

**APPLICATION OF CUBICALLY CONVERGENT ITERATIVE METHOD  
FOR ZERO – POLE ANALYSIS OF HIGH ORDER FILTER CIRCUITS**

**A THESIS SUBMITTED TO  
DEPARTMENT OF ELECTRONICS AND COMMUNICATION  
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**BY  
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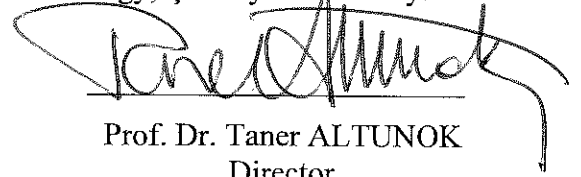
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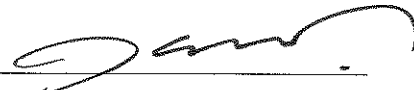
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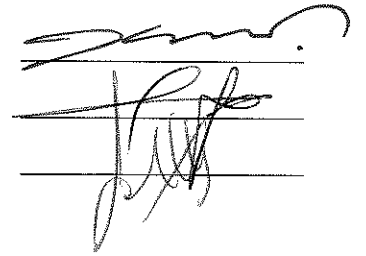
  
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## ABSTRACT

### APPLICATION OF CUBICALLY CONVERGENT ITERATIVE METHOD FOR ZERO – POLE ANALYSIS OF HIGH ORDER FILTER CIRCUITS

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In this thesis, a new method is proposed to find the roots of filter circuit transfer function using on iterative method. This method is very fast and simple iterative with cubically convergent. Additionally, this new method can find only one real root of the complex initial estimates. As the coupling coefficient changes (or root locus) shows how to move the pole-zero dynamics of poles and zeros in the complex frequency plane. The proposed method is compared to various methods in the literature, and it is concluded that the new method is more accurate and time efficient than the existing numerical methods.

**Keywords:** Cubically Convergent, Filter Circuit, High Order, Zero-Pole Analysis.

## ÖZ

### YÜKSEK DERECE FİLTRE DEVRELERİN DİREK ANALİZİ - SIFIR İÇİN KÜBİK YAKINSAK İTERATİF YÖNTEMİ UYGULAMASI

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Bu yazıda yeni bir yöntem kübik yakınsak iteratif yöntemi kullanılarak yüksek geçiren filtre transfer fonksiyonu köklerini bulmak için önerilmiştir. Bu yöntem çok hızlı ve kübik yakınsak ile tekrarlı basittir. Ayrıca, bu yeni yöntem, sadece bir gerçek başlangıç tahmin karmaşık köklerini bulabilirsiniz. Kavramanın katsayısı değişir gibi kutuplar ve sıfırlar karmaşık frekans düzleminde nasıl hareket kutup-sıfır dinamikleri (veya kök yer eğrisi) gösterir. Önerilen yöntem literatürde çeşitli yöntemlerle karşılaştırıldığında, ve yeni bir yöntem mevcut sayısal yöntemlere göre etkin daha doğru ve zaman sonucuna varılmıştır.

**Anahtar Kelimeler:** Kübik Yakınsak, Devre Filtre, Yüksek Derece, Sıfır-Kutup Analizi.

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## LIST OF ABBREVIATIONS

SBG	Simplification Before Generation
PZAC	Pole-Zero Assignment Controller

## CHAPTER 1

### INTRODUCTION

#### 1.1. Poles and Zeros

Poles and zeros are commonly employed by analog designers to characterize the linear behavior of analog integrated circuits [1]. Guerra et. al. introduces a methodology for symbolic pole/zero extraction based on the formulation of the time-constant matrix of the circuits. This methodology incorporates approximation techniques specifically devoted to achieve an optimum trade-off between accuracy and complexity of the symbolic root expressions.

Another approach allowing the determination of the approximate symbolic expressions for all poles/zeros of an arbitrary circuit is presented in [10]. This procedure uses the LR algorithm [11] converging to the eigenvalues of a circuit matrix (the state matrix in the case of pole computation).

The symbolic LR algorithm using simplification during LR iterations introduced in [3] for the computation of the approximate symbolic pole/zero expressions. It is shown that this approach is competitive or better with respect to the best known approaches. A two transistor amplifier is redesigned using these formulas.

In [4] they present a new matrix-based simplification before generation (SBG) method for pole/zero analysis which simplifies a symbolic generalized eigenvalue problem with respect to a selected root. The method uses a fast linear error estimation formula based on eigenvalue sensitivities to obtain a term ranking.

Accurate and efficient error control is achieved by tracking eigenvalue shifts numerically using an iterative generalized eigenvalue solver. The new algorithm is capable of computing real and complex dominant as well as unobservable poles and zeros.

Khodabakhshian [5] proposed an adaptive power system stabilizer New able to provide acceptable damping over a wide range of operating points. A new technique called adaptive control strategy is the zero-pole assignment control (PZAC) where the transfer function of the system ( $G_d(s) = \Delta\delta / \Delta P_m$ ) to a standard form of identification systems based on explicit correction. The controller design is mainly due to the continuous-time system using delta operator instead of the more usual operator. Simulation studies have been conducted in several models.

## 1.2. FIR Filter

The primary concern in FIR filter design is to find the minimum length finite filter coefficients for a given set of specifications. However, several other properties of FIR filters such as minimum phase, linear phase, maximally flat pass band, and equiripple frequency response are as important as primary concern in certain applications. Minimum phase FIR filters are desired in data communication systems, where the minimum delay is essential [13]. Maximally flatness of pass band response is desired in radar applications, where periodic amplitude modulations would be produced in case there exist ripples in the pass band [14]. Though these ripples are usually undesired in many applications, in applications requiring low complexity with less precision, ripples can be acceptable up to a specific limit. In those cases, equiripple filters provide the worst acceptable performance with minimal coefficient length [15, 16]. However, by all means, the linearity of phase is the mostly desired property in many filtering applications since linear phase systems provide constant group delay, which in return prevents any phase distortion caused by the filtering [15, 16, 13, 17]. A related property, group delay of an FIR filter is important in real time applications such as speech coding, digital modem synchronization [18] since the group delay of a linear phase FIR filter increases with the number of filter

coefficients [19]. In addition, linear phase filters with sharp transition regions that are usually used in narrow frequency band selection applications can have unacceptably high group delays. Thus, there has been a growing interest in finding filter coefficients with low group delays [20]. Since, in real time applications low group delay is required; group delay constraints are more desired compared to phase constraints in these applications.

### 1.2.1 Windowing Based FIR Filter Design

Windowing based FIR filter design techniques make use of frequency response of an ideal filter [21]. The design procedure starts with the selection of a desired frequency response, which is typically realizable by infinite length filter coefficients. Since in reality, only finite length coefficients can be used, a finite subset of infinite number of coefficients is selected using a window function in time domain. Hence, resulting frequency response becomes an approximation of desired frequency response. Though windowing based filter design techniques are fast, they do not satisfy any optimality criteria and filter coefficients satisfying the predefined specifications are found in an ad-hoc manner. Other than windowing, there exist different tools as Chebyshev approximation, FFT, least squares error design, and convex optimization, which can be combined to obtain filter design techniques with improved robustness and efficiency.

Unified linear phase FIR filters design, i.e., Parks-McClellan technique, relies on Chebyshev approximation and is one of the milestone techniques in terms of its efficiency and robustness [16]. In Parks-McClellan technique, well-known Chebyshev approximation and Remez exchange algorithm are combined to propose a unified equiripple linear phase FIR filter design procedure [22,23,24,25]. However, due to underlying Remez exchange algorithm, Parks-McClellan technique can not achieve the optimal filter response even though it can obtain the best equiripple approximation to the desired response [16]. In [26], METEOR, a technique based on limit approach is proposed. FIR filter design problem for a given set of mask constraints is formulated as a linearly constrained linear program and solved using

simplex algorithm. Hence, METEOR is capable of finding locally optimum solutions for any given set of constraints. Around the time METEOR was developed, it was considered a versatile technique as it is capable of designing linear phase, minimum phase, and flat pass band filters. On the other hand, METEOR is not capable of designing nonlinear phase filters and is not able to obtain globally optimal solutions. In [27, 14], Chebyshev approximation is used to model FIR filter design problem and solved using a multiple exchange algorithm, which is derived directly from Kuhn-Tucker conditions. In this formulation, objective is chosen to be the minimization of least-squares approximation error for given set of linear constraints. Hence, the technique is named as peak-constrained least-squares (PCLS) approach. Similar to Remez exchange algorithm, PCLS lacks the optimality. However, efficiency of PCLS outperformed the Parks-McClellan technique since Remez exchange algorithm is replaced with a multiple exchange algorithm. Even though, PCLS is capable of implementing phase and group delay constraints, for the convergence of PCLS, appropriate choice of its parameters is essential. In [15], based on projection on convex sets (POCS) [28], an efficient FFT based equiripple FIR filter design technique is proposed. In the proposed approach, iterations are performed between time and frequency domains using efficient FFT algorithm. Since Fourier transforms preserves convexity, algorithm is guaranteed to converge to a design that satisfies predefined constraints if a solution exists.

### **1.3. Butterworth Filter**

Butterworth filter frequency response of the signal processing filter designed as flat as possible in the pass band. This is also called a flat maximum size filter. This is the first time in 1930 by English engineer and physicist Stephen Butterworth in his article entitled "The Theory of Filter Amplifiers," described [6].

### **1.4. Radio Frequency Filter**

Radio frequency (RF) filter is a two-port network used to control the frequency response of a high frequency system. In the literature there exist different approaches

of filter design for high frequency applications. Most commonly used design methods are known as the image parameter method and the insertion loss method. Image parameter method consists of cascade of two-port filter sections to provide desired cutoff frequencies. Image filter design method is simple, however it must be iterated many times to achieve desired values. Second method is the insertion loss based design. This method uses network synthesis technique to design filter for specified frequency response. The design is simplified by low-pass filter prototypes. Transformation can be then applied to convert the other types of filters such as high-pass, band-pass and stop-band.

### **1.5. Schedule of Thesis**

In chapter 2 of the thesis, some fundamental theoretical concepts of filters are reviewed in a brief manner. Insertion loss based filter design for Butterworth and Chebyshev filters with only lumped and distributed elements are studied.

In the chapter 3 the method of finding root is described.

In the fourth section, the result and discussion is given. A conclusion is given in the last section.



## CHAPTER 2

### THE HIGH ORDER OF LOW-PASS FILTERS

#### 2.1 Fundamental Concepts on Filters

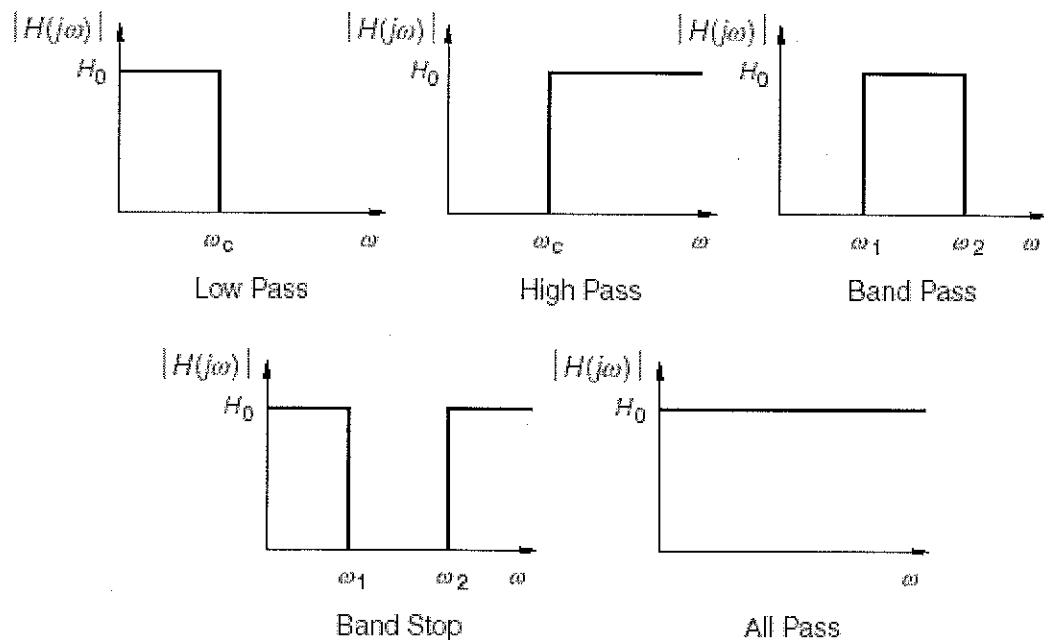
Input filter function to separate the different frequency components of the signal passes through a filter. Filters may be classified in a number of ways. For example, analog filters are used to process analog signals, which are a function of a continuous time variable. Digital filters, on the other hand, process digitized continuous waveforms. Analog passive filters may be classified as lumped element or distributed element devices. We may also classify filters as passive or active depending on the type of elements used in their construction. Five basic types of selective networks are commonly referred to in filter design. These band-stop, band pass, high pass, and low-pass filters includes all transition. Network capabilities  $p = q$  Laplace transfer function representing the complex frequency defined for the conversion of  $H(p)$  is determined by. The transfer function of the input signal, voltage or current output signal is the ratio of:

$$H(j\omega) = \frac{V_{out}}{V_{in}} = |H(j\omega)|e^{j\phi(\omega)} \quad (2.1)$$

Network input signal passes through a filter frequency and some others will stop and be linear circuit, this function is performed by add or creat a new frequency element. Ideal without loss, through identifying the passband frequency band, and ideally infinite loss, frequency of stops group, called the stop band. A passband corner in Figure 1. shows these losses represent an ideal low-pass filter. The frequency  $\omega_c$  is called the cut-off frequency of the filter. This element of the passband to stop due to

physical tape circuit requires a number of ideal low-pass filter is not infinite sudden changes that may take place. A real filter transfer characteristic, make a minimum loss passband transition to the maximum passband and stop band frequency range that separates.

As the selectivity defined by the transition band approaches the ideal steep characteristic, more complex and expensive filters. Similar considerations linear phase and / or group delay may be applied in the design of filters using plain. Passband, transition band and stop band of the concept of the transmission behavior of the filter as shown in Figure 1. are five main types of features allow.



**Figure 1** Transfer function characteristics for major filter types

So small it can easily be converted lost time frequency domain transfer function  $p = j\omega$ . The filter transfer function of the input signal voltage (or current) and can be written as a ratio of two polynomials output signal voltage ratio:

$$H(p) = \frac{P(p)}{Q(p)} = \frac{a_0 + a_1p + a_2p^2 + \dots + a_{m-1}p^m}{b_0 + b_1p + b_2p^2 + \dots + b_{n-1}p^n} \quad (2.2)$$

polynomials  $P(p)$  and  $S(p)$  where  $m$  and  $n$ , is order respectively. Within these polynomials, numerator and denominator polynomial order  $m$   $n$  must be equal to or less are Hurwitz stable.

Polynomial  $Q(p)$  as well as the order of the filters. Polynomial  $Q(p)$  and  $p(p)$  can be written as a-factor and

$$H(p) = \frac{(p-z_1)(p-z_2)(p-z_3)\dots(p-z_m)}{(p-k_1)(p-k_2)(p-k_3)\dots(p-k_n)} \quad (2.3)$$

Values,  $Z_3, Z_2, Z_1 \dots Z$  transfer function, or simply called zero-zero transmission.  $Q(s), K_1, K_2, K_3$  radicals.  $\dots, K_n$ , a transfer function poles. Poles and zeros, real or complex, but it should happen in case of complex conjugate pair of poles and may be zero. Represents loss or attenuation of the voltage transfer function of filter circuit size plot and given in dB.

$$L_{dB} = 20 \log |H(p)| \quad (2.4)$$

Poles and zeros realizable passive network, you must follow certain rules:

- The left half of all phases of a transfer function occurs in the  $p$ -plane. Left half-plane contains the imaginary  $-j\omega$ .
- Available in pairs of complex conjugate complex poles and zeros. Therefore, on the imaginary axis, zeros and poles could be single [31].

Insertion loss in the passband and a systematic method of control over amplitude and phase bandstop characteristics allow to synthesize highly desired response. Considered to meet the requirements necessary to design best practices. For example, the most important is the loss of at least one insert used a binomial response. Chebyshev response, I would like to meet the requirements for sharp cuts. The attenuation rate of the victim it is possible, a better phase response may be obtained using the linear phase filter design. In any case, the high-order filter insertion loss

filter performance method enables improved in a simple manner. Adding loss method, a filter response is well-defined by the additional power defeat or loss ratio  $P_{LR}$ ,

$$P_{LR} = \frac{P_s}{P_l} = \frac{1}{1 - |\Gamma(\omega)|^2} \quad (2.5)$$

where;  $P_l$  = Power distributed to the capacity,  $P_s$  = Power available from the source and  $\Gamma$  = the reflection coefficient at the input port. The insertion loss (IL) in dB is

$$IL = 10 \log P_{LR} \quad (2.6)$$

We know that  $|\Gamma(\omega)|^2$  is an even function of  $\omega$ , therefore it can be expressed as a polynomial in  $\omega^2$ . We can write

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)} \quad (2.7)$$

Where M and N are real polynomials in  $\omega^2$ . If we substitute this form in power loss ratio gives following form.

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)} \quad (2.8)$$

For a filter to be physically realizable its power loss ratio must be given in this form.

### 2.1.1 Maximally Flat

This property is also called binomial or Butterworth response. Provides the fastest possible response for a given passband filter complexity or order. For a low-pass

filter is indicated by

$$P_{LR} = 1 + k^2(\omega/\omega_c)^{2n} \quad (2.9)$$

The filter is of the order  $n$  and  $\omega_c$  cutoff frequency. Passband band  $\omega = 0$  to  $\omega = \omega_c$  power loss rate  $1+k^2$  extends to the edges. If you choose this -3 dB point, we have  $k = 1$ .

For  $\omega > \omega_c$ , the attenuation increase monotonically with frequency. For  $\omega \gg \omega_c$   $P_{LR} \cong k^2(\omega/\omega_c)^{2n}$  which shows that the insertion loss increases at the rate of 20 dB /decade.

### 2.1.2 Equal Ripple

This characteristic is also called the Chebyshev response. If the degree as a Chebyshev polynomials are used to determine the insertion loss of the low-pass filter

$$P_{LR} = 1 + k^2 T_n^2(\omega/\omega_c) \quad (2.10)$$

$T_n(x)$  oscillates between  $\pm 1$  since  $|x| \leq 1$  passband response amplitude will have the  $1 + k^2$  waves. Thus  $k^2$  determines the pass band ripple levels. For very large  $x$ ,  $T_n(x) \cong 1/2(2x)^n$  so for  $\omega \gg \omega_c$ , would insertion loss

$$P_{LR} \cong \frac{k^2}{4} \left(\frac{2\omega}{\omega_c}\right)^{2n} \quad (2.11)$$

The insertion loss for the Chebyshev case is  $(2^{2n})/4$  greater than Butterworth response, at any given frequency where  $\omega \gg \omega_c$ .

## 2.2 Low-Pass Filter

Insertion loss based design method is fully specified with a frequency response of a filter design uses network synthesis techniques. Impedance and frequency normalized design is simplified with respect, starting with a low-pass filter prototype. After low-pass prototype is obtained, after conversion into the desired frequency range and the impedance level is applied to convert the prototype design. Two fundamental low-pass characteristics are commonly preferred for the prototype filter design. These are Butterworth and Chebyshev type filters.

### 2.2.1 Butterworth Filter

The Butterworth, or "maximally flat" response Provides the fastest possible response for a given passband filter complexity. More closely approximate location of a filter with a very reactive element filter can be expected to have a right-angled shape with less reactive elements. N polarity (the reactive components) to a filter, low pass, Butterworth approach involves a transition-band maximum plateau close  $\omega = 0$ .

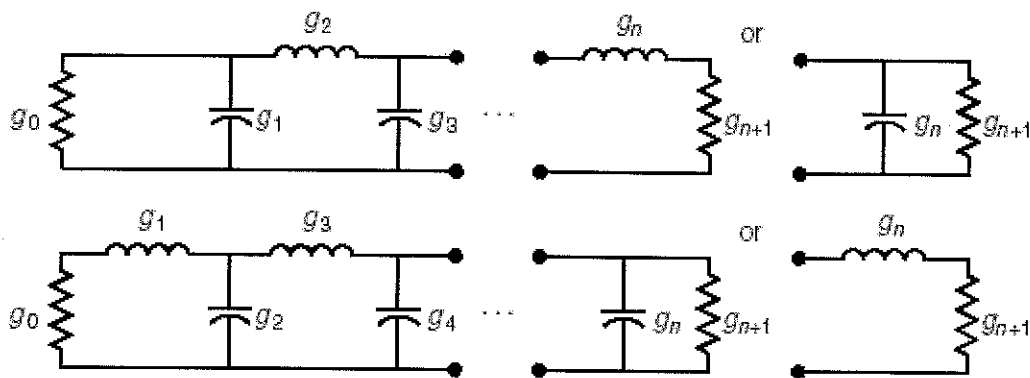


Figure 2 Low-pass filter network topology

Function to obtain this type of filter is

$$|H(j\omega)|^2 = G_T = \frac{H_0}{1 + (\omega/\omega_c)^{2n}} \quad (2.12)$$

Here  $H_0$  is less than and equal one. The first  $2n-1$ , the denominator of all derivatives of this function Zero  $\omega = 0$ , it implies that the maximum level. Pole of this function are all One size and are separated on the radiant unit circle. It also has no poles on the axis.

It is usually arranged in the transition-band minimum requirements. Herein For example, a small amount of poles required for yield the desired properties

$$n = \frac{\log[(10^{\alpha_{\min}/10} - 1)(10^{\alpha_{\min}/10} - 1)]}{2 \log(\omega_s / \omega_p)} \quad (2.13)$$

Maximum passband attenuation in this expression  $0 \leq \omega \leq \omega_p$  is  $\alpha_{\max}$ . Minimum diminution in stopband,  $\omega_s \leq \omega \leq \infty$ , is  $\alpha_{\min}$ .

Passband edge of time, will weaken the power of the filter  $\frac{1}{2}$  or -3 dB.

Iteration formulation for such filter fundamentals is shown in Figure 2. These reactions can produce a variety of references [29].

$$g_0 = g_{n+1} = 1, \quad g_k = 2 \sin \left[ \frac{(2k-1)\pi}{2n} \right] \quad k=1,2,3,\dots,n \quad (2.14)$$

Where is the source for the normalized low-pass design and load terminations are  $1 \Omega$ , if the cutoff frequency is set as  $\omega_c = 1$  rad/s and passband ripple is assumed 3 dB, the element values are calculated as in Table 1 [29].

It is possible to scale the response to have other attenuation levels at  $\omega = 1$  rad/s. For an attenuation of  $K_p$  in dB:

$$\omega_{k_p} = (10^{0.1K_p} - 1)^{1/(2n)} \quad (2.15)$$

In order for the filter to have  $K_p$  attenuation at  $\omega = 1$  rad/s, the 3 dB case pole positions or component values must be scaled  $\omega_{k_p}$  value.

**Table 1** Normalized Low Pass Butterworth Filter Element Values ( $\omega_c = 1$  rad/s; 3 dB Passband Ripple)

N	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$
1	2.000									
2	1.414	1.414								
3	1.000	2.000	1.000							
4	0.765	1.847	1.847	0.765						
5	0.618	1.618	2.000	1.618	0.618					
6	0.517	1.414	1.931	1.931	1.414	0.517				
7	0.445	1.246	1.801	2.000	1.801	1.246	0.445			
8	0.390	1.111	1.662	1.961	1.961	1.662	1.111	0.390		
9	0.347	1.000	1.532	1.879	2.000	1.879	1.532	1.000	0.347	
10	0.312	0.907	1.414	1.782	1.975	1.975	1.782	1.414	0.907	0.312
	$L_1$	$C_2$	$L_3$	$C_4$	$L_5$	$C_6$	$L_7$	$C_8$	$L_9$	$C_{10}$

Referring to Figure 2, where resources for normalized low-pass design and load terminations are  $1 \Omega$ , the cutoff frequency is  $\omega_c = 1$  rad/s and passband ripple 1 dB, the element values are calculated as in Table 2 [29].

**Table 2** Normalized Low Pass Butterworth Filter Element Values ( $\omega_c = 1$  rad/s; 1 dB Passband Ripple)

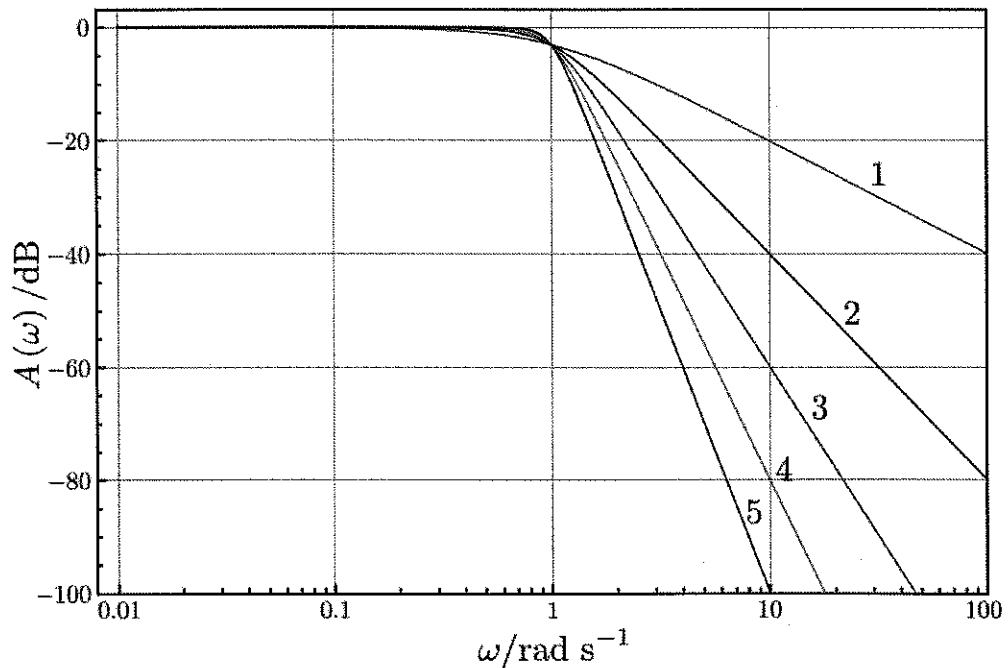
N	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$
1	1.017									
2	1.008	1.008								
3	0.798	1.596	0.798							
4	0.646	1.560	1.560	0.646						
5	0.539	1.413	1.747	1.413	0.539					
6	0.562	1.263	1.726	1.726	1.263	0.562				
7	0.404	1.132	1.636	1.815	1.636	1.132	0.404			
8	0.358	1.021	1.528	1.802	1.802	1.538	1.021	0.358		
9	0.322	0.927	1.421	1.743	1.855	1.743	1.421	0.927	0.322	
10	0.292	0.848	1.321	1.665	1.846	1.846	1.665	1.321	0.848	0.292
	$L_1$	$C_2$	$L_3$	$C_4$	$L_5$	$C_6$	$L_7$	$C_8$	$L_9$	$C_{10}$



### 2.3 Fifth Order of Butterworth Low-Pass Filter

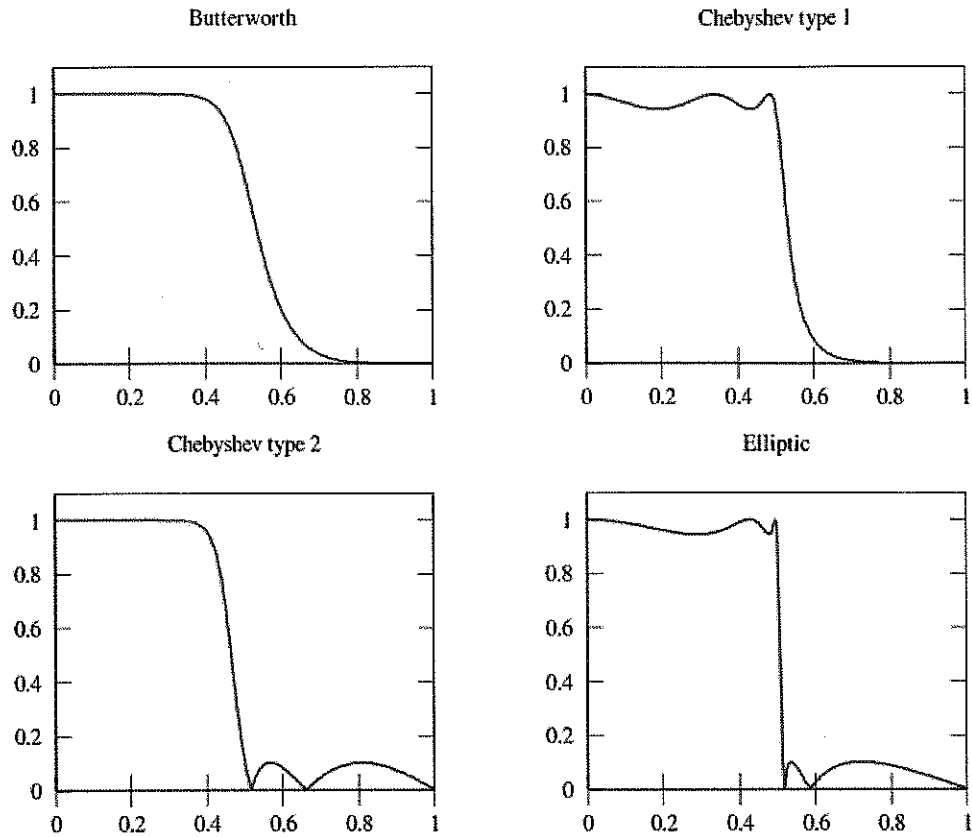
Butterworth filter's frequency response in the passband (ie there are no fluctuations) at the maximum level and stop band [7] right off the roll to zero. When viewed on a logarithmic Bode plot linearly in response to negative infinity to the right off the slopes. A first order filter response (All firstorder lowpass filter has same normalized frequency answer) -6 dB per octave (-20 dB per ten) roll off. -12 DB per octave second order filter, -18 dB, and therefore reduces the third order. Butterworth filter passband and / or non-stop band ripple monotonous, monotonous function of varying size with  $\omega$  Unlike other filter types. A Chebyshev Type II / Type I compared to filter or elliptical filter, Butterworth has slow roll-off (figure 4), thus requiring a higher order to implement a specific stop-band characteristics; but Butterworth pass filters, Chebyshev Type I is / Type II and elliptic filter passband can achieve a more linear phase response. In this study, five degree used this filter.

Figure 3 Butterworth is a discrete-time filter in addition to other common filter types an image showing the gain. All of these filters are of fifth order.



**Figure 3** Order with cut-off frequency from 1 to 5 Butterworth lowpass filters gain plot  $\omega_0 = 1$ .

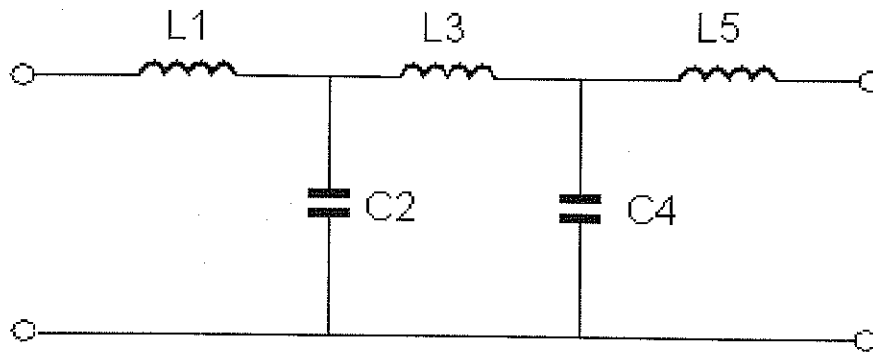
In figure 3, The filter order where the slope dB / on 20N note. Butterworth filter, Chebyshev filter or roll around slower than elliptic filter cutoff frequency, but without surge. This roles is illustrated in figure 2.



**Figure 4** Butterworth filter, Chebyshev type 1 filter, Chebyshev type 2 filter, Elliptic filter.

### Example 2.1

Here we are going to calculate the root of low-pass Butterworth filter. According to Table 1 normalized Butterworth filter structure and element values for order 5 are given in Figure 5.



**Figure 5** Low-pass Butterworth filter network topology for order 5

### 2.3.2 The Chebyshev Filter

Chebyshev filter design function provides a given passband ripple or the maximum bandwidth possible for possible low-pass band ripple for a given bandwidth. Chebyshev converter gain low-pass filter function

$$|H(j\omega)|^2 = G_T = \frac{H_0}{1 + \varepsilon^2 T_n^2(\omega/\omega_c)} \quad (2.16)$$

which  $\omega_c$  is low-pass cutoff frequency. An integer value of  $\varepsilon$  is less than 1 and is a measure of the transition waves. His argument is no axis least lie on an ellipse with the poles of the transfer function is 1-pole Chebyshev function  $T_n(x)$  oscillates between +1 and -1. Chebyshev function could be written in a method clearly indicating this feature:

$$\begin{aligned} T_n(x) &= \cos[n \times \arccos(x)] & 0 \leq x < 1 \\ T_n(x) &= \cosh[n \times \arccos h(x)] & x > 1 \end{aligned} \quad (2.17)$$

Then  $T_n(x)$  is less than 1 in the passband, passband transfer function is

$$\frac{1}{1 + \varepsilon^2} \leq |H(j\omega)|^2 \leq 1 \quad (2.18)$$

For an attenuation of  $K_p$  in dB,  $\varepsilon$  can be calculated as

$$\varepsilon = \sqrt{10^{0.1K_p} - 1} \quad (2.19)$$

Outside the passband,  $T_n(x)$  Almost exponential increase. Chebyshev functions could be find in terms of a polynomial formula to repeat his argument:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \quad (2.20)$$

The formula begins by setting  $T_0(x) = 1$  and  $T_1(x) = x$ . Also for  $n$  odd  $T_n(0) = 0$  and  $T_n(\pm 1) = \pm 1$ , while for  $n$  even  $T_n(0) = (-1)^{n/2}$  and  $T_n(\pm 1) = 1$ .

The next few Chebyshev functions are shown below:

$$\begin{aligned} T_2(x) &= 2x^2 - 1 \\ T_3(x) &= 4x^3 - 3x \\ T_4(x) &= 8x^4 - 8x^2 + 1 \\ T_5(x) &= 16x^5 - 20x^3 + 5x \\ T_6(x) &= 32x^6 - 48x^4 + 18x^2 - 1 \\ T_7(x) &= 64x^7 - 112x^5 + 56x^3 - 7x \\ T_8(x) &= 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1 \\ T_9(x) &= 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x \end{aligned} \quad (2.21)$$

Just like Butterworth approach has a number of iteration formula for Chebyshev filter. Filter first iteration formulas for  $g$  values find expression requires expanding the Chebyshev function with its own set. Correlation is made between the circuit and the function and so on (a certain number  $n$  of the reactive component) low-pass prototype filter structure is then th order filter functions are synchronized.

Butterworth and Chebyshev approach is a significant difference between the  $g_n + 1$ . Uneven levels even order Chebyshev impedance termination impedance is usually prevented by limiting the choices for the Chebyshev functions only one value. After

identifying these two filter functions pole transfer function of the circuit element values are found using the network synthesis techniques.

The recursive element value expression for Chebyshev filters with prescribed ripple and order are calculated as given below [30]

$$\begin{aligned}
 g_1 &= \frac{2a_1}{\sinh \beta/2N}, \\
 \beta &= \ln \frac{\sqrt{1+k^2} + 1}{\sqrt{1+k^2} - 1}, \\
 b_k &= \sinh^2 \frac{\beta}{2N} + \sin^2 \frac{k\pi}{N}, \\
 a_k &= \sin \frac{2k-1}{2N} \pi, \\
 g_k &= \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}} \quad k = 1, 2, 3, \dots, n
 \end{aligned} \tag{2.22}$$

For different passband ripple levels (which is a function of  $\epsilon$ ) there exist tabularized filter element values for a Chebyshev response [31]. A typical ripple case with 1 dB passband ripple is given in Table 3 [31].

**Table 3** Normalized Low Pass Chebyshev Filter Element Values ( $\omega_c = 1$  rad/s; 1 dB Pass Band Ripple)

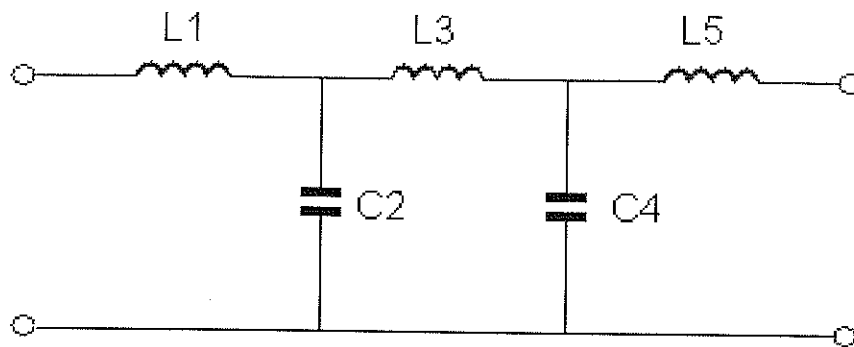
N	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$
1	1.017									
2	1.821	0.685								
3	2.023	0.994	2.023							
4	2.099	1.064	2.831	0.789						
5	2.134	1.091	3.000	1.091	2.134					
6	2.154	1.104	3.063	1.157	2.936	0.810				
7	2.166	1.111	3.093	1.173	3.093	1.111	2.166			
8	2.174	1.116	3.110	1.183	3.148	1.169	2.968	0.817		
9	2.179	1.119	3.121	1.189	3.174	1.189	3.121	1.119	2.179	
10	3.538	0.777	4.676	0.813	4.742	0.816	4.726	0.805	4.514	0.609
	$L_1$	$C_2$	$L_3$	$C_4$	$L_5$	$C_6$	$L_7$	$C_8$	$L_9$	$C_{10}$

If the passband frequency is maximum and the minimum stopband frequency yonder which the diminution is always larger than  $\alpha_{\min}$ , is  $\omega_c$ , than the required number of poles of the function is:

$$n = \frac{\arccos h \left[ \frac{1}{\epsilon} (10^{\alpha_{\max}/10} - 1)^{-1/2} \right]}{\arccos h(\omega_s / \omega_c)} \quad (2.23)$$

### Example 2.2

In this example the Chebyshev Low – pass filter is shown. Here we used fifth order of Chebyshev filter. According to Table 3 normalized Chebyshev element values for order 5 are given in Figure 2.5



**Figure 6** Low pass Chebyshev filter network topology for order 5

### 2.4 The 10-th Order FIR Filter

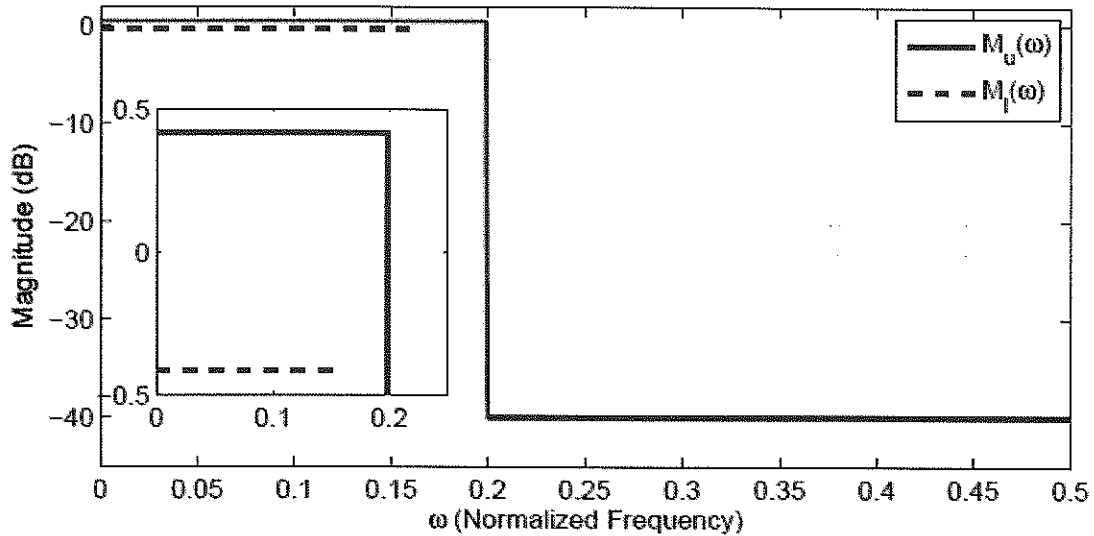
An FIR filter of length  $L$  is typically characterized by either its impulse response  $h_n$ ,  $0 \leq n \leq L - 1$  or its frequency response as:

$$H(\omega) = \sum_{n=0}^{L-1} h_n e^{-j\omega n}, \forall \omega \in [-\pi, \pi] \quad (2.24)$$

where  $\omega$  is the normalized frequency. In optimization based FIR filter design, filter coefficients are typically obtained by imposing desired constraints on the magnitude response of the filter as in the following feasibility problem:

$$\begin{aligned}
& \text{find } h_n \in R, 0 \leq n \leq L-1, \\
& \text{subject to } |H(\omega)|^2 \leq M_u(\omega), \forall \omega \in [0, \pi], \\
& |H(\omega)|^2 \geq M_l(\omega), \forall \omega \in [0, \omega_p]
\end{aligned} \tag{2.25}$$

where  $M_u(\omega)$  and  $M_l(\omega)$  are the upper and lower magnitude response masks shown in Fig. 7.



**Figure 7** A typical set of mask constraints on the magnitude response of an FIR filter.  $M_u(\omega)$  and  $M_l(\omega) = 1/1.1$  are the upper and lower mask constraints on the magnitude response with transition band  $\omega_p = 0.16 \leq \omega \leq 0.20 = \omega_s$ , respectively.

$M_u(\omega) = 1.1$  in the pass band and  $M_u(\omega) = 0.0001$  in the stop band.

The passband and stopband of the filter are defined as  $[0, \omega_p]$  and  $[\omega_s, \pi]$ , respectively. Since the magnitude response of an FIR filter with real coefficients is an even function, design constraints can be imposed on the nonnegative part of the spectrum. Then, (2.25) can be written in vector form as:

$$\begin{aligned}
& \text{find } h \in R^L, \\
& \text{subject to } h^T A(\omega) h \leq M_u(\omega), \forall \omega \in [0, \pi], \\
& h^T A(\omega) h \geq M_l(\omega), \forall \omega \in [0, \omega_p]
\end{aligned} \tag{2.26}$$

Here,  $\mathbf{h} = [h_0, h_1, \dots, h_{L-1}]^T$  and  $A(\omega)$  is given as:

$$\begin{aligned} A(\omega) &= \mathbf{v}(\omega)\mathbf{v}^H(\omega), \\ \mathbf{v}(\omega) &= [1, e^{j\omega}, \dots, e^{j\omega(L-1)}]^T \end{aligned} \quad (2.27)$$

Since its inception, digital finite impulse response (FIR) filters have been one of the prominent building blocks in digital signal processing (DSP) because of their benefits on hardware implementation in terms of cost, efficiency and design flexibility [12–13]. Accordingly, a diverse class of FIR filter design techniques including windowing, Fast Fourier Transform (FFT), Chebyshev approximation, least squares error, and convex optimization based techniques have been proposed in the literature [14–15]. Among prominent techniques, optimization based FIR filter design methods significantly flourished compared to others with the progression in computing resources in last 2 decades [12, 16]. Development of efficient convex optimization solvers, i.e., interior-point solvers, and introduction of flexible convex formulations such as Semidefinite Programming (SDP) and Second-Order Cone Programming (SOCP) triggered new possibilities in FIR filter design via convex optimization [17–18].

10-th degree transfer function of the FIR impulse response of the filter by z-transform Found:

$$\begin{aligned} H(z) &= \sum_{n=0}^{10} h(n)z^{-n} = 0 - 0.0127z^{-1} - 0.0248z^{-2} + \\ &0.0638z^{-3} + 0.2761z^{-4} + 0.4z^{-5} + 0.2761z^{-6} \\ &+ 0.0638z^{-7} - 0.0248z^{-8} - 0.0127z^{-9} \end{aligned} \quad (2.28)$$

Using the following expression:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = X(z = e^{j\omega}) \quad (2.29)$$



It is possible to obtain a constant frequency normalized transfer function. If for example  $\omega = 0.2\pi$  than:

$$\begin{aligned} H(e^{j0.2\pi}) = & -0.0127z^{-j0.2\pi} - 0.0248z^{-j0.4\pi} + \\ & 0.0638z^{-j0.6\pi} + 0.2761z^{-j0.8\pi} + 0.4z^{-j\pi} + 0.2761z^{-j1.2\pi} \\ & + 0.0638z^{-j1.4\pi} - 0.0248z^{-j1.6\pi} - 0.0127z^{-j1.8\pi} \end{aligned} \quad (2.30)$$

An example of a hardware implementation of this filter is shown in Figure 8.

Pole and zero of the transfer function are very essential for analysis and synthesis of discrete-time systems. According to their position, this is a discrete-time system stability test equipment due to rounding filter coefficients encountered during the implementation of the filter software application as well as mistakes made possible to detect errors.

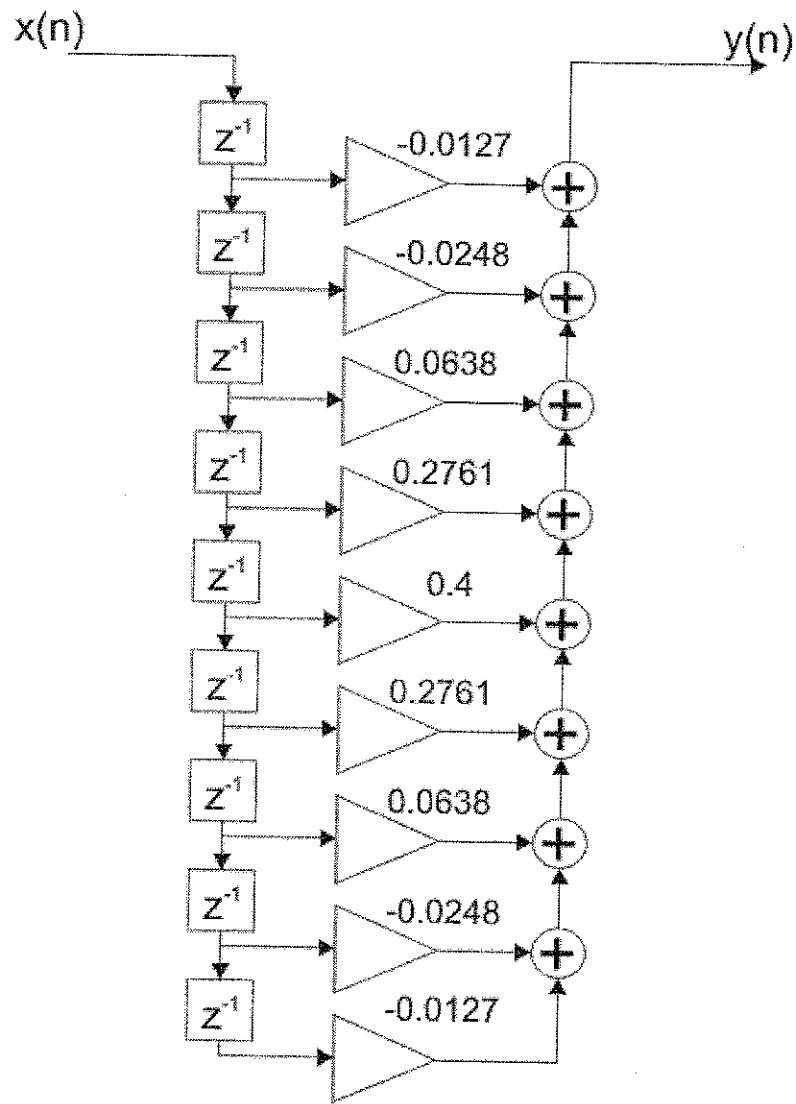


Figure 8 FIR filter realization

## CHAPTER 3

### PROPOSED METHOD

#### 3.1 Root Finding Algorithm

A non-linear function can be represented by  $F(x)$ . Thus, non-linear equations can be written more to hang [8]

$$F(x) = 0; \quad (3.1)$$

with a Taylor, we have  $G(x) = 0$ , which will be described later expressed arbitrary function in terms of a function.

$$\begin{aligned} F(x) = & F(x_n) + \frac{a_1(x_n)}{1!} (G(x) - G(x_n)) \\ & + \frac{a_2(x_n)}{2!} (G(x) - G(x_n))^2 + \dots \end{aligned} \quad (3.2)$$

Where

$$a_1(x_n) = \frac{F'(x_n)}{G'(x_n)} \quad (3.3)$$

$$a_{i+1}(x_n) = \frac{a'(x_n)_i}{G'(x_n)} \quad i = 2, 3, 4, \dots \quad (3.4)$$

Therefore

$$\begin{aligned}
 F(x) &= F(x_n) + \frac{F'(x_n)}{G'(x_n)}(G(x) - G(x_n)) \\
 &+ \left( \frac{F''(x_n)}{G'^2(x_n)} - \frac{F'(x_n)G''(x)}{G'^3(x_n)} \right) (G(x) - G(x_n))^2 + \dots
 \end{aligned}
 \tag{3.5}$$

Considering the overhead equation  $f(x)$  may be approaching:

$$F(x) \approx F(x_n) + \frac{F'(x_n)}{G'(x_n)}(G(x) - G(x_n))
 \tag{3.6}$$

suppose right hand of equation (3.5) be represented by  $H(x)$ :

$$H(x) = F(x_n) + \frac{F'(x_n)}{G'(x_n)}(G(x) - G(x_n))
 \tag{3.7}$$

In order for  $H(x)$  to be companionable with  $F(x)$  around  $x = x_n$  it must have:

$$H(x_n) = F(x_n)
 \tag{3.8}$$

$$H'(x_n) = F'(x_n)
 \tag{3.9}$$

$$H''(x_n) = F''(x_n)
 \tag{3.10}$$

Conditions are satisfied up automatically. Last case we should have to satisfy:

$$\frac{F''(x_n)}{F'(x_n)} = \frac{G''(x_n)}{G'(x_n)}
 \tag{3.11}$$

By providing Now,  $f(x)$  in Eq = 0. (3.6), we obtain:

$$x_{n+1} = G^{-1} \left( G(x_n) - G'(x_n) \frac{F(x_n)}{F'(x_n)} \right) \quad (3.12)$$

Note that this equation we have  $G(x) = x$ , will be equal to Newton's formula.

$$x_{n+1} = x_n + \frac{F'(x_n)}{F''(x_n)} \times \ln \left( 1 - \frac{F(x_n)F''(x_n)}{F'(x_n)^2} \right) \quad (3.13)$$

$n = 0, 1, 2, \dots$

### 3.2 Selection of $G(x)$

$G(x)$  is a function should be closely. In addition, the method of equations. Function (3.12) opposite  $G(x)$  can be obtained. Polynomials and exponential functions are usually suitable for this purpose. Thus,  $G(x)$  forms  $k^x$  or  $e^{kx}$  can be expressed by one.  $G(x) = x$  is from Eq. (3.11) and (3.12) we have:

$$\frac{F''(x_n)}{F'(x_n)} = \frac{(k-1)(k)x_n^{k-2}}{(k)x_n^{k-1}} \Rightarrow k = 1 + \frac{x_n F''(x_n)}{F'(x_n)} \quad (3.14)$$

$$x_{n+1} = \left( x_n^k - k \frac{x_n^{k-1} F(x_n)}{F'(x_n)} \right)^{1/k} \Rightarrow x_{n+1} = x_n \times \left( 1 - k \frac{F(x_n)}{x_n F'(x_n)} \right)^{1/k} \quad n = 0, 1, 2, \dots \quad (3.15)$$

If  $G(x) = k^x$ , then:

$$\frac{F''(x_n)}{F'(x_n)} = \frac{k^{x_n} \times (\ln(k))^2}{k^{x_n} (\ln(k))} \rightarrow k = e^{\frac{F''(x_n)}{F'(x_n)}} \quad (3.16)$$

$$x_{n+1} = \ln(k^{x_n} - \ln(k) \times k^{x_n} \frac{F(x_n)F''(x_n)}{F'(x_n)^2}) \quad (3.17)$$

Therefore:

$$x_{n+1} = x_n + \frac{F'(x_n)}{F''(x_n)} \times \ln\left(1 - \frac{F(x_n)F''(x_n)}{F'(x_n)^2}\right) \quad n=0, 1, 2, \dots \quad (3.18)$$

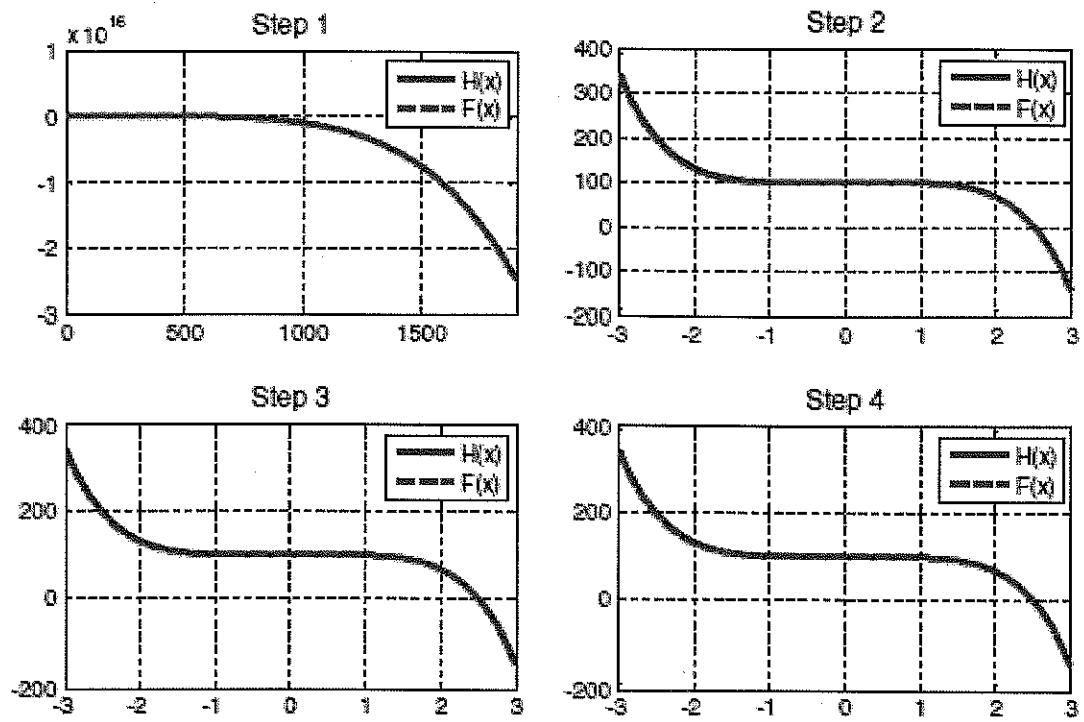
$G(x)$  as an appropriate choice,  $F(x)$  note is based.  $F(x)$  contains exponential terms,  $G(x) = k^x$  convergence can offer a good rate. On the other hand,  $G(x) = x^k$ , suitable for polynomial equations over the  $x^k$ . Also this set, I note  $(x)$  to be the same result as the  $x_{n+1} = \varphi(x_n)$  as  $G(x) = k^x$  (Eq. 3.18).

### Example 1

Also consider the following nonlinear equations to stand on the proposed method:

$$F(x) = \tan\left(\frac{x}{x^2 + 1}\right) \times e^{-x} - x^5 + 100 \quad (3.19)$$

To calculate the root of the equation, the initial value  $X_0 = 1800$  is arbitrary. In this example,  $G(x)$  is  $x^k$ . Table 4 shows the convergence of the date of the proposed method.  $F(x)$  and  $H(x)$  between the comparison procedure also describes the method shown in Fig. 9.



**Figure 9** Compatibility between  $F(x)$  and  $H(x)$  around the starting point in each step for Eq. (3.19).

This form of charts, each step around the start point,  $F(x)$  and  $H(x)$  shows the compatibility between  $H$ . Although this is far from the initial value in step 4 to find root root algorithm can be seen from Table 4  $|F(x_i)| = 4.26 \times 10^{-14}$ .

**Table 4**  
For the Convergence of the Proposed Method (3.19).

Step	$x_i$	$k$	$ F(x_i) $
1	1800	5.000000000000000	1.889567e+016
2	2.504806159963997	4.998552959678708	1.430728223
3	2.512032208495923	4.998581738776183	1.993302e-008
4	2.512032208395826	4.998581738775789	4.263256e-014

**Example 2.** Consider the following nonlinear equation:

$$F(z) = \sin(z) - z = 0 \quad (3.20)$$

Kepler's equation with this equation and  $M = 0$  [21] is a special case. This equation  $G(x)$  is set. Set different roots, private complex ones, to determine the ability of the proposed method to find the initial value for  $z$ , this and the real numbers and  $l$  ranged between increments from  $-20$  to  $20$ . Therefore algorithm 41<sup>2</sup> = running for 1681 different initial values. Each study is set for stopping by algorithm 13 different roots (real and complex roots of twelve) were found in Figure above. Found by Algorithm 3.20 shows the roots. Figure 3 shows the initial value of 13 different root zones of convergence. From Fig. 3 different starting values for the single convergent root can find harmony. The initial value of the field is extended towards the real axis, therefore, the algorithm (3.20) You can find more complex roots. Only you can find all the roots of the algorithm related domain actual initial values are interesting. So,  $[-20 \ 20]$  is the first value of the domain before No. 13 different root causes. Do not forget to examine specific cases of the Kepler equation in this example.  $E$  and  $M$  changes related to the initial values of the area and the roots will find all.

### 3.2.1 Rate of Convergence

Let  $\varepsilon_i = r - x_i$  be the actual error between  $x_i$  and the root  $r$ . By means of Eq. (3.5) and according to Eq. (3.11) we can write:

$$F(r) \approx F(x_i) + \frac{F'(x_i)}{G'(x_i)} (G(x_i) - G(r)) + \frac{a_3}{3!} (G(r) - G(x_i))^3 = 0 \quad (3.21)$$

We have:

$$x_{i+1} = G^{-1} \left( G(x_i) - G'(x_i) \frac{F(x_i)}{F'(x_i)} \right) = G^{-1} \left( G(r) + \frac{a_3}{3!} \frac{G'(x_i)}{F'(x_i)} (G(r) - G(x_i))^3 \right) \quad (3.22)$$



with Taylor series we have:

$$G(r) - G(x_i) \approx \varepsilon_i G'(x_i) \quad (3.23)$$

Therefore from Eqs. (3.21) and (3.22) we can conclude:

$$X_{i+1} = G^{-1}(G(r) + \frac{a_3 (G'(x_i))^4}{3! F'(x_i)} \varepsilon_i^3) \approx G^{-1}(G(r)) + (\frac{a_3 (G'(x_i))^4}{3! F'(x_i)} \varepsilon_i^3) \frac{1}{G'(G(r))} \quad (3.24)$$

Therefore:

$$r - x_{i+1} = \varepsilon_{i+1} = -(\frac{a_3 (G'(x_i))^4}{3! F'(x_i)} \frac{1}{G'(G(r))}) \varepsilon_i^3 \quad (3.25)$$

### 3.2.2 Numerical Examples

We now offer some examples to illustrate the effectiveness of the newly developed management. This section compares the proposed method with the MATLAB command `fsolv`, the methods of Sánchez ( $\psi_2^4$  and  $\psi_3^6$ ) [22], the method of Ujevic [23],

In JeSheng et al [24] and Newton's method, all calculations were performed using MATLAB. Computer programs used for the following stopping criteria:

$$|F(x_i)| < \varepsilon \quad (3.26)$$

$\varepsilon = 5 \times 10^{-12}$  was used.

The aim is to find the initial value of a large amount of radicals methods to compare the strength. The following examples NR method specifies the number of Roots found in the range prescribed initial value. Number indicates that failed, Ave F

evaluation function for a method to find a root shows the average number. (Note methods 30- repeat step one takes the punishment he fails to find any root).

## CHAPTER 4

### EXPERIMENTAL RESULT

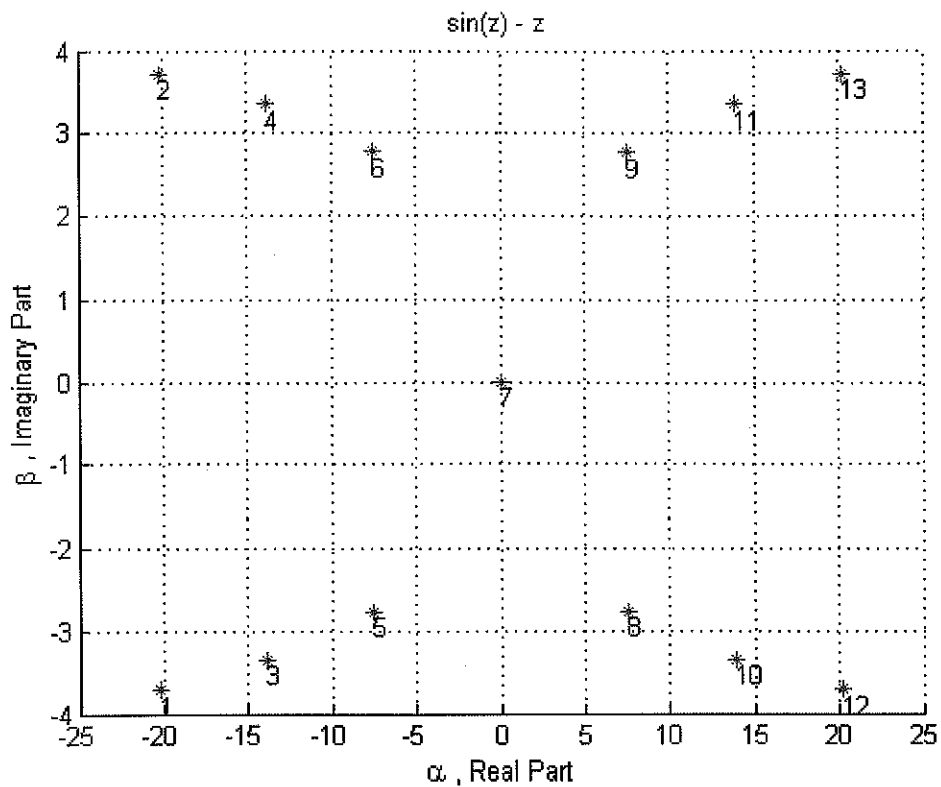
#### Example 1.

For testing of our method in first step we consider the following nonlinear equation:

$$F(z) = \sin(z) - z \quad (4.1)$$

This equation is a special case of the Kepler equation with  $\varepsilon = 1$  and  $M = 0$  [9]. For this equation  $G(x)$  is set to  $ekx$ . To determine the ability of the proposed method for finding different roots, specially complex ones, initial values for  $z$  are set to  $\alpha + \beta i$ , that  $\alpha$  and  $\beta$  are real numbers and vary from -20 to 20 with steps of 1. Therefore the algorithm runs for  $412 = 1681$  different initial values. The stopping criteria for each run is set to  $|F(x_i)| < 5 \times 10^{-12}$ . The algorithm found 13 different roots (one real and twelve complex roots). Figure 10. shows the roots found by the algorithm. The roots numbered from 1 to 13.

Also, equation (4.1) is calculate and result is shown in figure 10.



**Figure 10** Roots of Eq. (4.1) found by the algorithm (one real and twelve complex roots).

**Example 2.**

Analog filters place in analog interface circuits in digital systems as it can be seen in many applications. One of them is Bluetooth/Wi-Fi (Wireless Fidelity) receiver. Literature survey shows that 5th order Butterworth filter is appropriate for Bluetooth/Wi-Fi receiver [32, 33, 34].

In square root domain, high order filters are designed by cascading first and second order ones [34]. To design the 5th order Butterworth lowpass filter, its transfer function is decomposed to first a second order lowpass filters. The transfer function of normalized 5th order Butterworth lowpass filter is written as;

$$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1) \quad (4.2)$$

According to Equation (4.2), the filter can be designed by cascading one first order lowpass filter and two second order lowpass filters. The transfer functions of each part are defined as;

$$H_{lp1}(s) = \frac{\omega_0}{s + \omega_0} \quad (4.3)$$

$$H_{lp2}(s) = \frac{0.618\omega_0^2}{s^2 + 0.618\omega_0s + \omega_0^2} \quad (4.4)$$

$$H_{lp3}(s) = \frac{1.618\omega_0^2}{s^2 + 1.618\omega_0s + \omega_0^2} \quad (4.5)$$

A 5th-order, 1dB-ripple Butterworth low pass filter is constructed from two non-identical 2nd-order sections and an output RC network (see figure 11).

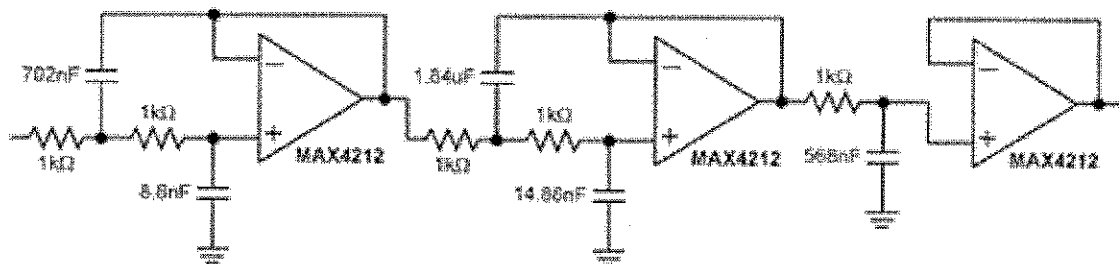
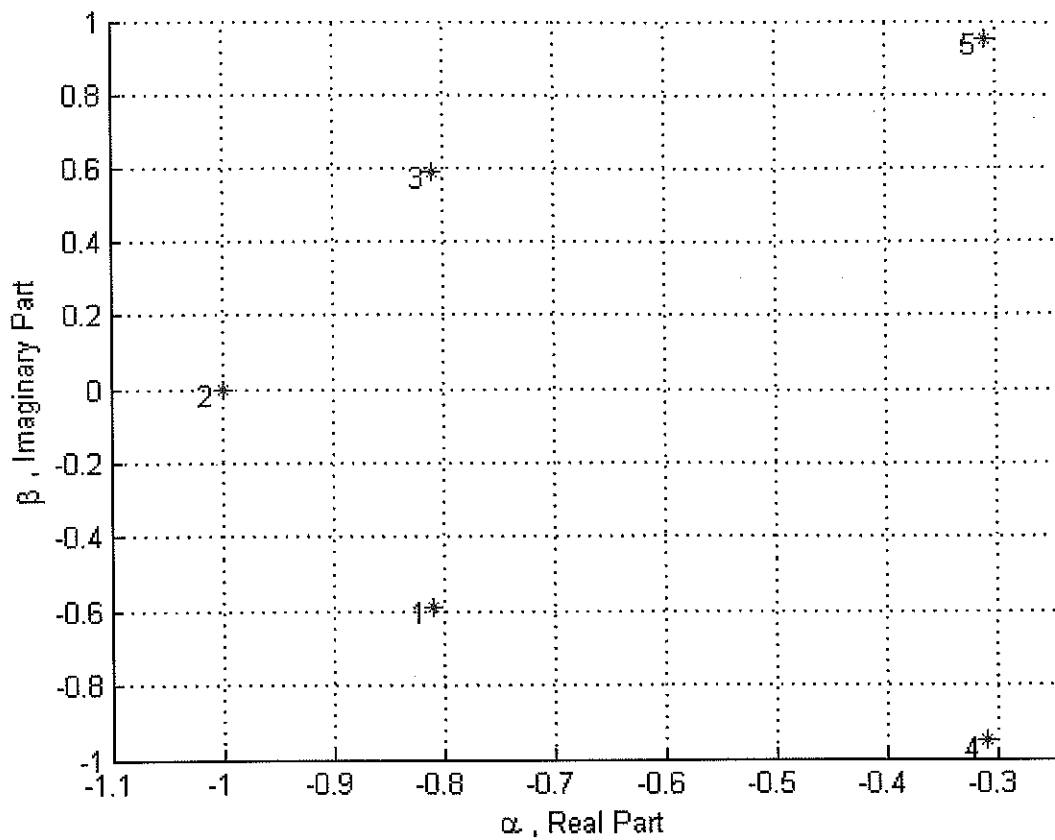


Figure 11 A 5th-order, 1dB-ripple Butterworth low pass filter

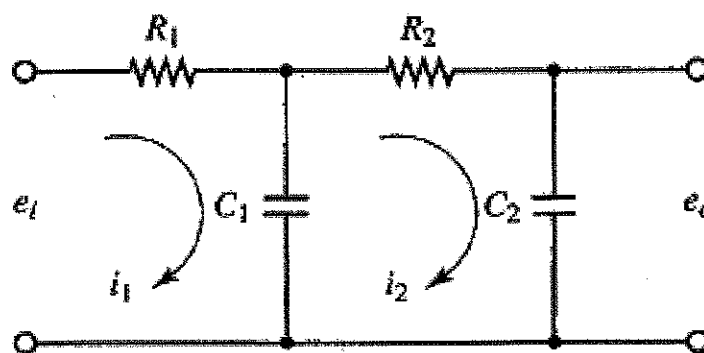


**Figure 12** Roots of Eq. (4.2) found by the algorithm

Figure 12. shows the result of equation (4.2).

**Example 3.**

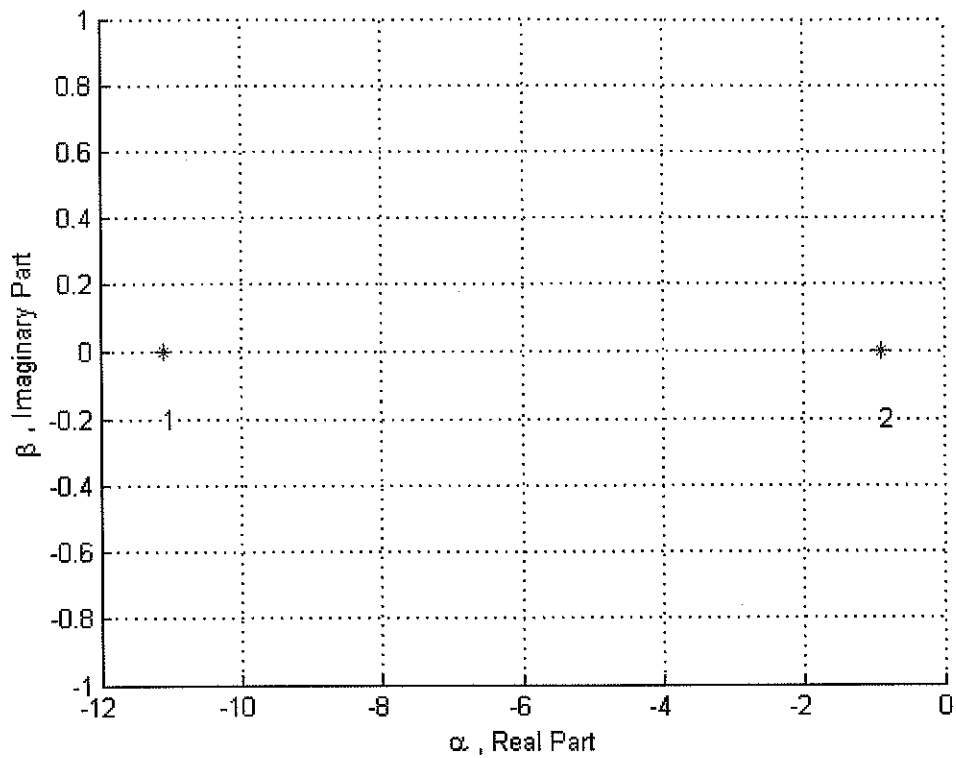
The transfer function of the cascade elements: Many feedback systems are load components to each other. Consider the system shown in Figure 13.



**Figure 13** Electrical system

The transfer function of figure 13 is:

$$\frac{H_1(s)}{Q(s)} = \frac{R_2}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) s + 1} \quad (4.6)$$



**Figure 14** Roots of Eq. (4.6) found by the algorithm

**Example 4.**

10-th degree FIR filter design using the Hamming window is impulse response:  
 $h(n) = \{0, -0.0127, -0.0248, 0.0638, 0.2761, 0.4, 0.2761, 0.0638, -0.0248, -0.0127, 0\}$

This filter has the impulse response of the transfer function by z transform:

$$H(z) = \sum_{n=0}^{10} h(n)z^{-n} = 0 - 0.0127z^{-1} - 0.0248z^{-2} + 0.0638z^{-3} + 0.2761z^{-4} + 0.4z^{-5} + 0.2761z^{-6} + 0.0638z^{-7} - 0.0248z^{-8} - 0.0127z^{-9}$$

For finding of the poles of this filter we apply the method and then we get the result (see figure 15).

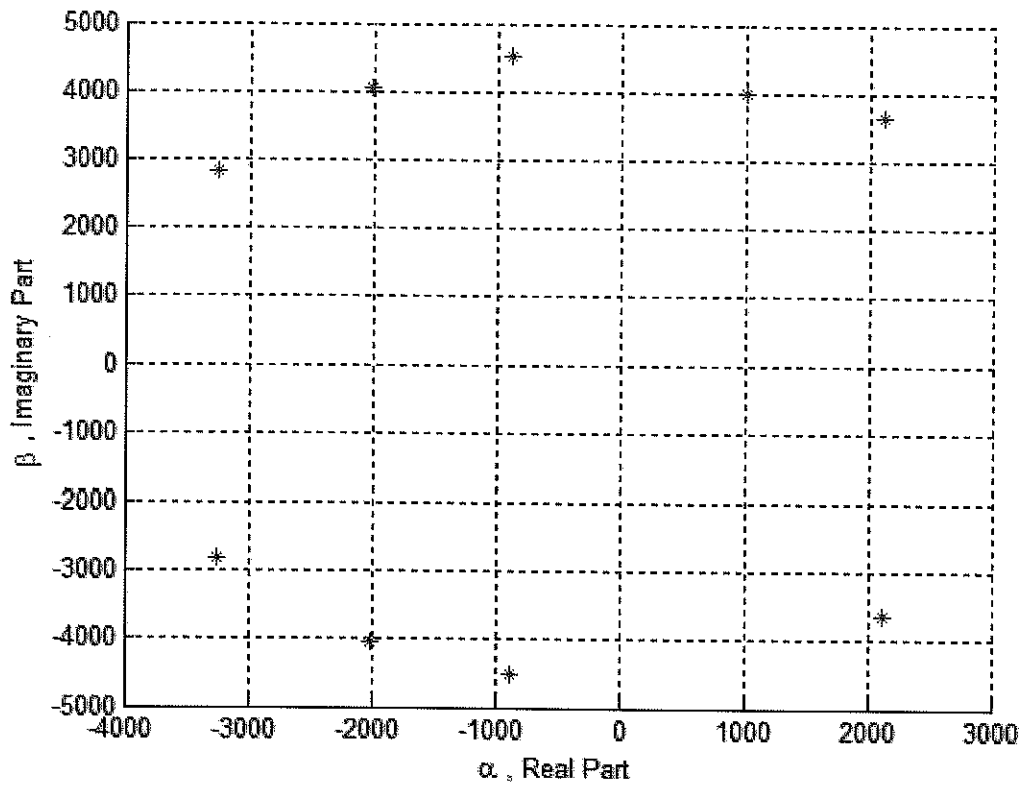


Figure 15 Roots of FIR filter found by the algorithm



**Table 5** Comparison Between Some Methods

Methods	Iteration Example 1	Iteration Example 2	Iteration Example 3	Iteration Example 4
Newton	21.7778	19.3580	6.4000	9.6790
Our method	4.9136	9.3086	3.9600	8.0741

In this chapter we did show the result of root finding method for a lot of example. As seen in the result our method is good to find the roots. for example in example 1, the classic Newton method get in 21.7778 average iteration value, but in our method same example is get in 4.9136 iteration average value. This show the robustness of our method than the other method.

## CHAPTER 5

### CONCLUSIONS

In this thesis, algorithms cubic nonlinear equations convergent iterative methods for calculating the complex roots modified, then we wrote down their programs by Matlab applications version 2014a by taking complex initial point to get the complex roots if it exists if not we real initial point to get real root. In this thesis we study initial point have real and imaginary part, as future work we modified these algorithm to compute the complex roots if the initial point is complex.

Finite impulse response filters have been one of the primary topics of digital signal processing since their inception. Consequently, diverse class of design techniques including Chebyshev approximation, Fast Fourier Transform and optimization based methods had been proposed in the literature. With developments in computational tools, new root finding technique tools and formulations on filters including interior-point solvers and semidefinite programming, emerged. Since FIR filter design problem can be modelled as a quadratically constrained quadratic program, filter design problem can be solved via interior-point based convex optimization methods such as semidefinite programming.

This thesis, presente a new, simple, effective and flexible method Butterworth filter to calculate the real and complex roots. You can find more roots compared to other existing methods proposed method numerical examples (especially complex roots) shows. The location of the initial value as the initial value so you can find the root of the method is pretty solid. In addition, the proposed method can handle complex starting values of other more efficient methods.

### **Future Work:**

1. In the future we can to employee this algorithm on hardware the advantage of our method is fast and also the complexity of our method is very low, for this reason we can use in hardware systems.
2. We used cubically convergent iterative method for zero – pole analysis of high order filter circuits method, in the future we will test and use the other methods for compression.
3. We used FIR filter, Butherworth, Chebishev in future we will use other low pass filter, and we will try to get high accuracy.

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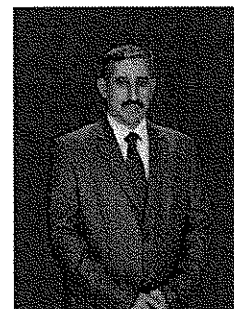
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## APPENDICES A

### CURRICULUM VITAE



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