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ORIGINAL ARTICLE

Analysis of a conformable generalized geophysical KdV equation with Coriolis effect

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Abstract In this manuscript, we study new solutions of generalized version of geophysical KdV equation which is called generalized perturbed KdV (gpKdV) under time–space conformable operator. We implement two methods to get some novel waves solution of the gpKdV equation. First, we use extended Tanh-method to extract new solutions of considered equations in the form of trigonometric hyperbolic functions. To achieve Sine and Cosine hyperbolic solutions, we use generalized Kudryashov (GK) technique with Riccati equation. We show the behaviour of solutions via 2D and 3D figures. Also, we analyze the Corioles effect on the evolution of waves solutions of the considered equation.

1. Introduction

From past few centuries, nonlinear PDEs have been frequently used in science and engineering problems [1,2]. There are several famous nonlinear PDEs, which have been analyzed by researchers in the current century [3]. Among them, KdV equation has gotten remarkable attention of the researchers from the past several decades. In 1877, Boussinesq proposed KdV equation and later reformulated by Korteweg and de Vries in 1895. KdV is a dispersive PDE, typically used in the study of waves produces in shallow water, dense oceans, plasma, and crystal lattice [4,5]. The classical KdV equation:

$$\frac{\partial}{\partial t}\psi(x,t)+\psi(x,t)\frac{\partial}{\partial x}\psi(x,t)+\frac{\partial^3}{\partial x^3}\psi(x,t)=0. \tag{1}$$

A KdV model has been utilized to explain several quantum mechanics-related scientific research events. It is utilized as a

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model in the study of hydrodynamics, aerodynamics, and continuum physics.

KdV equation has large family and its several generalized form has been introduced in the literature [3]. A few years ago, Geyer et al. formulated another version of KdV by taking into account equatorial two-dimensional one-layer ocean dynamics [6]. Further, they consider the so-called Coriolis effect, which becomes important for large-scale ocean waves, in contrast to well-known shallow water models like KdV. Geyer and his coauthor formulated the KdV equation as:

$$\frac{\partial}{\partial t}\psi(x,t) - \omega\frac{\partial}{\partial x}\psi(x,t) + \frac{3}{2}\psi(x,t)\frac{\partial}{\partial x}\psi(x,t) + \frac{1}{6}\frac{\partial^3}{\partial x^3}\psi(x,t) = 0. \tag{2}$$

The above equation is called geophysical KdV equation. This equation has been studied by several researchers [7,8]. Very recently, Alquran et al. formulated a modified version of the geophysical KdV Eq. (2), which they called gpKdV equation. We consider the nonlinear perturbed KdV equation in the form [9]

$$\psi_t + \varrho\psi_x + \Omega\psi\psi_x + \eta\psi_{xxx} = 0, \tag{3}$$

ψ is a function of x and t. In the gpKdV Eq. (3) the ϱ provides perturbation parameter representing Coriolis effect which is is the apparent deflection of moving objects, such as air and water, due to the Earth's rotation. Ω is the coefficient of non-linearity and η represents dispersion which is the process of separating a complex wave into its wavelengths etc. The Eq. (3) is generalized version of the geophysical KdV Eq. (2) and classical KdV Eq. (1). One can recover Eq. (2) by considering $\varrho = -\eta\omega, \Omega = \frac{3}{2}$ and $\eta = \frac{1}{6}$. Similarly one can get KdV Eq. (1) from Eq. (3) by considering $\varrho = 0, \Omega = 1$ and $\eta = 1$. The gpKdV equation has applications in areas of mechanics, medical engineering, acoustics and show the scientific explanations of transmission of sound in fluid. Saifullah et al. studied lump and its interacttions of the gpKdV Eq. (3) via Hirota bilinear technique [10].

The generalized geophysical KdV equation is a mathematical model that describes the propagation of long waves in geophysical systems, such as oceanic and atmospheric waves. Physically, it takes into account the effects of dispersion, nonlinearity, and dissipation in the wave dynamics, and it includes additional terms that account for the background flow and the effects of variable depth. The equation can be used to study the behavior of waves under various conditions, such as the interaction between waves of different frequencies, the generation of rogue waves, and the formation of coherent structures. Its solutions provide insights into the dynamics of geophysical systems and can be used to predict the occurrence of various phenomena, such as storm surges, tsunamis, and internal waves.

Nowadays, nonlocal operator have been used to study complete information of a physical processes during their heredity and memory features [11,12]. Nonlocal operators has several applications in control theory and delay differential equations [13–16], analysis [17–20], biomathematics and some other area of applied sciences [21–24]. Conformable operator got tremendous attention of the researchers of the current century. Some theoretical and computational works on conformable operator are listed in [25–29]. Mathematical physicist preferred to used conformable operator to study solitary waves solutions of nonlinear PDEs. The conformable operator is a natural extension of the integer order operators. The conformable operator has several important features such as linearity, Liebnitz rule, chain rule and quotient rule. Due to these features, we choose conformable operator to study the gpKdV (3). Consider the Eq. (3) under conformable operator as:

$$\mathscr{D}_t^{\gamma}\mathscr{P} + \varrho\mathscr{D}_x^{\beta}\mathscr{P} + \Omega\mathscr{P}\mathscr{D}_x^{\beta}\mathscr{P} + \eta\mathscr{D}_{xxx}^{3\beta}\mathscr{P} = 0, \tag{4}$$

where $0 < \gamma, \beta \leq 1$. The solution of conformable PDEs have been achieved by many methods in the literature. For instance, Jacobi elliptic method has been utilized to investigate exact solutions of conformable PDEs [30]. Similarly novel types of solutions are obtained with different techniques in [31–34]. Bilal et al. used sinh-Gordan technique to extract solitary solutions of Wazwaz-Benjamin-Bona-Mahony under conformable FO [35]. The Hirota bilinear approach has been used to derive lump and other rogue wave solutions of a nonlinear Schrodinger equation under conformable FO [36]. Dual wave solutions are studied in [37] of the Klein-Fock-Gordon equation under conformable Caputo operator. Explicit rational solution of equation describing the propagations of bidirectional waves in a low-pass electrical lines in space time fractional form is studied in [38]. Further, the solitonic and chaotic solutions for a novel time-fractional dual-mode KdV equation are reported in [39]. Various researchers have investigated soliton solutions of different integrable systems via various analytical methods [40–44]. The pgKdV is not studied under conformable FO. Therefore, in this paper, we use conformable FO to extract some solitary waves solutions of the gpKdV with help of extended tanh-method and GK method. Also we consider different values of the Coriolis effect and study its impact on the dynamics of the obtained exact solutions.

2. Preliminaries

Here, we outline the conformable derivative's fundamental definition and features.

Definition 1. Consider a function $\mathscr{G} : [0, \infty) \rightarrow \mathbb{R}$. $\mathscr{G}$ be ρ-order "conformable derivative" can be expressed in the form [45]:

$$\mathbb{K}_{\gamma}(\mathscr{G}(t)) = \lim_{\epsilon \to 0} \frac{\mathscr{G}(t + \epsilon t^{1-\gamma}) - \mathscr{G}(t)}{\epsilon}. \tag{5}$$

For each $t > 0, \gamma \in (0, 1]$. If $\mathscr{G}$ be γ-differentiable in nearly $(0, \gamma), \gamma > 0$ and $\lim_{\epsilon \to 0} \mathscr{G}^{(\gamma)}(t)$ be real, next $\mathscr{G}^{(\gamma)}(0) = \lim_{t \to 0^+} \mathscr{G}^{(\gamma)}(t)$.

Theorem 1. Let $\gamma \in (0, 1]$ and at $t > 0\, \mathscr{G}, \mathscr{H}$ be γ-differentiable. Then

1. $\mathbb{K}_{\gamma}(a\mathscr{G} + bF) = a\mathbb{K}_{\gamma}(\mathscr{G}) + b\mathbb{K}_{\gamma}(H), \forall a, b \in \mathbb{R}$.
2. $\mathbb{K}_{\gamma}(t^j) = jt^{j-1}, \forall j \in \mathbb{R}$
3. $\mathbb{K}_{\gamma}(p) = 0, \forall$ constant functions $\mathscr{G}(t) = p$.
4. $\mathbb{K}_{\gamma}(\mathscr{G}\mathscr{H}) = \mathscr{G}\mathbb{K}_{\gamma}(\mathscr{H}) + \mathscr{H}\mathbb{K}_{\gamma}(\mathscr{G})$.
5. $\mathbb{K}_{\gamma}\left(\frac{\mathscr{G}}{\mathscr{H}}\right) = \frac{H\mathbb{K}_{\gamma}(\mathscr{G}) + \mathscr{G}\mathbb{K}_{\gamma}(\mathscr{H})}{\mathscr{H}^2}$.
6. Furthermore, when $\mathscr{G}$ is differentiable, then we can write $\mathbb{K}_{\gamma}(\mathscr{G}(t)) = t^{1-\gamma}\frac{d\mathscr{G}}{dt}$.

The above properties and some other as well like Grönwall's inequality, chain law and Laplace transform etc are presented in [47].

Theorem 2. In the context of conformable differentiation, $\mathscr{G}$and $\mathscr{H}$ are γ-differentiable, then [46]

$$\mathbb{K}_{\gamma}(\mathscr{G} \circ \mathscr{H})(t) = t^{1-\gamma}\mathscr{G}\prime(t)\mathscr{H}_{\gamma}(t).$$

3. The extended tanh function technique

The extended tanh approach for getting numerous exact solutions of fractional nonlinear equations (FNLEs) is presented here. It was summarised by Wazwaz [48]. The essential concept behind the suggested approach is to obtain the solution called a polynomial with hyperbolic functions and solve the PDE first by solving the procedure, which includes first-order ODEs and the algebraic equations. In conformable sense this technique has been used by various mathematicians and researchers [49,50]. To start the technique, consider the FNLE related with $\mathscr{P} = \mathscr{P}(x,t)$ as:

$$\mathbb{W}(\mathscr{P}, \mathscr{P}_t^{\gamma}, \mathscr{P}_x^{\beta}, \mathscr{P}_x^{\beta}\mathscr{P}_t^{\gamma}, \mathscr{P}_{tt}^{\gamma}, \mathscr{P}_{xx}^{\beta}, \cdots) = 0, \quad 0 < \beta, \gamma \le 1, \tag{6}$$

where $\mathbb{W}$ is polynomial of $\mathscr{P}(x,t)$ and its derivatives of different orders in which the high order derivative and high order nonlinear terms are interrelated. Consider the wave transformation in the form

$$\zeta = k\frac{x^{\beta}}{\beta} + c\frac{t^{\gamma}}{\gamma}, \ \mathscr{P}(\zeta) = \mathscr{P}(x,t), \ \ 0 < \beta, \gamma \le 1, \tag{7}$$

where k and c are non-zero constants. Substituting Eq. (7) into Eq. (6), one can obtain the following system of ODEs.

$$\mathbb{W}(\mathscr{P}, \mathscr{P}', \mathscr{P}'', \mathscr{P}''', \mathscr{P}'''', \cdots) = 0, \tag{8}$$

where "$\prime$" represents the ordinary derivative. The integrate Eq. (8) as long as all terms have derivatives but to consider integration constants as zeros. Further consider the general solution in the following finite expansion

$$\mathscr{P}(x,t) = \mathscr{P}(\zeta) = \sum_{q=0}^{\varsigma} \alpha_q \psi^q + \sum_{q=1}^{\varsigma} \mathrm{Y}_q \psi^{-q}, \ \psi = \tanh(\Lambda\zeta), \tag{9}$$

where, Y represents wave number, leading to change of derivatives, ς represents positive integer, and its value can be found with the Homogeneous balance method. Eq. (9) is reduces to standard tanh method for $\mathrm{Y}_q \neq 0, 1 \leqslant q \leqslant \varsigma$. Plugging Eq. (9) into the obtained ODE and then collecting the coefficients of powers of ψ in the reduced equation, where coefficients need to be disappeared. After this, we achieve system of equations in the variables $\alpha_q, \mathrm{Y}_q, \zeta$, and Ω. Solving the attained system of equations, we achieve the exact solution $\mathscr{P}(x,t)$. The acquired analytical solutions can be the soliton solution in terms of $sech^2$, or kink solutions. Moreover, this method can also give the periodic and singular solution of the nonlinear PDE under consideration.

3.0.1. *Novel solutions of conformable geophysical KdV equation with extended* tanh *technique*

Here, we implement the suggested procedure to get some new exact solutions of the conformable suggested Eq. (3). Let

$$\zeta = k\frac{x^{\beta}}{\beta} - c\frac{t^{\gamma}}{\gamma}, \ \mathscr{P}(x,t) = \mathscr{P}(\zeta), \quad 0 < \beta, \gamma \le 1, \tag{10}$$

putting Eq. (10) into Eq. (3), we have

$$-c\mathscr{P}'(\zeta) + \eta\mathscr{P}'''(\zeta) - \Omega\mathscr{P}(\zeta)\mathscr{P}'(\zeta) + \varrho\mathscr{P}'(\zeta) = 0. \tag{11}$$

Next, we integrate Eq. (11) w.r.t ζ, one gets

$$-c\mathscr{P}(\zeta) + \eta\mathscr{P}''(\zeta) - \frac{1}{2}\Omega\mathscr{P}(\zeta)^2 + \varrho\mathscr{P}(\zeta) = 0. \tag{12}$$

To apply the suggested procedure, first we calculate the value of ς. From homogeneity principle, one may obtain $\varsigma = 2$. We get:

$$\mathscr{P}(\zeta) = \sum_{q=0}^{2} \alpha_q \psi^q + \sum_{q=1}^{2} \mathrm{Y}_q \psi^{-q}, \tag{13}$$

substituting Eq. (13) into Eq. (12), we obtain

$$\begin{aligned}
&-c\big(\alpha_0 + \alpha_1\tanh(\Lambda\zeta) + \alpha_2\tanh^2(\Lambda\zeta) + \mathrm{Y}_1\coth(\Lambda\zeta) + \mathrm{Y}_2\coth^2(\Lambda\zeta)\big)\\
&-\tfrac{1}{2}\Omega k\big(\alpha_0 + \alpha_1\tanh(\Lambda\zeta) + \alpha_2\tanh^2(\Lambda\zeta) + \mathrm{Y}_1\coth(\Lambda\zeta) + \mathrm{Y}_2\coth^2(\Lambda\zeta)\big)^2\\
&+k\varrho\big(\alpha_0 + \alpha_1\tanh(\Lambda\zeta) + \alpha_2\tanh^2(\Lambda\zeta) + \mathrm{Y}_1\coth(\Lambda\zeta) + \mathrm{Y}_2\coth^2(\Lambda\zeta)\big)\\
&+\big(-\alpha_1 + \alpha_2\coth(\Lambda\zeta) - 3\alpha_2 + \mathrm{Y}_1\coth^4(\Lambda\zeta) + 3\mathrm{Y}_2\coth^5(\Lambda\zeta) - \mathrm{Y}_2\coth^3(\Lambda\zeta)\big)\times\\
&2\eta k^3\Lambda^2\coth(\Lambda\zeta)\big(\tanh^2(\Lambda\zeta) - \tanh^4(\Lambda\zeta)\big) = 0,
\end{aligned} \tag{14}$$

consider the coefficients of each power of $\mathscr{Q}$ to zero in Eq. (14), we get the following algebraic system of equations

$$\begin{cases}
-\alpha_0 c + 2\alpha_2\eta k^3\Lambda^2 + 2\mathrm{Y}_2\eta k^3\Lambda^2 - \alpha_1\mathrm{Y}_1\Omega k - \alpha_2\mathrm{Y}_2\Omega k\\
\quad -\tfrac{1}{2}\alpha_0^2\Omega k + \alpha_0 k\varrho = 0,\\
-\mathrm{Y}_1 c - 2\mathrm{Y}_1\eta k^3\Lambda^2 - \alpha_0\mathrm{Y}_1\Omega k - \alpha_1\mathrm{Y}_2\Omega k + \mathrm{Y}_1 k\varrho = 0,\\
-\mathrm{Y}_2 c - 8\mathrm{Y}_2\eta k^3\Lambda^2 - \alpha_0\mathrm{Y}_2\Omega k - \tfrac{1}{2}\mathrm{Y}_1^2\Omega k + \mathrm{Y}_2 k\varrho = 0,\\
2\mathrm{Y}_1\eta k^3\Lambda^2 - \mathrm{Y}_1\mathrm{Y}_2\Omega k = 0,\\
6\mathrm{Y}_2\eta k^3\Lambda^2 - \tfrac{1}{2}\mathrm{Y}_2^2\Omega k = 0,\\
-\alpha_1 c - 2\alpha_1\eta k^3\Lambda^2 - \alpha_2\mathrm{Y}_1\Omega k - \alpha_0\alpha_1\Omega k + \alpha_1 k\varrho = 0,\\
-\alpha_2 c - 8\alpha_2\eta k^3\Lambda^2 - \tfrac{1}{2}\alpha_1^2\Omega k - \alpha_0\alpha_2\Omega k + \alpha_2 k\varrho = 0,\\
2\alpha_1\eta k^3\Lambda^2 - \alpha_1\alpha_2\Omega k = 0,\\
6\alpha_2\eta k^3\Lambda^2 - \tfrac{1}{2}\alpha_2^2\Omega k = 0.
\end{cases} \tag{15}$$

Via Software, we solve (15) and obtain following sets of non-trivial solutions of the suggested model with $\alpha_1 = \mathrm{Y}_1 = 0$ for each case as:

1. $\alpha_0 = \frac{3(k\varrho - c)}{\Omega k}, \ \alpha_2 = -\frac{3(k\varrho - c)}{\Omega k}, \ \mathrm{Y}_2 = 0, \ \Lambda = -\frac{\sqrt{c - k\varrho}}{2\sqrt{\eta}k^{3/2}}.$
2. $\alpha_0 = \frac{3(k\varrho - c)}{\Omega k}, \ \alpha_2 = 0, \ \mathrm{Y}_2 = -\frac{3(k\varrho - c)}{\Omega k}, \ \Lambda = -\frac{\sqrt{c - k\varrho}}{2\sqrt{\eta}k^{3/2}}.$
3. $\alpha_0 = \frac{3(k\varrho - c)}{2\Omega k}, \ \alpha_2 = -\frac{3(k\varrho - c)}{4\Omega k}, \ \mathrm{Y}_2 = -\frac{3(k\varrho - c)}{4\Omega k}, \ \Lambda = -\frac{\sqrt{c - k\varrho}}{4\sqrt{\eta}k^{3/2}}.$
4. $\alpha_0 = \frac{k\varrho - c}{2\Omega k}, \ \alpha_2 = \frac{3(k\varrho - c)}{4\Omega k}, \ \mathrm{Y}_2 = \frac{3(k\varrho - c)}{4\Omega k}, \ \Lambda = -\frac{i\sqrt{c - k\varrho}}{4\sqrt{\eta}k^{3/2}}.$
5. $\alpha_0 = \frac{c - k\varrho}{\Omega k}, \ \alpha_2 = \frac{3(k\varrho - c)}{\Omega k}, \ \mathrm{Y}_2 = 0, \ \Lambda = -\frac{i\sqrt{c - k\varrho}}{2\sqrt{\eta}k^{3/2}}.$
6. $\alpha_0 = \frac{c - k\varrho}{\Omega k}, \ \alpha_2 = 0, \ \mathrm{Y}_2 = \frac{3(k\varrho - c)}{\Omega k}, \ \Lambda = -\frac{i\sqrt{c - k\varrho}}{2\sqrt{\eta}k^{3/2}}.$

After putting the above values of parameters one by one into Eq. (13), we obtain the corresponding solution to each case as:

1. $\mathscr{P}_1(x,t) \ = \ \frac{3(k\varrho - c)}{\Omega k}\left(sech^2\left(\frac{\sqrt{c - k\varrho}\left(\frac{kx^{\beta}}{\beta} - \frac{ct^{\gamma}}{\gamma}\right)}{2\sqrt{\eta}k^{3/2}}\right)\right).$

2. $$\mathscr{P}_2(x,t) = \frac{3(k\varrho-c)}{\Omega k} - \frac{3(k\varrho-c)\coth^2\left(\frac{\sqrt{c-k\varrho}\left(\frac{kx^\beta}{\beta}-\frac{ct^\gamma}{\gamma}\right)}{2\sqrt{\eta}k^{3/2}}\right)}{\Omega k}.$$

3. $$\begin{cases} \mathscr{P}_3(x,t) = \frac{3(k\varrho-c)}{2\Omega k}\left(1-\frac{\tanh^2\left(\frac{\sqrt{c-k\varrho}\left(\frac{kx^\beta}{\beta}-\frac{ct^\gamma}{\gamma}\right)}{4\sqrt{\eta}k^{3/2}}\right)}{2}\right) \\ \quad -\frac{3(k\varrho-c)}{2\Omega k}\left(\frac{\coth^2\left(\frac{\sqrt{c-k\varrho}\left(\frac{kx^\beta}{\beta}-\frac{ct^\gamma}{\gamma}\right)}{4\sqrt{\eta}k^{3/2}}\right)}{2}\right) \end{cases}.$$

4. $$\begin{cases} \mathscr{P}_4(x,t) = \frac{k\varrho-c}{2\Omega k}\left(1-\frac{3\tan^2\left(\frac{\sqrt{c-k\varrho}\left(\frac{kx^\beta}{\beta}-\frac{ct^\gamma}{\gamma}\right)}{4\sqrt{\eta}k^{3/2}}\right)}{2}\right) \\ \quad -\frac{k\varrho-c}{2\Omega k}\left(\frac{3\cot^2\left(\frac{\sqrt{c-k\varrho}\left(\frac{kx^\beta}{\beta}-\frac{ct^\gamma}{\gamma}\right)}{4\sqrt{\eta}k^{3/2}}\right)}{2}\right) \end{cases}.$$

5. $$\mathscr{P}_5(x,t) = \frac{c-k\varrho}{\Omega k} - \frac{3(k\varrho-c)\tan^2\left(\frac{\sqrt{c-k\varrho}\left(\frac{kx^\beta}{\beta}-\frac{ct^\gamma}{\gamma}\right)}{2\sqrt{\eta}k^{3/2}}\right)}{\Omega k}.$$

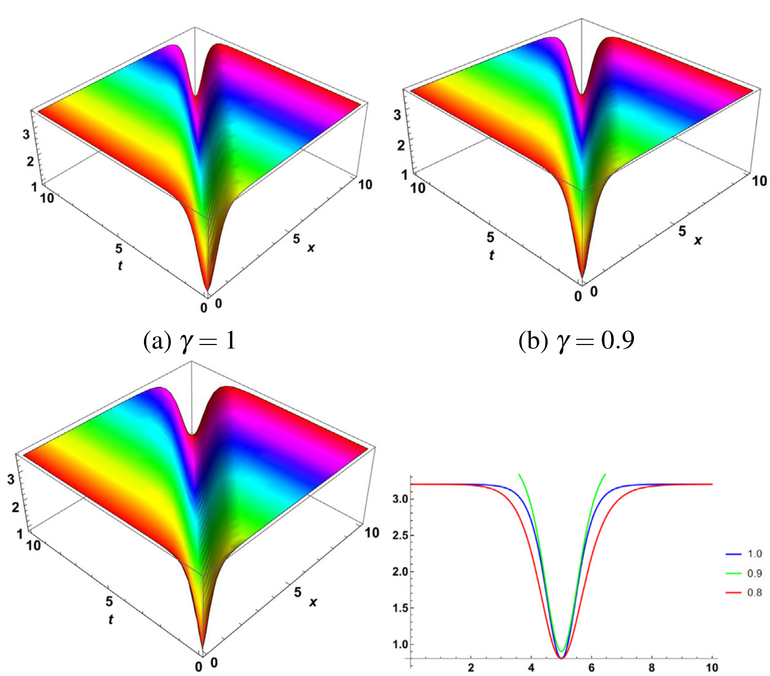
6. $$\mathscr{P}_6(x,t) = \frac{c-k\varrho}{\Omega k} - \frac{3(k\varrho-c)\cot^2\left(\frac{\sqrt{c-k\varrho}\left(\frac{kx^\beta}{\beta}-\frac{ct^\gamma}{\gamma}\right)}{2\sqrt{\eta}k^{3/2}}\right)}{\Omega k}.$$

3.0.2. Simulations and Discussion

In this section, we present numerical simulations of six exact solutions $[\psi_1(x,t) - \psi_6(x,t)]$ obtained through the extended tanh technique. These solutions exhibit various wave behaviors, including dark soliton, singular solutions, hyperbolic traveling wave solutions, and singular periodic type solutions. We consider $\beta = 1$ in the numerical simulations. The exact solution $\psi_1(x,t)$ is shown in Fig. 1 with varying fractional order γ, which displays dark soliton behavior. It should be noted that γ is inversely proportional to the separation of the dark soliton. Fig. 2 demonstrates the physical behavior of exact solution $\psi_2(x,t)$, where singular solutions are observed. Here, decreasing γ increases the gap between the waves. To further clarify, the convex hyperbolic traveling waves in $\psi_3(x,t)$ refer to wave shapes that are arched and bulging upwards, while the concave hyperbolic traveling waves in $\psi_4(x,t)$ are shaped like an inverted arch, bulging downwards. As the fractional order γ decreases, both the number and amplitude of these waves in $\psi_3(x,t)$ and $\psi_4(x,t)$ decrease, meaning that there are fewer and smaller waves. Moreover, the two singular solitons in $\psi_5(x,t)$ refer to wave patterns that resemble spikes or peaks, with only one peak present at higher values of γ, and two peaks at lower values of γ. As γ decreases, the distance between these two peaks increases, causing the periodic waves to move farther apart. (see Figs. 3–5).

4. Generalized Kudryashov Method

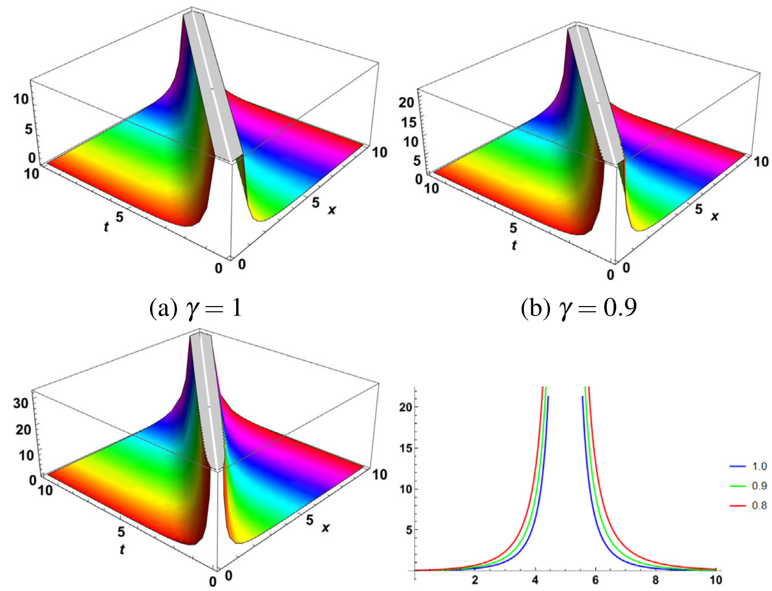
In determining the analytical soliton solutions to the NLEEs, the generalized Kudryashov (GK) technique is important and useful. In order to construct typical and diverse exact solu-



(a) $\gamma = 1$ (b) $\gamma = 0.9$

(c) $\gamma = 0.8$ (d) $\gamma = 1, 0.9, 0.8$

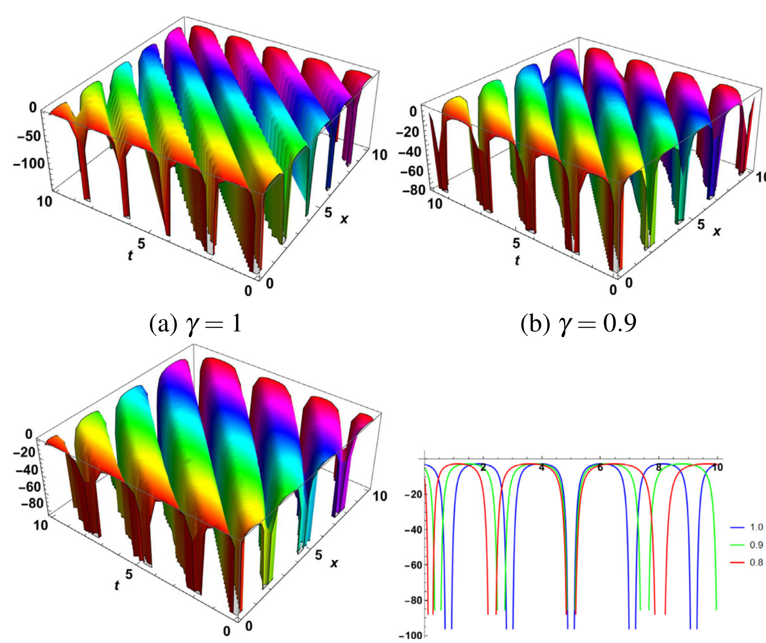
Fig. 1 Graphs of $\psi_1(x,t)$ for $k = 1, \eta = 0.1, \varrho = 0.1, \Omega = 1, c = 1$, presents the dark soliton.



(a) $\gamma = 1$ (b) $\gamma = 0.9$

(c) $\gamma = 0.8$ (d) $\gamma = 1, 0.9, 0.8$

Fig. 2 Graphs of $\psi_2(x,t)$ for $\mathtt{k} = 1, \eta = 0.6, \varrho = 0.1, \Omega = 1, \mathtt{c} = 1$, portrays the singular solution.



(a) $\gamma = 1$ (b) $\gamma = 0.9$

(c) $\gamma = 0.8$ (d) $\gamma = 1, 0.9, 0.8$

Fig. 3 Graphs of $\psi_3(x,t)$ for $\mathtt{k} = 1, \eta = -0.1, \varrho = 0.1, \Omega = 1, \mathtt{c} = 1$,demonstrates the convex hyperbolic waves.

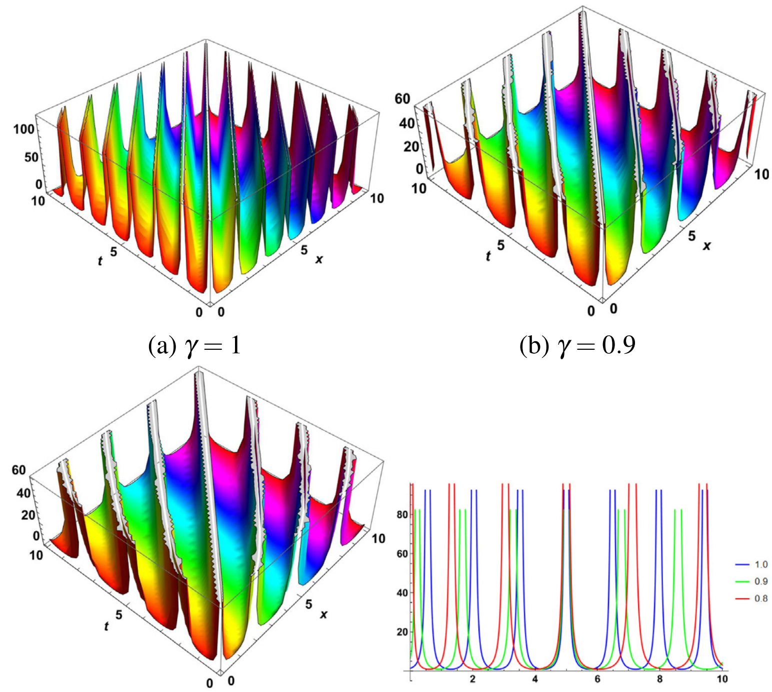
tions to NLEEs in terms of conformable in space and time, we present the general procedure of GK in this section. The solution general form is evaluated in accordance with GK method as follows:

$$\mathscr{P}(t) = \frac{\xi_0 + \sum_{r=1}^{s} \xi_r \mathscr{V}^r(\zeta)}{\varpi_0 + \sum_{r=1}^{\psi} \varpi_r \mathscr{V}^r(\zeta)}, \tag{16}$$

where s and $m \in Z^+, \xi_r(r = 1, 2, 3, \cdots, s)$ and $\varpi_r(r = 1, 2, 3, \cdots, m)$ are undetermined coefficients which are to be determined later and ζ is presented in Eq. (9). Further we have

$$\mathscr{V}(\zeta) = \frac{1}{1 + \mathscr{A} exp(\zeta)}, \tag{17}$$

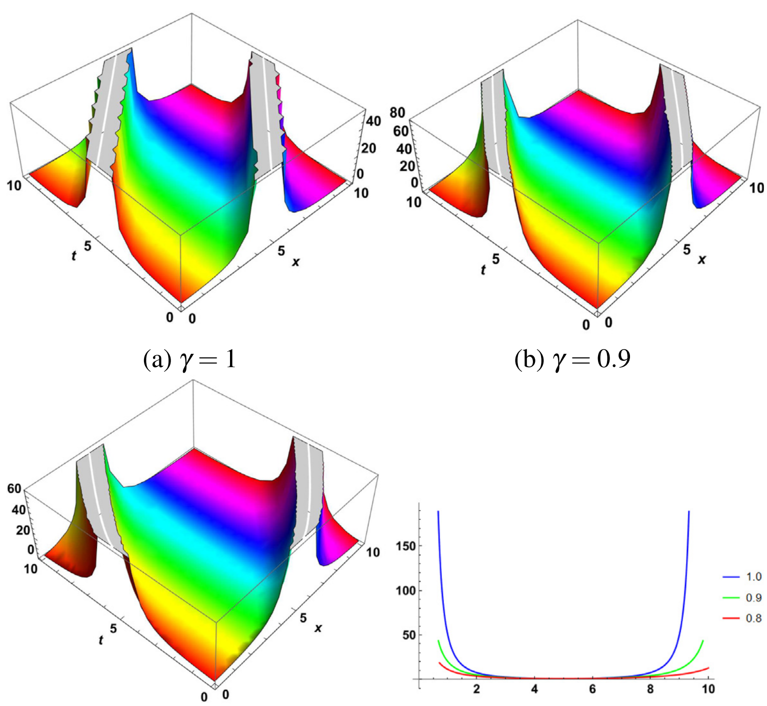
where $\mathscr{A}$ represents the integral constant and $\mathscr{V}(\zeta)$ is the general solution of the Riccati equation as



(a) $\gamma = 1$ (b) $\gamma = 0.9$

(c) $\gamma = 0.8$ (d) $\gamma = 1, 0.9, 0.8$

Fig. 4 Graphs of $\psi_4(x,t)$ for $\mathtt{k} = 1, \eta = 0.05, \varrho = 0.1, \Omega = 1, \mathtt{c} = 1$, demonstrates the concave hyperbolic waves.



(a) $\gamma = 1$ (b) $\gamma = 0.9$

(c) $\gamma = 0.8$ (d) $\gamma = 1, 0.9, 0.8$

Fig. 5 Graphs of $\psi_5(x,t)$ for $\mathtt{k} = 1, \eta = 2, \varrho = 0.1, \Omega = 1, \mathtt{c} = 1$, presents the two singular solitons.

$$\mathscr{V}'(\zeta) = \mathscr{V}^2(\zeta) - \mathscr{V}(\zeta), \tag{18}$$

where "′" represents the ordinary derivative with respect to ζ. With the help of homogeneous balance principle in Eq. (8), on can obtain the values of s and m. Then inserting solution (16) along with Eq. (18) into Eq. (8) a polynomial in the powers of $\mathscr{V}(\zeta)$ can be obtained. Further equating various powers of $\mathscr{V}(\zeta)$ into zero an algebraic system can be obtained. After solving the algebraic system and obtaining the values of ξ_r and ϖ_r and other parameters present in the considered model one may obtain exact solutions.

4.0.3. New solutions of conformable geophysical KdV equation with GK method

Here we apply the suggested GK method to the conformable perturbed KdV Eq. (3) to obtain different soliton solutions. Therefore, first we need to calculate the values of s and m. In Eq. (12), we balance the highest power of nonlinear term and the highest order derivative so we can write

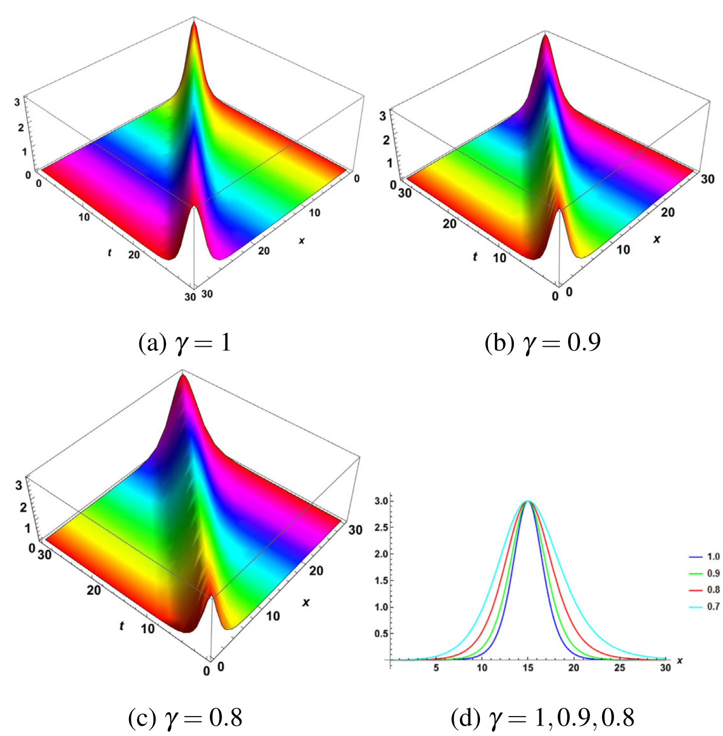
$$s = m + 2, \tag{19}$$

where m is a nonzero free parameter. Particularly, when $m = 1$, we obtain from Eq. (19), that $m = 3$. So the solution (16) turns out to be in the form

$$\mathscr{V}(\zeta) = \frac{\xi_0 + \xi_1\mathscr{V}(\zeta) + \xi_2\mathscr{V}^2(\zeta) + \xi_3\mathscr{V}^3(\zeta)}{\varpi_0 + \varpi_1\mathscr{V}(\zeta)}, \tag{20}$$

where $\xi_0, \xi_1, \xi_2, \xi_3, \varpi_0$ and $\varpi_1$are to be determined later. Now substituting Eq. (20) along with Eq. (18) into Eq. (8) one get the following polynomials in different powers of $\mathscr{V}(\zeta)$ in the form

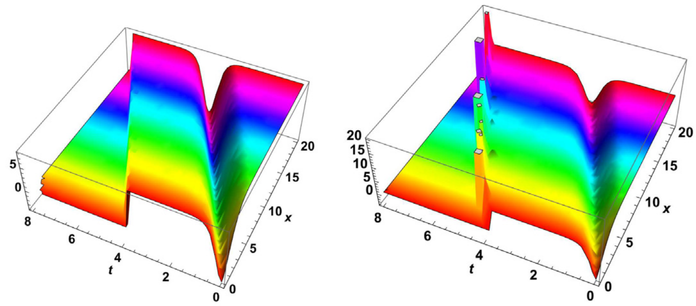
$$\begin{aligned}
&-\frac{c(\xi_1\mathscr{V}(\zeta)+\xi_2\mathscr{V}(\zeta)^2+\xi_3\mathscr{V}(\zeta)^3+\xi_0)}{\varpi_1\mathscr{V}(\zeta)+\varpi_0} - \eta k^3 [\frac{1}{(\varpi_1\mathscr{V}(\zeta)+\varpi_0)^3}\\
&(2\varpi_1^2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))^2(\xi_1\mathscr{V}(\zeta)+\xi_2\mathscr{V}(\zeta)^2+\xi_3\mathscr{V}(\zeta)^3+\xi_0))\\
&-\frac{\varpi_1}{(\varpi_1\mathscr{V}(\zeta)+\varpi_0)^2}-\mathscr{V}(\zeta)^2+2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))\mathscr{V}(\zeta)+\mathscr{V}(\zeta)(\xi_1\mathscr{V}(\zeta)+\xi_2\mathscr{V}(\zeta)^2+\xi_3\mathscr{V}(\zeta)^3+\xi_0)\\
&-2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))(\xi_1(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta)))) \quad -2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))(2\xi_2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))\mathscr{V}(\zeta))\\
&-2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))(3\xi_3(\mathscr{V}(\zeta)^4-\mathscr{V}(\zeta)^3)))\\
&+\frac{\xi_1}{\varpi_1\mathscr{V}(\zeta)+\varpi_0}(-\mathscr{V}(\zeta)^2+2(\mathscr{V}(\zeta)^3-\mathscr{V}(\zeta)^2)+\mathscr{V}(\zeta))+2\xi_2\mathscr{V}(\zeta)^2\\
&-\mathscr{V}(\zeta)^2+2\xi_2(-\mathscr{V}(\zeta)^2+2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))\mathscr{V}(\zeta)+\mathscr{V}(\zeta))\mathscr{V}(\zeta)\\
&+3\xi_3(-\mathscr{V}(\zeta)^2+2(\mathscr{V}(\zeta)^2-\mathscr{V}(\zeta))\mathscr{V}(\zeta)+\mathscr{V}(\zeta))\mathscr{V}(\zeta)^2+6\xi_3\mathscr{V}(\zeta)^2\\
&-\mathscr{V}(\zeta)^2\mathscr{V}(\zeta)-\frac{\Omega k(\xi_1\mathscr{V}(\zeta)+\xi_2\mathscr{V}(\zeta)^2+\xi_3\mathscr{V}(\zeta)^3+\xi_0)^2}{2(\varpi_1\mathscr{V}(\zeta)+\varpi_0)^2}+\frac{k\varrho(\xi_1\mathscr{V}(\zeta)+\xi_2\mathscr{V}(\zeta)^2+\xi_3\mathscr{V}(\zeta)^3+\xi_0)}{\varpi_1\mathscr{V}(\zeta)+\varpi_0}=0.
\end{aligned} \tag{21}$$



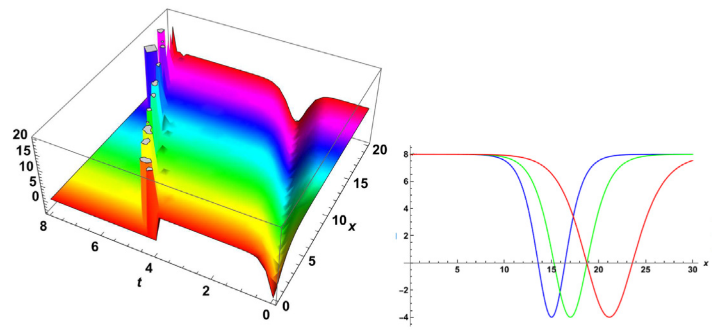
(a) $\gamma = 1$ (b) $\gamma = 0.9$

(c) $\gamma = 0.8$ (d) $\gamma = 1, 0.9, 0.8$

Fig. 6 Dynamics of $\psi_{1GK}(x,t)$ for $k = 1, \eta = 1, \varrho = 2, \Omega = 1, c = 1, \mathscr{A} = 1$, showing the bright soliton.

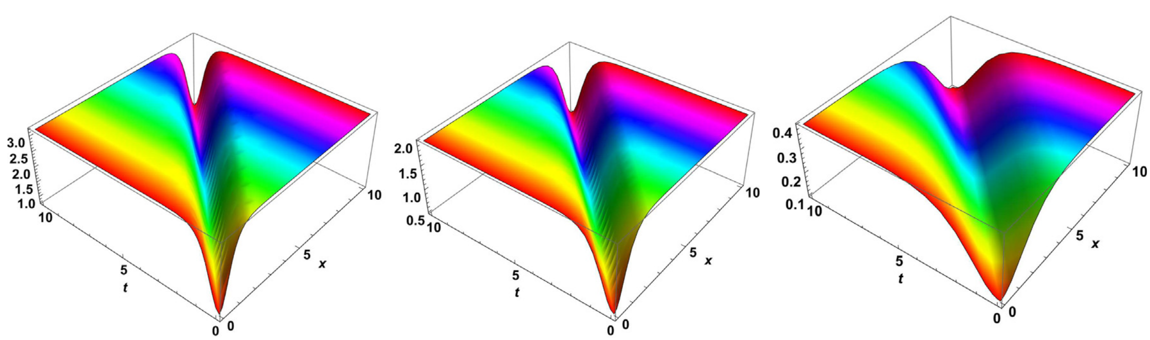


(a) $\gamma = 1$ (b) $\gamma = 0.9$



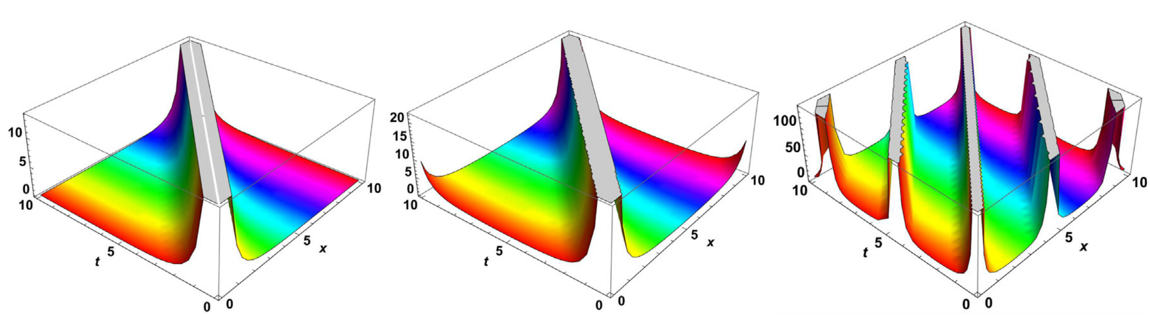
(c) $\gamma = 0.8$ (d) $\gamma = 1, 0.9, 0.8$

Fig. 7 Hybrid solitonic behavior of $\psi_{2GK}(x,t)$ for $\Bbbk = 1, \eta = 4, \varrho = 1, \Omega = -1, \mathtt{c} = 1, \mathscr{A} = 2,$.



(a) $\rho = 0.2$ (b) $\rho = 0.5$ (c) $\rho = 0.9$

Fig. 8 Affects of ϱ on exact solution $\psi_1(x,t)$ with $\Bbbk = 1, \eta = 0.1, \gamma = 1, \Omega = 1, \mathtt{c} = 1$.



(a) $\rho = 0.2$ (b) $\rho = 1.2$ (c) $\gamma = 2.2$

Fig. 9 Affects of ϱ on exact solution $\psi_2(x,t)$ with $\Bbbk = 1, \eta = 0.6, \gamma = 0.99, \Omega = 1, \mathtt{c} = 1$.

After putting the coefficients of distinct exponents of $\mathscr{V}(\zeta)$ to zero in Eq. (21), one can obtain:

$$
\begin{gathered}
-2c\xi_0\varpi_0^2+\Omega(-k)\xi_0^2\varpi_0+2k\xi_0\varpi_0^2\varrho=0,\\
-4c\xi_0\varpi_0\varpi_1-2c\xi_1\varpi_0^2+2\eta k^3\varpi_0(\xi_0\varpi_1-\xi_1\varpi_0)\\
-\Omega k\xi_0^2\varpi_1-2\Omega k\xi_0\xi_1\varpi_0+4k\xi_0\varpi_0\varpi_1\varrho+2k\xi_1\varpi_0^2\varrho=0,\\
-2c\xi_0\varpi_1^2-4c\xi_1\varpi_0\varpi_1-2c\xi_2\varpi_0^2-2\eta k^3\xi_0\\
-2\eta k^3\varpi_0(\xi_0\varpi_1-\xi_1\varpi_0)\varpi_1(2\varpi_0+\varpi_1)+2k\xi_2\varpi_0^2\varrho\\
-8\eta k^3\xi_2\varpi_0^2-2\Omega k\xi_0\xi_1\varpi_1-2\Omega k\xi_0\xi_2\varpi_0+2k\xi_0\varpi_1^2\varrho\\
-\Omega k\xi_1^2\varpi_0+4k\xi_1\varpi_0\varpi_1\varrho+2\eta k^3\xi_1\varpi_0(2\varpi_0+\varpi_1)=0,\\
2k\xi_1\varpi_1^2\varrho-2c\xi_1\varpi_1^2-2c\xi_3\varpi_0^2+2\eta k^3\xi_0\varpi_1(2\varpi_0+\varpi_1)\\
-2\Omega k\xi_1\xi_2\varpi_0+6\eta k^3\varpi_0(2\xi_2\varpi_0-\xi_2\varpi_1-3\xi_3\varpi_0)\\
+8\eta k^3\xi_2\varpi_0^2-2\Omega k\xi_0\xi_2\varpi_1-4c\xi_2\varpi_0\varpi_1+4k\xi_2\varpi_0\varpi_1\varrho\\
+2k\xi_3\varpi_0^2\varrho-2\eta k^3\xi_1\varpi_0(2\varpi_0+\varpi_1)-2\Omega k\xi_0\xi_3\varpi_0\\
-\Omega k\xi_1^2\varpi_1=0,\\
-2c\xi_2\varpi_1^2-4c\xi_3\varpi_0\varpi_1-6\eta k^3\varpi_0(2\xi_2\varpi_0-\xi_2\varpi_1-3\xi_3\varpi_0)\\
+2\eta k^3\xi_3\varpi_0(12\varpi_0-11\varpi_1)-2\Omega k\xi_0\xi_3\varpi_1-2\Omega k\xi_1\xi_2\varpi_1\\
-2\Omega k\xi_1\xi_3\varpi_0-\Omega k\xi_2^2\varpi_0+2\eta k^3\xi_2\varpi_1(6\varpi_0-\varpi_1)\\
+2k\xi_2\varpi_1^2\varrho+4k\xi_3\varpi_0\varpi_1\varrho=0,\\
-2c\xi_3\varpi_1^2-2\eta k^3\xi_2\varpi_1(6\varpi_0-\varpi_1)+4\eta k^3\xi_2\varpi_1^2\\
+32\eta k^3\xi_3\varpi_0\varpi_1-2\eta k^3\xi_3\varpi_0(12\varpi_0-11\varpi_1)-8\eta k^3\xi_3\varpi_1^2\\
-2\Omega k\xi_1\xi_3\varpi_1-\Omega k\xi_2^2\varpi_1-2\Omega k\xi_2\xi_3\varpi_0+2k\xi_3\varpi_1^2\varrho=0,\\
-4\eta k^3\xi_2\varpi_1^2-32\eta k^3\xi_3\varpi_0\varpi_1+20\eta k^3\xi_3\varpi_1^2\\
-2\Omega k\xi_2\xi_3\varpi_1-\Omega k\xi_3^2\varpi_0=0,\\
-12\eta k^3\xi_3\varpi_1^2-\Omega k\xi_3^2\varpi_1=0,
\end{gathered}
\tag{22}
$$

solving the above system of algebraic equation we obtain the following different cases of parameters values

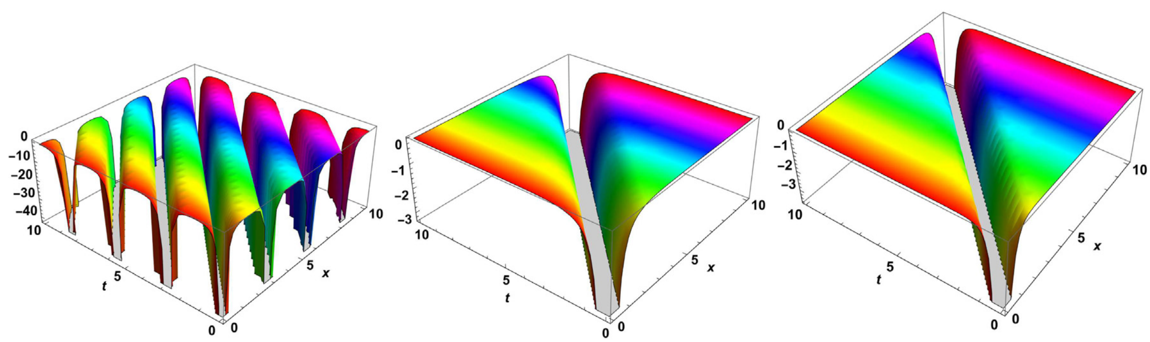
1. $c=k\varrho-\eta k^3,\ \xi_0=0,\ \xi_1=\frac{12\eta k^2\xi_0}{\Omega},$
   $\xi_2=-\frac{12\left(\eta k^2\xi_0-\eta k^2\varpi_1\right)}{\Omega},\ \xi_3=-\frac{12\eta k^2\varpi_1}{\Omega}.$
2. $c=k\left(\eta k^2+\varrho\right),\ \xi_0=-\frac{2\eta k^2\xi_0}{\Omega},\ \xi_1=\frac{2\left(6\eta k^2\xi_0-\eta k^2\varpi_1\right)}{\Omega},$
   $\xi_2=-\frac{12\left(\eta k^2\xi_0-\eta k^2\varpi_1\right)}{\Omega},\ \xi_3=-\frac{12\eta k^2\varpi_1}{\Omega}.$

The solutions obtained by putting the above values in Eq. (16) are presented as follows.

1. $$\psi_{1GK}(x,t)=\frac{12\eta k^2\mathscr{A}\left(sinh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma\left(k\varrho-\eta k^3\right)}{\gamma}\right)+cosh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma\left(k\varrho-\eta k^3\right)}{\gamma}\right)\right)}{\Omega\left(\mathscr{A}sinh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma(k\varrho-\eta k^3)}{\gamma}\right)+\mathscr{A}cosh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma(k\varrho-\eta k^3)}{\gamma}\right)+1\right)^2}.\quad ,$$
2. $$\begin{aligned}\psi_{2GK}(x,t)=&-\frac{2\eta k^2\left((-\mathscr{A}^2+4\mathscr{A})sinh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma\left(k\varrho-\eta k^3\right)}{\gamma}\right)\right)}{\Omega\left(-\mathscr{A}sinh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma(k\varrho-\eta k^3)}{\gamma}\right)+\mathscr{A}cosh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma(k\varrho-\eta k^3)}{\gamma}\right)+1\right)^2}\\&+\frac{2\eta k^2\left((\mathscr{A}^2-4\mathscr{A})cosh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma\left(k\varrho-\eta k^3\right)}{\gamma}\right)\right)}{\Omega\left(-\mathscr{A}sinh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma(k\varrho-\eta k^3)}{\gamma}\right)+\mathscr{A}cosh\left(\frac{kx^\beta}{\beta}-\frac{t^\gamma(k\varrho-\eta k^3)}{\gamma}\right)+1\right)^2}.\end{aligned}\quad .$$

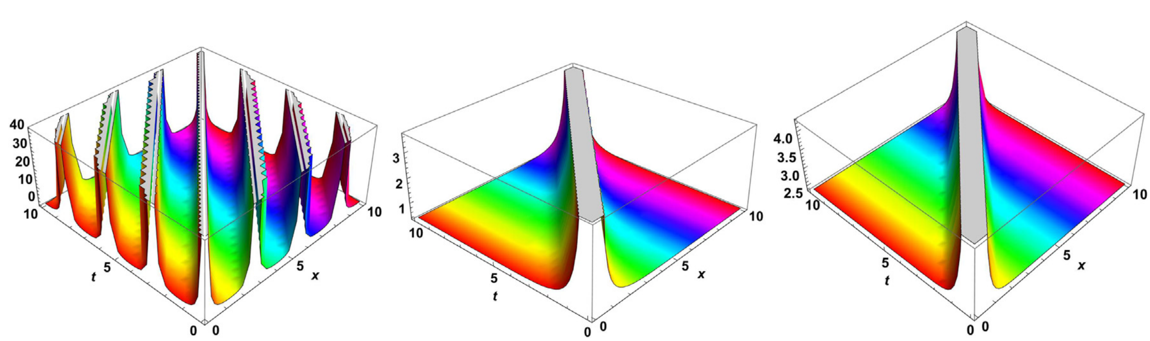
4.1. Simulations and Discussion

In this section, we provide graphical representations of the exact solutions $\psi_{1GK}(x,t)$ and $\psi_{2GK}(x,t)$ for confirmable pKdV Eq.(3), while considering different values of γ and fixing $\beta=1$. We employ the generalized GK method, which allows us to observe soliton and hybrid type solutions. Fig. 6 displays the dynamics of the exact solution $\psi_{1GK}(x,t)$ with varying γ,



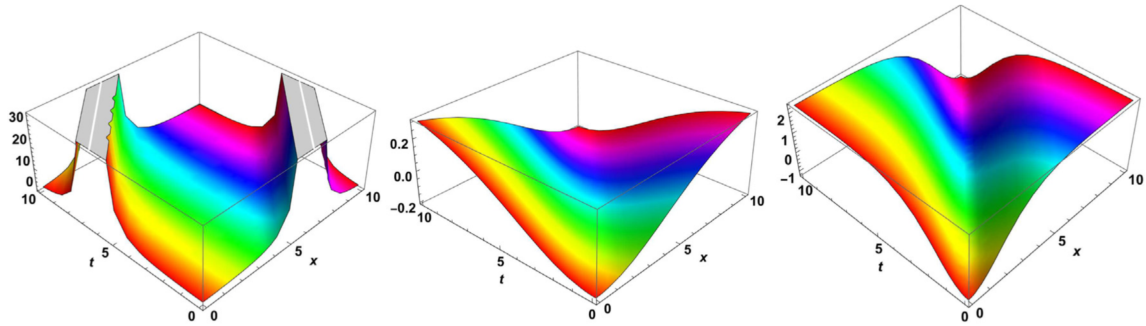
(a) $\rho=0.5$ (b) $\rho=1.2$ (c) $\rho=2.2$

Fig. 10 Affects of ϱ on exact solution $\psi_3(x,t)$ with $\mathtt{k}=1,\eta=-0.1,\gamma=1,\Omega=1,\mathtt{c}=1$.



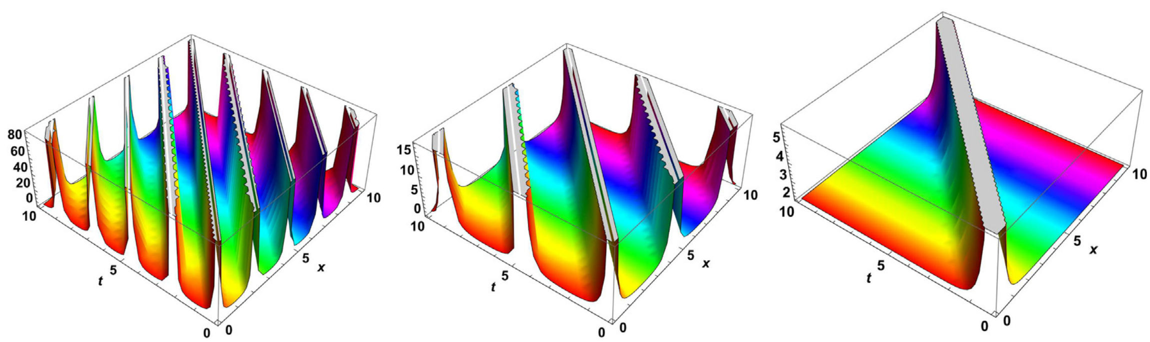
(a) $\rho=0.5$ (b) $\rho=1.2$ (c) $\rho=2.2$

Fig. 11 Affects of ϱ on exact solution $\psi_4(x,t)$ with $\mathtt{k}=1,\eta=0.1,\gamma=1,\Omega=1,\mathtt{c}=1$.



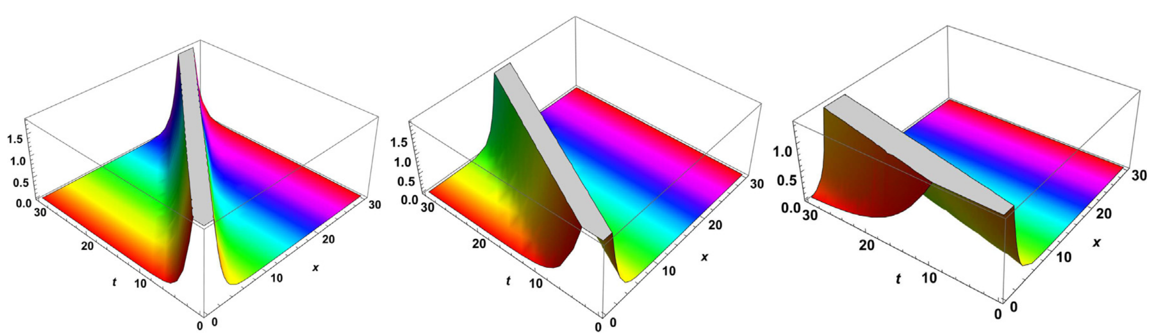
(a) $\rho = 0.5$ (b) $\rho = 1.2$ (c) $\rho = 2.2$

Fig. 12 Affects of ϱ on exact solution $\psi_5(x,t)$ with $\mathtt{k} = 1, \eta = 2, \gamma = 1, \Omega = 1, \mathtt{c} = 1$.



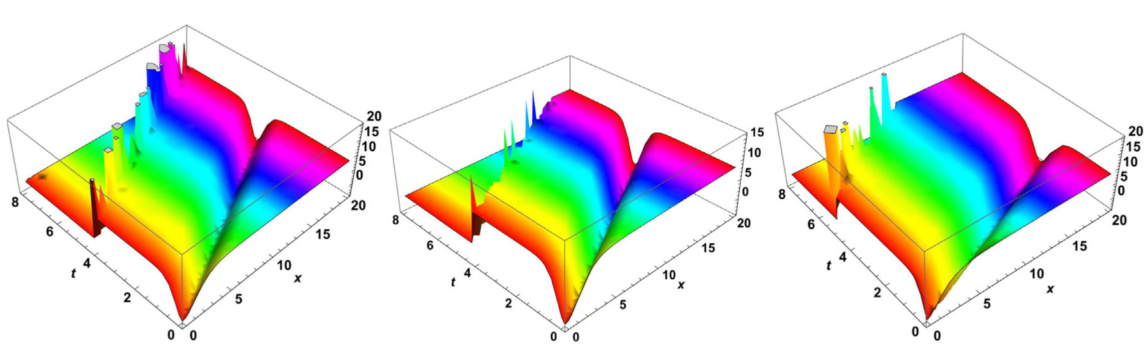
(a) $\rho = 0.2$ (b) $\rho = 0.8$ (c) $\rho = 2$

Fig. 13 Affects of ϱ on exact solution $\psi_6(x,t)$ with $\mathtt{k} = 1, \eta = 0.1, \gamma = 1, \Omega = 1, \mathtt{c} = 1$.



(a) $\rho = 1.9$ (b) $\rho = 1.5$ (c) $\rho = 1.2$

Fig. 14 Affects of ϱ on exact solution $\psi_{1GK}(x,t)$ with $\mathtt{k} = 1, \eta = 1, \gamma = 1, \Omega = 1, \mathtt{c} = 1, \mathscr{A} = 1$.



(a) $\rho = 0.8$ (b) $\rho = 0.5$ (c) $\rho = 0.2$

Fig. 15 Affects of ϱ on exact solution $\psi_{2GK}(x,t)$ with $\mathtt{k} = 1, \eta = 4, \gamma = 1, \Omega = -1, \mathtt{c} = 1, \mathscr{A} = 2$.

revealing an increase in the separation between waves at small fractional order. Similarly, Fig. 6 illustrates the behavior of the exact solution $\psi_{2GK}(x,t)$ with varying γ, where the hybrid solution exhibits a kink and a dark soliton when $\gamma = 1$. Upon decreasing the value of γ, we observe a breather wave on top of the kink while the dark soliton moves away from its initial position. The use of the generalized GK method allows us to gain insights into the behavior of these solutions and their dependence on the value of γ. (see Fig. 7).

5. Dynamics of the exact solutions with varying Coriolis effect

The purpose of this part of the article is to observe the impact of the Coriolis effect ϱ on the evolution of different exact solutions obtained with both extended tanh and generalized GK method. In Fig. 8 $\psi_1(x,t)$ is presented with varying ϱ, where we see that with increase in Coriolis effect increase the separation of the wave. Further, in Fig. 9 $\psi_2(x,t)$ is presented with varying ϱ, where increase in ϱ shows the transition from one to many hyperbolic waves. Fig. 10 presents $\psi_3(x,t)$, here we see that increasing ϱ deacreses the number of hyperbolic waves to single hyperbolic wave. Also Fig. 11 and 13 depicts the dynamics of $\psi_4(x,t)$ and $\psi_6(x,t)$ respectively where it can be observed that the Coriolis effect ϱ plays important rule in reduction of the waves from many to one. Furthermore, Fig. 12 presents $\psi_5(x,t)$, where the transion from two periodic to one dark soliton solution is observed. In the similar way, Fig. 14 and 15 demonstrates the physical behaviour of the solutions $\psi_{1GK}(x,t)$ and $\psi_{2GK}(x,t)$ respectively with varying ϱ.

6. Conclusion

The conformable operators have been extensively used by mathematical physicists in the analysis of solitary wave solutions of nonlinear PDEs. In this work, conformable operators were applied to study the gpKdV equation, which is the generalization of geophysical and standard KdV equations. Two reliable and easy analytical methods, the extended tanh-method and generalized GK method, were used to extract some novel solutions of the considered gpKdV equation in the form of trigonometric hyperbolic functions. The acquired solutions were simulated via 2D and 3D graphs using Mathematica. Important solutions such as bright and dark solitons, singular solutions, hyperbolic travelling wave solutions, singular periodic type solutions, and other hybrid solutions were observed. The Coriolis effect on the dynamics of the travelling wave solutions was also shown. Furthermore, the conformable operator's great effect on the solitons behavior was demonstrated. Variation in the wave solutions was observed by changing the space and time fractional order.

Future directions in this area of research could involve the application of the conformable operator to study other nonlinear PDEs, as well as exploring new analytical methods for extracting novel solutions. Additionally, investigating the physical implications of the observed solutions and their potential practical applications in fields such as oceanography, coastal engineering, and fluid mechanics could be an interesting direction for future research. Finally, incorporating experimental data and comparing the results of simulations with real-world observations could further validate the effectiveness of the conformable operator in analyzing wave behavior.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] P. Tandel, H. Patel, T. Patel, Tsunami wave propagation model: A fractional approach, J. Ocean Eng. Sci. 7 (6) (2022) 509–520.
[2] Y. Wang, Z. Tang, L. Wang, X. Wang, Linear and quadratic damping coefficients of a single module of a very large floating structure over variable bathymetry: Physical and numerical free-decay experiments, J. Ocean Eng. Sci. 7 (6) (2022) 607–618.
[3] A.M. Wazwaz, Partial differential equations and solitary waves theory, Springer Science & Business Media, 2010.
[4] N. Cheemaa, A.R. Seadawy, T.G. Sugati, D. Baleanu, Study of the dynamical nonlinear modified Korteweg-de Vries equation arising in plasma physics and its analytical wave solutions, Result Phy. 19 (2020) 103480.
[5] A. Ankiewicz, Mahyar Bokaeeyan, N. Akhmediev, Shallow-water rogue waves: An approach based on complex solutions of the Korteweg-de Vries equation, Phys. Rev. E 99 (2019) 050201 (R).
[6] A. Geyer, R. Quirchmayr, Shallow water equations for equatorial tsunami waves, Phil. Trans. R. Soc. A 376 (2017) 20170100.
[7] S.T.R. Rizvi, A.R. Seadawy, F. Ashraf, M. Younis, H. Iqbal, D. Baleanu, Lump and Interaction solutions of a geophysical Korteweg-de Vries equation, Results Phys. 19 (2020) 103661.
[8] K. Hosseini1, A. Akbulut, D. Baleanu, S. Salahshour, M. Mirzazadeh, L. Akinyemi, The geophysical KdV equation: its solitons, complexiton, and conservation laws, GEM Int. J. Geomath., 13, (2022) 12.
[9] M. Alquran, R. Alhami, Analysis of lumps, single-stripe, breather-wave, and two-wave solutions to the generalized perturbed-KdV equation by means of Hirota's bilinear method, Nonlinear Dyn. 109 (2022) 1985–1992.
[10] S. Saifullah, S. Ahmad, M.A. Alyami, M. Inc, Analysis of interaction of lump solutions with kink-soliton solutions of the generalized perturbed KdV equation using Hirota-bilinear approach, Phys. lett., A 454 (2022) 128503.
[11] A. Khan, T. Akram, A. Khan, S. Ahmad, K. Nonlaopon, Investigation of time fractional nonlinear KdV-Burgers equation under fractional operators with nonsingular kernels, AIMS Math. 8 (1) (2023) 1251–1268.
[12] M. Arfan, K. Shah, A. Ullah, M. Shutaywi, P. Kumam, Z. Shah, On fractional order model of tumor dynamics with drug interventions under nonlocal fractional derivative, Results Phys. 21 (2021) 103783.
[13] Kavitha Williams, W., V. Vijayakumar, R. Udhayakumar, Sumati Kumari Panda, and Kottakkaran Sooppy Nisar.

Existence and controllability of nonlocal mixed Volterra-Fredholm type fractional delay integro-differential equations of order $1<r<2$. Numerical Methods for Partial Differential Eqs. (2020).

[14] Yong Zhou, V. Vijayakumar, C. Ravichandran, R. Murugesu, Controllability results for fractional order neutral functional differential inclusions with infinite delay, Fixed Point Theory 18 (2) (2017) 773–798.

[15] K. Kavitha, V. Vijayakumar, R. Udhayakumar, C. Ravichandran, Results on controllability of Hilfer fractional differential equations with infinite delay via measures of noncompactness, Asian J. Control 24 (3) (2022) 1406–1415.

[16] M. Raja, V. Mohan, R. Udhayakumar Vijayakumar, Yong Zhou, A new approach on the approximate controllability of fractional differential evolution equations of order $1<r<2$ in Hilbert spaces, Chaos, Solitons & Fractals 141 (2020) 110310.

[17] Samiha Belmor, Chokkalingam Ravichandran, Fa.hd. Jarad, Nonlinear generalized fractional differential equations with generalized fractional integral conditions, Journal of Taibah University for Science 14 (1) (2020) 114–123.

[18] Kasthurisamy Jothimani, Kalimuthu Kaliraj, Sumati Kumari Panda, Kotakkaran Sooppy Nisar, Chokkalingam Ravichandran, Results on controllability of non-densely characterized neutral fractional delay differential system, Evol. Eqs. Control Theory 10 (3) (2021).

[19] Emile Franc Goufo, Chokkalingam Ravichandran Doungmo, Gunvant A. Birajdar, Self-similarity techniques for chaotic attractors with many scrolls using step series switching, Mathematical Modelling and Analysis 26 (4) (2021) 591–611.

[20] Chokkalingam Ravichandran, Kasilingam Munusamy, Kottakkaran Sooppy Nisar, Natarajan Valliammal, Results on neutral partial integrodifferential equations using Monch-Krasnoselskii fixed point theorem with nonlocal conditions, Fractal and Fractional 6 (2) (2022) 75.

[21] C. Xu, W. Zhang, C. Aouiti, Z. Liu, L. Yao, Bifurcation insight for a fractional-order stage-structured predator- prey system incorporating mixed time delays, Mathematical Methods in the Applied Sciences (2023), https://doi.org/10.1002/mma.8981.

[22] C. Xu, D. Mu, Z. Liu, Y. Pang, M. Liao, C. Aouiti, New insight into bifurcation of fractional-order 4D neural networks incorporating two different time delays, Commun. Nonlinear Sci. Numer. Simul. 118 (2023) 107043.

[23] C. Xu, Z. Liu, P. Li, J. Yan, L. Yao, Bifurcation mechanism for fractional-order three-triangle multi-delayed neural networks, Neural Process. Lett. (2022), https://doi.org/10.1007/s11063-022-11130-y.

[24] C. Xu, M. Liao, P. Li, Y. Guo, Z. Liu, Bifurcation properties for fractional order delayed BAM neural networks, Cognitive Computation 13 (2021) 322–356.

[25] Thabet Abdeljawad, On conformable fractional calculus, Journal of computational and Applied Mathematics 279 (2015) 57–66.

[26] Mohammed Al-Refai, Thabet Abdeljawad, Fundamental results of conformable Sturm-Liouville eigenvalue problems, Complexity 2017 (2017).

[27] Abdeljawad, Thabet, Mohammed AL Horani, and Roshdi Khalil. Conformable fractional semigroups of operators. J. Semigroup Theory Appl. 2015 (2015): Article-ID.

[28] Kamal Shah, Thabet Abdeljawad, Fa.hd. Jarad, Qasem Al-Mdallal, On Nonlinear Conformable Fractional Order Dynamical System via Differential Transform Method, Computer Modeling in Engineering & Sciencesm 136 (2) (2023) 1457–1472.

[29] Aziz Khan, Muhammad Sher, Kamal Shah, Thabet Abdeljawad, Existence and Stability Theory of Pantograph Conformable Fractional Differential Problem, Thermal Science 27 (1) (2023) 10.

[30] Q. Feng, A new approach for seeking coefficient function solutions of conformable fractional partial differential equations based on the Jacobi elliptic equation, Chinese J. Phys. 56 (6) (2018) 2817–2828.

[31] Marwan Alquran, Rahaf Alhami, Convex-periodic, kink-periodic, peakon-soliton and kink bidirectional wave-solutions to new established two-mode generalization of Cahn-Allen equation, Results in Physics 34 (2022) 105257.

[32] Marwan Alquran, New interesting optical solutions to the quadratic-cubic Schrodinger equation by using the Kudryashov-expansion method and the updated rational sine-cosine functions, Opt. Quant. Electron. 54 (10) (2022) 666.

[33] Marwan Alquran, New symmetric bidirectional progressive surface-wave solutions to a generalized fourth-order nonlinear partial differential equation involving second-order time-derivative, Journal of Ocean Engineering and Science (2022).

[34] Marwan Alquran, Physical properties for bidirectional wave solutions to a generalized fifth-order equation with third-order time-dispersion term, Results in Physics 28 (2021) 104577.

[35] M. Bilal, J. Ren, Dynamics of exact solitary wave solutions to the conformable time-space fractional model with reliable analytical appraoches, Opt. Quantum Electron. 54 (2022) 40.

[36] S.T.R. Rizvi, A.R. Seadawy, K. Ali, M. Younis, M.A. Ashraf, Multiple lump and rogue wave for time fractional resonant nonlinear Schrödinger equation under parabolic law with weak nonlocal nonlinearity, Opt. Quantum Electron. 54 (2022) 212.

[37] Marwan Alquran, Feras Yousef, Farah Alquran, Tukur A. Sulaiman, Abdullahi Yusuf, Dual-wave solutions for the quadratic-cubic conformable-Caputo time-fractional Klein-Fock-Gordon equation, Mathematics and Computers in Simulation 185 (2021) 62–76.

[38] Fares Bekhouche, Marwan Alquran, Iryna Komashynska, Explicit rational solutions for time-space fractional nonlinear equation describing the propagation of bidirectional waves in low-pass electrical lines, Rom. J. Phys 66 (2021) 114.

[39] Marwan Alquran, Imad Jaradat, Shaher Momani, Dumitru Baleanu, Chaotic and solitonic solutions for a new time-fractional two-mode Korteweg-de Vries equation, Rom. Rep. Phys 72 (3) (2020) 117.

[40] M. Ali Akbar, Abdul-Majid Wazwaz, Forhad Mahmud, Dumitru Baleanu, Ripan Roy, Hemonta Kumar Barman, W. Mahmoud, Mohammed A. Al Sharif, M.S. Osman, Dynamical behavior of solitons of the perturbed nonlinear Schrodinger equation and microtubules through the generalized Kudryashov scheme, Results in Physics 43 (2022) 106079.

[41] U.H.M. Zaman, Mohammad Asif Arefin, M. Ali Akbar, M. Hafiz Uddin, Analytical behavior of soliton solutions to the couple type fractional-order nonlinear evolution equations utilizing a novel technique, Alexandria Eng. J. 61 (12) (2022) 11947–11958.

[42] Most Shewly Aktar, M. Ali Akbar, Kottakkaran Sooppy Nisar, Haifa I. Alrebdi, A. Abdel-Aty, Steeping and dispersive effects analysis of a couple of long-wave equations in dispersive media, Alexandria Eng. J. 61 (12) (2022) 9457–9470.

[43] S.M. Rayhanul Islam, S.M. Shahansha Khan, Yiasir Arafat, M. Ali Akbar, Diverse analytical wave solutions of plasma physics and water wave equations, Results Phys. 40 (2022) 105834.

[44] Khalid K. Ali, M.S. Mehanna, M. Ali Akbar, Approach to a (2 + 1)-dimensional time-dependent date-Jimbo-Kashiwara-Miwa

equation in real physical phenomena, Appl. Comput. Math. 21 (2) (2022) 193–206.
[45] Roshdi Khalil, Mohammed Al Horani, Abdelrahman Yousef, Mohammad Sababheh, A new definition of fractional derivative, J. Comput. Appl. Math. 264 (2014) 65–70.
[46] M. Eslami, H. Rezazadeh, The first integral method for Wu–Zhang system with conformable time-fractional derivative, Calcolo 53 (3) (2016) 475–485.
[47] R. Khalil, M.A. Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, J. Comput. Appl. Math. 264 (2014) 65–70.
[48] A.M. Wazwaz, The extended tanh method for abundant solitary wave solutions of nonlinear wave equations, Appl. Math. Comput. 187 (2007) 1131–1142.
[49] M.A. Arefin, U. Sadiya, M. Inc, M.H. Uddin, Adequate soliton solutions to the space–time fractional telegraph equation and modified third-order KdV equation through a reliable technique, Opt. Quantum Electron 54 (5) (2022) 1–18.
[50] Z. Dahmani, A. Anber, I. Jebril, Solving conformable evolution equations by an extended numerical method, Jordan J. Math. Stat. (JJMS) 15 (2) (2022) 363–380.