	32.53	14.14	35.30
58	40.31	31.61	29.49	13.71	3.43	34.46	0.00	34.46	7.67	32.53	14.14
67	10.29	26.11	23.00	38.33	34.96	6.86	34.46	0.00	31.05	7.67	32.53
70	38.47	34.29	31.61	20.86	10.84	32.53	7.67	31.05	0.00	27.64	6.86
85	17.93	31.61	28.27	39.09	33.77	14.14	32.53	7.67	27.64	0.00	27.64
86	41.13	39.98	37.09	27.64	17.48	35.30	14.14	32.53	6.86	27.64	0.00

$P_{ijk}$  is a parameter which indicates 2 if drone launches from node  $i$  to serve customer location  $k$  over path  $(i,j)$ , 1 if drone launches from a point on path  $(i,j)$  to serve customer location  $k$ , and 0 if drone service is not possible. As in this parameter, calculations were very difficult in 3-index parameters. To find out what value they would take, the possible return distances of the drones were calculated for all possibilities. This was obtained by subtracting the distance between the two locations of the drone's return from the empty drone range. After that, possible returning distance is multiplied by  $\beta$  which is 0.7 to find remaining leaving range based on loaded drone. After this process, we can assign these values with the logic formulation we have established. According to this logic, if remaining leaving distance is equal or smaller than zero, it takes value of 0. If remaining leaving distance is equal or larger than distance between these two customer locations, it takes value of 2. Finally, if remaining leaving distance is between zero and distance between these two customer locations, it takes value of 1. For only instance-1,  $P_{ijk}$  values are given in Appendix G.

$T_{ijk}^{drn}$  represents the flight times of drone if the truck is on path  $(i,j)$  and the drone is launched to serve customer location  $k$ . This parameter is shaped depending on

$P_{ijk}$ . In this parameter, a similar procedure is followed in calculating the flight times of drones as in  $P_{ijk}$ . If  $P_{ijk}$  is 2; the corresponding flight time of drone is found by dividing the returning distance by the empty drone speed, the leaving distance which is the distance between two customer locations by the loaded drone speed and adding these two values  $((\text{Returning distance}/60 + \text{Leaving distance}/45) * 60)$ . If  $P_{ijk}$  is 1; the corresponding flight time of drone is found by dividing the returning distance by the empty drone speed, the leaving distance which is possible leaving distance between a point on path and customer location by the loaded drone speed and adding these two values. If  $P_{ijk}$  is 0; we give large value because the flight of drone is not possible. For only instance-1,  $T_{ijk}^{drn}$  values are given in Appendix H.

$S_{ijk}^{trk}$  represents the travel time of truck up to launch of drone to serve customer location  $k$  over path  $(i,j)$  when  $P_{ijk}^B = 1$ . In this parameter, we calculate for cases with  $P_{ijk}^B = 1$ . We try to find out how long the truck traveled before launching the drone. Since this calculation is not possible for each case, we have given reasonable values. For only instance-1,  $S_{ijk}^{trk}$  values are given in Appendix I.

$T^{max}$  represents the upper bound on total operation time. We give the value 10000 minutes for this parameter, which we think is large enough, but this value can be changed.

$T^{trk}$  represents the service time of the truck to serve a customer. We give the value of 1 minute for this parameter. While the truck is serving to a customer, it takes one minute to serve according to this setting.

$C^{trk}$  represents the operating and fuel cost of truck and taken as 8.39  $\text{¢}/\text{km}$ .  $C^{drn}$  represents the operating and battery usage cost of drone and taken as 8.75  $\text{¢}/\text{min}$ .  $C^{opt}$  represents the opportunity cost of time and taken as 14.13  $\text{¢}/\text{min}$ . These three cost parameter values are taken from Özbilge (2021). Since how the opportunity cost is found is a bit more complicated, we can explain its calculation as follows. It was used <https://insights.tirport.com/istatistikler/> website on 11.01.2021 for all cost calculations. Here, it is first found the gross profit margin and annual revenue in the transportation and warehousing area. Accordingly, the annual revenue for transportation and storage was 539.66 billion TL and the gross profit margin was 14.9%. Multiplying these two values and dividing by the total number of trucks, it is calculated the gross profit per truck. Then, annual working hours are calculated. For

this, it is assumed that the truck driver could drive a truck for 9 hours a day. Besides, it is assumed that there are 220 working days in a year, and the product of these two values gives the yearly working hours which is 1980 hours. To find the hourly gross profit per truck, the gross profit per truck was divided by the annual working hours and the result was 48.09 billion TL. When this value is written in dollars, the result is \$8.48. This value in hours became \$0.1413 when converted to minutes. Since it is evaluated in cents within the scope of the problem, it is converted the dollar value into cents and found the final result, 14.13 ¢. If the reader is interested details of how these numbers are assumed, he/she may be referred to Özbilge (2021).

## 5.2. NUMERICAL SETTINGS

Numerical settings are simply the determination of a few problem parameters according to the database structure of the numerical study. We explain the methodology that we take run of the numerical study. As I have shown in the Table 3 earlier, these specific ratio parameters  $\alpha$ ,  $\beta$  and  $\xi$  could be used at several setting values, but we took one value for each.  $\alpha$  is the speed ratio of loaded drone to empty drone and this value is 0.8 which is come from 45/60.  $\beta$  is the range ratio of loaded drone to empty drone and this value is 0.7 which is come from 11.05/15.79.  $\xi$  is the speed ratio of truck in city to empty drone and this value is 0.6 which is come from 35/60. For now, we keep only one value of these parameters and run them. We want to take runs in different settings in the future.

For our cost parameters which are  $C^{trk}$ ,  $C^{opt}$  and  $C^{drn}$ , we added one setting apart from the values taken from Özbilge (2021) and created two settings for each in total. We calculated the other value we determined for the truck cost as 10.49 by adding a 25 percent increase, assuming that the fuel price will increase in the future. The opportunity cost is simply the time difference between the implementation of the two systems. The company can serve more customer and earn more money in this time difference. We calculated the other value we determined for the opportunity cost as 10.6 by subtracting a 25 percent decrease, assuming that the competitive market will show itself end of the day and demand intensity will bring new customers. Finally, the other value of operating and battery usage cost of drone is determined as 6.56. Again, 25 percent cost reduction was implemented because we thought in the future, battery costs would be more affordable. As a result, our numerical settings are composed of three cost parameters with two different values for each. We generate 4 random

instances and according to that, there are total of 32 instances in HMR model. Also, CMR model has 16 instances. To sum up, we study on total of 48 instances. The numerical settings of the models are given in the Table 8 below.

Table 8. Numerical Settings in the Model Runs

Parameter	HMR MODEL		CMR MODEL	
	Assumed Values	#	Assumed Values	#
$\alpha$	0.8	1	N.A	N.A
$\beta$	0.7	1	N.A	N.A
$\xi$	0.6	1	N.A	N.A
$C^{trk}$	8.39, 10.49	2	8.39, 10.49	2
$C^{opt}$	14.13, 10.6	2	14.13, 10.6	2
$C^{drn}$	8.75, 6.56	2	N.A	N.A
Total number of cases	8		4	
Random instance per case	4		4	
Total number of instances	32		16	

We will use the phrase high (H) or low (L) according to the numerical setting values of cost parameters we give to name the instances. For example, in  $C^{trk}$  parameter, value of 8.39 is taken as L and value of 10.49 is taken as H.

### 5.3. RESULTS

In Chapter 4, two different models are introduced. The first one is the classical milk run model (CMR) and the second is the hybrid milk run (HMR) model. These two models are run using the numerical settings we have shown so far in Section 5.2. We will now present our results.

The test instances are generated by a random selection of 10 customers. Each test instance is run for three pairs for HMR and two pairs for CMR. In each pair, 4 test

instances are generated. As we said before, we solved the model in the GAMS optimization program. The following can be said for the model size as a result of the run for an instance. Our model consists of 5813 variables and 1782 of them are binary variables. The remaining 4031 variables are continuous variables. The total number of constraints in the model is 118760. In addition, the solution time of the model is found to be 4037.110 seconds, which equates to approximately 67.3 minutes.

#### 5.4. COMPARISON OF HMR WITH CMR

We calculated the costs and % improvement ( $\Delta$ ) of cost between these two models and all results of study is given in Appendix K. Besides the values of  $\Delta$ , we also kept a record of values of some decision variables like  $\tau$ , sum of  $w_i$ ,  $y_i$  and  $r_{dk}$ . Since drone cost parameters and variables are not important in the CMR model, the first two letters in the case name we give based on three letters define the CMR. Accordingly, if the first two letters of the case names are the same, i.e., HHH (high-high-high) and HHL (high-high-low), we wrote those sections of the results in light colors because the run results were the same.

The  $\Delta$  is a percentage cost benefit if the HMR model is used instead of the CMR model. The  $\Delta$  percentages are calculated using equation (36).

$$\Delta = \frac{z_{cmr} - z_{dmr}}{z_{cmr}} \times 100 \quad (36)$$

$\Delta$  percentages are calculated for each of the 8 cases and 4 instances of these cases. In this study, we performed  $\Delta$  analysis for each case and in this context, we compiled the average, minimum and maximum values in the Table 9 as you see on next page.

Table 9. Average  $\Delta$  Values for All Cases

Case			Average $\Delta$	Min	Max
Ctrk	Copt	Cdrn			
L	L	L	<b>34.99%</b>	31.44%	38.31%
L	L	H	<b>34.64%</b>	31.19%	37.99%
L	H	L	<b>38.22%</b>	31.70%	48.53%
L	H	H	<b>35.02%</b>	31.49%	38.22%
H	L	L	<b>34.95%</b>	31.40%	38.35%
H	L	H	<b>34.62%</b>	31.16%	38.06%
H	H	L	<b>35.26%</b>	31.65%	38.53%
H	H	H	<b>37.73%</b>	31.46%	47.50%
<b>Average</b>			<b>35.68%</b>	<b>31.44%</b>	<b>40.69%</b>

As you see the average  $\Delta$  value is 35.68%. This means that using HMR model instead of the CMR model is 35.68% more beneficial in terms of costs. Also, the overall minimum and maximum values of  $\Delta$  are found as 31.44% and 40.69%. This means that our values of  $\Delta$  are between these two values.

By separating the  $C^{drn}$  parameter, we compared all other parameters and the  $\Delta$ . For this, we took values of  $C^{drn}$  parameter as 6.56 (L) and 8.75 (H) separately. First,  $C^{drn} = L$  (low) based graphical results are shown in the Figure 20 below.

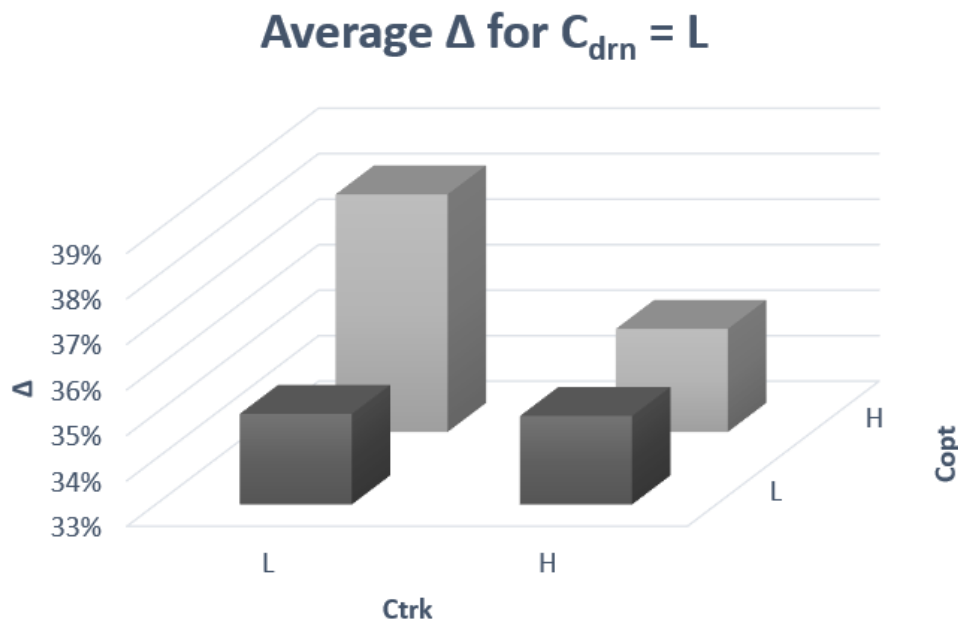


Figure 20. Average  $\Delta$  of HMR Models with  $C_{drn}=L$  with respect to the CMR Model



HMR model resulted in lower costs compared to the CMR model and the average  $\Delta$  is 35.85 % for our setting. There are slight differences between  $C^{opt}$  and  $C^{trk}$  setting pairs. We obviously see that when the  $C^{trk}$  is taken as low value,  $C^{opt}$  is increasing. This provides more benefits in terms of cost in using the HMR model than using the CMR model.

On every instance basis, the route does not change regardless of the parameters, only the objective changes.  $C^{opt}$  parameter increases when  $C^{trk}$  is low, but actually there is no change in total since the route does not change. This is due to the fact that nothing changes on instance bases.

Now let's take the high value of  $C^{drn}$  parameter. Accordingly, appropriate graphical results are given in the Figure 21 below.

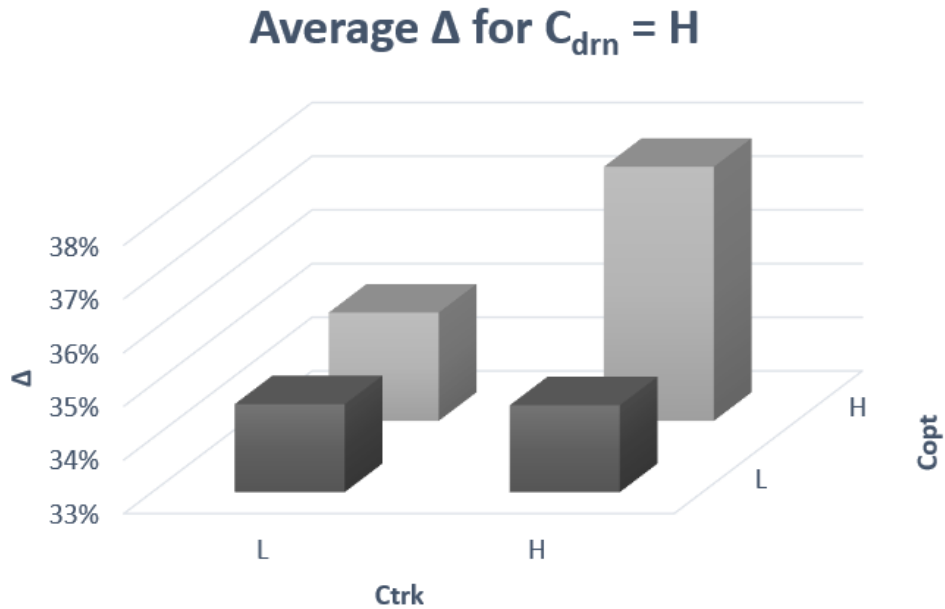


Figure 21. Average  $\Delta$  of HMR Models with  $C_{drn}=H$  with respect to the CMR Model

HMR model resulted in lower costs compared to the CMR model and the average  $\Delta$  is 35.50 % for our setting. There are slight differences between  $C^{opt}$  and  $C^{trk}$  setting pairs. We see that when the  $C^{trk}$  is taken as high value, to increase  $C^{opt}$  provides more benefits in terms of cost in using the HMR model than using the CMR model.

As a result of the comparison, we determined that the cost parameters we determined on instance basis did not change the route created regardless of the value they received. We observed that only the objective value changes. We see that  $C^{opt}$  is

increasing when  $C^{trk}$  is high, but actually there is no change in total since the route does not change. This is due to the fact that nothing changes on instance bases.

Here, we investigated the effects of cost parameters on  $\Delta$  in the scope of the  $\Delta$  comparison analysis. As a result, it was revealed that they had no effect. The reason why this effect did not occur may be due to the small size of the problem or the fact that the 25% change amount we applied in costs was not sufficient. Maybe, ratio of 50% increase or decrease would have revealed the impact of cost parameters better.



## CHAPTER 6

### CONCLUSIONS AND FUTURE WORK

In this thesis, we have proposed a milk run distribution system model which includes truck and drones and analyzed the benefits of this system. According to this model, the truck and the drones which are inside of the truck, deliver demands to the customer locations in milk run delivery model. The abbreviation for the model we design is HMR, which we call hybrid milk run. The model we compare the hybrid milk run (HMR) model with is the classical milk run and its abbreviation is CMR. A truck follows its predetermined route and delivers to customers who have demands in classical milk run (CMR) model.

After introducing both models, we work on both in separate numerical settings designed to each model. The settings we use for the CMR model are the two cost parameter values which are  $C^{trk}$  and  $C^{opt}$ . For both, two cost settings are determined as one of them is high and the other is low. On the other hand,  $C^{trk}$ ,  $C^{opt}$  and  $C^{drn}$  are used as numerical settings in the HMR model. Again, all these cost parameters are consisting of two setting values. Both models consist of 10 customer locations, and these locations are randomly taken from Özbilge's (2021) study with 96 locations for each instance.

We can share the findings we obtained at the end of this study as follows. In the first step, we calculated and interpreted a cost  $\Delta$  measure, which is described as the cost benefit as a percentage if the hybrid milk run model is used instead of the classical milk run model based on all cases and instances. Here, the average  $\Delta$  value was found as 35.68% and this value shows that using HMR model instead of the CMR model is 35.68% more beneficial in terms of costs. In addition, the average minimum and maximum values of the  $\Delta$  were determined for all cases. Accordingly, while the worst value average is 31.44%, the best value average is found as 40.69%. Secondly, when we look at the comparison of all results for HMR and CMR, we see that the HMR model dominates the CMR model in terms of cost. In other words, the costs required by the HMR model we propose are much more affordable. In the light of these data, it

seems that we have ensured a serious improvement in terms of cost as a result of this study.

This study is the second part of the Scientific Research Project (BAP) of Çankaya University. The following ideas may be suggested to assist researchers working in this field or latter parts in this study. First, as you know, the number of customer locations we used in this study is 10. Since we wanted the problem to run on a larger instance, we originally identified 20 customer locations and prepared all input data for this goal. Our model, which we designed according to this input data, worked for approximately 188.3 hours and the relative  $\Delta$  was 70.7%. For this reason, we realized that we cannot solve the instances which consist of 20 customer locations. For future studies, we think that heuristics can be developed to solve the problem in larger instances. We can suggest this as an extension of our work. Lastly, the benefit analysis of the hybrid milk run (HMR) model against the classical milk run (CMR) model we have presented in this study can be expanded and a triple comparison and benefit analysis study can be conducted including HMR, CMR, and Özbilge's (2021) study which is A Milk-Run Distribution System Design for Integrating Drones.

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## APPENDIX A. GAMS CMR MODEL

```
Set i/  
$call =xls2gms r=SETS!d4:d1000 i=input_CMR_Ctrk-L_Copt-  
L_Cust10a_inst01.xlsx o=set.inc  
$include set.inc  
/;  
  
alias(i,j,k);  
  
parameter Dem(i)/  
$call=xls2gms r=Demand!c4:d1000 i=input_CMR_Ctrk-L_Copt-  
L_Cust10a_inst01.xlsx o=parameter.inc  
$include parameter.inc  
/;  
  
parameter Zeta/  
$call =xls2gms r=Scalars!d4 i=input_CMR_Ctrk-L_Copt-  
L_Cust10a_inst01.xlsx o=parameter.inc  
$include parameter.inc  
/;  
  
parameter Ttrkserv/  
$call =xls2gms r=Scalars!h4 i=input_CMR_Ctrk-L_Copt-  
L_Cust10a_inst01.xlsx o=parameter.inc  
$include parameter.inc  
/;  
  
parameter Ctrk/  
$call =xls2gms r=Scalars!j4 i=input_CMR_Ctrk-L_Copt-  
L_Cust10a_inst01.xlsx o=parameter.inc  
$include parameter.inc  
/;
```



```

parameter Copt/
$call =xls2gms r=Scalars!k4 i=input_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.xlsx o=parameter.inc
$include parameter.inc
/;

```

```

table L(i,j)
$call=xls2gms r=Lij!h6:ac1000 i=input_CMR_Ctrk-
L_Copt-L_Cust10a_inst01.xlsx o=parameter.inc
$include parameter.inc
;

```

```

table Ttrk(i,j)
$call=xls2gms r=Tij!i4:ad1000 i=input_CMR_Ctrk-
L_Copt-L_Cust10a_inst01.xlsx o=parameter.inc
$include parameter.inc
;

```

```

variable
z
z_trk
z_opt
;

```

positive variable

t(i) The departure time of truck from location i

tau The completion time of the tour

u(i)

binary variable

x(i,j) ONE if truck moves from location i to location j  
and ZERO otherwise

```
;
```

```

equations
obj
objtrk
objopt
c1
c2
c3
c4
c5
c6
c7
;

*obj.. z=e=tau;
obj.. z=e=z_trk + z_opt;
objtrk.. z_trk=e=Ctrk*((sum(i, (sum(j,
L(i,j)*x(i,j))))));
objopt.. z_opt=e=Copt*tau ;
c1(j).. sum(i, x(i,j))=e=1;
c2(j).. sum(k, x(j,k))=e=1;

c3.. t("1")=e=0;
*c4(j)$ (ord(j) ne 1).. sum(i, x(i,j))=e=sum(k, x(j,k));

*c6.. sum(i, x(i,"1"))=e=1;
*c7.. sum(k, x("1",k))=e=1;
c4(i,j)$ ((ord(i) ne ord(j)) and (ord(i) ge 2) and (ord(j)
ge 2)).. u(i)-u(j)+card(i)*x(i,j)=l=card(i)-1;

c5(i,j)$ (ord(j) ne 1).. t(j)=g=(t(i)+ Ttrk(i,j)+
Ttrkserv)* x(i,j);

c6(i).. x(i,"1")*t(i)+x(i,"1")*Ttrk(i, "1")=l=tau;
c7(i).. x(i,i)=e=0;

```

```
Model LH_CM_10a_inst01 /all/;
```

```
Solve LH_CM_10a_inst01 using MIQCP minimizing z;
```

```
*$onEcho > descriptions.txt
```

```
*text="OF"      rng=z!b4
```

```
*text="1 if drone d serves location j, 0 otherwise"
```

```
rng=z!b3
```

```
*$offEcho
```

```
$onEcho > ToWriteZ.txt
```

```
epsout=0
```

```
var=z.L          rng=z!D4:Z1000
```

```
$offEcho
```

```
$onEcho > ToWriteZ_trk.txt
```

```
epsout=0
```

```
var=z_trk.L      rng=z!D6:Z1000
```

```
$offEcho
```

```
$onEcho > ToWriteZ_opt.txt
```

```
epsout=0
```

```
var=z_opt.L      rng=z!D7:Z1000
```

```
$offEcho
```

```
$onEcho > ToWriteT.txt
```

```
epsout=0
```

```
var=t.L          rng=t!D4:Z1000
```

```
$offEcho
```

```
$onEcho > ToWriteTau.txt
```

```
epsout=0
```

```
var=tau.L        rng=tau!D4:Z1000'
```

```
$offEcho
```

```

$onEcho > ToWriteX.txt
epsout=0
var=x.L                rng=x!D4:Z1000
$offEcho

execute_unload          'output_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.gdx'
execute                 'gdxxrw          output_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.gdx @ToWriteZ.txt '
execute                 'gdxxrw          output_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.gdx @ToWriteZ_trk.txt '
execute                 'gdxxrw          output_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.gdx @ToWriteZ_opt.txt '
execute                 'gdxxrw          output_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.gdx @ToWriteT.txt '
execute                 'gdxxrw          output_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.gdx @ToWriteTau.txt '
execute                 'gdxxrw          output_CMR_Ctrk-L_Copt-
L_Cust10a_inst01.gdx @ToWriteX.txt '
;

```

## APPENDIX B. GAMS HMR MODEL

```
Set i/  
$call=xls2gms r=SETS!d4:d1000 i=input_Ctrk-L_Copt-H_Cdrn-  
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=set.inc  
$include set.inc  
/;  
  
alias(i,j,k);  
  
Set d/  
$call=xls2gms r=SETS!h4:h1000 i=input_Ctrk-L_Copt-H_Cdrn-  
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=set.inc  
$include set.inc  
/;  
  
parameter Dem(i)/  
$call=xls2gms r=Demand!c4:d1000 i=input_Ctrk-L_Copt-  
H_Cdrn-H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=parameter.inc  
$include parameter.inc  
/;  
  
parameter DB(i)/  
$call=xls2gms r=Demand!g4:h1000 i=input_Ctrk-L_Copt-  
H_Cdrn-H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=parameter.inc  
$include parameter.inc  
/;
```

```
parameter Zeta/  
$call=xls2gms r=Scalars!d4 i=input_Ctrk-L_Copt-H_Cdrn-  
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=parameter.inc  
$include parameter.inc  
/;
```

```
parameter Tmax/  
$call=xls2gms r=Scalars!f4 i=input_Ctrk-L_Copt-H_Cdrn-  
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=parameter.inc  
$include parameter.inc  
/;
```

```
parameter Ttrkserv/  
$call=xls2gms r=Scalars!h4 i=input_Ctrk-L_Copt-H_Cdrn-  
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=parameter.inc  
$include parameter.inc  
/;
```

```
parameter Ctrk/  
$call=xls2gms r=Scalars!j4 i=input_Ctrk-L_Copt-H_Cdrn-  
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=parameter.inc  
$include parameter.inc  
/;
```

```
parameter Copt/  
$call=xls2gms r=Scalars!k4 i=input_Ctrk-L_Copt-H_Cdrn-  
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx  
o=parameter.inc  
$include parameter.inc  
/;
```

```

parameter Cdrn/
$call=xls2gms r=Scalars!l4 i=input_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx
o=parameter.inc
$include parameter.inc
/;

table L(i,j)
$call=xls2gms r=Lij!h6:ac1000 i=input_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx
o=parameter.inc
$include parameter.inc
;

table Ttrk(i,j)
$call=xls2gms r=Tij!i4:ad1000 i=input_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx
o=parameter.inc
$include parameter.inc
;

table Tdrn(i,j,k)
$call=xls2gms r=Tijk!a12:x1000 i=input_Ctrk-L_Copt-
H_Cdrn-H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx
o=parameter.inc
$include parameter.inc
;

table P(i,j,k)
$call=xls2gms r=Pijk!a12:x1000 i=input_Ctrk-L_Copt-
H_Cdrn-H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx
o=parameter.inc
$include parameter.inc
;

```

```

table Strk(i,j,k)
$call=xls2gms    r=Sijk2!a12:x1000    i=input_Ctrk-L_Copt-
H_Cdrn-H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.xlsx
o=parameter.inc
$include parameter.inc
;

display i, d, Dem, DB, Zeta, Tmax, Ttrkserv, Ctrk, Copt,
Cdrn, L, Ttrk, Tdrn, P, Strk;

variable
z
z_trk
z_opt
z_drn
;

positive variable
t(i) The departure time of truck from location i
f(d,k) The departure time of drone d to serve location k
tau The completion time of the tour
node
u(i)

binary variable
x(i,j) ONE if truck moves from location i to location j
and ZERO otw
w(i) ONE if customer location i is served by truck and
ZERO otw
y(i) ONE if location i's demand is delivered by drone(s)
and ZERO otw
v(i,j,k) ONE if flight(s)of drone(s) for location k is(are)
associated with truck movement from location i to j and
ZERO otw

```



r(d,k) ONE if customer location k is served by drone d  
and ZERO otherwise

;

equations

obj

objtrk

objopt

objdrn

c0

c1

c2

c3

c4

c5

c6

c7

c8

c9

c10

c11

c12

c13

c14

c15

c16

c17

c18

c19

;

obj.. z=e=z\_trk + z\_opt + z\_drn;

objtrk.. z\_trk=e=Ctrk\*((sum(i,(sum(j,  
L(i,j)\*x(i,j))))));

objopt.. z\_opt=e=Copt\*tau ;

```

objdrn..          z_drn=e=Cdrn*((sum(i,(sum(j,(sum(k,
Tdrn(i,j,k)*v(i,j,k)*sum(d,r(d,k)))))))))/100;

c0.. node=e=sum(i,w(i));
c1(i)$ (ord(i) ne 1).. w(i)+y(i)=e=DB(i);
c2(i,j,k).. v(i,j,k)=l=x(i,j);
c3(k).. y(k)=e=((sum(i,(sum(j,v(i,j,k))))));
c4(j).. sum(i,x(i,j))=e=w(j);
c5(j).. sum(k,x(j,k))=e=w(j);
c6.. sum(i,x(i,"1"))=e=1;
c7.. sum(k,x("1",k))=e=1;
c8(i,j)$((ord(i) ne ord(j)) and (ord(i) ge 2) and (ord(j)
ge 2)).. u(i)-u(j)+node*x(i,j)=l=node-1;
c9(k).. (sum(d,r(d,k)))=e=Dem(k)*y(k);
c10(d,k).. f(d,k)=l=Tmax*r(d,k);
c11(i,j)$ (ord(j) ne 1).. t(j)=g=(t(i)+ Ttrk(i,j)+
Ttrkserv)* x(i,j);
variable fr(d,k); equation deffr(d,k); deffr(d,k)..
fr(d,k) =e= f(d,k)*r(d,k);
c12(d,i,j,k)..
fr(d,k)*v(i,j,k)+Tdrn(i,j,k)*v(i,j,k)*r(d,k)=l=t(j);
variable tr(i,d,k); equation deftr(i,d,k); deftr(i,d,k)..
tr(i,d,k) =e= t(i)*r(d,k);
c13(d,i,j,k).. f(d,k)=g=tr(i,d,k)*v(i,j,k)*P(i,j,k)-
Ttrkserv*v(i,j,k)*P(i,j,k)*r(d,k);
*c14(d,i,j,k).. f(d,k)=g=tr(i,d,k)*v(i,j,k)-
tr(i,d,k)*v(i,j,k)*P(i,j,k)+Strk(i,j,k)*v(i,j,k)*r(d,k)-
Strk(i,j,k)*v(i,j,k)*P(i,j,k)*r(d,k);
*variable vr(i,j,k,d); equation defvr(i,j,k,d);
defvr(i,j,k,d).. vr(i,j,k,d)=e= v(i,j,k)*r(d,k);
*c14(d,i,j,k).. f(d,k)=g=vr(i,j,k,d)*(2-
P(i,j,k))*t(i)+Strk(i,j,k)*vr(i,j,k,d)*(2-P(i,j,k));
c14(d,i,j,k).. f(d,k)=g=tr(i,d,k)*v(i,j,k)*(2-P(i,j,k))
+Strk(i,j,k)*v(i,j,k)*r(d,k)*(2-P(i,j,k));

```

```

c15(i,j,k).. t(i)+(Strk(i,j,k)*v(i,j,k))=l=tau;
c16(i).. x(i,"1")*t(i)+x(i,"1")*Ttrk(i,"1")=l=tau;
c17(i)$ (ord(i) ne 1).. w("1")=e=1;
c18.. t("1")=e=0;
c19(i).. x(i,i)=e=0;

Model LHH_Cust10a_3NC_inst01 /all/;

*option MIQCP = GUROBI;

Solve LHH_Cust10a_3NC_inst01 using MIQCP minimizing z;

*$onEcho > descriptions.txt
*text="OF"          rng=z!b4
*text="1 if drone d serves location j, 0 otherwise"
rng=z!b3
*$offEcho

$onEcho > ToWriteZ.txt
epsout=0
var=z.L              rng=z!D4:Z1000
$offEcho

$onEcho > ToWriteZ_trk.txt
epsout=0
var=z_trk.L          rng=z!D6:Z1000
$offEcho

$onEcho > ToWriteZ_opt.txt
epsout=0
var=z_opt.L          rng=z!D7:Z1000
$offEcho

```

```
$onEcho > ToWriteZ_drn.txt  
epsout=0  
var=z_drn.L          rng=z!D8:Z1000  
$offEcho
```

```
$onEcho > ToWriteF.txt  
epsout=0  
var=f.L              rng=f!D4:DZ1000  
$offEcho
```

```
$onEcho > ToWriteT.txt  
epsout=0  
var=t.L              rng=t!D4:Z1000  
$offEcho
```

```
$onEcho > ToWriteTau.txt  
epsout=0  
var=tau.L            rng=tau!D4:Z1000'  
$offEcho
```

```
$onEcho > ToWriteX.txt  
epsout=0  
var=x.L              rng=x!D4:Z1000  
$offEcho
```

```
$onEcho > ToWriteW.txt  
epsout=0  
var=w.L              rng=w!D4:Z1000  
$offEcho
```

```
$onEcho > ToWriteY.txt  
epsout=0  
var=y.L              rng=y!D4:Z1000  
$offEcho
```

```
$onEcho > ToWriteV.txt
epsout=0
var=v.L          rng=v!D4:Z1000
$offEcho
```

```
$onEcho > ToWriteR.txt
epsout=0
var=r.L          rng=r!D4:Z1000
$offEcho
```

```
execute_unload          'output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx'
execute      'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteZ.txt '
execute      'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteZ_trk.txt '
execute      'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteZ_opt.txt '
execute      'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteZ_drn.txt '
execute      'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteF.txt '
execute      'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteT.txt '
execute      'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteTau.txt '
```

```
execute          'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteX.txt '
execute          'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteW.txt '
execute          'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteY.txt '
execute          'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteV.txt '
execute          'gdxxrw          output_Ctrk-L_Copt-H_Cdrn-
H_alfa0.8_beta0.7_zeta0.6_Cust10a_3NC_inst01.gdx
@ToWriteR.txt '
;
```

## APPENDIX C. CUSTOMER LOCATIONS

Table 10. Center Locations of Customers for Instance-2

Locations	Xc	Yc
1	14.32	0
9	-1	11
14	-9	9
16	-5	9
17	-3	9
34	-7	5
41	9	3
50	-13	-1
72	5	-7
74	9	-7
96	1	-13

Table 11. Center Locations of Customers for Instance-3

Locations	Xc	Yc
1	14.32	0
2	-3	13
6	-7	11
23	9	9
24	-11	7
46	-9	1
48	11	1
68	-11	-7
77	-7	-9
84	7	-9
97	3	-13

Table 12. Center Locations of Customer for Instance-4

Locations	Xc	Yc
1	14.32	0
22	7	9
26	-7	7
29	7	7
31	11	7
32	-11	5
59	9	-3
60	11	-3
73	7	-7
74	9	-7
80	-1	-9



## APPENDIX D. CUSTOMER DEMANDS

*Table 13. Demands of the Locations for Instance-2*

Locations	# of Demands
1	0
9	1
14	3
16	1
17	7
34	6
41	2
50	2
72	4
74	4
96	2

*Table 14. Demands of the Locations for Instance-3*

Locations	# of Demands
1	0
2	3
6	2
23	1
24	2
46	2
48	2
68	2
77	2
84	2
97	5

*Table 15. Demands of the Locations for Instance-4*

Locations	# of Demands
1	0
22	2
26	3
29	3
31	3
32	4
59	6
60	2
73	6
74	4
80	4

## APPENDIX E. DISTANCE MATRICES

*Table 16. Distance Matrix for the Instance-2*

EUCLIDEAN DISTANCES		x	14.32	-1	-9	-5	-3	-7	9	-13	5	9	1
		y	0	11	9	9	9	5	3	-1	-7	-7	-13
x	y	Locations	1	9	14	16	17	34	41	50	72	74	96
14.32	0	1	0.00	18.86	25.00	21.32	19.52	21.90	6.11	27.34	11.66	8.79	18.62
-1	11	9	18.86	0.00	8.25	4.47	2.83	8.49	12.81	16.97	18.97	20.59	24.08
-9	9	14	25.00	8.25	0.00	4.00	6.00	4.47	18.97	10.77	21.26	24.08	24.17
-5	9	16	21.32	4.47	4.00	0.00	2.00	4.47	15.23	12.81	18.87	21.26	22.80
-3	9	17	19.52	2.83	6.00	2.00	0.00	5.66	13.42	14.14	17.89	20.00	22.36
-7	5	34	21.90	8.49	4.47	4.47	5.66	0.00	16.12	8.49	16.97	20.00	19.70
9	3	41	6.11	12.81	18.97	15.23	13.42	16.12	0.00	22.36	10.77	10.00	17.89
-13	-1	50	27.34	16.97	10.77	12.81	14.14	8.49	22.36	0.00	18.97	22.80	18.44
5	-7	72	11.66	18.97	21.26	18.87	17.89	16.97	10.77	18.97	0.00	4.00	7.21
9	-7	74	8.79	20.59	24.08	21.26	20.00	20.00	10.00	22.80	4.00	0.00	10.00
1	-13	96	18.62	24.08	24.17	22.80	22.36	19.70	17.89	18.44	7.21	10.00	0.00

*Table 17. Distance Matrix for the Instance-3*

EUCLIDEAN DISTANCES		x	14.32	-3	-7	9	-11	-9	11	-11	-7	7	3
		y	0	13	11	9	7	1	1	-7	-9	-9	-13
x	y	Locations	1	2	6	23	24	46	48	68	77	84	97
14.32	0	1	0.00	21.66	23.99	10.46	26.27	23.35	3.47	26.27	23.15	11.60	17.24
-3	13	2	21.66	0.00	4.47	12.65	10.00	13.42	18.44	21.54	22.36	24.17	26.68
-7	11	6	23.99	4.47	0.00	16.12	5.66	10.20	20.59	18.44	20.00	24.41	26.00
9	9	23	10.46	12.65	16.12	0.00	20.10	19.70	8.25	25.61	24.08	18.11	22.80
-11	7	24	26.27	10.00	5.66	20.10	0.00	6.32	22.80	14.00	16.49	24.08	24.41
-9	1	46	23.35	13.42	10.20	19.70	6.32	0.00	20.00	8.25	10.20	18.87	18.44
11	1	48	3.47	18.44	20.59	8.25	22.80	20.00	0.00	23.41	20.59	10.77	16.12
-11	-7	68	26.27	21.54	18.44	25.61	14.00	8.25	23.41	0.00	4.47	18.11	15.23
-7	-9	77	23.15	22.36	20.00	24.08	16.49	10.20	20.59	4.47	0.00	14.00	10.77

Table 17 (Continued)

7	-9	84	11.60	24.17	24.41	18.11	24.08	18.87	10.77	18.11	14.00	0.00	5.66
3	-13	97	17.24	26.68	26.00	22.80	24.41	18.44	16.12	15.23	10.77	5.66	0.00

Table 18. Distance Matrix for the Instance-4

EUCLIDEAN DISTANCES		x	14.32	7	-7	7	11	-11	9	11	7	9	-1
		y	0	9	7	7	7	5	-3	-3	-7	-7	-9
x	y		1	22	26	29	31	32	59	60	73	74	80
14.32	0	1	0.00	11.60	22.44	10.13	7.75	25.81	6.11	4.48	10.13	8.79	17.77
7	9	22	11.60	0.00	14.14	2.00	4.47	18.44	12.17	12.65	16.00	16.12	19.70
-7	7	26	22.44	14.14	0.00	14.00	18.00	4.47	18.87	20.59	19.80	21.26	17.09
7	7	29	10.13	2.00	14.00	0.00	4.00	18.11	10.20	10.77	14.00	14.14	17.89
11	7	31	7.75	4.47	18.00	4.00	0.00	22.09	10.20	10.00	14.56	14.14	20.00
-11	5	32	25.81	18.44	4.47	18.11	22.09	0.00	21.54	23.41	21.63	23.32	17.20
9	-3	59	6.11	12.17	18.87	10.20	10.20	21.54	0.00	2.00	4.47	4.00	11.66
11	-3	60	4.48	12.65	20.59	10.77	10.00	23.41	2.00	0.00	5.66	4.47	13.42
7	-7	73	10.13	16.00	19.80	14.00	14.56	21.63	4.47	5.66	0.00	2.00	8.25
9	-7	74	8.79	16.12	21.26	14.14	14.14	23.32	4.00	4.47	2.00	0.00	10.20
-1	-9	80	17.77	19.70	17.09	17.89	20.00	17.20	11.66	13.42	8.25	10.20	0.00

## APPENDIX F. TRAVEL TIMES OF TRUCK

*Table 19. Total Travel Times of Truck for Instance-2*

Locations	1	9	14	16	17	34	41	50	72	74	96
1	0.00	32.34	42.86	36.54	33.47	37.55	10.48	46.87	19.99	15.08	31.91
9	32.34	0.00	14.14	7.67	4.85	14.55	21.95	29.09	32.53	35.30	41.29
14	42.86	14.14	0.00	6.86	10.29	7.67	32.53	18.46	36.45	41.29	41.43
16	36.54	7.67	6.86	0.00	3.43	7.67	26.11	21.95	32.35	36.45	39.09
17	33.47	4.85	10.29	3.43	0.00	9.70	23.00	24.24	30.67	34.29	38.33
34	37.55	14.55	7.67	7.67	9.70	0.00	27.64	14.55	29.09	34.29	33.77
41	10.48	21.95	32.53	26.11	23.00	27.64	0.00	38.33	18.46	17.14	30.67
50	46.87	29.09	18.46	21.95	24.24	14.55	38.33	0.00	32.53	39.09	31.61
72	19.99	32.53	36.45	32.35	30.67	29.09	18.46	32.53	0.00	6.86	12.36
74	15.08	35.30	41.29	36.45	34.29	34.29	17.14	39.09	6.86	0.00	17.14
96	31.91	41.29	41.43	39.09	38.33	33.77	30.67	31.61	12.36	17.14	0.00

*Table 180. Total Travel Times of Truck for Instance-3*

Locations	1	2	6	23	24	46	48	68	77	84	97
1	0.00	37.13	41.13	17.93	45.04	40.02	5.95	45.04	39.68	19.89	29.55
2	37.13	0.00	7.67	21.68	17.14	23.00	31.61	36.93	38.33	41.43	45.74
6	41.13	7.67	0.00	27.64	9.70	17.48	35.30	31.61	34.29	41.85	44.57
23	17.93	21.68	27.64	0.00	34.46	33.77	14.14	43.91	41.29	31.05	39.09
24	45.04	17.14	9.70	34.46	0.00	10.84	39.09	24.00	28.27	41.29	41.85
46	40.02	23.00	17.48	33.77	10.84	0.00	34.29	14.14	17.48	32.35	31.61
48	5.95	31.61	35.30	14.14	39.09	34.29	0.00	40.13	35.30	18.46	27.64
68	45.04	36.93	31.61	43.91	24.00	14.14	40.13	0.00	7.67	31.05	26.11
77	39.68	38.33	34.29	41.29	28.27	17.48	35.30	7.67	0.00	24.00	18.46
84	19.89	41.43	41.85	31.05	41.29	32.35	18.46	31.05	24.00	0.00	9.70
97	29.55	45.74	44.57	39.09	41.85	31.61	27.64	26.11	18.46	9.70	0.00

Table 21. Total Travel Times of Truck for Instance-4

Locations	1	22	26	29	31	32	59	60	73	74	80
1	0.00	19.89	38.47	17.37	13.28	44.25	10.48	7.68	17.37	15.08	30.47
22	19.89	0.00	24.24	3.43	7.67	31.61	20.86	21.68	27.43	27.64	33.77
26	38.47	24.24	0.00	24.00	30.86	7.67	32.35	35.30	33.94	36.45	29.29
29	17.37	3.43	24.00	0.00	6.86	31.05	17.48	18.46	24.00	24.24	30.67
31	13.28	7.67	30.86	6.86	0.00	37.87	17.48	17.14	24.96	24.24	34.29
32	44.25	31.61	7.67	31.05	37.87	0.00	36.93	40.13	37.09	39.98	29.49
59	10.48	20.86	32.35	17.48	17.48	36.93	0.00	3.43	7.67	6.86	19.99
60	7.68	21.68	35.30	18.46	17.14	40.13	3.43	0.00	9.70	7.67	23.00
73	17.37	27.43	33.94	24.00	24.96	37.09	7.67	9.70	0.00	3.43	14.14
74	15.08	27.64	36.45	24.24	24.24	39.98	6.86	7.67	3.43	0.00	17.48
80	30.47	33.77	29.29	30.67	34.29	29.49	19.99	23.00	14.14	17.48	0.00

## APPENDIX G. VALUES OF $P_{ijk}$ FOR INSTANCE-1

*Table 22. Pijk Values for Instance-1*

i	.	j	k
---	---	---	---

			1	21	28	33	52	54	58	67	70	85	86
1	.	1	0	0	0	0	0	2	0	2	0	0	0
1	.	21	2	1	1	1	0	1	0	1	0	0	0
1	.	28	2	1	1	1	0	2	0	1	0	0	0
1	.	33	0	1	1	1	1	0	1	0	1	0	0
1	.	52	0	0	0	1	1	0	1	0	1	0	1
1	.	54	2	1	1	0	0	2	0	2	0	1	0
1	.	58	0	0	0	1	1	0	1	0	1	0	1
1	.	67	2	1	1	0	0	2	0	2	0	1	0
1	.	70	0	0	0	1	1	0	1	0	1	0	1
1	.	85	2	0	0	0	0	2	0	2	0	2	0
1	.	86	0	0	0	0	1	0	1	0	1	0	1
21	.	1	1	2	2	0	0	1	0	1	0	1	0
21	.	21	0	2	2	0	0	0	0	0	0	0	0
21	.	28	1	2	2	1	0	0	0	1	0	0	0
21	.	33	0	2	1	1	1	0	1	0	1	0	0
21	.	52	0	0	0	1	1	0	1	0	1	0	1
21	.	54	1	2	2	0	0	1	0	1	0	1	0
21	.	58	0	0	0	1	1	0	1	0	1	0	1
21	.	67	1	2	1	0	0	1	0	1	0	1	0
21	.	70	0	0	0	1	1	0	1	0	1	0	1
21	.	85	1	0	0	0	0	1	0	1	0	1	0
21	.	86	0	0	0	0	1	0	1	0	1	0	1
28	.	1	1	1	2	0	0	1	0	1	0	1	0
28	.	21	1	2	2	1	0	1	0	1	0	0	0

Table 22 (Continued).

28	.	28	0	2	2	0	0	0	0	0	0	0	0
28	.	33	0	1	2	1	0	0	1	0	0	0	0
28	.	52	0	0	0	1	1	0	1	0	1	0	1
28	.	54	1	2	2	0	0	2	0	1	0	1	0
28	.	58	0	0	0	1	1	0	1	0	1	0	1
28	.	67	1	1	2	0	0	1	0	1	0	1	0
28	.	70	0	0	0	1	1	0	1	0	1	0	1
28	.	85	1	0	0	0	0	1	0	1	0	1	0
28	.	86	0	0	0	0	1	0	1	0	1	0	1
33	.	1	1	1	1	0	0	1	0	1	0	1	0
33	.	21	1	1	1	2	0	1	0	1	0	0	0
33	.	28	1	1	1	2	0	1	0	1	0	0	0
33	.	33	0	0	0	2	2	0	0	0	0	0	0
33	.	52	0	0	0	2	0	0	0	0	0	0	1
33	.	54	1	1	1	0	0	1	0	1	0	1	0
33	.	58	0	0	0	2	2	0	2	0	1	0	1
33	.	67	1	1	1	0	0	1	0	1	0	1	0
33	.	70	0	0	0	2	2	0	1	0	1	0	1
33	.	85	1	0	0	0	0	1	0	1	0	1	0
33	.	86	0	0	0	0	1	0	1	0	1	0	1
52	.	1	1	1	1	0	0	1	0	1	0	1	0
52	.	21	1	1	1	1	0	1	0	1	0	0	0
52	.	28	1	1	1	1	0	1	0	1	0	0	0
52	.	33	0	1	1	2	2	0	2	0	1	0	0
52	.	52	0	0	0	2	2	0	2	0	2	0	0
52	.	54	1	0	0	0	0	1	0	0	0	1	0
52	.	58	0	0	0	1	2	0	2	0	2	0	1
52	.	67	1	1	1	0	0	1	0	1	0	1	0
52	.	70	0	0	0	1	2	0	2	0	2	0	1
52	.	85	1	0	0	0	0	1	0	1	0	1	0



Table 22 (Continued).

52	.	86	0	0	0	0	2	0	2	0	2	0	2
54	.	1	2	1	1	0	0	2	0	2	0	1	0
54	.	21	1	1	1	1	0	2	0	1	0	0	0
54	.	28	1	1	2	1	0	2	0	1	0	0	0
54	.	33	0	1	1	1	1	0	1	0	1	0	0
54	.	52	0	0	0	1	1	0	1	0	1	0	1
54	.	54	2	0	0	0	0	2	0	2	0	0	0
54	.	58	0	0	0	0	1	0	0	0	1	0	1
54	.	67	2	1	1	0	0	2	0	2	0	1	0
54	.	70	0	0	0	1	1	0	1	0	1	0	1
54	.	85	2	0	0	0	0	2	0	2	0	2	0
54	.	86	0	0	0	0	1	0	1	0	1	0	1
58	.	1	1	1	1	0	0	1	0	1	0	1	0
58	.	21	1	1	1	1	0	1	0	1	0	0	0
58	.	28	1	1	1	1	0	1	0	1	0	0	0
58	.	33	0	1	1	2	2	0	2	0	1	0	0
58	.	52	0	0	0	1	2	0	2	0	2	0	1
58	.	54	1	1	1	0	0	1	0	1	0	1	0
58	.	58	0	0	0	0	2	0	2	0	2	0	0
58	.	67	1	1	1	0	0	1	0	1	0	1	0
58	.	70	0	0	0	1	2	0	2	0	2	0	2
58	.	85	1	0	0	0	0	1	0	1	0	1	0
58	.	86	0	0	0	0	2	0	2	0	2	0	2
67	.	1	2	1	1	0	0	2	0	2	0	1	0
67	.	21	1	1	1	1	0	1	0	2	0	0	0
67	.	28	1	1	1	1	0	2	0	2	0	0	0
67	.	33	0	1	1	1	1	0	1	0	1	0	0
67	.	52	0	0	0	1	1	0	1	0	1	0	2
67	.	54	2	1	1	0	0	2	0	2	0	2	0
67	.	58	0	0	0	1	1	0	1	0	1	0	2

Table 22 (Continued).

67	.	67	2	0	0	0	0	2	0	2	0	2	0
67	.	70	0	0	0	1	1	0	1	0	1	0	2
67	.	85	1	0	0	0	0	2	0	2	0	2	0
67	.	86	0	0	0	0	1	0	1	0	1	0	1
70	.	1	1	1	1	0	0	1	0	1	0	1	0
70	.	21	1	1	1	1	0	1	0	1	0	0	0
70	.	28	1	1	1	1	0	1	0	1	0	0	0
70	.	33	0	1	1	1	2	0	2	0	2	0	0
70	.	52	0	0	0	1	2	0	2	0	2	0	1
70	.	54	1	1	1	0	0	1	0	1	0	1	0
70	.	58	0	0	0	1	2	0	2	0	2	0	2
70	.	67	1	1	1	0	0	1	0	1	0	1	0
70	.	70	0	0	0	0	2	0	2	0	2	0	2
70	.	85	1	0	0	0	0	1	0	1	0	1	0
70	.	86	0	0	0	0	1	0	2	0	2	0	2
85	.	1	2	1	1	0	0	2	0	2	0	2	0
85	.	21	1	1	1	1	0	1	0	1	0	0	0
85	.	28	1	1	1	1	0	1	0	1	0	0	0
85	.	33	0	1	1	1	1	0	1	0	1	0	0
85	.	52	0	0	0	1	1	0	1	0	1	0	1
85	.	54	1	1	1	0	0	2	0	2	0	2	0
85	.	58	0	0	0	1	1	0	1	0	1	0	1
85	.	67	1	1	1	0	0	2	0	2	0	2	0
85	.	70	0	0	0	1	1	0	1	0	1	0	1
85	.	85	1	0	0	0	0	1	0	2	0	2	0
85	.	86	0	0	0	0	1	0	1	0	1	0	1
86	.	1	1	1	1	0	0	1	0	1	0	1	0
86	.	21	1	1	1	1	0	1	0	1	0	0	0
86	.	28	1	1	1	1	0	1	0	1	0	0	0
86	.	33	0	1	1	1	1	0	1	0	1	0	0

Table 22 (Continued).

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86	.	52	0	0	0	1	2	0	2	0	2	0	2
86	.	54	1	1	1	0	0	1	0	1	0	1	0
86	.	58	0	0	0	1	1	0	2	0	2	0	2
86	.	67	1	1	1	0	0	1	0	1	0	1	0
86	.	70	0	0	0	1	1	0	1	0	2	0	2
86	.	85	1	0	0	0	0	1	0	1	0	1	0
86	.	86	0	0	0	0	1	0	1	0	2	0	2

## APPENDIX H. FLIGHT TIMES OF DRONES FOR INSTANCE-1

*Table 23. Flight Times of Drones for Instance-1*

i	.	j	k										
			1	21	28	33	52	54	58	67	70	85	86
1	.	1	1000	1000	1000	1000	1000	8.099	1000	14.01	1000	1000	1000
1	.	21	12.96	14.73	14.87	15.7	1000	15.51	1000	15.75	1000	1000	1000
1	.	28	11.66	14.87	14.73	15.68	1000	14.63	1000	15.63	1000	1000	1000
1	.	33	1000	15.4	15	15.11	15.21	15.7	15.68	14.73	14.87	1000	15.21
1	.	52	1000	15.69	15.4	15.45	15.51	1000	1000	15.13	15.15	1000	15
1	.	54	3.471	1000	1000	1000	1000	15.51	15.4	1000	1000	12.62	1000
1	.	58	1000	1000	15.53	15.58	15.63	1000	1000	15.27	15.28	1000	15.03
1	.	67	37.81	1000	1000	1000	1000	15.75	15.63	1000	1000	15.28	1000
1	.	70	1000	1000	1000	1000	1000	1000	1000	15.54	15.53	1000	15.3
1	.	85	10.46	1000	1000	1000	1000	1000	1000	1000	1000	15.53	1000
1	.	86	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	15.51
21	.	1	14.73	12.96	14.33	1000	1000	14.96	1000	15.13	1000	15.43	1000
21	.	21	1000	0	4.667	1000	1000	1000	1000	1000	1000	1000	1000
21	.	28	15.51	2	2.667	15.68	1000	15.4	1000	15.63	1000	1000	1000
21	.	33	1000	14.56	15.68	14.73	15.13	1000	15.27	1000	15.54	1000	1000
21	.	52	1000	1000	1000	15.13	14.73	1000	14.87	1000	15.15	1000	15.41
21	.	54	14.96	11.66	12.67	1000	1000	14.73	1000	15	1000	15.28	1000
21	.	58	1000	1000	1000	15.27	14.87	1000	14.73	1000	15.03	1000	15.28
21	.	67	15.13	15.23	15.63	1000	1000	15	1000	14.73	1000	15.03	1000
21	.	70	1000	1000	1000	15.54	15.15	1000	15.03	1000	14.73	1000	15
21	.	85	15.43	1000	1000	1000	1000	15.28	1000	15.03	1000	14.73	1000
21	.	86	1000	1000	1000	1000	15.41	1000	15.28	1000	15	1000	14.73
28	.	1	14.73	15.6	11.66	1000	1000	14.96	1000	15.13	1000	15.43	1000

Table 23 (Continued).

28	.	21	15.6	2.667	2	15.7	1000	15.51	1000	15.75	1000	1000	1000
28	.	28	1000	4.667	0	1000	1000	1000	1000	1000	1000	1000	1000
28	.	33	1000	15.7	14.14	14.73	15.13	1000	15.27	1000	15.54	1000	1000
28	.	52	1000	1000	1000	15.13	14.73	1000	14.87	1000	15.15	1000	15.41
28	.	54	14.96	14.33	10	1000	1000	13.33	1000	15	1000	15.28	1000
28	.	58	1000	1000	1000	15.27	14.87	1000	14.73	1000	15.03	1000	15.28
28	.	67	15.13	15.75	13.42	1000	1000	15	1000	14.73	1000	15.03	1000
28	.	70	1000	1000	1000	15.54	15.15	1000	15.03	1000	14.73	1000	15
28	.	85	15.43	1000	1000	1000	1000	15.28	1000	15.03	1000	14.73	1000
28	.	86	1000	1000	1000	1000	15.41	1000	15.28	1000	15	1000	14.73
33	.	1	14.73	15.6	15.51	1000	1000	14.96	1000	15.13	1000	15.43	1000
33	.	21	15.6	14.73	14.87	14.56	1000	15.51	1000	15.75	1000	1000	1000
33	.	28	15.51	14.87	14.73	14.14	1000	15.4	1000	15.63	1000	1000	1000
33	.	33	1000	1000	1000	0	14	1000	1000	1000	1000	1000	1000
33	.	52	1000	1000	1000	6	8	1000	12.67	1000	15.15	1000	15.41
33	.	54	14.96	15.51	15.4	1000	1000	14.73	1000	15	1000	15.28	1000
33	.	58	1000	1000	1000	8	10	1000	10.67	1000	15.03	1000	15.28
33	.	67	15.13	15.75	15.63	1000	1000	15	1000	14.73	1000	15.03	1000
33	.	70	1000	1000	1000	12.17	14.32	1000	15.03	1000	14.73	1000	15
33	.	85	15.43	1000	1000	1000	1000	15.28	1000	15.03	1000	14.73	1000
33	.	86	1000	1000	1000	1000	15.41	1000	15.28	1000	15	1000	14.73
52	.	1	14.73	15.6	15.51	1000	1000	14.96	1000	15.13	1000	15.43	1000
52	.	21	15.6	14.73	14.87	15.7	1000	15.51	1000	15.75	1000	1000	1000
52	.	28	15.51	14.87	14.73	15.68	1000	15.4	1000	15.63	1000	1000	1000
52	.	33	1000	15.7	15.68	8	6	1000	10.67	1000	15.54	1000	1000
52	.	52	1000	1000	1000	14	0	1000	4.667	1000	14.76	1000	1000
52	.	54	14.96	15.51	15.4	1000	1000	14.73	1000	15	1000	15.28	1000
52	.	58	1000	1000	1000	15.27	2	1000	2.667	1000	12.9	1000	15.28

Table 23 (Continued).

52	.	67	15.13	15.75	15.63	1000	1000	15	1000	14.73	1000	15.03	1000
52	.	70	1000	1000	1000	15.54	6.325	1000	7.139	1000	8.433	1000	15
52	.	85	15.43	1000	1000	1000	1000	15.28	1000	15.03	1000	14.73	1000
52	.	86	1000	1000	1000	1000	10.2	1000	10.91	1000	12.43	1000	13.6
54	.	1	4.628	15.6	15.51	1000	1000	3.471	1000	11.34	1000	15.43	1000
54	.	21	15.6	14.73	14.87	15.7	1000	11.66	1000	15.75	1000	1000	1000
54	.	28	15.51	14.87	13.33	15.68	1000	10	1000	15.63	1000	1000	1000
54	.	33	1000	15.7	15.68	14.73	15.13	1000	15.27	1000	15.54	1000	1000
54	.	52	1000	1000	1000	15.13	14.73	1000	14.87	1000	15.15	1000	15.41
54	.	54	8.099	1000	1000	1000	1000	0	1000	9.333	1000	1000	1000
54	.	58	1000	1000	1000	15.27	14.87	1000	14.73	1000	15.03	1000	15.28
54	.	67	10.63	15.75	15.63	1000	1000	4	1000	5.333	1000	15.03	1000
54	.	70	1000	1000	1000	15.54	15.15	1000	15.03	1000	14.73	1000	15
54	.	85	15.08	1000	1000	1000	1000	8.246	1000	9.805	1000	10.99	1000
54	.	86	1000	1000	1000	1000	15.41	1000	15.28	1000	15	1000	14.73
58	.	1	14.73	15.6	15.51	1000	1000	14.96	1000	15.13	1000	15.43	1000
58	.	21	15.6	14.73	14.87	15.7	1000	15.51	1000	15.75	1000	1000	1000
58	.	28	15.51	14.87	14.73	15.68	1000	15.4	1000	15.63	1000	1000	1000
58	.	33	1000	15.7	15.68	10.67	8.667	1000	8	1000	15.54	1000	1000
58	.	52	1000	1000	1000	15.13	2.667	1000	2	1000	12.29	1000	15.41
58	.	54	14.96	15.51	15.4	1000	1000	14.73	1000	15	1000	15.28	1000
58	.	58	1000	1000	1000	1000	4.667	1000	0	1000	10.43	1000	1000
58	.	67	15.13	15.75	15.63	1000	1000	15	1000	14.73	1000	15.03	1000
58	.	70	1000	1000	1000	15.54	8.991	1000	4.472	1000	5.963	1000	14.99
58	.	85	15.43	1000	1000	1000	1000	15.28	1000	15.03	1000	14.73	1000
58	.	86	1000	1000	1000	1000	12.86	1000	8.246	1000	9.963	1000	10.99
67	.	1	8.005	15.6	15.51	1000	1000	8.804	1000	6.004	1000	15.43	1000
67	.	21	15.6	14.73	14.87	15.7	1000	15.51	1000	15.23	1000	1000	1000
67	.	28	15.51	14.87	14.73	15.68	1000	15.33	1000	13.42	1000	1000	1000

Table 23 (Continued).

67	.	33	1000	15.7	15.68	14.73	15.13	1000	15.27	1000	15.54	1000	1000
67	.	52	1000	1000	1000	15.13	14.73	1000	14.87	1000	15.15	1000	15.41
67	.	54	11.48	15.51	15.4	1000	1000	5.333	1000	4	1000	14.21	1000
67	.	58	1000	1000	1000	15.27	14.87	1000	14.73	1000	15.03	1000	15.28
67	.	67	14.01	1000	1000	1000	1000	9.333	1000	0	1000	10.43	1000
67	.	70	1000	1000	1000	15.54	15.15	1000	15.03	1000	14.73	1000	15
67	.	85	15.43	1000	1000	1000	1000	13.58	1000	4.472	1000	5.963	1000
67	.	86	1000	1000	1000	1000	15.41	1000	15.28	1000	15	1000	14.73
70	.	1	14.73	15.6	15.51	1000	1000	14.96	1000	15.13	1000	15.43	1000
70	.	21	15.6	14.73	14.87	15.7	1000	15.51	1000	15.75	1000	1000	1000
70	.	28	15.51	14.87	14.73	15.68	1000	15.4	1000	15.63	1000	1000	1000
70	.	33	1000	15.7	15.68	14.73	14.43	1000	13.96	1000	12.17	1000	1000
70	.	52	1000	1000	1000	15.13	8.433	1000	7.963	1000	6.325	1000	15.41
70	.	54	14.96	15.51	15.4	1000	1000	14.73	1000	15	1000	15.28	1000
70	.	58	1000	1000	1000	15.27	10.43	1000	5.963	1000	4.472	1000	13.58
70	.	67	15.13	15.75	15.63	1000	1000	15	1000	14.73	1000	15.03	1000
70	.	70	1000	1000	1000	1000	14.76	1000	10.43	1000	0	1000	9.333
70	.	85	15.43	1000	1000	1000	1000	15.28	1000	15.03	1000	14.73	1000
70	.	86	1000	1000	1000	1000	15.41	1000	14.21	1000	4	1000	5.333
85	.	1	13.94	15.6	15.51	1000	1000	14.47	1000	11.97	1000	10.46	1000
85	.	21	15.6	14.73	14.87	15.7	1000	15.51	1000	15.75	1000	1000	1000
85	.	28	15.51	14.87	14.73	15.68	1000	15.4	1000	15.63	1000	1000	1000
85	.	33	1000	15.7	15.68	14.73	15.13	1000	15.27	1000	15.54	1000	1000
85	.	52	1000	1000	1000	15.13	14.73	1000	14.87	1000	15.15	1000	15.41
85	.	54	14.96	15.51	15.4	1000	1000	10.99	1000	9.963	1000	8.246	1000
85	.	58	1000	1000	1000	15.27	14.87	1000	14.73	1000	15.03	1000	15.28
85	.	67	15.13	15.75	15.63	1000	1000	14.99	1000	5.963	1000	4.472	1000
85	.	70	1000	1000	1000	15.54	15.15	1000	15.03	1000	14.73	1000	15
85	.	85	1000	1000	1000	1000	1000	1000	1000	10.43	1000	0	1000

Table 23 (Continued).

85	.	86	1000	1000	1000	1000	15.41	1000	15.28	1000	15	1000	14.73
86	.	1	14.73	15.6	15.51	1000	1000	14.96	1000	15.13	1000	15.43	1000
86	.	21	15.6	14.73	14.87	15.7	1000	15.51	1000	15.75	1000	1000	1000
86	.	28	15.51	14.87	14.73	15.68	1000	15.4	1000	15.63	1000	1000	1000
86	.	33	1000	15.7	15.68	14.73	15.13	1000	15.27	1000	15.54	1000	1000
86	.	52	1000	1000	1000	15.13	13.6	1000	12.99	1000	11.66	1000	10.2
86	.	54	14.96	15.51	15.4	1000	1000	14.73	1000	15	1000	15.28	1000
86	.	58	1000	1000	1000	15.27	14.87	1000	10.99	1000	9.805	1000	8.246
86	.	67	15.13	15.75	15.63	1000	1000	15	1000	14.73	1000	15.03	1000
86	.	70	1000	1000	1000	15.54	15.15	1000	15.03	1000	5.333	1000	4
86	.	85	15.43	1000	1000	1000	1000	15.28	1000	15.03	1000	14.73	1000
86	.	86	1000	1000	1000	1000	1000	1000	1000	1000	9.333	1000	0



## APPENDIX I. VALUES OF $S_{ijk}^{trk}$ FOR INSTANCE-1

*Table 24. Sijk Values for Instance-1*

i	.	j	k										
			1	21	28	33	52	54	58	67	70	85	86
1	.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	.	21	0.00	1.06	6.89	23.49	0.00	15.04	0.00	25.45	0.00	0.00	0.00
1	.	28	0.00	6.89	1.06	22.27	0.00	0.00	0.00	20.16	0.00	0.00	0.00
1	.	33	0.00	23.49	22.27	1.06	18.54	0.00	4.37	0.00	16.51	0.00	0.00
1	.	52	0.00	0.00	0.00	18.54	1.06	0.00	6.89	0.00	19.49	0.00	10.78
1	.	54	0.00	15.04	10.20	0.00	0.00	0.00	0.00	0.00	0.00	5.09	0.00
1	.	58	0.00	0.00	0.00	4.37	6.89	0.00	1.06	0.00	14.09	0.00	5.09
1	.	67	0.00	25.45	20.16	0.00	0.00	0.00	0.00	0.00	0.00	14.09	0.00
1	.	70	0.00	0.00	0.00	16.51	19.49	0.00	14.09	0.00	1.06	0.00	12.71
1	.	85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	.	86	0.00	0.00	0.00	0.00	10.78	0.00	5.09	0.00	12.71	0.00	1.06
21	.	1	1.06	0.00	0.00	0.00	0.00	11.17	0.00	18.55	0.00	11.53	0.00
21	.	21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	.	28	15.04	0.00	0.00	22.27	0.00	10.20	0.00	20.16	0.00	0.00	0.00
21	.	33	0.00	0.00	22.27	1.06	18.54	0.00	4.37	0.00	16.51	0.00	0.00
21	.	52	0.00	0.00	0.00	18.54	1.06	0.00	6.89	0.00	19.49	0.00	10.78
21	.	54	11.17	0.00	0.00	0.00	0.00	1.06	0.00	12.71	0.00	5.09	0.00
21	.	58	0.00	0.00	0.00	4.37	6.89	0.00	1.06	0.00	14.09	0.00	5.09
21	.	67	18.55	0.00	20.16	0.00	0.00	12.71	0.00	1.06	0.00	14.09	0.00
21	.	70	0.00	0.00	0.00	16.51	19.49	0.00	14.09	0.00	1.06	0.00	12.71
21	.	85	11.53	0.00	0.00	0.00	0.00	5.09	0.00	14.09	0.00	1.06	0.00
21	.	86	0.00	0.00	0.00	0.00	10.78	0.00	5.09	0.00	12.71	0.00	1.06
28	.	1	1.06	18.82	0.00	0.00	0.00	11.17	0.00	18.55	0.00	11.53	0.00
28	.	21	18.82	0.00	0.00	23.49	0.00	15.04	0.00	25.45	0.00	0.00	0.00

Table 24 (Continued).

28	.	28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	.	33	0.00	23.49	0.00	1.06	18.54	0.00	4.37	0.00	16.51	0.00	0.00
28	.	52	0.00	0.00	0.00	18.54	1.06	0.00	6.89	0.00	19.49	0.00	10.78
28	.	54	11.17	0.00	0.00	0.00	0.00	0.00	0.00	12.71	0.00	5.09	0.00
28	.	58	0.00	0.00	0.00	4.37	6.89	0.00	1.06	0.00	14.09	0.00	5.09
28	.	67	18.55	25.45	0.00	0.00	0.00	12.71	0.00	1.06	0.00	14.09	0.00
28	.	70	0.00	0.00	0.00	16.51	19.49	0.00	14.09	0.00	1.06	0.00	12.71
28	.	85	11.53	0.00	0.00	0.00	0.00	5.09	0.00	14.09	0.00	1.06	0.00
28	.	86	0.00	0.00	0.00	0.00	10.78	0.00	5.09	0.00	12.71	0.00	1.06
33	.	1	1.06	18.82	15.04	0.00	0.00	11.17	0.00	18.55	0.00	11.53	0.00
33	.	21	18.82	1.06	6.89	0.00	0.00	15.04	0.00	25.45	0.00	0.00	0.00
33	.	28	15.04	6.89	1.06	0.00	0.00	10.20	0.00	20.16	0.00	0.00	0.00
33	.	33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
33	.	52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	19.49	0.00	10.78
33	.	54	11.17	15.04	10.20	0.00	0.00	1.06	0.00	12.71	0.00	5.09	0.00
33	.	58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	14.09	0.00	5.09
33	.	67	18.55	25.45	20.16	0.00	0.00	12.71	0.00	1.06	0.00	14.09	0.00
33	.	70	0.00	0.00	0.00	0.00	0.00	0.00	14.09	0.00	1.06	0.00	12.71
33	.	85	11.53	0.00	0.00	0.00	0.00	5.09	0.00	14.09	0.00	1.06	0.00
33	.	86	0.00	0.00	0.00	0.00	10.78	0.00	5.09	0.00	12.71	0.00	1.06
52	.	1	1.06	18.82	15.04	0.00	0.00	11.17	0.00	18.55	0.00	11.53	0.00
52	.	21	18.82	1.06	6.89	23.49	0.00	15.04	0.00	25.45	0.00	0.00	0.00
52	.	28	15.04	6.89	1.06	22.27	0.00	10.20	0.00	20.16	0.00	0.00	0.00
52	.	33	0.00	23.49	22.27	0.00	0.00	0.00	0.00	0.00	16.51	0.00	0.00
52	.	52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
52	.	54	11.17	15.04	10.20	0.00	0.00	1.06	0.00	12.71	0.00	5.09	0.00
52	.	58	0.00	0.00	0.00	4.37	0.00	0.00	0.00	0.00	0.00	0.00	5.09
52	.	67	18.55	25.45	20.16	0.00	0.00	12.71	0.00	1.06	0.00	14.09	0.00
52	.	70	0.00	0.00	0.00	16.51	0.00	0.00	0.00	0.00	0.00	0.00	12.71
52	.	85	11.53	0.00	0.00	0.00	0.00	5.09	0.00	14.09	0.00	1.06	0.00

Table 24 (Continued).

52	.	86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
54	.	1	0.00	18.82	15.04	0.00	0.00	0.00	0.00	0.00	11.53	0.00
54	.	21	18.82	1.06	6.89	23.49	0.00	0.00	0.00	25.45	0.00	0.00
54	.	28	15.04	6.89	0.00	22.27	0.00	0.00	0.00	20.16	0.00	0.00
54	.	33	0.00	23.49	22.27	1.06	18.54	0.00	4.37	0.00	16.51	0.00
54	.	52	0.00	0.00	0.00	18.54	1.06	0.00	6.89	0.00	19.49	0.00
54	.	54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
54	.	58	0.00	0.00	0.00	4.37	6.89	0.00	1.06	0.00	14.09	0.00
54	.	67	0.00	25.45	20.16	0.00	0.00	0.00	0.00	0.00	14.09	0.00
54	.	70	0.00	0.00	0.00	16.51	19.49	0.00	14.09	0.00	1.06	0.00
54	.	85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
54	.	86	0.00	0.00	0.00	0.00	10.78	0.00	5.09	0.00	12.71	0.00
58	.	1	1.06	18.82	15.04	0.00	0.00	11.17	0.00	18.55	0.00	11.53
58	.	21	18.82	1.06	6.89	23.49	0.00	15.04	0.00	25.45	0.00	0.00
58	.	28	15.04	6.89	1.06	22.27	0.00	10.20	0.00	20.16	0.00	0.00
58	.	33	0.00	23.49	22.27	0.00	0.00	0.00	0.00	0.00	16.51	0.00
58	.	52	0.00	0.00	0.00	18.54	0.00	0.00	0.00	0.00	0.00	10.78
58	.	54	11.17	15.04	10.20	0.00	0.00	1.06	0.00	12.71	0.00	5.09
58	.	58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
58	.	67	18.55	25.45	20.16	0.00	0.00	12.71	0.00	1.06	0.00	14.09
58	.	70	0.00	0.00	0.00	16.51	0.00	0.00	0.00	0.00	0.00	0.00
58	.	85	11.53	0.00	0.00	0.00	0.00	5.09	0.00	14.09	0.00	1.06
58	.	86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
67	.	1	0.00	18.82	15.04	0.00	0.00	0.00	0.00	0.00	11.53	0.00
67	.	21	18.82	1.06	6.89	23.49	0.00	15.04	0.00	0.00	0.00	0.00
67	.	28	15.04	6.89	1.06	22.27	0.00	0.00	0.00	0.00	0.00	0.00
67	.	33	0.00	23.49	22.27	1.06	18.54	0.00	4.37	0.00	16.51	0.00
67	.	52	0.00	0.00	0.00	18.54	1.06	0.00	6.89	0.00	19.49	0.00
67	.	54	0.00	15.04	10.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
67	.	58	0.00	0.00	0.00	4.37	6.89	0.00	1.06	0.00	14.09	0.00

Table 24 (Continued).

67	.	67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
67	.	70	0.00	0.00	0.00	16.51	19.49	0.00	14.09	0.00	1.06	0.00	12.71
67	.	85	11.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
67	.	86	0.00	0.00	0.00	0.00	10.78	0.00	5.09	0.00	12.71	0.00	1.06
70	.	1	1.06	18.82	15.04	0.00	0.00	11.17	0.00	18.55	0.00	11.53	0.00
70	.	21	18.82	1.06	6.89	23.49	0.00	15.04	0.00	25.45	0.00	0.00	0.00
70	.	28	15.04	6.89	1.06	22.27	0.00	10.20	0.00	20.16	0.00	0.00	0.00
70	.	33	0.00	23.49	22.27	1.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
70	.	52	0.00	0.00	0.00	18.54	0.00	0.00	0.00	0.00	0.00	0.00	10.78
70	.	54	11.17	15.04	10.20	0.00	0.00	1.06	0.00	12.71	0.00	5.09	0.00
70	.	58	0.00	0.00	0.00	4.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00
70	.	67	18.55	25.45	20.16	0.00	0.00	12.71	0.00	1.06	0.00	14.09	0.00
70	.	70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
70	.	85	11.53	0.00	0.00	0.00	0.00	5.09	0.00	14.09	0.00	1.06	0.00
70	.	86	0.00	0.00	0.00	0.00	10.78	0.00	0.00	0.00	0.00	0.00	0.00
85	.	1	0.00	18.82	15.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
85	.	21	18.82	1.06	6.89	23.49	0.00	15.04	0.00	25.45	0.00	0.00	0.00
85	.	28	15.04	6.89	1.06	22.27	0.00	10.20	0.00	20.16	0.00	0.00	0.00
85	.	33	0.00	23.49	22.27	1.06	18.54	0.00	4.37	0.00	16.51	0.00	0.00
85	.	52	0.00	0.00	0.00	18.54	1.06	0.00	6.89	0.00	19.49	0.00	10.78
85	.	54	11.17	15.04	10.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
85	.	58	0.00	0.00	0.00	4.37	6.89	0.00	1.06	0.00	14.09	0.00	5.09
85	.	67	18.55	25.45	20.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
85	.	70	0.00	0.00	0.00	16.51	19.49	0.00	14.09	0.00	1.06	0.00	12.71
85	.	85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
85	.	86	0.00	0.00	0.00	0.00	10.78	0.00	5.09	0.00	12.71	0.00	1.06
86	.	1	1.06	18.82	15.04	0.00	0.00	11.17	0.00	18.55	0.00	11.53	0.00
86	.	21	18.82	1.06	6.89	23.49	0.00	15.04	0.00	25.45	0.00	0.00	0.00
86	.	28	15.04	6.89	1.06	22.27	0.00	10.20	0.00	20.16	0.00	0.00	0.00
86	.	33	0.00	23.49	22.27	1.06	18.54	0.00	4.37	0.00	16.51	0.00	0.00

Table 24 (Continued).

86	.	52	0.00	0.00	0.00	18.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00
86	.	54	11.17	15.04	10.20	0.00	0.00	1.06	0.00	12.71	0.00	5.09	0.00
86	.	58	0.00	0.00	0.00	4.37	6.89	0.00	0.00	0.00	0.00	0.00	0.00
86	.	67	18.55	25.45	20.16	0.00	0.00	12.71	0.00	1.06	0.00	14.09	0.00
86	.	70	0.00	0.00	0.00	16.51	19.49	0.00	14.09	0.00	0.00	0.00	0.00
86	.	85	11.53	0.00	0.00	0.00	0.00	5.09	0.00	14.09	0.00	1.06	0.00
86	.	86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

## APPENDIX J. DETERMINATION OF THE EFFECTIVE SPEED

Table 25. Analysis Study on Effective Speed of Truck

From		To		Google Maps			Our Approximation			Time Deviation
Name	Coordinates	Name	Coordinates	Road Distance (km)	Real Time (min)	Average Speed (km/h)	Euclidean Distance (km)	Approximation Time (min)	Constant Average Speed (km/h)	(Approx-Real)/Real
Antares	39.96917596968125, 32.82228771427618	Kızılay	39.92219306471676, 32.850783502835384	8,0	20,0	24,0	5,9	10,0	35,0	-50%
Acity	39.945859000993195, 32.760832613957426	Panora AVM	39.84892957205567, 32.832587069487204	17,7	26,0	40,8	12,2	21,0	35,0	-19%
Ankamall	39.950954575030714, 32.83082274454951	Eryaman	39.976742262538856, 32.610066213900474	21,7	33,0	39,5	19,9	34,2	35,0	4%
Kuğulu	39.90211356185844, 32.85972668986185	Gordion	39.90000651247426, 32.690983556403054	15,9	27,0	35,3	14,5	24,8	35,0	-8%
Bağlıca	39.90071697294597, 32.6442542823942	Cepa	39.90967139250738, 32.77866514047989	14,8	22,0	40,4	11,5	19,7	35,0	-10%
Via Green	39.907912177335234, 32.743800025636446	Evo Kule	39.874853173451264, 32.649214606983556	10,6	10,0	63,6	8,9	15,2	35,0	52%
İvedik OSB	39.990686765024996, 32.74279733933528	Forum Ankara AVM	40.01724826970993, 32.82313486370834	10,0	10,0	60,0	7,5	12,8	35,0	28%
									AVERAGE	-1%

## APPENDIX K. RESULTS OF HMR AND CMR MODELS

*Table 26. All Results for HMR and CMR Models*

Case-Instance				CMR				HMR								
Ctrk	Copt	Cdrn	Instance	ztrk	zopt	zcmr	$\tau$	ztrk	zopt	zdrn	zhmr	$\tau$	$\Sigma w$	$\Sigma y$	$\Sigma r$	$\Delta$
L	L	L	1	610.5	1428.1	2038.6	134.7	395.1	876.8	22.1	1294.0	82.7	2	8	22	36.53%
L	L	L	2	664.1	1544.2	2208.3	145.7	405.4	936.9	20.9	1362.4	88.3	2	8	21	38.31%
L	L	L	3	684.4	1588.2	2272.6	149.8	473.7	1067.3	17.1	1558.0	100.7	3	7	17	31.44%
L	L	L	4	589.7	1383.3	1973.6	130.5	397.9	882.9	27.9	1308.7	83.3	2	8	28	33.69%
L	L	H	1	610.5	1428.1	2038.6	134.7	395.6	876.8	29.5	1301.4	82.7	2	8	22	36.16%
L	L	H	2	664.1	1544.2	2208.3	145.7	405.4	936.9	27.9	1369.4	88.3	2	8	21	37.99%
L	L	H	3	684.4	1588.2	2272.6	149.8	473.7	1067.3	22.8	1563.7	100.7	3	7	17	31.19%
L	L	H	4	589.7	1383.3	1973.6	130.5	397.9	882.9	37.2	1318.1	83.3	2	8	28	33.21%
L	H	L	1	610.5	1903.7	2514.2	134.7	395.6	876.8	22.1	1294	82.7	2	8	22	48.53%
L	H	L	2	664.1	2058.5	2722.5	145.7	405.4	1247.8	20.9	1674.1	88.3	2	8	21	38.51%
L	H	L	3	684.4	2117.1	2801.5	149.8	473.7	1422.7	17.1	1913.5	100.7	3	7	17	31.70%
L	H	L	4	589.7	1843	2433.7	130.5	397.9	1177	27.9	1602.8	83.3	2	8	28	34.14%
L	H	H	1	610.5	1903.7	2514.2	134.7	395.6	1168.8	29.5	1593.4	82.7	2	8	22	36.62%
L	H	H	2	664.1	2058.5	2722.5	145.7	405.4	1247.8	27.9	1681.9	88.3	2	8	21	38.22%
L	H	H	3	684.4	2117.1	2801.5	149.8	473.7	1422.7	22.8	1919.2	100.7	3	7	17	31.49%
L	H	H	4	589.7	1843	2433.7	130.5	397.9	1176	37.2	1612.9	83.3	2	8	28	33.73%
H	L	L	1	763.2	1428.1	2191.4	134.7	493.9	876.8	22.1	1392.9	82.7	2	8	22	36.44%



Table 26 (Continued)

H	L	L	2	830.3	1544.2	2374.5	145.7	506.8	936.1	20.9	1463.8	88.3	2	8	21	38.35%
H	L	L	3	855.7	1588.2	2443.9	149.8	592.2	1067.3	17.1	1676.6	100.7	3	7	17	31.40%
H	L	L	4	737.4	1383.3	2120.7	130.5	497.5	882.9	27.9	1408.3	83.3	2	8	28	33.59%
H	L	H	1	763.2	1428.1	2191.4	134.7	493.9	876.8	29.5	1400.3	82.7	2	8	22	36.10%
H	L	H	2	830.3	1544.2	2374.5	145.7	506.8	936.1	27.9	1470.8	88.3	2	8	21	38.06%
H	L	H	3	855.7	1588.2	2443.9	149.8	592.2	1067.3	22.8	1682.3	100.7	3	7	17	31.16%
H	L	H	4	737.4	1383.3	2120.7	130.5	497.5	882.9	37.2	1417.6	83.3	2	8	28	33.15%
H	H	L	1	763.2	1903.7	2667	134.7	493.9	1168.8	22.1	1684.9	82.7	2	8	22	36.82%
H	H	L	2	830.3	2058.5	2888.7	145.7	506.8	1247.8	20.9	1775.6	88.3	2	8	21	38.53%
H	H	L	3	855.7	2117.1	2972.8	149.8	592.2	1422.7	17.1	2032	100.7	3	7	17	31.65%
H	H	L	4	737.4	1844	2581.3	130.5	497.5	1177	27.9	1702.4	83.3	2	8	28	34.05%
H	H	H	1	763.2	1903.7	2667	134.7	493.9	876.8	29.5	1400.3	82.7	2	8	22	47.50%
H	H	H	2	830.3	2058.5	2888.7	145.7	506.8	1247.8	27.9	1782.6	88.3	2	8	21	38.29%
H	H	H	3	855.7	2117.1	2972.8	149.8	592.2	1422.7	22.8	2037.7	100.7	3	7	17	31.46%
H	H	H	4	737.4	1844	2581.3	130.5	497.5	1177	37.2	1711.7	83.3	2	8	28	33.69%