THE APPLICATION OF BDW METHOD TO THE PLANE WAVE DIFFRACTION PROBLEM FROM PEC HALF PLANE

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ABSTRACT

THE APPLICATION OF BDW METHOD TO THE PLANE WAVE DIFFRACTION PROBLEM

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Diffraction is a phenomenon by which wavefronts of propagating waves bend in the neighborhood of obstacles. Diffraction around apertures is described approximately by the diffraction wave method. Diffraction problems are among the difficult encountered in optics, and exact rigorous solutions are quite rare.

A new procedure for calculating the scattered fields from a perfectly conducting body is introduced. The boundary diffraction wave method is used to evaluate the scattered field at an observation point after a certain obstacle with an aperture. The scattered field is combination of the diffracted and transmitted fields. The diffracted field is originated from the boundary of the aperture whereas the transmitted field originates from the aperture itself.

The boundary diffraction wave method defines a new vector potential which is associated with any incident scalar wave field, and the integral of this vector potential expressing the field scattered by a perfect conducting screen with an aperture whose dimensions are larger than the wavelength. The Helmholtz-Kirchhoff formula is expressed in terms of the new vector potential to evaluate the disturbance, from certain points Q on a surface S, at an observation point. The method demands that the location of the point Q always depends on the location of the observation point.

Key words: Boundary diffraction wave, Scattered field and perfectly conducting body.

ÖZ

DÜZLEMSEL DALGALARIN DAGILMA PROBLEMİNE BDW METODUNUN UYGULANMASI

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Dağılma, yayılan dalgaların dalga yüzlerinin engellerin mahallinde eğildiği bir fenomendir. Açıklıkların etrafındaki dağılma, dağılma dalga metodu ile yaklaşık olarak betimlenir. Dağılma problemleri optikte karşılaşılan zorlukların arasında yer alır ve tam kesin çözümleri oldukça nadirdir.

Mükemmel iletkenlikteki gövdeden saçılan alanların hesaplanmasına ilişkin yeni bir prosedür ortaya konmaktadır. Bir açıklığa sahip belirli bir engelden sonra bir gözlem noktasında saçılan alan değerlendirmesi yapmak için sınır dağılma dalga metodu kullanılmaktadır. Saçılan alan dağılan ve iletilen alanların bir kombinasyonudur. İletilen alan açıklığın kendisinden kaynaklanmakta iken, saçılan alan açıklığın sınırından kaynaklanmaktadır.

Sınır dağılma dalga metodu herhangi bir tesadüfi sayıl dalga alanı ile ilgili olan yeni bir vektör potansiyeli ile bu vektör potansiyelinin, ölçüleri dalga boyundan daha büyük olan bir açıklığa sahip mükemmel iletkenlikteki ekran ile saçılan alanı ifade eden entegralini tanımlar. Helmholtz-Kirchoff formülü bir gözlem noktasındaki S yüzeyi üzerinde bulunan Q belirli noktalarından bozukluğu değerlendirmek üzere yeni vektör potansiyeli açısından ifade edilmektedir. Metot bir S yüzeyinin üzerinde yer alan Q noktasının yerinin her zaman gözlem noktasına bağlı olmasını gerektirmektedir.

Anahtar kelimeler: Sınır dağılma dalgası, saçılan alan ve mükemmel iletkenlikteki gövde.

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CHAPTER 1

INTRODUCTION

Any monochromatic scalar wave field has a vector potential associated with it, for any typical observation point (P), there is disturbance point (Q) effects on it. The vector potential associated with scalar wave field has the property that the normal component of its curl, taken with respect to the coordinates of (Q), where the source point (Q) is located on a closed surface (S) surrounding the observation point(P). The curl of the vector potential with respect to the coordinates of (Q) always equals to the integrand of the Helmholtz-Kirchhoff integral.

The Boundary Diffraction Wave Method considers some points (Qi) on (S) at which the vector potential has singularities, and by the summation of these contributions, the field at the observation point (P) can be evaluated.

The method is investigated by studying the field generated from diffraction of the monochromatic wave by an aperture in an opaque screen. The field which is evaluated at the observation point is called the scattered field (Kirchhoff`s field) and is equal to two parts, the first one is the contribution from each point in the boundary of the aperture, since, the method said that there are associated vector potential ($\vec{W}$) in each element (dl) of the boundary, by taking the integral of these disturbance along the boundary (Γ) of the aperture, the first contribution can be evaluated. The second part represents the disturbances propagated from infinite number of points in the aperture, if the incident wave lights upon the aperture is plane or spherical wave, the last part of the Kirchhoff`s field is changed to be obeyed to the geometrical optics.

According to the principle of Huygens and fresnel, when an incident ray diffracted by an obstacle, where the wavelength of this incident wave is smaller than the linear dimensions of the obstacle, and the incident wave considered to be a primary wave. Huygens and fresnel said in this principle that, each point of the unobstructed part of the primary wave is considered to be a center of a secondary disturbance, these secondary disturbance generate a wavelets, the interference of these wavelets make a superpositions which are considered to be points to generate the transmitted field which is part of the diffracted field or the Kirchhoff`s field that can be measured at some typical observation points after the obstacle. Another suggestion of the physical model for diffraction, by young, considered that the diffraction arises from the interference between the unobstructed light and the light reflected from some points in the boundary of the aperture of the obstacle, but the suggestion was only formulated in a rough qualitative manner. The theory by Maggi and Rubinowicz showed that the Kirchhoff diffraction integral can be decomposed to boundary diffraction wave and geometrical wave, the analysis of Maggi and Rubinowicz interested the diffraction when the incident wave is plane or spherical.

We mathematically analyze, in the second chapter of our thesis, the generalization of BDWM which contains the evaluation of the vector potential associated with the Helmholtz-Kirchhoff integral, the vector potential associated with the a homogenous plane wave . The third chapter explains the physical analysis of the generalization of the BDWM. The fourth chapter discuses the application of the BDWM on a half plane screen. The fifth chapter in our thesis explains the comparison between our solution and the solution of Ganci. In addition this chapter illustrates the comparison between our solution and the exact solution.

Our contribution is the analysis of the generalization of the boundary diffraction wave method and the application of this method on half plane problem. In addition, we compared our solution with Ganci solution for the same problem (half plane problem). Moreover, we compared our solution with the exact solution for the same problem by plotting graphs illustrating the comparison between the two solutions at chosen incident angles.

In the Literature review, there is no body solved a diffraction problem by usage of the BDWM.

CHAPTER 2

THE MATHEMATICAL ANALYSIS OF THE GENERALIZATION OF THE BOUNDARY DIFFRACTION WAVE METHOD

2.1. Introduction

The first step of the generalization of the Maggi-Rubinowics theory of the boundary diffraction wave is the defining of a new vector potential $\vec{W}(Q, P)$ which is associated with any monochromatic scalar wavefield U(P). Where P denotes the observation point and Q denotes the secondary source point.

The normal component of the curl of the monochromatic scalar wavefield U(P) with respect to the coordinates of Q on the surface S surrounding P is equal to the integrand of the Helmholtz-Kirchhoff integral.

$$curl_Q \vec{W}(Q, P) . \vec{n} = \frac{1}{4\pi}\left[U(Q)\frac{\partial}{\partial n}\left(\frac{e^{iks}}{s}\right) - \frac{e^{iks}}{s}\frac{\partial}{\partial n}(U(Q)) \right] \qquad (2.1)$$

Where $\frac{\partial}{\partial n}$ denotes the differentiation along the inward unit vector $\vec{n}$ normal to S, and s is the distance QP.

According to the Huygens and Fresnel principle which studies the diffraction of the light by an obstacle whose linear dimensions are large compared to the wavelength of the incident wave, each point of unobstructed part of the primary wave is assumed to be a center of a secondary wave. The superpositions of these secondary waves are considered to be the sources of the diffracted field.

Maggi and Rubinowicz showed that the diffraction by an aperture in an opaque screen may be decomposed into the sum of two terms. The first term represents the wave originating in every point of the boundary of the aperture (Boundary Diffracted Wave), the second one represents a wave propagated through the aperture in accordance with the laws of geometrical optics (Geometrical Wave). The vector potential $\vec{W}(Q,P)$ has always singularities at some points Qi on the surface S and the field at P inside S may be expressed as the sum of the disturbances propagated from these points.

2.2. The Vector Potential Associated With The Helmholtz-Kirchhoff Integral.

For any monochromatic scalar wave filed V(x, y, z, t) in free space, we can write

$$V(x, y, z) = U(x, y, z) \exp(-i\omega t) \tag{2.2}$$

where (x, y, z) are the Cartesian rectangular coordinates at any typical point P in the wave field, ω denotes the angular frequency and t denotes the time. U is the space dependent part satisfies the Helmholtz equation.

$$\left(\nabla^2 + k^2\right)U = 0 \tag{2.3}$$

where k = ω/c, c denotes the velocity of light. The disturbance at any point P within a volume **v** bounded by surface S according to Helmholtz-Kirchhoff integral is expressed as:

$$U(P) = \iint_s \vec{V}(Q,P)\,.\,\vec{n}\, dS \tag{2.4}$$

where S is a closed surface bounding **v** and containing P, and the vector $\vec{V}(Q,P)$ can be written as

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[U(Q)\,\mathrm{grad}_Q \frac{e^{iks}}{s} - \frac{e^{iks}}{s}\,\mathrm{grad}_Q U(Q)\right] \tag{2.5}$$

The vector $\vec{V}(Q,P)$ equals to a suitable chosen vector potential $\vec{W}(Q,P)$. By taking the divergence of $\vec{V}(Q,P)$ with respect to a chosen point Q on the surface S we will get

$$\mathrm{div_Q}\vec{V}(Q,P) = \frac{1}{4\pi}\left[U(Q)\nabla_Q^2 \frac{e^{iks}}{s} - \frac{e^{iks}}{s}\nabla_Q^2 U(Q) \right] \tag{2.6}$$

where $\mathrm{div_Q}\ \mathrm{grad_Q} = \nabla_Q^2$.

The functions $\frac{e^{iks}}{s}$ and U(Q) satisfy the Helmholtz equation, so from Eq. (2.3) we can write the following expression for the function $\frac{e^{iks}}{s}$

$$\nabla_Q^2 \frac{e^{iks}}{s} = -k^2 \frac{e^{iks}}{s} \tag{2.7.1}$$

Also, the wave function U(Q) can be expressed with the same expression, then

$$\nabla_Q^2 U(Q) = -k^2 U(Q) \tag{2.7.2}$$

where k is constant, by substituting from Eqs. (2.7.1) and (2.7.2) into Eq. (2.6), it follows that

$$\mathrm{div_Q}\vec{V}(Q,P) = 0. \tag{2.8}$$

Here, whatever the nature of U is, $\vec{V}(Q,P)$ can expressed in terms of vector potential

$$\vec{V}(Q,P) = curl_Q \vec{W}(Q,P) \tag{2.9}$$

where, $\vec{W}(Q,P)$ is a vector potential which has always singularities at some points Qi on the surface S, the Helmholtz-Kirchhoff formula in Eq. (2.4) becomes

$$U(P) = \iint_{S} curl_Q \vec{W}(Q,P) . \vec{n}\, ds \qquad (2.10)$$

Let`s now consider P is a fixed point, so the above equation will be a function of Q, and the vector potential $\vec{W}(Q,P)$ must have singularities on the surface S, if $\vec{W}(Q,P)$ has no singularities on S then U(p) = 0.

All the singularities of $\vec{W}(Q,P)$ on the surface S occur at discrete points Q_1, Q_2, ………, Q_n, which are surrounded by a small circles with radii σ_1, σ_2, ……, σ_n , and the boundaries of these circles are Γ_1, Γ_2, …….., Γ_n

$$\iint_{S} curl_Q\, W(Q,P) . n\, ds = \sum_{i} \int_{\Gamma i} W.\vec{l}\, dl \qquad (2.11)$$

where, $\vec{l}$ is the unit vector along the tangent to Γ_i and dl is an element of Γ_i, let`s assume that each point Qi effects on the typical point P by the disturbance $F_i(p)$, so the total disturbance at p can be expressed as

$$U(P) = \sum_{i} F_i(P). \qquad (2.12)$$

The disturbance $F_i(P)$ can be expressed as the limit of the integral of the vector potential $\vec{W}(Q,P)$ associated with each point Q_i along Γ_i when $\sigma_i \rightarrow 0$

$$Fi(P) = \lim_{\sigma \rightarrow 0} \int_{\Gamma i} \vec{W} . \vec{l} dl. \qquad (2.13)$$

With the help of stokes theorem[1] the surface integral can be reduced to a set of line integrals. Since the general formula of Eq. (2.12) is

$$U(P) = \sum_{i} F_i(P) + \sum_{i} G_i(P) \qquad (2.14)$$

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[1]Stokes' theorem in differential geometry is a statement about the integration of differential forms which generalizes several theorems from vector calculus. Taken from "www.answers.com"

$G_i(P)$ are the contributions from all the singularities of U inside the volume v bounded by S, and equals to

$$Gi(P) = \lim_{\rho_i \to 0} \iint_s \vec{V}(Q,P) \,.\, \vec{n}\, dS \tag{2.15}$$

In the above equation, s is the small sphere with radius ρ_i containing point O_i, and the integration is taken throughout the volume inside the surface S, which excludes these regular points O_i.

2.3. Vector Potential for a Homogenous Plane Wave

The incident homogeneous plane wave propagated in the direction specified by the unit vector $\vec{m}$ is expressed as:

$$U(P) = A\, e^{ik\vec{m}\,.\,\vec{r}} \tag{2.16}$$

Here, $\vec{r}$ is the position vector of p and A is a constant.

Since $\vec{r}_1$ is the position vector of Q the vector $\vec{V}(Q,P)$ in Eq. (2.5) can be written as:

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[U(\vec{r}_1) \mathrm{grad}_Q \frac{e^{iks}}{s} - \frac{e^{iks}}{s} \mathrm{grad}_Q U(\vec{r}_1) \right] \tag{2.17}$$

where, $U(\vec{r}_1)$ is the incident homogeneous plane wave propagated in the direction which is specified by the unit vector $\vec{m}$ at the typical point Q with position vector $\vec{r}_1$, this wave may be written as:

$$U(Q) = A\, e^{ik\vec{m}\,.\,\vec{r}_1} \tag{2.18}$$

Then, the vector $\vec{V}(Q,P)$ associated with the plane wave U(Q) can be expressed as:

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[A\, e^{ik\vec{m}.\vec{r}_1} \nabla_Q \frac{e^{iks}}{s} - \frac{e^{iks}}{s} \nabla_Q (A\, e^{ik\vec{m}.\vec{r}_1}) \right] \tag{2.19}$$

s denotes the distance between the secondary source point Q and the observation point P, and ∇_Q represents the partial differential with respect to the position vector $\vec{r}_1$,

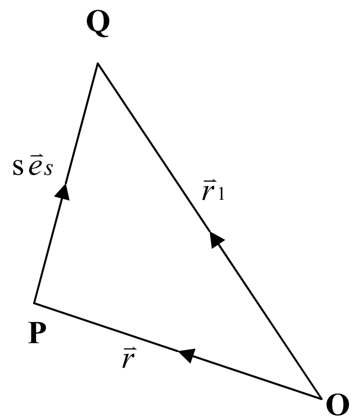

Figure 2.1: Illustrating the position of the points Q and P

Since, as it is shown in Fig. 2.1 the vector $s\vec{e}_s$ is a function of $\vec{r}_1$, and equals to $\vec{r}_1 - \vec{r}$, where $\vec{r}$ is the position vector of the observation point P, so ∇_Q can be also taking with respect to s, and the vector $\vec{V}(Q,P)$ will be

$$\vec{V}(Q,P) = \frac{A}{4\pi}\left[e^{ik\vec{m}.\vec{r}_1} \frac{\partial}{\partial s} \frac{e^{iks}}{s} \vec{e}_s - \frac{e^{iks}}{s} \frac{\partial}{\partial r_1} e^{ik\vec{m}.\vec{r}_1} \vec{e}_s \right]. \tag{2.20}$$

Let`s solve the differential in Eq. (2.20) for the vector $\vec{V}(Q,P)$, to get

$$\vec{V}(Q,P) = \frac{A}{4\pi}\left[e^{ik\vec{m}.\vec{r}_1} \left(\frac{1}{s} e^{iks} (ik) - \frac{e^{iks}}{s^2} \right) \vec{e}_s - \frac{e^{iks}}{s} e^{ik\vec{m}.\vec{r}_1} (ik\vec{m}) \vec{e}_s \right]. \tag{2.21}$$

By taking $e^{ik\vec{m}\vec{r}_1} \frac{e^{iks}}{s}$ out of the parenthesis, the vector $\vec{V}(Q,P)$ will be

$$\vec{V}(Q,P)=\frac{A}{4\pi}e^{ik\vec{m}\vec{r}_1}\frac{e^{iks}}{s}\left[ik\,\vec{e}_s-\frac{1}{s}\vec{e}_s-ik\vec{m}\vec{e}_s\right] \quad (2.22)$$

hence, the second term in the right hand side of Eq. (2.22) is in the direction of $\vec{s}$, so the final expression of vector $\vec{V}(Q,P)$ is

$$\vec{V}(Q,P)=\frac{A}{4\pi}e^{ik\vec{m}\vec{r}_1}\frac{e^{iks}}{s}\left[\left(ik-\frac{1}{s}\right)\vec{e}_s-ik\vec{m}\vec{e}_s\right] \quad (2.23)$$

$\vec{e}_s$ denotes the unit vector in the direction PQ, and equals

$$\vec{e}_s=\frac{\vec{s}}{s} \quad (2.24)$$

Where s is the magnitude of $\vec{s}$. Eq. (2.23) can be expressed by means of the vector potential [1]

$$\vec{W}(Q,P)=A\,e^{ik\vec{m}.\vec{r}_1}\frac{e^{iks}}{s}\frac{1}{4\pi}\frac{\vec{e}_s\times\vec{m}}{1+\vec{e}_s\cdot\vec{m}} \quad (2.25)$$

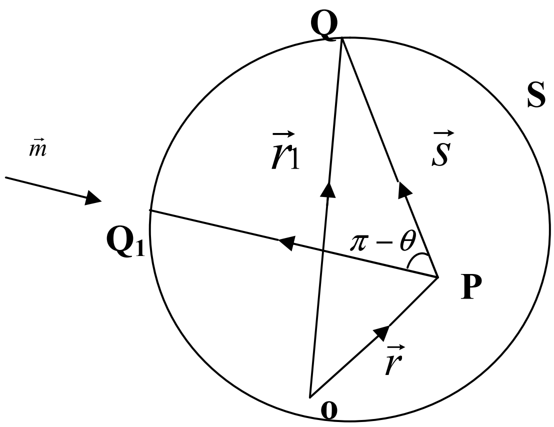

Figure 2.2: Illustrates the first point on the aperture that the incident ray hits it

The cross and dot product in Eq. (2.25) can be written as a function of the angle θ Fig. 2.2, it can be expressed as:

$$\frac{\vec{e}_s \times \vec{m}}{1 + \vec{e}_s \cdot \vec{m}} = \frac{\sin\theta}{1 - \cos\theta} = \frac{1}{\tan\left(\frac{\theta}{2}\right)} \tag{2.26.1}$$

where θ is the angle between $\vec{s}$ and the direction of the incident wave $\vec{m}$ and Q_1 is the first point at which the incident wave intersects the surface S Fig. 2.2, assumed that the point Q_1 is surrounded by a small circle Γ_i with radius σ_i as shown in Fig. 2.3, that φ is the azimuthal angle of the point Q_1 so

$$\mathrm{dl} = \sigma_i \, \mathrm{d}\varphi \tag{2.26.2}$$

where dl is an element from the small circle Γi and $\mathrm{d}\varphi$ is an element from the azimuthal angle φ. The integral of the vector potential $\vec{W}(Q, P)$ along Γ_i is equal to

$$\int_{\Gamma i} \vec{W}.\vec{1}\mathrm{dl} = \frac{A}{4\pi} \int_{\Gamma i} e^{ik\vec{m}.\vec{r}_1} \frac{e^{iks}}{s} \frac{(\vec{e}_s \times \vec{m}).\vec{1}}{1 + \vec{e}_s \cdot \vec{m}} \mathrm{dl} \tag{2.27}$$

Since, Fig. 2.1, the position vector $\vec{r}_1$ is equal to the summation of the vector $\vec{s}$ and the position vector $\vec{r}$

$$\vec{r}_1 = \vec{s} + \vec{r} \, . \tag{2.28}$$

Multiply each side in Eq. (2.28) with the unit vector $\vec{m}$ as a dot product, to get

$$\vec{m} \, . \, \vec{r}_1 = \vec{m}.\vec{s} + \vec{m}.\vec{r} \tag{2.29.1}$$

in the exponent of (e) in Eq. (2.27) substitute by means of Eq. (2.29.1)

$$e^{ik\vec{m}.\vec{r}_1} = e^{ik\vec{m}.\vec{s}} \; e^{ik\vec{m}.\vec{r}} \tag{2.29.2}$$

by substituting from Eqs. (2.26.1), (2.26.2) and (2.29.2) into Eq. (2.27), the integral of the vector potential $\vec{W}(Q, P)$ along Γ_i will be

$$\int_{\Gamma i} \vec{\mathrm{W}}.\vec{1}\mathrm{dl} \approx \frac{\mathrm{A}}{4\pi} e^{ik\vec{m}.\vec{r}} \frac{e^{ik(\vec{m}.\vec{s}+\vec{s})}}{s} \frac{\sigma_1}{\tan\left(\theta/2\right)} \int_0^{2\pi} d\varphi \tag{2.30}$$

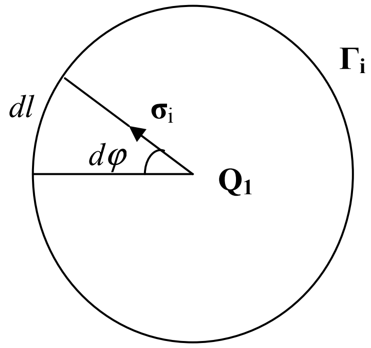

Figure 2.3: Illustrates the small circle Γi surrounding Q_1

since $\sigma_i << s$ and the angle θ between $\vec{s}$ and the direction of the incident wave $\vec{m}$ is equal to π when Q is closed to Q_1 we can say $\theta = \sigma_i / s$ and $\tan(\theta/2)=(\theta/2)$ then:

$$\frac{\theta}{2} = \frac{\sigma_i}{2s} \tag{2.31.1}$$

At the intersection point Q_1, the vector $\vec{s}$ is applied at the direction of the incident wave and in the opposite direction, so the angle between $\vec{s}$ and $\vec{m}$ approximately equals π.

$$\vec{\mathrm{m}}.s\vec{e}_s + s\vec{e}_s = s\cos(\pi) + s = 0 \tag{2.31.2}$$

Where, $\vec{m}$ is a unit vector, from Eqs. (2.31.1) and (2.31.2), Eq. (2.30) can be expressed as:

$$\int_{\Gamma i} \vec{\mathrm{W}}.\vec{1}\mathrm{dl} \approx \frac{\mathrm{A}}{4\pi} e^{ik\vec{m}.\vec{r}} \int_0^{2\pi} \frac{e^0}{s} \frac{\sigma_1}{\sigma_1/2\mathrm{s}} d\varphi = \frac{2\mathrm{A}}{4\pi} e^{ik\vec{m}.\vec{r}} \int_0^{2\pi} d\varphi \tag{2.32}$$

Since, the limit of the integral of the vector potential $\vec{\mathrm{W}}(\mathrm{Q},\mathrm{P})$ when σi goes to zero is equal to the contribution $F_1(P)$from the point of the intersection (Q_1) at the observation point (P).

$$F_1(\mathrm{P}) = \lim_{\sigma i \to \infty} \int_{\Gamma i} \vec{W}.\vec{1}\mathrm{dl}. \qquad (2.33)$$

Then, the contribution from the point (Q_1) at the observation point (P) can be expressed as:

$$F_1(\mathrm{P}) = \lim_{\sigma \to 0} \left[\frac{A}{4\pi} e^{ik\vec{m}.\vec{r}} \int_0^{2\pi} 2\, d\varphi \right] = \mathrm{A}\ \mathrm{e}^{ik\vec{m}\ .\vec{r}} \qquad (2.34)$$

Thus $F_1(\mathrm{P})$ is the value of the field which is generated from point Q_1, and measured at the observation point p. This field can be expressed as:

$$F_1(\mathrm{P}) = \mathrm{U(P)}. \qquad (2.35)$$

2.4. General Expression for The Vector Potential Associated With Any Given Wave Field.

Let`s consider that the associated vector potential is $\vec{W}(\vec{r}_1, \vec{r})$, which is associated with the wave field U($\vec{r}_1$), where $\vec{r}_1$ and $\vec{r}$ are the position vectors of the disturbance source point Q and the observation point P, respectively.

The wave field may be considered as an angular spectrum of plane wave and expressed as:

$$\mathrm{U}(\vec{r}_1) = U_0^{+}(\vec{r}_1) + U_0^{-}(\vec{r}_1) + Ui^{+}(\vec{r}_1) + Ui^{-}(\vec{r}_1), \qquad (2.36)$$

The wave field U($\vec{r}_1$) also can be expressed as a compact form at the singular point Q with position vector $\vec{r}_1$

$$U(\vec{r}_1) = \iint A(p) e^{ik\vec{m}.\vec{r}_1} dp_x dp_y \qquad (2.37)$$

the singular point Q is located in the plan z = p_z in the place where this plane intersects the surface S which, surrounds the observation point p. From the Eq. (2.25) the vector potential $\vec{W}$ (Q , P) is equal to

$$\vec{W}(\vec{r}_1,\vec{r}) = A\, e^{ik\vec{m}.\vec{r}_1} \frac{e^{iks}}{s} \frac{1}{4\pi} \frac{\vec{e}_s \times \vec{m}}{1 + \vec{e}_s \cdot \vec{m}}. \tag{2.38}$$

Since $\vec{r}_1$ and $\vec{r}$ are the position vectors of the disturbance source point Q and the observation point P, respectively, the vector potential may be expressed as:

$$\vec{W}(\vec{r}_1,\vec{r}) = \frac{1}{4\pi} \frac{e^{iks}}{s} \vec{e}_s \times \vec{W}(\vec{r}_1,\vec{e}_s), \tag{2.39}$$

Comparison with Eq. (2.25), the part $\vec{W}(\vec{r}_1,\vec{e}_s)$ of the vector potential in Eq. (2.39) can be expressed as:

$$\vec{W}(\vec{r}_1,\vec{e}_s) = \iint A(p) e^{ik\vec{m}.\vec{r}_1} \frac{\vec{m}}{1+\vec{e}_s.\vec{m}} dp_x dp_y \tag{2.40}$$

$\vec{e}_s$ is the unit vector of the vector $\vec{s}$. By taking the gradient of Eq. (2.37) with respect to $\vec{r}_1$, Assume that, $grad^1$ means that the gradient is taken with respect to $\vec{r}_1$.

$$grad^1 U(\vec{r}_1) = \iint A(p) grad^1 e^{ik\vec{m}.\vec{r}_1} dp_x dp_y \tag{2.41}$$

the integral in the right hand side of Eq. (2.41) can be written as:

$$grad^1 U(\vec{r}_1) = ik \iint \vec{m} A(p) e^{ik\vec{m}.\vec{r}_1} dp_x dp_y. \tag{2.42}$$

By comparing Eqs. (2.37) and (2.42) we find that $grad^1$ is equal to the multiplication of $ik\vec{m}$. So we can write $\vec{m}$ as

$$\vec{m} = \frac{grad^1}{ik} = (ik)^{-1} grad^1. \tag{2.43}$$

The part $\frac{\vec{m}}{1+\vec{e}_s.\vec{m}}$ in Eq. (2.40) can be expressed as a function of grad^1, to get

$$\frac{\vec{m}}{1+\vec{e}_s.\vec{m}}=\frac{(ik)^{-1}grad^1}{1+\vec{e}_s.(ik)^{-1}grad^1}. \tag{2.44}$$

By substituting the above expression into Eq. (2.40)

$$\vec{W}(\vec{r}_1,\vec{e}_s)=\frac{(ik)^{-1}grad^1}{1+\vec{e}_s.(ik)^{-1}grad^1}\iint A(p)e^{ik\vec{m}.\vec{r}_1}dpxdpy\ . \tag{2.45}$$

From the Eq. (2.37) the integral in the above equation equals $U(\vec{r}_1)$ by substitute the value of the integral by the wave function $U(\vec{r}_1)$, to get

$$\vec{W}(\vec{r}_1,\vec{e}_s)=\frac{(ik)^{-1}grad^1}{1+\vec{e}_s.(ik)^{-1}grad^1}U(\vec{r}_1) \tag{2.46}$$

In Eq. (2.46) the term $\vec{e}_s.grad^1$ is equal to the operator represents the differential along P A_∞ Fig. 2.4 [2]

$$\vec{e}_s.grad^1=\frac{\partial}{\partial\tau} \tag{2.47}$$

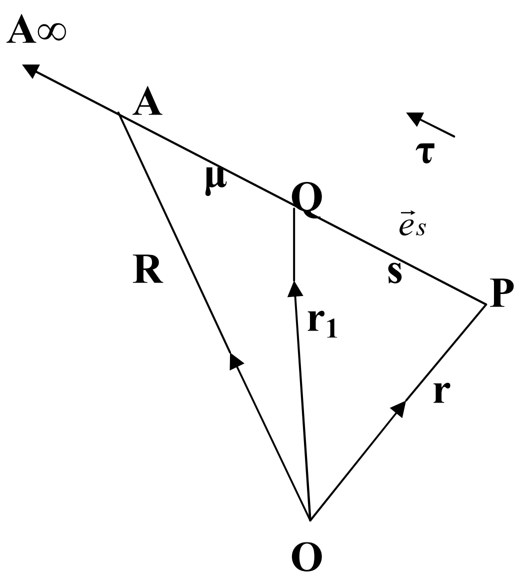

Figure 2.4: Illustrating the meaning of the position vectors $\vec{r}$ and $\vec{r}_1$

It's concluded from Fig. 2.4, that, the position vector $\vec{R}$ of the point A, which is a typical point in the line PA_∞, this line represents the contribution from the infinity at the observation point P, can be written as:

$$\vec{R} = \vec{r} + \tau\vec{e}_s \tag{2.48}$$

$\tau\vec{e}_s$ represents the vector between P and A_∞, by substituting from Eq. (2.47) into Eq. (2.46)

$$\vec{W}(\vec{r}_1, \vec{e}_s) = \frac{(ik)^{-1}}{1 + (ik)^{-1}\dfrac{\partial}{\partial \tau}} grad^1 U(\vec{r}_1). \tag{2.49}$$

By taking $(ik)^{-1}$ as a common factor from the dominator in Eq. (2.49) to get

$$\vec{W}(\vec{r}_1, \vec{e}_s) = \frac{(ik)^{-1}}{(ik)^{-1}\left(\dfrac{1}{(ik)^{-1}} + \dfrac{\partial}{\partial \tau}\right)} grad^1 U(\vec{r}_1). \tag{2.50}$$

the factor $(ik)^{-1}$ can be canceled from numerator and dominator

$$\vec{W}(\vec{r}_1, \vec{e}_s) = \frac{1}{ik + \dfrac{\partial}{\partial \tau}} grad^1 U(\vec{r}_1). \tag{2.51}$$

Eq. (2.51) can be written as:

$$\frac{\partial}{\partial \tau}\vec{W}(\vec{r}_1, \vec{e}_s) + ik\vec{W}(\vec{r}_1, \vec{e}_s) = grad^1 U(\vec{r}_1). \tag{2.52}$$

to be more general along PA_∞, Eq. (2.52) is valid also for $U(\vec{R})$, so it may be applied for $\vec{R}$ (the position vector of a typical point A on the line PA_∞) instead of $\vec{r}_1$ (the position vector of the point Q)

$$\frac{\partial}{\partial \tau}\vec{W}(\vec{R},\vec{e}_s)+ik\vec{W}(\vec{R},\vec{e}_s)= grad U(\vec{R}). \tag{2.53}$$

to solve Eq. (2.53), let`s multiply both sides by $e^{ik\tau}$

$$\frac{\partial}{\partial \tau}\vec{W}(\vec{R},\vec{e}_s)\, e^{ik\vec{\tau}}+ik\vec{W}(\vec{R},\vec{e}_s)\, e^{ik\vec{\tau}} = e^{ik\vec{\tau}} grad U(\vec{R}). \tag{2.53.1}$$

It is clear seen that the left side of Eq. (2.53.1) is the differentiation of $\frac{\partial}{\partial \tau}\left[\vec{W}(\vec{R},\vec{e}_s)\, e^{ik\vec{\tau}}\right]$, Eq. (2.53.2) verify this

$$\frac{\partial}{\partial \tau}\vec{W}(\vec{R},\vec{e}_s)\, e^{ik\vec{\tau}} = \frac{\partial}{\partial \tau}\vec{W}(\vec{R},\vec{e}_s)\, e^{ik\vec{\tau}}+ik\vec{W}(\vec{R},\vec{e}_s)\, e^{ik\vec{\tau}}. \tag{2.53.2}$$

It's concluded from Eq. (2.53.1) and Eq. (2.53.2), that

$$\frac{\partial}{\partial \tau}\left[\vec{W}(\vec{R},\vec{e}_s)\, e^{ik\vec{\tau}}\right]= e^{ik\vec{\tau}} grad U(\vec{R}). \tag{2.54}$$

Let`s taking the integral of Eq. (2.54) from ($\tau = s_0$ to $\tau = s$) for both sides, after applying it again for $\vec{r}_1$

$$\int_{\tau=s_0}^{\tau=s}\frac{\partial}{\partial \tau}\left[\vec{W}(\vec{r}_1,\vec{e}_s)\, e^{ik\vec{\tau}}\right]= \int_{\tau=s_0}^{\tau=s} e^{ik\vec{\tau}} grad U(\vec{r}_1)\partial\tau. \tag{2.55}$$

By solving the integral of the left hand side of Eq. (2.55), and substituting $\vec{r}_1 = \vec{r}+\vec{s}$ since, at $\tau = s_0$ the vector $\vec{s} = s_0\vec{e}_s$ and $\tau = s$ the vector $\vec{s} = s_0\vec{e}_s$, the Eq. (2.55) will be

$$\vec{W}(\vec{r}+s\vec{e}_s,\vec{e}_s)\, e^{iks} - \vec{W}(\vec{r}+s_0\vec{e}_s,\vec{e}_s)\, e^{iks_0} = \int_{\tau=s_0}^{\tau=s} e^{ik\vec{\tau}} grad U(\vec{r}+\tau\vec{e}_s)\partial\tau. \tag{2.56}$$

In Eq. (2.56), take the second term of the left hand side to the right hand side, to get

$$\vec{W}(\vec{r}+s\vec{e}_s,\vec{e}_s)\,e^{iks} = \int_{\tau=s_0}^{\tau=s} e^{ik\tau}\,grad\,U(\vec{r}+\tau\vec{e}_s)\partial\tau + \vec{W}(\vec{r}+s_0\vec{e}_s,\vec{e}_s)\,e^{iks_0} \qquad (2.56.1)$$

From Eq. (2.39), the vector potential $\vec{W}(\vec{r}_1,\vec{r})$ can be rewritten in terms of $\vec{r}$ and $s\vec{e}_s$, since $\vec{r}_1 = \vec{r} + \vec{s}$

$$\vec{W}(\vec{r}_1,\vec{r}) = \vec{W}(\vec{r}+s\vec{e}_s,\vec{r}) = \frac{1}{4\pi}\frac{e^{iks}}{s}\vec{e}_s \times \vec{W}(\vec{r}+s\vec{e}_s,\vec{e}_s)\ . \qquad (2.56.2)$$

From Eq. (2.56.2), $\vec{W}(\vec{r}+s\vec{e}_s,\vec{e}_s)$ can be rewritten as

$$\vec{W}(\vec{r}+s\vec{e}_s,\vec{e}_s) = \frac{4\pi\vec{W}(\vec{r}_1,\vec{r})}{\frac{e^{iks}}{s}\vec{e}_s}\ . \qquad (2.56.3)$$

Substitute the value of $\vec{W}(\vec{r}+s\vec{e}_s,\vec{e}_s)$ from Eq. (2.56.3) in to Eq. (2.56.1)

$$\frac{4\pi\vec{W}(\vec{r}_1,\vec{r})}{\frac{e^{iks}}{s}\vec{e}_s}e^{iks} = \int_{\tau=s_0}^{\tau=s} e^{ik\tau}\,grad\,U(\vec{r}+\tau\vec{e}_s)\partial\tau + W(\vec{r}+s_0\vec{e}_s,\vec{e}_s)\,e^{iks_0}\ . \qquad (2.57)$$

By doing some changes in the above equation the vector potential $\vec{W}(\vec{r}_1,\vec{r})$ can be expressed as:

$$\vec{W}(\vec{r}_1,\vec{r}) = \frac{\vec{e}_s}{4\pi s}\left[\int_{\tau=s_0}^{\tau=s} e^{ik\tau}\,grad\,U(\vec{r}+\tau\vec{e}_s)\partial\tau + W(\vec{r}+s_0\vec{e}_s,\vec{e}_s)\,e^{iks_0}\right]. \qquad (2.58)$$

From Fig. 2.4, the following expressions can be concluded

$$\tau\vec{e}_s = \vec{\mu} + \vec{s} \qquad (2.59.1)$$

$$\vec{r} + \tau\vec{e}_s = \vec{r}_1 + \mu\vec{e}_s \qquad (2.59.2)$$

Then $\partial\tau = \partial\mu$ and from Fig. 2.4 at τ = s_0 the vector $\vec{\mu} = \infty$ and $\vec{\tau} = \vec{s}$ the vector $\vec{\mu} = 0$, the formula of the vector potential $\vec{W}(\vec{r}_1, \vec{r})$, by substituting in Eq. (2.58), can be expressed as:

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s}{4\pi s}\left[\int_{\infty}^{0} e^{ik[\vec{\mu}+\vec{s}]} grad U(\vec{r}_1 + \mu\vec{e}_s)\partial\mu + \vec{W}(\vec{r} + s_0\vec{e}_s, \vec{e}_s)\, e^{iks_0}\right] \qquad (2.60)$$

Since the second term of the right hand side of Eq. (2.60) represents the vector potential associated with the infinity, so let`s take the limit of this term when $s0 \to \infty$.

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s}\int_{\infty}^{0} e^{ik\mu} grad U(\vec{r}_1 + \mu\vec{e}_s)\partial\mu + \frac{\vec{e}_s}{4\pi s} \times \lim_{s0\to\infty} \vec{W}(\vec{r} + s_0\vec{e}_s, \vec{e}_s)\, e^{iks_0} \qquad (2.61)$$

In general there is a contribution from infinity, denoted by the term $\vec{W}\infty$, which is represented by the second term in Eq. (2.61).

$$\vec{W}\infty = \frac{\vec{e}_s}{4\pi s} \times \lim_{s0\to\infty} \vec{W}(\vec{r} + s_0\vec{e}_s, \vec{e}_s)\, e^{iks_0}. \qquad (2.62)$$

$\vec{W}(\vec{r}_1, \vec{r})$ is a vector potential which has the form of a spherical wave and is also apart from $\vec{W}\infty$. The amplitude of $\vec{W}(\vec{r}_1, \vec{r})$ is a right angle to the direction joins the two points Q($\vec{r}_1$) and P($\vec{r}$). This vector amplitude equals to the summation of the contributions from each point A on the half line A∞Q.

$e^{ik\mu}$ in Eq. (2.61) is the effective retardation term and equals to $e^{i\varpi(\mu/c)}$, where μ/c represents the time needed for the light disturbance to propagate from A to Q.

It appears difficult to obtain a general closed expression for the residual contribution $\vec{W}\infty$. Since the field obeys the sommerfeld radiation condition[1] in half of the space (assume the

[1]Sommerfeld (1949) was the first to introduce the terminology ``radiation boundary condition". Sommerfeld defined the condition of radiation as ``the sources must be sources, not sinks of energy. The energy which is radiated from the sources must scatter to infinity; no energy may be radiated from infinity into ... the field." Taken from "www.answers.com"

half of the space along the z_direction), to determine this contribution, let`s consider the asymptotic behavior of the integral in Eq. (2.40) as the point Q moves to infinity in the direction specified by a unit vector $\vec{u}$. From the principle of stationary phase and from Eq. (2.40), the approximation of $\vec{W}(R\vec{u}, \vec{e}_s)$ is $\vec{W}_0^+$ and $\vec{W}_0^-$ with amplitudes A_0^+ and A_0^-, respectively, appendix A.

Because of the value of e^{ikR}, it is evident from the behavior of $\vec{W}_i^+$ and $\vec{W}_i^-$ that with increasing R, $\vec{W}_i^+$ increasing exponentially to ∞ or decreasing exponentially to zero according to u_z, and the opposite is the case for each plane wave contributing to $\vec{W}_i^-$, where $\vec{W}_i^-$ decreased as ($u_z>0$) and increased as ($u_z<0$). Then in the representation in Eq. (2.36), if Eq. (2.36) is to be valid for $z \to -\infty$ then $U_i^- = 0$, and if it is to be valid for $z \to +\infty$ then $U_i^- = 0$.

Returning to Eq. (2.62), let`s decompose $\vec{W}\infty$ into two parts.

$$\vec{W}\infty = \vec{W}_\infty^+ + \vec{W}_\infty^- \tag{2.63}$$

$\vec{W}_\infty^+$ is associated with spectral amplitude A_∞^+ and $\vec{W}_\infty^-$ is associated with spectral amplitude A_∞^-, when $\vec{R} \to \infty$ where ($u_z<0$) or ($u_z>0$) . It can be found, in Eq. (A1) in appendix A, $\vec{W}_\infty^+ = 0$ when ($u_z>0$) and $\vec{W}_\infty^- = 0$ when ($u_z<0$) , because $\vec{R} \to \infty$.

For $\vec{W}_\infty^-$, when ($u_z>0$) the spectral amplitude $A_\infty^- = 0$, then $\vec{W}_\infty^- = 0$, and for $\vec{W}_\infty^+$ when ($u_z<0$) the spectral amplitude $A_\infty^+ = 0$, then $\vec{W}_\infty^+ = 0$.

It is concluded from the above that if ($u_z>0$) or ($u_z<0$) and the incident field obeys the sommerfeld radiation condition as $\vec{R} \to \infty$ in the half space ($z > 0$) or in the half space ($z < 0$) then the residual term $\vec{W}\infty = 0$.

2.5. Diverging Spherical Wave

In this section we evaluate the vector potential which is associated with the diverging spherical wave, let`s consider the following diverging spherical wave

$$U(\vec{r}) = \frac{e^{ikr}}{r}, \tag{2.64}$$

which represents the wave field, where $\bar{r}$ is the position vector of the observation point P, r is the distance OP let`s take the gradient for the above spherical wave with respect to $\vec{R}$ (the position vector of a certain point on the line PA∞),

$$\mathrm{gradU}(\vec{R}) = \mathrm{grad}\left[\frac{e^{ikR}}{R}\right]. \tag{2.65}$$

By solving the above gradient with respect to the position vector $\vec{R}$, to get

$$\mathrm{gradU}(\vec{R}) = \frac{1}{R} \times ik\, e^{ikR} - e^{ikR} \times \frac{1}{R^2}. \tag{2.66}$$

Take the term e^{ikR} as a common factor of the right hand side of the above equation

$$\mathrm{gradU}(\vec{R}) = \left(\frac{ik}{R} - \frac{1}{R^2}\right) e^{ikR}. \tag{2.67}$$

Multiply and divide the right hand side of Eq. (2.67) by R, to get the spherical wave at the end of the equation

$$\mathrm{gradU}\,(\vec{R}) = R\left(\frac{ik}{R} - \frac{1}{R^2}\right)\frac{e^{ikR}}{R}. \tag{2.68}$$

It is inferred from Fig. 2.4, that the position vector $\vec{R}$ can be expressed in terms of $\vec{r}_1$ and $\mu\vec{e}_s$, where $\mu\vec{e}_s$ is the vector between the source point Q and a certain point A on the line PA_∞, this vector $\mu\vec{e}_s$ can be also expressed in terms of $\bar{\tau}$ and $\bar{s}$.

$$\vec{R} = (\vec{r}_1 + \mu\vec{e}_s) \tag{2.69.1}$$

$$\bar{\mu} = \bar{\tau} - \bar{s} \tag{2.69.2}$$

Substitute the value of $\vec{R}$ from Eq. (2.69) into Eq. (2.61) instead of $(\vec{r}_1 + \mu\vec{e}_s)$

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s} \int_{\infty}^{0} e^{ik\mu}\, grad U(\vec{R}) \partial\mu + \vec{W}_\infty \tag{2.70}$$

From Eq. (2.68) get the value of $grad U(\vec{R})$, the Eq. (2.70) will be

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s} \int_{\infty}^{0} e^{ik\mu} R\left(\frac{ik}{R} - \frac{1}{R^2}\right) \frac{e^{ikR}}{R} \partial\mu + \vec{W}_\infty \tag{2.71}$$

Take the value $\frac{1}{R}$ as a common factoring the integrand and multiply and divide the integrand by R, to get

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s} \int_{\infty}^{0} e^{ik\mu} R\left(ik - \frac{1}{R}\right) \frac{e^{ikR}}{R^2} \partial\mu + \vec{W}_\infty \tag{2.72}$$

The exponent of e can be written as the summation of the two exponents (ikμ and ikR)

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s} \int_{\infty}^{0} R\left(ik - \frac{1}{R}\right) \frac{e^{ik(R+\mu)}}{R^2} \partial\mu + \vec{W}_\infty \tag{2.73}$$

The first case $\left[\vec{R} = (\vec{r}_1 + \mu\vec{e}_s) \text{and} \quad \mu = 0\right] \Rightarrow \vec{R} = \vec{r}_1.$ Then

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{e^{iks}}{4\pi s} \vec{e}_s \times \vec{r}_1 \int_{\infty}^{0} \left(ik - \frac{1}{R}\right) \frac{e^{ik(R+\mu)}}{R^2} \partial\mu + \vec{W}_\infty \tag{2.74}$$

To evaluate the integral in Eq. (2.74), we have to verify that the integrand of the integral in Eq. (2.74) is equal to the differential D.

$$D = \frac{\partial}{\partial\mu}\left[\frac{1}{\vec{R} + \vec{r}_1 . \vec{e}_s + \vec{\mu}} \frac{e^{ik(R+\mu)}}{R}\right]. \tag{2.75}$$

Solve the above differential with respect to μ to get

$$D = \frac{1}{\vec{R} + \vec{r}_1 . \vec{e}_s + \vec{\mu}} \left[-\frac{e^{ik(R+\mu)}}{R} \frac{\partial R}{\partial \mu} + \frac{ik}{R} e^{ik(R+\mu)} \left(\frac{\partial R}{\partial \mu} + 1 \right) \right] + \frac{e^{ik(R+\mu)}}{R} \left[\frac{-1}{\left(\vec{R} + \vec{r}_1 . \vec{e}_s + \vec{\mu} \right)^2} \left(\frac{\partial R}{\partial \mu} + 1 \right) \right] \tag{2.76}$$

Get the square value of $\vec{R}$ from Eq. (2.69), and get the differential of $\vec{R}$ with respect to μ

$$\vec{R} = \left(\vec{r}_1 + \mu \vec{e}_s \right) \Rightarrow \vec{R}^2 = \left(\vec{r}_1{}^2 + 2 \vec{r}_1 \mu \vec{e}_s + \mu^2 \right) \tag{2.77.1}$$

$$\frac{\partial \vec{R}}{\partial \mu} = 1 \tag{2.77.2}$$

From Fig. 24 and the Eq. (2.77), the following equation can be proven

$$\vec{r}_1 + \mu \vec{e}_s = r_1 . \vec{e}_s + \vec{\mu} \tag{2.78.1}$$

$$\frac{\partial \vec{R}}{\partial \mu} = \frac{r_1 . \vec{e}_s + \vec{\mu}}{\vec{R}} = \frac{\vec{R}}{\vec{R}} \tag{2.78.2}$$

By substituting from Eq. (2.78) in to the result of the differential D, the value of the differential D will be

$$D = \frac{1}{2\vec{R}} \left[-\frac{e^{ik(R+\mu)}}{\vec{R}^2} \frac{\vec{R}}{\vec{R}} + \frac{ik}{\vec{R}} e^{ik(R+\mu)} \left(\frac{\vec{R}}{\vec{R}} + 1 \right) \right] + \frac{e^{ik(R+\mu)}}{\vec{R}} \left[\frac{-1}{4\vec{R}^2} \left(\frac{\vec{R}}{\vec{R}} + 1 \right) \right] \tag{2.79}$$

By entering the term $\frac{1}{2\vec{R}}$ into the parenthesis, and since $\frac{\vec{R}}{\vec{R}}$ is equal one, the value of D will be

$$D = -\frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{2\vec{R}^{3}} + \frac{2ik}{2\vec{R}^{2}}\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)} - \frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{4\vec{R}^{3}} \times 2. \tag{2.80}$$

With some changing in Eq. (2.80) the value of D can be expressed as:

$$D = -\frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{2\vec{R}^{3}} + \frac{ik}{\vec{R}^{2}}\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)} - \frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{2\vec{R}^{3}}. \tag{2.81}$$

The above expression of the differential D in Eq. (2.81) can be written as:

$$D = \frac{ik}{\vec{R}^{2}}\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)} - \frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{\vec{R}^{3}} \tag{2.82}$$

By taking the term $\frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{\vec{R}^{2}}$ as a common factor from the right hand side in Eq. (2.82), the solution of the differential in Eq. (2.75) equals

$$\frac{\partial}{\partial\mu}\left[\frac{1}{\vec{R}+\vec{r}_{1}.\vec{e}_{s}+\vec{\mu}}\frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{\vec{R}}\right] = \left(ik - \frac{1}{\vec{R}}\right)\frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{\vec{R}^{2}} \tag{2.83}$$

It's concluded from Eq. (2.83) that the integrand in Eq. (2.74) is equal to the differential D, by substituting from Eq. (2.83) into Eq. (2.74), the vector potential $\vec{\mathrm{W}}\left(\vec{\mathrm{r}}_{1},\vec{\mathrm{r}}\right)$ can be written as:

$$\vec{\mathrm{W}}\left(\vec{\mathrm{r}}_{1},\vec{\mathrm{r}}\right) = \frac{\mathrm{e}^{\mathrm{iks}}}{4\pi s}\vec{e}_{s}\times\vec{r}_{1}\int_{\infty}^{0}\frac{\partial}{\partial\mu}\left[\frac{1}{\vec{R}+\vec{r}_{1}.\vec{e}_{s}+\vec{\mu}}\frac{\mathrm{e}^{\mathrm{ik}\left(R+\mu\right)}}{\vec{R}}\right]\partial\mu + \vec{\mathrm{W}}_{\infty}\ . \tag{2.84}$$

Now, the differentiation cancels the integration, so the vector potential in the above equation will be

$$\vec{W}(\vec{r}_1,\vec{r}) = \frac{e^{iks}}{4\pi s}\vec{e}_s \times \vec{r}_1 \left[\frac{1}{\vec{R} + \vec{r}_1 . \vec{e}_s + \vec{\mu}} \frac{e^{ik(R+\mu)}}{\vec{R}} \right] + \vec{W}_\infty . \quad (2.85)$$

Concluded from Fig. 2.4 that when $\vec{\mu} = 0 \Rightarrow \vec{R} = \vec{r}_1$ and $\vec{\mu} = \infty \Rightarrow \vec{R} = \infty$, now let`s get the vector potential $\vec{W}(\vec{r}_1,\vec{r})$ when $\vec{\mu} = 0$.

$$\vec{W}(\vec{r}_1,\vec{r}) = \frac{e^{iks}}{4\pi s}\vec{e}_s \times \left[\frac{\vec{r}_1}{\vec{r}_1 + \vec{r}_1 . \vec{e}_s} \frac{e^{ik(r_1)}}{r_1} \right] + \vec{W}_\infty \quad (2.86)$$

Since the divergent spherical wave obeys the sommerfeld radiation condition then

$$\vec{W}_\infty = 0 \quad (2.87)$$

Now substituting $\frac{\vec{s}}{s}$ *for* $\vec{e}_s$

$$\vec{W}(\vec{r}_1,\vec{r}) = \frac{e^{iks}}{4\pi s}\frac{\vec{s}}{s} \times \left[\frac{\vec{r}_1}{\vec{r}_1 + \vec{r}_1 . \frac{\vec{s}}{s}} \frac{e^{ik(r_1)}}{r_1} \right] \quad (2.88)$$

Rearrange the above equation to get the new expression for the vector potential $\vec{W}(\vec{r}_1,\vec{r})$

$$\vec{W}(\vec{r}_1,\vec{r}) = \frac{1}{4\pi}\frac{e^{ik(r_1)}}{r_1}\frac{e^{iks}}{s} \left[\frac{\vec{s}}{s} \times \frac{\vec{r}_1}{\vec{r}_1 + \vec{r}_1 . \frac{\vec{s}}{s}} \right] \quad (2.98)$$

Remove the term $\frac{\vec{s}}{s}$ from the above equation to get the vector potential associated with a diverging spherical wave

$$\vec{W}(\vec{r}_1,\vec{r})=\frac{1}{4\pi}\frac{e^{ik(r_1)}}{r_1}\frac{e^{iks}}{s}\frac{\vec{s}\times\vec{r}_1}{s\vec{r}_1+\vec{r}_1.\vec{s}} \tag{2.90}$$

From Eq. (2.90) the singularities are given by the condition

$$s\vec{r}_1+\vec{r}_1.\vec{s}=0 \tag{2.91.1}$$

$$\vec{W}(\vec{r}_1,\vec{r})=\infty \tag{2.91.2}$$

This implies that the angle (ϴ) between $\vec{r}_1$ and $\vec{s}$ equals π Fig. 2.5, here we have two cases

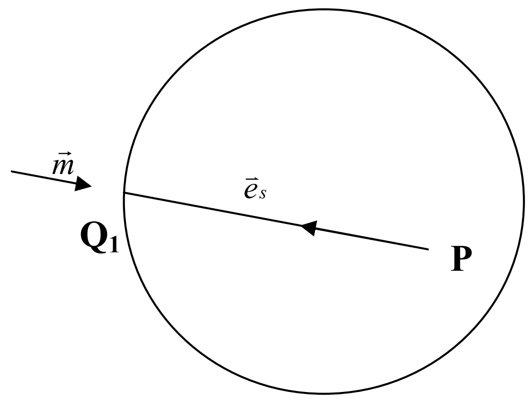

Figure 2.5: Illustrating the directions of the incident ray and the vector $\vec{s}$ between the points P and Q_1

1. The surface S does not enclose the singularity O of the spherical wave, then $\vec{W}$ has only one singular point on S, this singular point is Q_1.

a. $\vec{W}$ has only one singularity on S.

b. The field is regular within the volume bounded by S , the disturbance $F_1(P)$ associated with Q_1 equals to U(p)

$$F_1(P) = U(P) = \frac{e^{ikr}}{r} \tag{2.92}$$

where r is the distance OP

2. The surface S encloses the singularity O of the spherical wave and no point Q1 on S obeys the condition (2.91). Then $\vec{W}_\infty$ has no singularity on the surface, and the field U has one singularity at O within the volume bounded by S. According to Eq. (2.15) the disturbance $G_1(P)$ associated with O must be equal to U(p).

2.6. Converging Spherical Wave

In this section we evaluate the vector potential which is associated with the converging spherical wave, let`s consider the following converging spherical wave

$$U(\vec{r}) = \frac{e^{(-ikr)}}{r} \tag{2.93}$$

$\vec{r}$ is the position vector of the observation point P, Eq. (2.93) can be written with respect to $\vec{R}$ (the position vector of the point A) Fig. 2.5

$$U(\vec{R}) = \frac{e^{(-ikR)}}{R} \tag{2.94}$$

By taking the gradient for the converging spherical wave in Eq. (2.94) with respect to $\vec{R}$, then the result of the differential will be

$$\text{grad}U(\vec{R}) = \frac{-1}{R}(ik)e^{(-ikR)} + e^{(-ikR)}\left(\frac{-1}{R^2}\right) \tag{2.95}$$

Take the term $e^{(-ikR)}$ as a common factor and multiply and divide the right hand side of Eq. (2.95) by R to get

$$\text{gradU}(\vec{R}) = R\left(\frac{-ik}{R} - \frac{1}{R^2}\right)\frac{e^{(-ikR)}}{R} \tag{2.96}$$

It is inferred from Fig. 2.4, that the position vector $\vec{R}$ can be expressed in terms of $\vec{r}_1$ and $\mu\vec{e}_s$, where $\mu\vec{e}_s$ is the vector between the source point Q and a certain point A on the line PA_∞, this vector $\mu\vec{e}_s$ can be also expressed in terms of $\vec{\tau}$ and $\vec{s}$. From Eqs. (2.61) and (2.69) the vector potential $\vec{W}(\vec{r}_1, \vec{r})$ can be written as:

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s}\int_{\infty}^{0} e^{ik\mu}\, grad\text{U}(\vec{r}_1 + \mu\vec{e}_s)\partial\mu + \vec{W}_\infty \tag{2.97}$$

By substituting from Eq. (2.96) into the above equation, since $\vec{R} = (\vec{r}_1 + \mu\vec{e}_s)$, the above equation will be

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s} \times \int_{\infty}^{0} e^{ik\mu} R\left(-ik - \frac{1}{R}\right)\frac{e^{-ikR}}{R^2}\partial\mu + \vec{W}_\infty \tag{2.98}$$

The exponent of e in the above equation can be written as the summation of the two exponents (ikμ and -ikR)

$$\vec{W}(\vec{r}_1, \vec{r}) = \frac{e^{iks}}{4\pi s}\vec{e}_s \times \vec{r}\int_{\infty}^{0}\left(-ik - \frac{1}{R}\right)\frac{e^{ik(-R+\mu)}}{R^2}\partial\mu + \vec{W}_\infty \tag{2.99}$$

Similar to the derivation in section (2.5) we can verify that the integrand in Eq. (2.99) is equal to the differential

$$\frac{\partial}{\partial\mu}\left[\frac{1}{-\vec{R} + \vec{r}\,.\vec{e}_s + \vec{\mu}}\,\frac{e^{ik(-R+\mu)}}{R}\right] \tag{2.100}$$

Then, by substituting in the integral in Eq. (2.99) where the limits if the integration from μ_0 to 0

$$\int_{\mu_0}^{0}\left(-ik-\frac{1}{R}\right)\frac{e^{ik(-R+\mu)}}{R^2}\partial\mu=\left[\frac{1}{-\vec{R}+\vec{r}\,.\vec{e}_s+\vec{\mu}}\frac{e^{ik(-R+\mu)}}{R}\right]_{\mu_0}^{0} \tag{2.101}$$

when $\vec{\mu}=0 \Rightarrow \vec{R}=\vec{r}_1$ and when $\vec{\mu}=\vec{\mu}_0 \Rightarrow \vec{R}=\vec{R}_0$

Since $\vec{\mu}_0 << \vec{R}_0$ Then

$$\int_{\mu_0}^{0}\left(-ik-\frac{1}{R}\right)\frac{e^{ik(-R+\mu)}}{R^2}\partial\mu=\frac{1}{-\vec{r}_1+\vec{r}_1.\vec{e}_s}\frac{e^{-ik\vec{r}_1}}{\vec{r}_1}-\frac{1}{-R_0+\vec{r}_1.\vec{e}_s+\vec{\mu}_0}\frac{e^{-ik\vec{R}_0}}{\vec{R}_0} \tag{2.102}$$

The next expression can be inferred from Fig. 2.4

$$\vec{\mu}_0=\left[\vec{R}_0^{\,2}-(\vec{r}\times\vec{e}_s)^2\right]^{\frac{1}{2}}-(\vec{r}_1.\vec{e}_s). \tag{2.103}$$

Take $\vec{R}_0^{\,2}$ as a common factor and out of the square root from the square root in Eq.(2.103)

$$\vec{\mu}_0=\vec{R}_0\left[1-\frac{(\vec{r}\times\vec{e}_s)^2}{\vec{R}_0^{\,2}}\right]^{\frac{1}{2}}-(\vec{r}_1.\vec{e}_s) \tag{2.104}$$

$$\vec{\mu}_0=\vec{R}_0-\frac{1}{2}\frac{(\vec{r}\times\vec{e}_s)^2}{\vec{R}_0}+0\left(\frac{1}{\vec{R}_0^{\,3}}\right)-(\vec{r}_1.\vec{e}_s). \tag{2.105}$$

Then the second term in the right side of Eq. (2.102) will be

$$\frac{1}{-\vec{R}_0+\vec{r}_1.\vec{e}_s+\vec{\mu}_0}\frac{e^{ik(-\vec{R}_0+\vec{\mu})}}{\vec{R}_0}=-\frac{1}{-\vec{R}_0+\vec{r}_1.\vec{e}_s+\vec{R}_0-\frac{1}{2}\frac{(\vec{r}\times\vec{e}_s)^2}{\vec{R}_0}-(\vec{r}_1.\vec{e}_s)}\frac{e^{-ik(\vec{r}_1.\vec{e}_s)}}{\vec{R}_0} \tag{2.106}$$

By doing some cut short operations in the right side of the above equation

$$\frac{1}{-\vec{R}_0+\vec{r}_1.\vec{e}_s+\vec{\mu}_0}\frac{e^{ik(-\vec{R}_0+\vec{\mu})}}{\vec{R}_0}=-\frac{2}{(\vec{r}\times\vec{e}_s)^2}e^{-ik(\vec{r}_1.\vec{e}_s)} \tag{2.107}$$

By substituting from Eqs. (2.102) and (2.107) into Eq. (2.99)

$$\vec{W}(\vec{r}_1,\vec{r})=\frac{e^{iks}}{4\pi s}\vec{e}_s\times\vec{r}\left[\frac{1}{-\vec{r}_1+\vec{r}_1.\vec{e}_s}\frac{e^{-ik\vec{r}_1}}{\vec{r}_1}+\frac{2}{(\vec{r}\times\vec{e}_s)^2}e^{-ik(\vec{r}_1.\vec{e}_s)}\right]+\vec{W}_\infty \qquad (2.108)$$

Multiply the value $\frac{e^{iks}}{4\pi s}\vec{e}_s\times\vec{r}$ by each individual term from the two terms in the parenthesis

$$\vec{W}(\vec{r}_1,\vec{r})=\frac{e^{iks}}{4\pi s}\vec{e}_s\times\vec{r}\left[\frac{1}{-\vec{r}_1+\vec{r}_1.\vec{e}_s}\frac{e^{-ik\vec{r}_1}}{\vec{r}_1}\right]+\frac{e^{iks}}{4\pi s}\vec{e}_s\times\vec{r}\frac{2}{(\vec{r}\times\vec{e}_s)^2}e^{-ik(\vec{r}_1.\vec{e}_s)}+\vec{W}_\infty \qquad (2.109)$$

By doing some changing operations in the second term of the right side of the above equation, to get

$$\vec{W}(\vec{r}_1,\vec{r})=\frac{e^{iks}}{4\pi s}\vec{e}_s\times\vec{r}\left[\frac{1}{-\vec{r}_1+\vec{r}_1.\vec{e}_s}\frac{e^{-ik\vec{r}_1}}{\vec{r}_1}\right]+\frac{e^{(-ik(\vec{r}_1.\vec{e}_s)+iks)}}{2\pi s}\frac{(\vec{e}_s\times\vec{r})}{(\vec{r}\times\vec{e}_s)^2}+\vec{W}_\infty \qquad (2.110)$$

According to the Fig. 2.4

$$\begin{aligned}\vec{s}-\vec{r}_1.\vec{e}_s&=-\vec{e}_s.\vec{r}\\-\vec{s}+\vec{r}_1.\vec{e}_s&=\vec{e}_s.\vec{r}\end{aligned} \qquad (2.111)$$

the second term of the right side of Eq. (2.110) can be written as:

$$\frac{e^{(-ik(\vec{r}_1.\vec{e}_s)+iks)}}{2\pi s}\frac{(\vec{e}_s\times\vec{r})}{(\vec{e}_s\times\vec{r})^2}=\frac{1}{2\pi s}\frac{(\vec{e}_s\times\vec{r})}{(\vec{e}_s\times\vec{r})^2}e^{[-ik(\vec{e}_s.\vec{r})]} \qquad (2.112)$$

then the associated vector potential in Eq. (2.110) will be

$$\vec{W}(\vec{r}_1,\vec{r})=\frac{e^{iks}}{4\pi s}\vec{e}_s\times\vec{r}\left[\frac{1}{-\vec{r}_1+\vec{r}_1.\vec{e}_s}\frac{e^{-ik\vec{r}_1}}{\vec{r}_1}\right]+\frac{1}{2\pi s}\frac{(\vec{e}_s\times\vec{r})}{(\vec{e}_s\times\vec{r})^2}e^{[-ik(\vec{e}_s.\vec{r})]}+\vec{W}_\infty. \qquad (2.113)$$

CHAPTER 3

THE PHYSICAL ANALYSIS OF THE GENERALIZATION OF THE BOUNDARY DIFFRACTION WAVE METHOD

3.1. Introduction

The diffracted field $U_k(P)$ is equal to the summation of $U^{(B)}(P)$ the disturbance originated at each point in the boundary of the aperture, and $\sum_i F_i(P)$ the total effect of the disturbance propagated from certain specific points in the aperture.

$$U_k(P) = U^{(B)}(P) + \sum_i F_i(P). \tag{3.1}$$

When the wave incident upon the aperture is plane or spherical wave, the last term in right side of Eq. (3.1) is found to represent a wave disturbance $U^{(G)}(P)$.

$\vec{W}(Q, P)$ may be associated with any given monochromatic wave field U(p) . The Helmholtz_ Kirchhoff integral represents the disturbance at a point **P** inside a volume bounded by a closed surface S or the sum of line integrals taken along small circles surrounding certain special points Qi on S, these points depend on the location of **P** and represent the singularities of the potential on S.

In this part the analysis will be extended to the case when the Helmholtz_ Kirchhoff integral is taken over an open surface, and the this theory treats the diffraction in media containing material obstacles whose linear dimensions are large compared to the wavelength, and to be more understandable, let`s verify it on considered aperture in an opaque screen, since when the field incident upon the aperture is plane or spherical Eq. (3.1) becomes

$$U_k(P) = U^{(B)}(P) + U^{(G)}(p) \quad (3.2)$$

Where $U^{(G)}(p)$ represents the geometrical optics field generated by Q_i, and obeys the laws of geometrical optics.

3.2. Transformation of The Basic Integral of Kirchhoff`s Diffraction Theory.

To understand and analysis Eq. (3.1), let`s begin this section with some assumptions and considerations

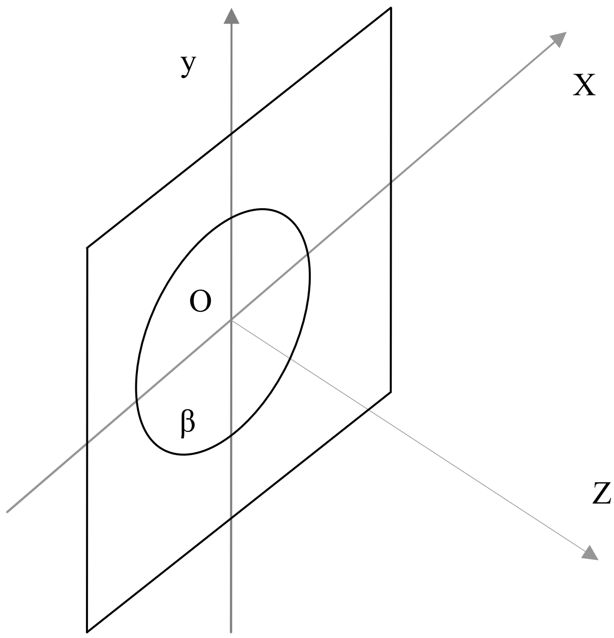

Figure 3.1: Illustrating the obstacle with an aperture

Consider the wave incidents on an aperture **β** in plane opaque screen Fig. 3.1, the diffraction of this monochromatic scalar wave $\vec{V}(p,t) = U(P)e^{-iwt}$, and the evaluated diffracted field at an observation point ***P*** in the half space z>0. With the help of Green's theorem and Helmholtz-Kirchhoff integral, the Kirchhoff field can be expressed as:

$$U_k(P) = \iint_{\beta} \vec{V}(Q,P).\vec{n}\,dS \quad (3.3)$$

the vector $\vec{V}(Q,P)$ can expressed as:

$$\vec{V}(\vec{r}_1,\vec{r}) = \frac{1}{4\pi}\left[U(\vec{r}_1)\text{grad}_Q\frac{e^{iks}}{s} - \frac{e^{iks}}{s}\text{grad}_Q U(\vec{r}_1) \right] \tag{3.4}$$

$\vec{r}_1$ is the position vector of the point Q which is a typical point in the aperture **β**, s denotes the distance QP, n is the unit vector normal to the plane of the screen and pointing into the half space (z >0), and k= w/c, c is the velocity of light.

As illustrated before the scalar wave $\vec{V}(Q,P)$ equals to the curl_Q of specific vector potential $\vec{W}(Q,P)$, so Eq. (3.4) can be written as:

$$\vec{V}(Q,P) = \vec{V}(\vec{r}_1,\vec{r}) = \text{curl}_Q\vec{W}(Q,P) \tag{3.5}$$

Returning to Eq. (2.61), the vector potential is

$$\vec{W}(Q,P) = \vec{W}(\vec{r}_1,\vec{r}) = \frac{\vec{e}_s\, e^{iks}}{4\pi s}\int_{\infty}^{0} e^{ik\mu}\, grad U(\vec{r}_1 + \mu\vec{e}_s)\partial\mu + \vec{W}_\infty \tag{3.6}$$

s is the distance PQ, $\vec{e}_s$ is the unit vector pointing in the direction from P to Q, $\vec{r}_1$ is the position vector of Q Fig. 3.2, and $\vec{W}_\infty$ is a certain residual contribution from infinity, since $\vec{W}_\infty$ = 0 if the incident field obeys the sommerfeld radiation condition in the half space.

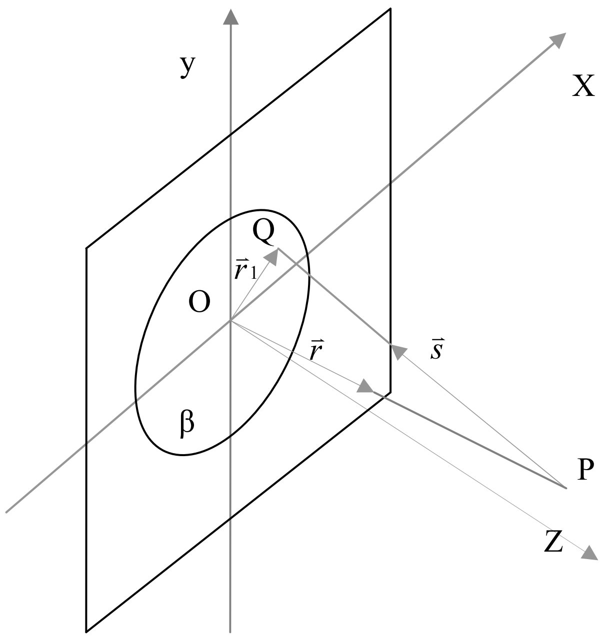

Figure 3.2: Illustrating the position of the point P and Q

Let`s assume that the field U is regular at each point Q in the aperture plane β, the vector potential $\vec{W}(Q,P)$ is a function of Q and has singularities at some points from Q_1, .. to Qn, each point Qi is surrounded by small circle of radius σ_i. So the Kirchhoff field at a typical point P can be expressed as:

$$U_k(P) = U_{k1}(P) + U_{k2}(P) \tag{3.7}$$

$U_{k1}(P)$ is the contribution field at the observation point P generated from the boundary of the aperture, and $U_{k2}(P)$ is the contribution field at the observation point P generated from the aperture.

From Eqs. (3.5) and (3.3) the second term of the Kirchhoff field can be written as:

$$U_{k2}(p) = \iint_{\beta} culQ\vec{W}(Q,P).\vec{n}\, dS = \sum_{i} F_i(P) \tag{3.8}$$

Since, the contribution $F_i(P)$ from a certain point Q_i on the observation point can be written as the limit, when the radius, of small circle Γ_i centered at Q, goes to Zero, of the integration along Γ_i of the vector potential.

$$F_i(P) = \lim_{\sigma i \to 0} \int_{\Gamma i} \vec{W}.\vec{l}\, dl \tag{3.9}$$

then the Kirchhoff field is equal to

$$U_k(P) = \int_{\Gamma} \vec{W}.\vec{l}\, dl + \sum_{\beta} \lim_{\sigma i \to 0} \int_{\Gamma i} \vec{W}.\vec{l}\, dl \tag{3.10}$$

The first term of the right hand side of Eq. (3.10) is considered to be the first part of the Kirchhoff field and written as the following expression:

$$U_{k1}(P) = \int_{\Gamma} \vec{W}.\vec{l}\, dl = U^B(P) \tag{3.11}$$

Γ denotes the boundary of the aperture, Γ_i denotes the boundary of a small circle around Q_i with radius σ_i, l is the unit vector tangential to boundary curves, and $\sum_{\beta}$ is the summation over all the singularities Q_i in the aperture β. In addition there are two points to note

1. In the Kirchhoff formula Eq. (3.3) only values of the incident field and its normal derivatives at points in the aperture enter as it illustrated in the previous part.
2. The expression in Eq. (3.11) for the boundary wave requires the knowledge of the potential W(Q,P) at each point Q of the boundary of the aperture.

Returning to Eq. (2.14)

$$U(P) = \sum_{S} F_i(P) + \sum_{V} G_i(P) \tag{3.12}$$

F_i are the contributions from all the singularities of $\vec{W}$ on any closed convex surface S surrounding P, and G_i are the contributions from all the singularities of U inside the volume v bounded by S.

$$G_i(P) = \lim_{\rho i \to 0} \iint_{Si} \vec{V}(Q,P) \, . \, \vec{n} \, dS \tag{3.13}$$

where ρ_i denotes the radius of sphere s_i surrounding a point Qi inside the volume v bounded by S, and n is the unit vector inward normal into the sphere. Here the surface S consisting of the aperture β, apportion C of the screen around the boundary of the aperture, and a large hemisphere D of radius R centered on some point in the aperture β Fig. 3.3, so U(P) can be expressed as:

$$U(P) = \sum_{\beta} F_i(P) + \sum_{C} F_i(P) + \sum_{D} F_i(P) + \sum_{V} G_i(P) \tag{3.14}$$

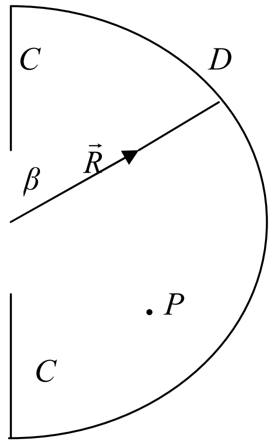

Figure 3.3: Shows the regions of the surface S

Now assume $\vec{R} \to \infty$ and the incident field satisfies sommerfeld radiation condition, the last two terms in Eq. (3.14) will be absent, so Eq. (3.14) can be written as:

$$\mathrm{U(P)} = \sum_{\beta} \mathrm{F_i}(P) + \sum_{C} \mathrm{F_i}(P) \tag{3.15}$$

One of the following must arise

I. All of the singularities of $\vec{\mathrm{W}}$ in the aperture plane are inside β, so no singularities of $\vec{\mathrm{W}}$ in the a portion C, and Eq. (3.10) can be expressed as:

$$\mathrm{U_k(P) = U^B(P) + U(P)} \tag{3.16}$$

the second term of the right side of Eq. (3.16) U(P) represents the disturbance from the aperture on the observation point (p)

$$\mathrm{U(P)} = \sum_{\beta} \mathrm{F_i}(P) \tag{3.17}$$

II. No singularities of $\vec{\mathrm{W}}$ are inside β, so Eq. (3.10) will be written as

$$\mathrm{U_k}(P) = \mathrm{U^B}(P) \tag{3.18}$$

III. Some of the singularities of $\vec{W}$ of the aperture plane are inside β, so Eq. (3.10) will be

$$U(P)=\sum_{\beta}F_i(P)+\sum_{C}F_i(P) \tag{3.19}$$

3.3. The Maggi-Rubinowics Representation

3.3.1 The Diffraction of a Plane Wave

Consider the diffraction of plane wave at an aperture in an opaque screen is

$$U(P)=Ae^{ik\vec{m}.\vec{r}} \tag{3.20}$$

From Eq. 2.25 , the vector potential $\vec{W}(Q,P)$ can be written as:

$$\vec{W}(\vec{r}_1,\vec{r})=A\,e^{ik\vec{m}.\vec{r}_1}\frac{e^{iks}}{s}\frac{1}{4\pi}\frac{\vec{e}_s\times\vec{m}}{1+\vec{e}_s\cdot\vec{m}} \tag{3.21}$$

$\vec{r}_1$ is the position vector of Q, s denotes the distance from Q to P, and p is the unit vector of the incident wave. The vector potential now has only one singularity Q_1in the plane of the aperture at the point given by $(1+\vec{e}_s\cdot p)$, this is the point in which the line through P in the direction of propagation of the plane wave intersects the plane of the aperture Fig. 3.4.

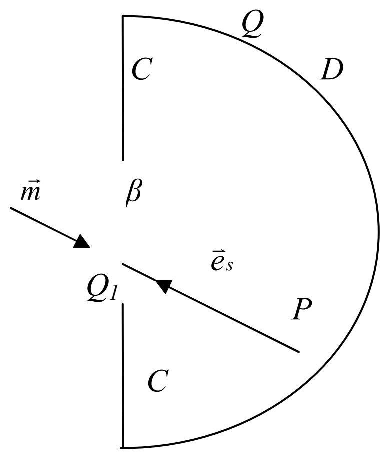

Figure 3.4: Illustrating the position of Q_1 on the surface S

"Q_i lies inside the aperture or outside it according to the position of P, as it lies in the direct beam of light or in the geometrical shadow"

$$U_k(P) = U^B(P) + U(P) \tag{3.22}$$

where $U^G(P)$ equals $U(P)$, and Eq. (3.22) is the Maggi-Rubinowicz representation of the Kirchhoff diffraction integral. According to Maggi-Rubinowicz, the second term of Eq. (3.22) has two forms depending on the position of the observation point P with respect to the beam of light direction.

1. When the observation point P lies on the direct beam, then

$$U^G(P) = U(P) = \sum_{\beta} F_i(P) + \sum_{C} F_i(P) \tag{3.23}$$

2. If the observation point P lies on the geometrical shadow, then

$$U^G(P) = 0 \tag{3.24}$$

According to Eqs. (3.11) and (3.21) the boundary wave $U^{(B)}(P)$ is now given by the integral

$$U^{(B)}(P) = \frac{A}{4\pi} \int_{\Gamma} e^{ik\vec{m}.\vec{r}_1} \frac{e^{iks}}{s} \frac{\vec{e}_s \times \vec{m} \ .\vec{1}}{1 + \vec{e}_s \cdot \vec{m}} dl \tag{3.25}$$

$\vec{r}_1$ is the position vector of Q, Γ is the boundary of the aperture where Q is located, and l denotes the unit vector tangential to the boundary of the aperture.

Eq. (3.22) expresses the diffracted field as a superposition of the boundary wave $U^B(P)$ with the geometrical wave $U^G(P)$. Notice that

1. In the direct beam the field is the interference of the unperturbed incident field U(P) with the boundary field $U^B(P)$.
2. In the shadow region the field arises from the boundary wave only if the wave incidents upon the aperture is a spherical wave (diverging or converging), the above results also valid, but with the followed changing.

3.3.2. Diffraction of The Divergent Spherical Wave

Consider the Kirchhoff`s diffraction of divergent spherical wave is

$$U(p) = A\frac{e^{ikr}}{r} \tag{3.26}$$

the vector potential associated with the diverging spherical wave can be written as:

$$\vec{W}(Q,P) = A\frac{e^{ik\vec{m}.\vec{r}_1}}{\vec{r}_1}\frac{e^{iks}}{s}\frac{1}{4\pi}\frac{\vec{e}_s \times \vec{r}_1}{\vec{s}\,\vec{r}_1 + \vec{e}_s \cdot \vec{r}_1} \tag{3.27}$$

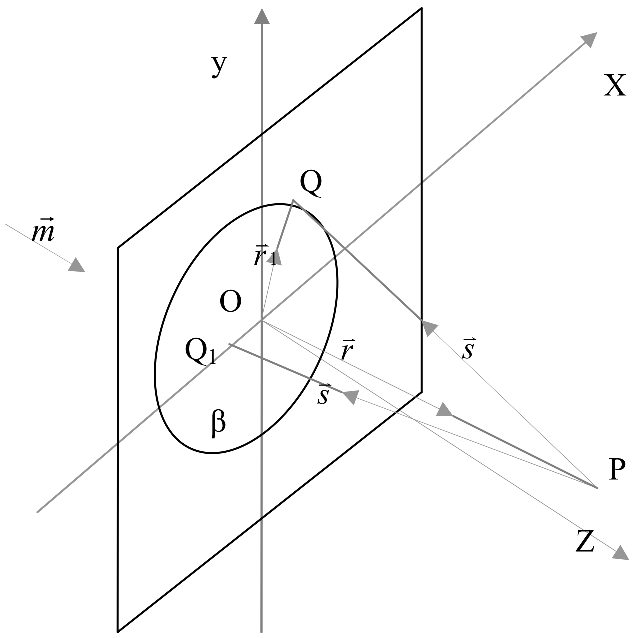

Figure 3.5: Showing the position of the intersection point Q_1 in 3-Dimensions.

Where A is the amplitude, $\vec{r}_1$ is the position vector of Q, and $\vec{W}(Q,P)$ has only one singularity Q_1 in the plane of the aperture, this point is (Q_1), which is the first point that the incident wave intersect the plane of the aperture Fig. 3.5. So the diffracted field of divergent spherical wave generated from the boundary of the aperture at the observation point P can be expressed as:

$$U^{(B)}(P) = \int_{\Gamma} \vec{W} . \vec{1} dl = \frac{A}{4\pi} \int_{\Gamma} \frac{e^{ik\vec{m}.\vec{r}_1}}{\vec{r}_1} \frac{e^{iks}}{s} \frac{\vec{e}_s \times \vec{r}_1 . \vec{1}}{\vec{s}\,\vec{r}_1 + \vec{e}_s \cdot \vec{r}_1} dl \tag{3.28}$$

3.3.3. Diffraction of a Convergent Spherical Wave

Consider the Kirchhoff`s diffraction of a convergent spherical wave is

$$U(p) = A\frac{e^{-ikr}}{r} \tag{3.29}$$

with it's singularity (r=0) on the same side of the aperture plane where the point of observation P is located. From Eq. (3.10) the Kirchhoff`s field is

$$U_K(P) = U^{(B)}(P) + \sum_{\beta} F_i(P) \tag{3.30}$$

The vector potential associated with the convergent spherical wave can be written as:

$$\vec{W}(Q,P) = -A\frac{e^{-ik\vec{m}.\vec{r}_1}}{\vec{r}_1} \frac{e^{iks}}{s} \frac{1}{4\pi} \frac{\vec{e}_s \times \vec{r}_1}{\vec{s}\,\vec{r}_1 + \vec{e}_s \cdot \vec{r}_1} \tag{3.31}$$

$\vec{r}_1$ denotes the position vector OQ, and Q_1 is the singularity point of the vector potential in Eq. (3.31), this point represent the point of intersection of the plan of the aperture with the line OP. The contribution from the point Q_1 depending on the location of the observation point P will be as the following: Fig. 2.5

1. If the observation point P lies in region I, then

$$F_i(P) = A\frac{e^{-ikr}}{r} \tag{3.32.1}$$

2. If the observation point P lies in region II, then

$$F_i(P) = -A\frac{e^{ikr}}{r} \tag{3.32.2}$$

Where, $\vec{r}$ is the position vector of P.

From Eqs. (3.23), (3.30), (3.32.1), and (3.32.2) the diffracted field from the aperture can be expressed as a geometrical field according to the position of the observation point P, so the Kirchhoff's field can be written as:

$$U_K(P) = U^{(B)}(P) + \sum_{\beta} F_i(P) + \sum_{C} F_i(P) \tag{3.33}$$

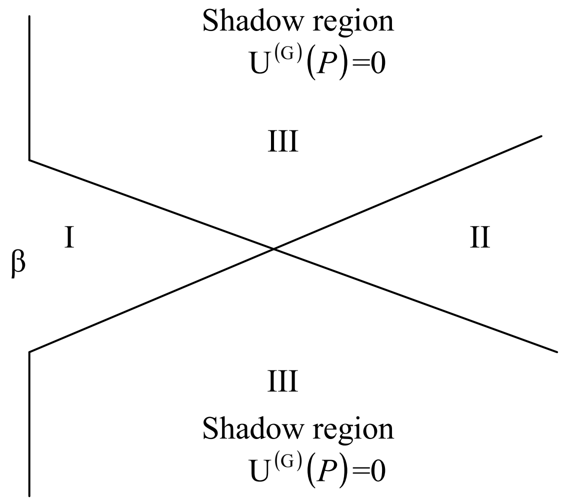

Figure 3.6: Illustrating the position of point P in C S W case

Depending on the location of P Fig. 3.6, the value of the geometrical wave field $U^{(G)}(P)$ is variant as the following

1. If the observation point P lies in region I, then

$$U^{(G)}(P) = A\frac{e^{-ikr}}{r} \tag{3.34.1}$$

2. If the observation point P lies in region II, then

$$U^{(G)}(P) = -A\frac{e^{ikr}}{r} \tag{3.34.2}$$

3. If the observation point P lies in region III, then

$$U^{(G)}(P) = 0 \tag{3.34.3}$$

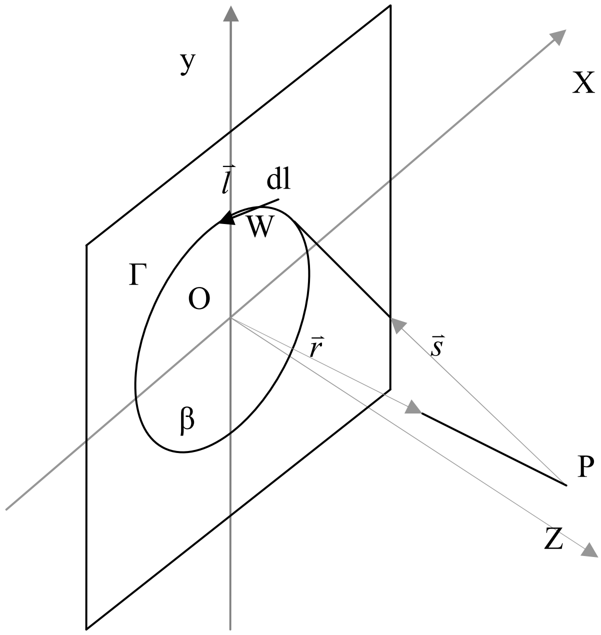

Figure 3.7: Illustrating the contribution from the boundary of the aperture

The boundary wave $U^{(B)}(P)$ is given by the integral of the vector potential around the edge of the aperture Fig. 3.7 as:

$$U^{(B)}(P) = \int_{\Gamma} \vec{W}.\vec{l}\,dl = -\frac{A}{4\pi}\int_{\Gamma} \frac{e^{ik\vec{m}.\vec{r}_1}}{\vec{r}_1}\,\frac{e^{iks}}{s}\,\frac{\vec{e}_s \times \vec{r}_1 . \vec{l}}{\vec{s}\,\vec{r}_1 + \vec{e}_s \cdot \vec{r}_1}\,dl \tag{3.35}$$

3.3.4. An Approximate Expression for The Vector Potential

There is a vector potential associated with any field, which is represented by geometrical optics, In this section we will derive this vector potential according to boundary–diffraction theory. For unperturbed incident field, let`s assume the amplitude $A(\vec{r}_1)$ and the phase $k\varphi(\vec{r}_1)$ for this unperturbed incident field, which can be expressed as:

$$U(\vec{r}_1) = A(\vec{r}_1)e^{k\phi(\vec{r}_1)} \tag{3.36}$$

where Ø and A are real.

As it is illustrated in Eqs. (2.37) and (2.35), the vector potential is written as:

$$\vec{W}(\vec{r}_1, \vec{r}\,) = \frac{e^{iks}}{4\pi s}\vec{e}_s \frac{(ik)^{-1} grad^1}{1 + \vec{e}_s.(ik)^{-1} grad^1} U(\vec{r}_1) \tag{3.37}$$

Let`s take the unperturbed incident field at Eq. (3.36)

$$U(\vec{r}_1) = A(\vec{r}_1) e^{k\phi(\vec{r}_1)} \tag{3.38}$$

By taking the gradient for the unperturbed incident field in Eq. (3.36) with respect to $\vec{r}_1$ the position vector of Q

$$grad^1 U(\vec{r}_1) = A(\vec{r}_1) e^{k\phi(\vec{r}_1)} \times ikgrad^1\phi + e^{k\phi(\vec{r}_1)} grad^1 A(\vec{r}_1) \tag{3.39}$$

In the right hand side of the above equation, take the value $A(\vec{r}_1) e^{k\phi(\vec{r}_1)}$ as a common factor, to be

$$grad^1 U(\vec{r}_1) = \left(ikgrad^1\phi + \frac{grad^1 A(\vec{r}_1)}{A(\vec{r}_1)} \right) A(\vec{r}_1) e^{k\phi(\vec{r}_1)} \tag{3.40}$$

Comparison with Eq. (3.36) the value $A(\vec{r}_1) e^{k\phi(\vec{r}_1)}$ in the previous equation is equal to the unperturbed incident field $U(\vec{r}_1)$, substitute $U(\vec{r}_1)$ instead of $A(\vec{r}_1) e^{k\phi(\vec{r}_1)}$ in the above equation to get, and take ik as a common factor

$$grad^1 U(\vec{r}_1) = ik\left(grad^1\phi + \frac{1}{ik}\frac{grad^1 A(\vec{r}_1)}{A(\vec{r}_1)} \right) U(\vec{r}_1) \tag{3.41}$$

Transfer the factor ik to the left hand side

$$ik^{-1} grad^1 U(\vec{r}_1) = \left(grad^1\phi + \frac{1}{ik}\frac{grad^1 A(\vec{r}_1)}{A(\vec{r}_1)} \right) U(\vec{r}_1) \tag{3.42}$$

Where "the second term on the right hand side of Eq. (3.42) is very much smaller than the first term"[2].

then Eq. (3.42) can be written as:

$$ik^{-1}grad^1 U(\vec{r}_1) = grad^1\phi\, U(\vec{r}_1) \qquad (3.43.1)$$

By comparison the both side of Eq. (3.43.1), we can say

$$ik^{-1}grad^1 \approx grad^1\phi \qquad (3.43.2)$$

From Eq. (3.43.1) the gradient of the unperturbed incident field $grad^1 U(\vec{r}_1)$ can be written as:

$$grad^1 U(\vec{r}_1) = ik\; grad^1\phi\, U(\vec{r}_1) \qquad (3.43.3)$$

Substitute the value of $grad^1 U(\vec{r}_1)$ from Eq. (3.43.3) into Eq. (3.37), to get a new form for the vector potential $\vec{W}(\vec{r}_1, \vec{r}\,)$

$$\vec{W}(\vec{r}_1, \vec{r}\,) = \frac{e^{iks}}{4\pi s}\vec{e}_s \frac{(ik)^{-1}}{1+\vec{e}_s.(ik)^{-1}grad^1}\, ik\; grad^1\phi\, U(\vec{r}_1) \qquad (3.44)$$

Rearrange the above equation, to get the following form for the vector potential $\vec{W}(\vec{r}_1, \vec{r}\,)$

$$\vec{W}(\vec{r}_1, \vec{r}) = U(\vec{r}_1)\frac{e^{iks}}{s}\frac{1}{4\pi}\vec{e}_s \frac{grad^1\phi(\vec{r}_1)}{1+\vec{e}_s.(ik)^{-1}grad^1} \qquad (3.45)$$

Substitute the valve of ($ik^{-1}grad^1$) from Eq. (3.43.2) into the previous equation since Ø is a function of $\vec{r}_1$, so the vector potential $\vec{W}(\vec{r}_1, \vec{r}\,)$ will be

$$\vec{W}(\vec{r}_1, \vec{r}) = U(\vec{r}_1)\frac{e^{iks}}{s}\frac{1}{4\pi}\vec{e}_s \frac{grad^1\phi(\vec{r}_1)}{1+\vec{e}_s.\,grad^1\phi(\vec{r}_1)} \qquad (3.46)$$

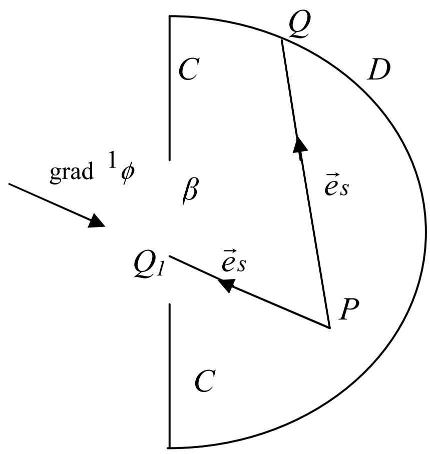

Figure 3.8: Illustrating the direction of grad $^1\phi$

The vector potential in Eq. (3.46) is seen to be pointing in the direction at right angles to the line specified in direction by the unit vector $\vec{e}_s$, and to the normal of $\mathrm{grad}^1\phi(\vec{r}_1)$ Fig. 3.8, the vector potential in Eq. (3.46) consists of

1. The incident wave

$$U(\vec{r}_1)\ .$$

2. A secondary spherical wave

$$\frac{e^{iks}}{s}\ .$$

3. A vectorial inclination factor

$$\frac{1}{4\pi}\vec{e}_s\frac{\mathrm{grad}^1\phi(\vec{r}_1)}{1+\vec{e}_s\,.\,grad^1\phi(\vec{r}_1)}\ .$$

.

Notice that, from the Fig. 3.8, the singularities of the of the vector potential $\vec{W}(\vec{r}_1,\vec{r})$ are given by

$$1+\vec{e}_s\,.\,grad^1\phi(\vec{r}_1)=0 \tag{3.47}$$

$grad^1\phi$ represents the unit vector in the direction of the incident ray. Hence, the singularities in the aperture of the vector potential $\vec{W}(Q,P)$ are those points Q_i, which lie on the rays that pass through the point (P), Fig. 3.8, depending on the location of P and the chosen direction of the incident ray.

3.3.5. An Approximate Generalization of The Maggi-Rubinowics Representation

The Maggi-Rubinowics representation, which expresses the Kirchhoff`s field as:

$$U_K(\vec{r}) = U^{(B)}(P) + U^{(G)}(P) \qquad (3.48)$$

Where $U^{(B)}(P)$ denotes the boundary wave, $U^{(G)}(P)$ is the geometrical wave, and Eq. (3.48) is corresponding to the plane incident wave or spherical incident wave.
The general representation of Kirchhoff`s field is

$$U_K(r) = U^{(B)}(P) + \sum_{\beta} Fi(P) \qquad (3.49)$$

where Fi(P) represents the contribution from a certain points Qi in the aperture, and if the incident wave is plane or spherical, then

$$\sum_{\beta} Fi(P) = U^{(G)}(P) \qquad (3.50)$$

Let`s assume that the field incident upon the aperture represented by the form

$$U(\vec{r}_1) = A(\vec{r}_1) e^{k\phi(\vec{r}_1)} \qquad (3.51)$$

and if we use the approximated vector potential in Eq. (3.46)

$$\vec{W}(\vec{r}_1, \vec{r}) = U(\vec{r}_1) \frac{e^{iks}}{s} \frac{1}{4\pi} \vec{e}_s \frac{grad^1 \phi(\vec{r}_1)}{1 + \vec{e}_s \,.\, grad^1 \phi(\vec{r}_1)} \qquad (3.52)$$

.

As what was illustrated in Fig. 3.8, $grad^1 \phi(\vec{r}_1)$ represents the unit vector in the direction of the incident ray, so we can say

$$grad^1 \phi = \vec{m} \qquad (3.53)$$

From Eq. (2.25), the vector potential $\vec{W}(Q,P)$ equals to

$$\vec{W}(Q,P) = e^{ik\vec{m}.\vec{r}_1} \frac{e^{iks}}{s} \frac{1}{4\pi} \frac{\vec{e}_s \times \vec{m}}{1 + \vec{e}_s \cdot \vec{m}} \tag{3.54}$$

Since, $grad^1\phi = \vec{m}$, then the wave $e^{ik\vec{m}.r_1}$ in the vector potential $\vec{W}(Q,P)$ can be expressed as $e^{ikgrad^1\phi.r_1}$ and approximately equals $e^{ik\phi(r_1)} = U(Q)$, then the above expression of the vector potential will be

$$\vec{W}(Q,P) = U(Q) \frac{e^{iks}}{s} \frac{1}{4\pi} \frac{\vec{e}_s \times \vec{m}}{1 + \vec{e}_s \cdot \vec{m}} \tag{3.55}$$

From Eq. (3.28), the contribution from the boundary of the aperture is

$$U^{(B)}(P) = \int_\Gamma \vec{W}.\vec{1}dl = \frac{1}{4\pi}\int_\Gamma U(Q) \frac{e^{iks}}{s} \frac{\vec{e}_s \times \vec{m}.\vec{1}}{1 + \vec{e}_s \cdot \vec{m}} dl \tag{3.56}$$

and like what was illustrated in section 3.2 the disturbance from any point Qi at the aperture is equal to

$$F_i(P) = \lim_{\sigma i \to 0} \int_{\Gamma_\circ} \vec{W}.\vec{1}dl \tag{3.57}$$

Substitute the value of the vector potential $\vec{W}(Q,P)$ into the Eq. (3.57), so the contribution $F_i(P)$ from (Qi) will be

$$F_i(P) = \lim_{\sigma i \to 0} \int_{\Gamma i} \vec{W}.\vec{1}dl = \frac{1}{4\pi} \lim_{\sigma i \to 0} \int_{\Gamma i} U(Q) \frac{e^{iks}}{s} \frac{\vec{e}_s \times \vec{m}.\vec{1}}{1 + \vec{e}_s \cdot \vec{m}} dl. \tag{3.58}$$

The singularities are those points Qi, which lie on the geometrical rays that pass through P. The contribution Fi(P) from (Qi) can be written as [3]

$$Fi(P) = U(Qi)\, e^{iks_i} \Lambda_j. \tag{3.59}$$

Where Fi(P) gives the contribution at P from point Qi at the aperture, s_i is the distance Q_iP, Fig. 3.9, and

$$\Lambda_j = \left(\frac{r_i \, r_i'}{R_i \, R_i'} \right)^{\frac{1}{2}} \varepsilon_i \qquad (3.60)$$

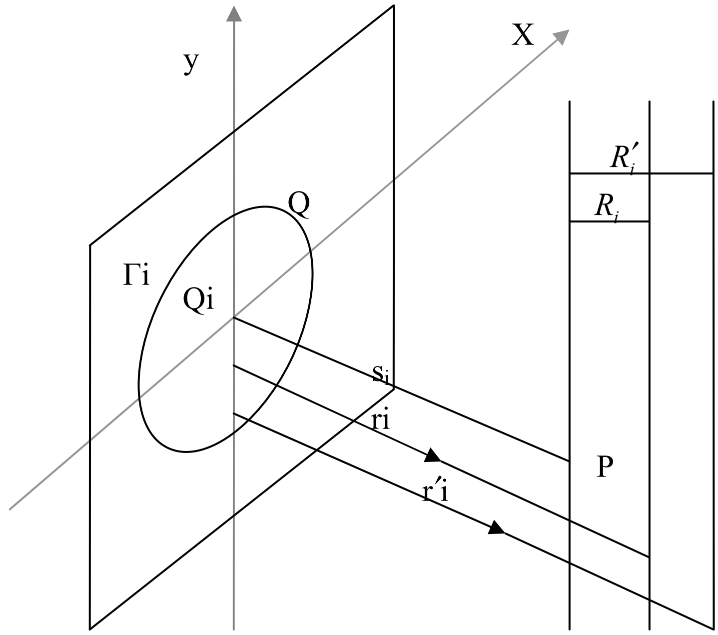

Figure 3.9: Illustrating the notations relating to the evaluation of Λ_j

where, $r_i \, r_i'$ and $R_i \, R_i'$ are the principal of curvature of the wave front of the incident at Qi and P, and

$$\varepsilon_i = \begin{cases} +1 \text{ if } R_i R_i' > 0,\ R_i > 0 \\ -1 \text{ if } R_i R_i' > 0,\ R_i < 0 \\ -1 \text{ if } R_i R_i' < 0 \end{cases} \qquad (3.61)$$

e^{iks_i} accounts for the change in phase associated with the passage of light from Qi to P, $\left(\frac{r_i \, r_i'}{R_i \, R_i'} \right)^{\frac{1}{2}}$ expresses the corresponding change in the amplitude of the light in accordance with the geometrical intensity law, and the factor ε_i accounts for the phase changes at foci (the phase anomaly).

The above explanation obtained the contribution from the aperture, which represents the second term of Eq. (3.49) when the incident wave is not plane wave or spherical wave.

To get the contribution of the boundary wave, let`s consider Eq. (3.56), which represents the boundary wave arises from the superposition of contributions from each element (dl) of the boundary of the aperture, this contribution of (dl) at a point ($Q(r_1)$) of Γ is proportional to the field at this point and considered to be propagated in the form of a spherical wavelet.

The directional behavior of the secondary wavelet is represented by the inclination factor

$$\frac{1}{4\pi}\frac{\vec{e}_s \times \vec{m}.\vec{1}}{1 + \vec{e}_s \cdot \vec{m}} = \frac{1}{4\pi}\frac{\sin(\theta)}{1 + \cos(\theta)}\cos(\varphi) \qquad (3.62)$$

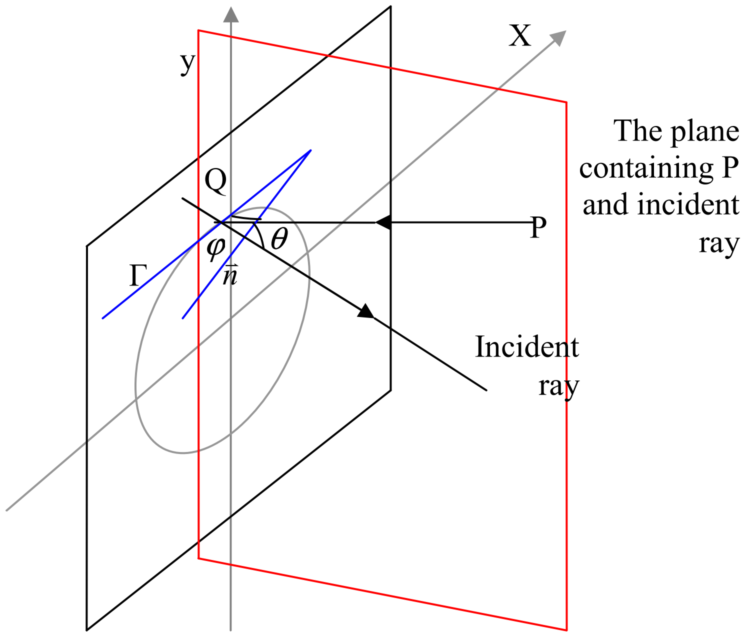

Figure 3.10: Showing the position of angles θ and φ

where θ is the angle between the direction of the vector $\vec{s}$ and the direction of the incident ray through Q, and φ is the angle between the tangent to boundary Γ at Q and the normal to the plane containing PQ and the incident ray Fig. 3.10.

"Here there are two terms for the asymptotic expression of the boundary diffraction wave $U^{(B)}(P)$ at Eq. (3.56)"[3].

The first term is

$$U_I^{(B)}(P) = \frac{1}{k^{\frac{1}{2}}} \sum_i U(Q_i) \frac{e^{iks_i}}{4\pi s_i} \left[\frac{2\pi}{\left| \partial^2 (s+\phi) / \partial l^2 \right|} \right]_i^{\frac{1}{2}} \times \alpha_i \frac{(\vec{e}_s \times \vec{m}).\vec{1}}{1 + \vec{e}_s \cdot \vec{m}} \tag{3.63}$$

The value of αi is changing according to the condition $\partial^2 (s+\phi) / \partial l^2$ as the following

$$\alpha i = e^{i\pi/4} \quad \text{if} \quad \partial^2 (s+\phi) / \partial l^2 > 0$$
$$\alpha i = e^{-i\pi/4} \quad \text{if} \quad \partial^2 (s+\phi) / \partial l^2 < 0 \tag{3.64}$$

The second term of the asymptotic expression of the boundary diffraction wave $U^{(B)}(P)$ at Eq. (3.56) is

$$U_{II}^{(B)}(P) = \frac{1}{ik} \sum_i U(Qi) \frac{e^{iks_i}}{4\pi s_i} \times \left[\frac{\sin\varphi(\vec{m} - \vec{e}_s).\vec{n}}{[(\vec{m} + \vec{e}_s).1^+][(\vec{m} + \vec{e}_s).\vec{1}^-]} \right]_i \tag{3.65}$$

where n_i denotes the unit normal to the aperture at Q_i pointing into the half space containing P, φ_i is the angle between the two tangents at Q_i, if superscripts (+, -) denote the limiting values at the two sides of Qi when Q^+_i and Q^-_i refer to the positive and negative sides of Q_i , respectively.

From Eq. (3.36), unperturbed incident field is

$$U(\vec{r}_1) = A(Q) e^{ik\phi(\vec{r}_1)} \tag{3.66}$$

Where Ø is the phase angle, and from Eq. (3.3) the Kirchhoff field was expressed as

$$U_k(P) = \iint_\beta \vec{V}(Q,P).\vec{n} dS \tag{3.67}$$

where, β is the aperture of the obstacle. Let`s write the Kirchhoff integral of the above equation as the following expression:

$$U_k(P) = \iint_{\beta} g(Q,P) e^{ikf(Q,P)} ds \tag{3.68}$$

To verify the Eq. (3.68), starting with Eqs. (3.36) and (3.3), we have to verify that

$$V(Q,P) = g(Q,P) e^{ikf(Q,P)} \tag{3.68.1}$$

the function f(Q,P) is equal

$$f(Q,P) = s + \phi(Q) \tag{3.68.2}$$

the function g(Q,P) is equal

$$g(Q,P) = \left[\left(ik - \frac{1}{s} \right) (\vec{e}_s . \vec{n}) - \left(ik\vec{m} + \frac{grad^1 A}{A} \right) . \vec{n} \right] A(Q) \tag{3.68.3}$$

The vector $\vec{V}(Q,P)$ associated with the plane wave U(Q) can be expressed as:

$$\vec{V}(Q,P) = U(Q) grad_Q \frac{e^{iks}}{s} - \frac{e^{iks}}{s} grad_Q U(Q) \tag{3.69}$$

the plane wave U(Q) is equal

$$U(Q) = A(Q) e^{ik\phi(Q)} \tag{3.70}$$

Then, the vector $\vec{V}(Q,P)$ associated with the plane wave U(Q) will be

$$\vec{V}(Q,P) = A(Q) e^{ik\phi(Q)} grad_Q \frac{e^{iks}}{s} - \frac{e^{iks}}{s} grad_Q \left(A(Q) e^{ik\phi(Q)} \right) \tag{3.71}$$

Solve the above gradient with respect to $\vec{r}_1$ in the vector $\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P})$, since Q is a secondary source point with position vector $\vec{r}_1$, and s is a function of $\vec{r}_1$ as it illustrated before, to get a new form of the vector $\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P})$

$$\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P}) = \mathrm{A}(\mathrm{Q}) e^{\mathrm{ik}\phi(\mathrm{Q})}\left(\frac{\mathrm{ik}}{\mathrm{s}}\mathrm{e}^{\mathrm{iks}} - \frac{\mathrm{e}^{\mathrm{iks}}}{s^2}\right) - \frac{\mathrm{e}^{\mathrm{iks}}}{\mathrm{s}}\left[\mathrm{A}(\mathrm{Q})\,\mathrm{ik}\phi'(\mathrm{Q}) e^{\mathrm{ik}\phi(\mathrm{Q})} + e^{\mathrm{ik}\phi(\mathrm{Q})}\,\mathrm{grad}_{\mathrm{Q}}\mathrm{A}(\mathrm{Q})\right] \quad (3.72)$$

take the summation of the exponents of e, and take the value $\mathrm{A}(\mathrm{Q})\mathrm{e}^{\mathrm{ik}(\mathrm{s}+\phi(\mathrm{Q}))}$ as a common factor from the above equation

$$\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P}) = \left(\frac{\mathrm{ik}}{\mathrm{s}} - \frac{1}{s^2}\right) - \frac{1}{\mathrm{s}}\left(\mathrm{ik}\phi'(\mathrm{Q}) + \frac{\mathrm{grad}_{\mathrm{Q}}\mathrm{A}(\mathrm{Q})}{\mathrm{A}(\mathrm{Q})}\right)\mathrm{A}(\mathrm{Q})\mathrm{e}^{\mathrm{ik}(\mathrm{s}+\phi(\mathrm{Q}))} \quad (3.73)$$

Since, $\frac{1}{s} = \vec{e}_{s}.\vec{n}$ and $\phi'(\mathrm{Q}) = \vec{\mathrm{P}}$. [1]

The previous equation will be

$$\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P}) = \left[\left(\mathrm{ik} - \frac{1}{s}\right)\vec{e}_{s}.\vec{n} - \left(\mathrm{ik}\vec{\mathrm{m}} + \frac{\mathrm{grad}_{\mathrm{Q}}\mathrm{A}}{\mathrm{A}}\right)\vec{e}_{s}.\vec{n}\right]\mathrm{A}(\mathrm{Q})\mathrm{e}^{\mathrm{ik}(\mathrm{s}+\phi(\mathrm{Q}))} \quad (3.74)$$

Compare the value of the vector $\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P})$ in Eq. (3.74) with the assumptions in Eqs. (3.68.2) and (3.68.3) to conclude that

$$\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P}) = \mathrm{g}(\mathrm{Q},\mathrm{P})\mathrm{e}^{\mathrm{ik}f(\mathrm{Q},\mathrm{P})}. \quad (3.75)$$

Substitute the value of $\vec{\mathrm{V}}(\mathrm{Q},\mathrm{P})$ from the previous equation in to Eq. (3.68), to get the Kirchhoff field as a function of the two functions g(Q,P) and f(Q,P)

$$U_k(P) = \iint_{\beta} \vec{V}(Q,P).\vec{n}dS = \iint_{\beta} g(Q,P)e^{ikf(Q,P)}.\vec{n}dS \quad (3.76)$$

Here we have three cases

1. Points inside the aperture at which the phase f(Q,P) of the integrand is stationary $(U_k(I))$.
2. Points on the boundary of the aperture Γ at which f(Q,P) is stationary with respect to a small displacement along Γ $(U_k(II))$.
3. Points where the boundary of the aperture Γ has a discontinuously changing tangent $(U_k(III))$.

It`s concluded from case (1), that the singular points Qi in the aperture of the vector potential $W(\bar{r}_1, \bar{r})$ given by Eq. (3.46) are the critical points of the first case, then

$$U_k(I) = \sum_{\beta} F_i(P) \quad (3.77)$$

The second contribution associated with the second case from the boundary of the aperture and equals to, Eq. (3.63),

$$U_k(II) = U_I(B) \quad (3.78)$$

The third contribution associated with each points Qi at which the tangential derivative of the phase will change discontinuously at the corner of the boundary of the aperture and equals to, Eq. (3.65),

$$U_k(III) = U_{II}(B) \quad (3.79)$$

CHAPTER 4

APPLICATION OF THE BOUNDARY DIFFRACTION WAVE METHOD

4.1 Introduction

The boundary diffraction wave method is used to determine the diffracted field associated with a certain wave, when this wave hits an obstacle with an aperture. The diffracted field or Kirchhoff`s field is composed of two fields, one of them originated from the aperture when the incident wave transmitted through the aperture with out diffraction, these waves, according to Huygens principle, the front wave of this incident wave generate a secondary wave, the interference of the wavelets associated with this wave make a disturbance on an observation point after the aperture. The second one is the contribution from the boundary of the aperture, here the incident wave is diffracted, and the diffracted field originated by this boundary can be measured at the observation point.

4.2 The Application of The Method on a Half Plane Screen

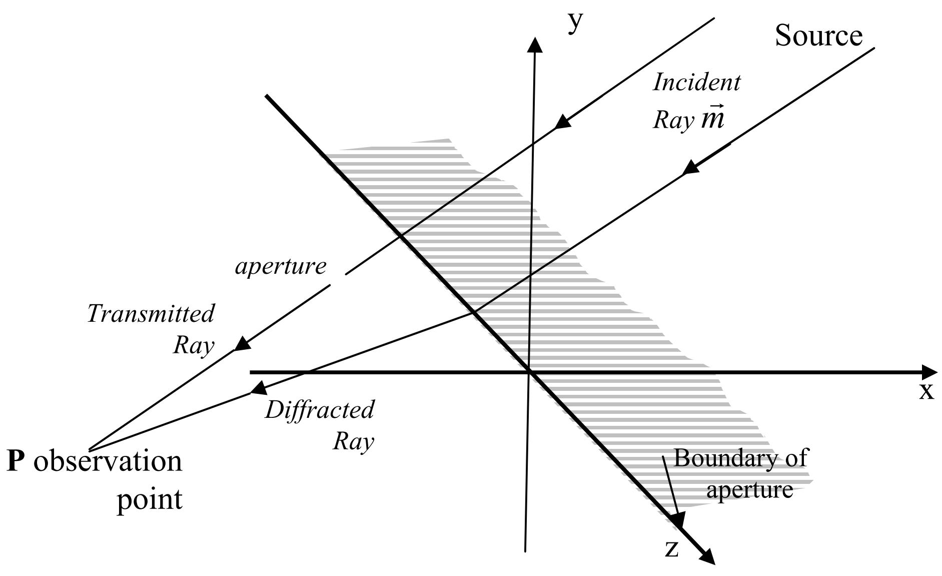

Figure 4.1: An obstacle with a half plane aperture with infinity boundary

Our study is the application of the boundary diffraction wave Method on a half plane aperture, the boundary of this aperture is located along z_direction, and applied on it, so the dimensions of the aperture are from (- ∞) to (0) in the x_direction, and from (-∞) to (∞) in the z_direction, Fig. 4.1.

The incident ray is diffracted when it hits the boundary of the aperture and transmitted when passes through the aperture itself. The diffracted field at the observation point (P) can be evaluated by applying the boundary diffraction wave theory, this diffracted field (Kirchhoff`s field) is the summation of the contribution of the field associated with the aperture and the field associated with the boundary of the aperture.

4.2.1 The Analysis of The Diffracted Ray

The incident ray hits the boundary of the aperture at point (Q) by an angle (ϕ_0) with the positive side of x-axis, let`s consider the point Q is located at the origin, Fig. 4.2, then the ray diffracted by the boundary, the diffracted field can be measured by assume that the diffracted ray is diffracted by an angle (ϕ).

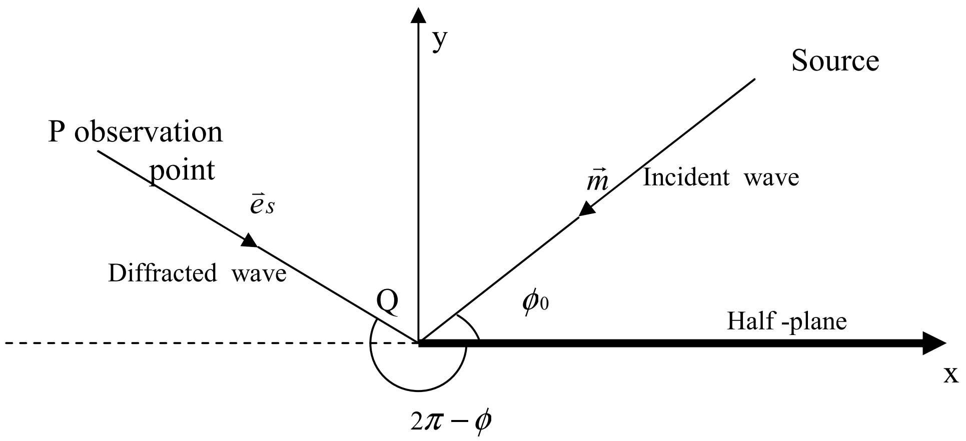

Figure 4.2: Illustrating the incident ray and the reflection boundary of the diffracted ray

The point Q can be considered as secondary source with an associated contribution which can be evaluated as:

$$U^{(B)}(P) = \int_{\Gamma} \vec{W} . \vec{1} dl \tag{4.1}$$

$\vec{l}$ denotes the unit vector tangential to the boundary (here $\vec{l} \equiv \vec{e}_z$), dl is an element from the boundary Γ, and from Eq.(2.25), $\vec{W}$ is the vector potential associated with the plane wave incident at the point (Q) on the boundary of the aperture:

From Fig. 4.2, we can find ($\vec{e}_s \times \vec{m}$) and ($\vec{e}_s \cdot \vec{m}$), since $\vec{m}$ denotes the unit vector of the incident wave, and $\vec{e}_s$ denotes the unit vector of the vector $\vec{s}$ between the observation point P and Q. Since we have two boundary of the diffracted ray, so let`s analysis both of them.

I. The Reflection Boundary of The Diffracted Ray

Analysis the two unit vectors in Fig. 4.2, the first unit vector is the unit vector $\vec{m}$ which represents the unit vector of the incident ray and can be analyzed in (X, Y) plane as:

$$\vec{m} = -\cos(\phi_0)\vec{e}_x - \sin(\phi_0)\vec{e}_y . \tag{4.2}$$

The second unit vector is the unit vector $\vec{e}_s$ which represents the unit vector of diffracted ray and can be analyzed in x-y plane as:

$$\vec{e}_s = \cos(\pi - \phi)\vec{e}_x - \sin(\pi - \phi)\vec{e}_y . \tag{4.3}$$

From the relations of trigonometric functions the unit vector of the diffracted ray $\vec{e}_s$ may be written as:

$$\vec{e}_s = -\cos(\phi)\vec{e}_x - \sin(\phi)\vec{e}_y . \tag{4.4}$$

Take the cross product of the two unit vectors $\vec{e}_s$ and $\vec{m}$

$$\vec{e}_s \times \vec{p} = [-\cos(\phi)\vec{e}_x - \sin(\phi)\vec{e}_y] \times [-\cos(\phi_0)\vec{e}_x - \sin(\phi_0)\vec{e}_y]. \tag{4.5}$$

Solve the above cross product of the two unit vectors $\vec{e}_s$ and $\vec{m}$, to get

$$\vec{e}_s \times \vec{p} = (\cos\phi \sin\phi_0 - \cos\phi_0 \sin\phi)\vec{e}z . \tag{4.6}$$

In Eq. (4.6) the cross product of $\vec{e}_s$ and $\vec{m}$ can be rewritten as:

$$\vec{e}_s \times \vec{p} = (\sin\phi_0 \cos\phi - \cos\phi_0 \sin\phi)\vec{e}z\,. \tag{4.7}$$

From the relations of trigonometric functions the cross product above of the two unit vectors $\vec{e}_s$ and $\vec{m}$ may be expressed as:

$$\vec{e}_s \times \vec{m} = \sin(\phi_0 - \phi)\vec{e}z\,. \tag{4.8}$$

Let`s find the dot product of the two unit vectors $\vec{e}_s$ and $\vec{m}$, Fig. 4.2, after analyze them in the x-y plane

$$\vec{e}_s \cdot \vec{m} = [-\cos(\phi)\vec{e}_x - \sin(\phi)\vec{e}_y].[-\cos(\phi 0)\vec{e}_x - \sin(\phi 0)\vec{e}_y]. \tag{4.9}$$

Take the dot product of the unit vectors $\vec{e}_s$ and $\vec{m}$ in the above equation to get

$$\vec{e}_s \cdot \vec{m} = \cos(\phi)\cos(\phi_0) + \sin(\phi)\sin(\phi_0). \tag{4.10}$$

From the relations of trigonometric functions the dot product above of the two unit vectors $\vec{e}_s$ and $\vec{m}$ may be expressed as:

$$\vec{e}_s \cdot \vec{m} = \cos(\phi - \phi_0). \tag{4.11}$$

II The Shadow Boundary of The Diffracted Ray

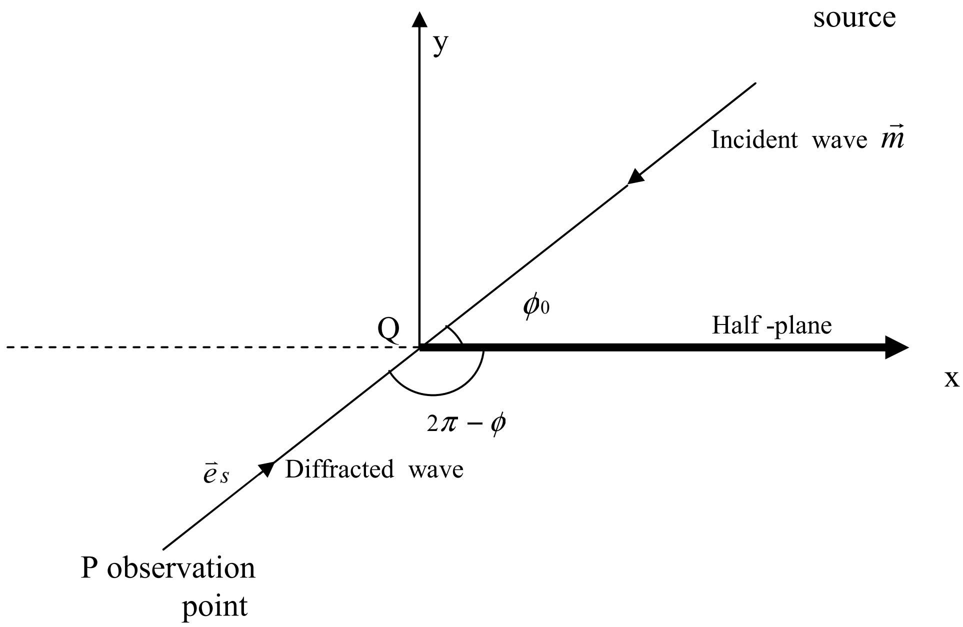

Figure 4.3: Illustrating the incident ray and the shadow boundary of the diffracted ray

The cross product and dot product of the unit vectors $\vec{e}_s$ *and* $\vec{m}$ in the second boundary case of the diffracted ray can be determine as the following:

Analysis the two unit vector in Fig. 4.3, the first unit vector is the unit vector $\vec{m}$ which represents the unit vector of the incident ray and can be analyzed in x-y plane. This vector is equal to Eq. (4.2)

The second unit vector is the unit vector $\vec{e}_s$ which represents the unit vector of diffracted ray and can be analyzed in x-y plane as

$$\vec{e}_s = \cos(\phi - \pi)\vec{e}_x + \sin(\phi - \pi)\vec{e}_y . \qquad (4.12)$$

From the relations of trigonometric functions the unit vector of the diffracted ray $\vec{e}_s$ may be written as

$$\vec{e}_s = -\cos(\phi)\vec{e}_x - \sin(\phi)\vec{e}_y . \qquad (4.13)$$

Take the cross product of the two unit vectors $\vec{e}_s$ and $\vec{m}$

$$\vec{e}_s \times \vec{m} = [-\cos(\phi)\vec{e}_x - \sin(\phi)\vec{e}_y] \times [-\cos(\phi_0)\vec{e}_x - \sin(\phi_0)\vec{e}_y]. \tag{4.14}$$

Solve the above cross product of the two unit vectors $\vec{e}_s$ and $\vec{m}$, to get

$$\vec{e}_s \times \vec{m} = (\cos\phi \sin\phi_0 - \cos\phi_0 \sin\phi)\vec{e}_z . \tag{415}$$

In the previous equation the cross product of $\vec{e}_s$ and $\vec{m}$ can be rewritten as:

$$\vec{e}_s \times \vec{m} = (\sin\phi_0 \cos\phi - \cos\phi_0 \sin\phi)\vec{e}_z . \tag{4.16}$$

From the relations of trigonometric functions the cross product above of the two unit vectors $\vec{e}_s$ and $\vec{m}$ may be expressed as:

$$\vec{e}_s \times \vec{m} = \sin(\phi_0 - \phi)\vec{e}_z . \tag{4.17}$$

Let`s find the dot product of the two unit vectors $\vec{e}_s$ and $\vec{m}$ in Fig. 4.3 after analyze them in the x-y plane

$$\vec{e}_s \cdot \vec{m} = [-\cos(\phi)\vec{e}_x - \sin(\phi)\vec{e}_y].[-\cos(\phi 0)\vec{e}_x - \sin(\phi 0)\vec{e}_y]. \tag{4.18}$$

Take the dot product of the unit vectors $\vec{e}_s$ and $\vec{m}$ in the above equation to get

$$\vec{e}_s \cdot \vec{m} = \cos(\phi)\cos(\phi_0) + \sin(\phi)\sin(\phi_0). \tag{4.19}$$

From the relations of trigonometric functions the dot product above of the two unit vectors $\vec{e}_s$ and $\vec{m}$ can be expressed as:

$$\vec{e}_s \cdot \vec{m} = \cos(\phi - \phi_0). \tag{4.20}$$

From the above analysis, it is clear from Eqs. (4.8) and (4.17) that the cross products $\vec{e}_s \times \vec{m} = \sin(\phi_0 - \phi)\vec{e}z$ in both sides of the boundary conditions of the diffracted ray are the same and from Eqs. (4.11) and (4.20) that the dot products of $\vec{e}_s \cdot \vec{m} = \cos(\phi - \phi_0)$ in both sides of the boundary conditions of the diffracted ray are the same, so we will consider only one boundary to find the disturbance from the boundary of the aperture.

4.3 The Kirchhoff`s Field

As it is mentioned before in our case study, the Kirchhoff field composed to two parts, the contribution from aperture and the contribution from the aperture itself. This section evaluates the field at the observation point, which is originated by these two contributions.

4.3.1 The Contribution from The Boundary Of The Aperture

To evaluate this part, let`s substitute from Eqs. (2.25), (4.8) and (4.11) into Eq.(4.1) to get the contribution from the boundary of the aperture as:

$$U^{(B)}(P)= Ae^{ik\vec{m}.\vec{r}_1}\frac{1}{4\pi}\frac{\sin(\phi_0-\phi)\vec{e}z}{1\ +\ \cos(\phi-\phi_0)}\int_{\Gamma}\frac{e^{iks}}{s}\vec{1}dl. \qquad (4.21)$$

Because the boundary of the aperture is applied on z-axis, and $\vec{1}$ is the unit vector tangent to the boundary and dl is an element from the boundary, so we can say ($\vec{1}=\vec{e}_z$) and $dl = dz'$. Since the secondary source point Q is located at the origin, then r_1 the position vector of(Q) is equal to (0), so $e^{ik\vec{m}.\vec{r}_1}=1$, and the above equation can be written as:

$$U^{(B)}(P)= A\frac{1}{4\pi}\frac{\sin(\phi_0-\phi)\vec{e}_z}{1\ +\ \cos(\phi-\phi_0)}\vec{e}_z\int_{\Gamma}\frac{e^{iks}}{s}dz'. \qquad (4.22)$$

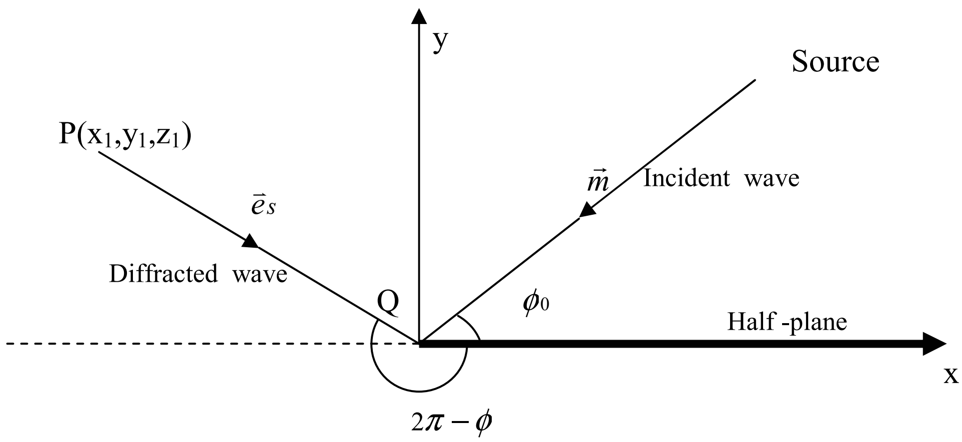

Figure 4.4: Illustrating the incident and diffracted angles

From Fig. 4.4, $s=\sqrt{\left(x_1^2+y_1^2+(z_1-z')^2\right)}$ the limits of the integral according to z-axis is from (-∞) to (∞), and $\vec{e}z\,.\,\vec{e}z=1$. By substituting in Eq. (4.22)

$$U^{(B)}(P)=\frac{A}{4\pi}\frac{\sin(\phi_0-\phi)}{1\ +\ \cos(\phi-\phi_0)}\int_{-\infty}^{\infty}\frac{e^{ik\left(\sqrt{\left(x_1^2+y_1^2+\left(z_1-z'\right)^2\right)}\right)}}{\sqrt{\left(x_1^2+y_1^2+(z_1-z')^2\right)}}\,dz' \tag{4.23}$$

to be more easy to evaluate Eq. (4.23), let`s assume that

$$C=\frac{A}{4\pi}\frac{\sin(\phi_0-\phi)}{1\ +\ \cos(\phi-\phi_0)}, \tag{4.24}$$

at assume that the square root in Eq. (4.23) is equal to R

$$R=\sqrt{\left(x_1^2+y_1^2+(z_1-z')^2\right)}, \tag{4.25.1}$$

and the first and second terms of the square root R is equal to R_1, so

$$R_1^2=x_1^2+y_1^2, \tag{4.25.2}$$

and the third term of the square root R (z_1-z') equals to

$$z_1-z'=R_1 sh(\alpha) \tag{4.26.1}$$

$$-dz'=R_1 ch(\alpha)d\alpha\ . \tag{4.26.2}$$

From Eqs. (4.25.1), (4.26.1) and (4.26.2) the value of R will be

$$R=\left[R_1^2+R_1^2 sh^2(\alpha)\right]^{1/2}. \tag{4.27}$$

Take R_1^2 as a common factor from the previous equation to get

$$R = \left[R_1^2\left(1 + sh^2(\alpha)\right)\right]^{1/2}. \tag{4.28}$$

From the relations of trigonometric functions the square root R can be written as

$$R = \left[R_1^2 ch^2(\alpha)\right]^{1/2}. \tag{4.29}$$

The final expression of the square root R will be

$$R = R_1 ch(\alpha) . \tag{4.30}$$

By substituting from Eqs. (4.24), (4.26.1), (4.26.2) and (4.30) into Eq. (4.23)

$$U^{(B)}(P) = C \int_{-\infty}^{\infty} \frac{e^{ik\left(R_1 ch\alpha\right)}}{R_1 ch\alpha} (\text{-} R_1 ch\alpha) d\alpha = -C \int_{-\infty}^{\infty} e^{ikR_1 ch\alpha} d\alpha \tag{4.31}$$

the phase function

$$\Psi(\alpha) = ch\alpha \tag{4.32}$$

let`s take the two first terms in Taylor series[1] for $\Psi(\alpha)$

$$\Psi(\alpha) = \Psi(\alpha_0) + \frac{\Psi'(\alpha_0)}{1!}(\alpha - \alpha_0) + \frac{1}{2!}\Psi^2(\alpha_0)(\alpha - \alpha_0)^2 + \frac{1}{3!}\Psi^3(\alpha_0)(\alpha - \alpha_0)^3........ \tag{4.33}$$

where $\alpha_0 = 0$ and $\Psi(\alpha_0) = 1$.

---

[1] If $n \geq 0$ is an integer and f is a function which is n times continuously differentiable on the closed interval $[a, x]$ and $n + 1$ times differentiable on the open interval (a, x), then we have

$$f(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f^{(2)}(a)}{2!}(x - a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x - a)^n + R_n$$

taken from (www.answers.com)

Then the three first terms of the Taylor series will be

$$\Psi(\alpha) = 1 + \frac{1}{2}\alpha^2 . \tag{4.34}$$

Substituting from Eq. (4.32) into Eq. (4.31)

$$\mathrm{U}^{(\mathrm{B})}(\mathrm{P}) = -C\int_{-\infty}^{\infty} e^{ikR_1 ch\alpha} d\alpha = -C\int_{-\infty}^{\infty} e^{ikR_1 \Psi(\alpha)} d\alpha . \tag{4.35}$$

Now substitute from Eq. (4.34) into the above equation, to get

$$\mathrm{U}^{(\mathrm{B})}(\mathrm{P}) = -C\int_{-\infty}^{\infty} e^{ikR_1\left(1 + \frac{1}{2}\alpha^2\right)} d\alpha . \tag{4.36}$$

Separate the exponents of e in the integrand of the above equation

$$\mathrm{U}^{(\mathrm{B})}(\mathrm{P}) = -C\int_{-\infty}^{\infty} e^{ikR_1} e^{\frac{ikR_1\alpha^2}{2}} d\alpha \tag{4.37}$$

since, the value e^{ikR_1} is a constant, so the previous equation can be written as

$$\mathrm{U}^{(\mathrm{B})}(\mathrm{P}) = -Ce^{ikR_1}\int_{-\infty}^{\infty} e^{\frac{ikR_1\alpha^2}{2}} d\alpha . \tag{4.38}$$

By Hankel function, the above integration can be evaluated

$$\mathrm{U}^{(\mathrm{B})}(\mathrm{P}) = -Ce^{ikR_1} . \frac{\sqrt{2\pi}\ \mathrm{e}^{\left(\frac{-\mathrm{i}\pi}{4}\right)}}{\sqrt{kR_1}} \tag{4.39}$$

By substitute the value of C from Eq. (4.24) into the above equation

$$U^{(B)}(P) = -\frac{A}{2\sqrt{2\pi}} \frac{\sin(\phi_0 - \phi)}{1 + \cos(\phi - \phi_0)} \frac{e^{\left(ikR_1 - \frac{i\pi}{4}\right)}}{\sqrt{kR_1}}. \qquad (4.40)$$

It is apparent from Eq. (4.40) that the field approaches infinity at the shadow region for $\phi = \pi + \phi_0$. A function will be defined to cancel this defect of Eq. (4.40). The related function must be equal to one out of the shadow boundary. This function can be defined as [6]

$$f(\phi, \pi \mp \phi_0) = p(\phi, \pi \mp \phi_0) \times \left[1 - \exp\left(-\sqrt{2\pi kR_1}\left|\cos\frac{\phi \pm \phi_0}{2}\right|\right)\right] \qquad (4.41)$$

where the phase function $p(\phi, \pi \mp \phi_0)$ can be defined as

$$p(\phi, \pi \mp \phi_0) = \exp\left[i\left(\pi/4\right)\exp\left[-\left|\phi - (\pi \mp \phi_0)\right|\right]\right]. \qquad (4.42)$$

A comparison between the exact solution and the solution of BDWM is illustrated in section 5.4.

4.3.2. The Contribution from The Aperture

Consider the incident wave is a homogenous plane wave, and the incident wave is propagated in the direction specified by the unit vector $\vec{m}$, so it can be written as

$$U(P) = Ae^{ik\vec{m}.\bar{r}} \qquad (4.43)$$

$\bar{r}$ denotes the position vector of the observation point P and A is a constant
By returning to equation (2.5) which is

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[U(Q)\text{grad}_Q\frac{e^{iks}}{s} - \frac{e^{iks}}{s}\text{grad}_Q U(Q)\right] \qquad (4.44)$$

where s is the distance between the observation point P and the point Q at the aperture, and the vector $\vec{s}$ is equal to $\vec{r}_1 - \vec{r}$, where $\vec{r}_1$ and $\vec{r}$ are the position vector of the point Q at the aperture and the observation point P, respectively. From Eq. (2.26), the incident wave at a specific point Q at the aperture can be written as:

$$U(Q) = Ae^{ik\vec{m}.\bar{r}_1}. \tag{4.45}$$

By substituting from Eq. (4.45) into Eq. (4.44), the vector V(Q,P) can be expressed as:

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[e^{ik\vec{m}.\bar{r}_1} grad_Q \frac{e^{iks}}{s} - \frac{e^{iks}}{s} grad_Q e^{ik\vec{m}.\bar{r}_1} \right] \tag{4.46}$$

since the gradient in the above equation is taking with respect to the point (Q), and the vector $\vec{s}$ can be expressed as a function on $\vec{r}_1$ the position vector of Q, then the $grad_Q$ can be written as $\frac{\partial}{\partial s}$ or $\frac{\partial}{\partial \bar{r}_1}$, and the above equation can be written as:

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[e^{ik\vec{m}.\bar{r}_1} \frac{\partial}{\partial s} \frac{e^{iks}}{s} - \frac{e^{iks}}{s} \frac{\partial}{\partial \bar{r}_1} e^{ik\vec{m}.\bar{r}_1} \right]. \tag{4.47}$$

By solving the above differential, we get

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[e^{ik\vec{m}.\bar{r}_1} \left(\frac{e^{iks}}{s} ik - \frac{e^{iks}}{s^2} \right) - \frac{e^{iks}}{s} e^{ik\vec{m}.\bar{r}_1} ik\vec{m} \right] \tag{4.48}$$

with some changing in the above equation we get

$$\vec{V}(Q,P) = \frac{1}{4\pi}\left[e^{ik\vec{m}.\bar{r}_1} \frac{e^{iks}}{s} \left(ik - \frac{1}{s} \right) - \frac{e^{iks}}{s} e^{ik\vec{m}.\bar{r}_1} ik\vec{m} \right] \tag{4.49}$$

Since, the first term in the above equation is in the direction of the vector $\vec{s}$, and by taking $e^{ik\vec{m}.\vec{r}_1}\frac{e^{iks}}{s}$ out of the parenthesis the vector $\vec{V}(Q,P)$ will be

$$\vec{V}(Q,P) = \frac{1}{4\pi} e^{ik\vec{m}.\vec{r}_1} \frac{e^{iks}}{s}\left[\left(ik - \frac{1}{s}\right)\vec{e}_s - ik\vec{m}\right] \tag{4.50}$$

$\vec{e}_s$ denotes the unit vector of the vector $\vec{s}$. According to the boundary diffraction wave theory, and like what is illustrated in Eq. (2.4), the disturbance from the aperture at the observation point p may be expressed as:

$$U(P) = \iint_s \vec{V}(Q,P).\vec{n}\, dS \ . \tag{4.51}$$

In Eq. (4.51), S is any closed surface bounding a volume v containing the observation point P and $\vec{n}$ is the unit vector inward normal to S. "Regardless the nature of U, the vector $\vec{V}(Q,P)$ can always be expressed in terms of a vector potential $\vec{W}(Q,P)$"[2]

$$\vec{V}(Q,P) = curl_Q\, \vec{W}\,(Q\,,P)\,. \tag{4.52}$$

Let`s now consider P is a fixed point, so the above equation will be a function of Q. The vector potential $\vec{W}(Q,P)$ must have singularities on the surface S, if $\vec{W}(Q,P)$ has no singularities on S then U(p) = 0.

All the singularities of $\vec{W}(Q,P)$ on the surface S occur at discrete points Q_1, Q_2, ………, Q_n, which are surrounded by a small circles with radii σ_1, σ_2, ……, σ_n , and the boundaries of these circles are Γ_1, Γ_2, …….., Γ_n , then from Stokes theorem and from Eq. (2.11) the Eq. (4.51) can be written as:

$$U(P) = \iint_{s-} curl_Q\, \vec{W}\,(Q\,,P).\vec{n}\, ds = \sum_i \int_{\Gamma i} \vec{W}.\vec{l}\, dl \tag{4.53}$$

$\vec{l}$ is the unit vector along the tangent to Γ_i, dl is an element of Γ_i and s_ denotes the region of S which excludes the small circle. Let`s assume that each point Qi effects on the typical point P by the disturbance F_i(p), so the total disturbance at p can be expressed as:

$$U(P) = \sum_i F_i(P) . \quad (4.54)$$

The contribution from each point Qi at the aperture on the observation point P is equal to the other disturbance from the other points in the aperture, so we can evaluate the disturbance from one point Q_1, and this disturbance represents the evaluation of the other disturbance originated at the other points and measured at the a certain observation points. The disturbance Fi(P) from (Qi) can be expressed as the limit of the integral of the vector potential W(Q,P) associated with each point Qi along Γ_i when $\sigma_i \rightarrow 0$

$$Fi(P) = \lim_{\sigma \rightarrow 0} \int_{\Gamma i} \vec{W} . \vec{l} dl. \quad (4.55)$$

Now consider (Q_1) is the first point at which, the incident wave with the unit vector $\bar{p}$ intersects the plane of the aperture Fig. 4.5 and at the point of intersection (Q_1), the angle between the unit vector of the distance (s) between the observation point P and Q_1 ($\bar{e}_s$) and the unit vector of the incident wave $\vec{m}$ approximately equals π, i.e. The angle between $\bar{e}s$ and $\vec{m} \cong \pi$.

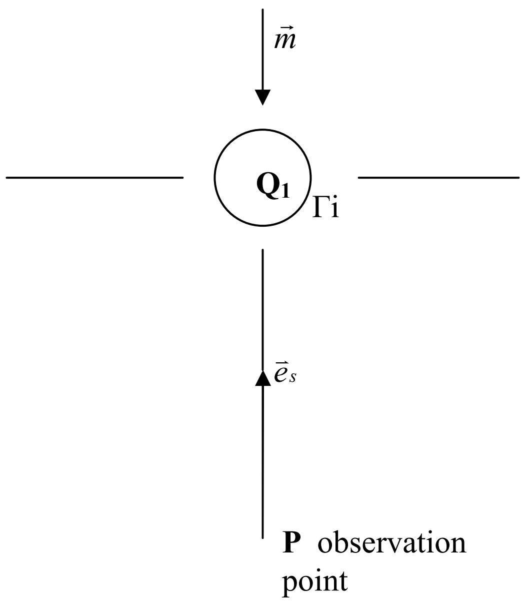

Figure 4.5: Illustrating the directions of $\bar{e}_s$ and $\vec{m}$ at point Q_1

The vector potential associated with the plane wave at point (Q_1) equals

$$\vec{W}(Q,P) = A\, e^{ik\vec{m}.\vec{r}_1} \frac{e^{iks}}{s} \frac{1}{4\pi} \frac{\vec{e}_s \times \vec{m}}{1 + \vec{e}_s \cdot \vec{m}} \quad . \tag{4.56}$$

By substituting from Eq. (4.56) into Eq. (4.55) the contribution from (Q1) at the observation point P can be evaluated as:

$$F_1(p) = \lim_{\sigma \to 0} \int_{\Gamma i} \vec{W}.\vec{1}dl. = \frac{A}{4\pi} \int_{\Gamma i} e^{ik\vec{m}.\vec{r}_1} \frac{e^{iks}}{s} \frac{\vec{e}_s \times \vec{m}.\vec{1}}{1 + \vec{e}_s \cdot \vec{m}} dl. \tag{4.57}$$

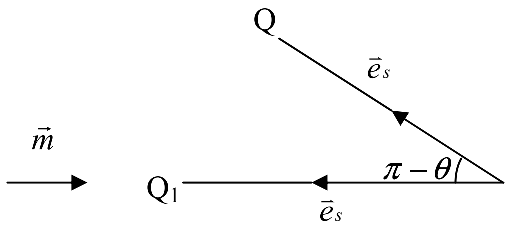

Figure 4.6: The angle θ between $\bar{e}_s$ and $\vec{m}$

Since θ is the angle between the unit vector of the distance (s) (between the observation point P and Q_1) ($\bar{e}_s$) and the unit vector of the incident wave ($\vec{m}$), Fig. 4.6, so the inclination factor in Eq. (4.57) can be expressed as:

$$\frac{\vec{e}_s \times \vec{m}}{1 + \vec{e}_s \cdot \vec{m}} = \frac{\sin\theta}{1 - \cos\theta} = \frac{1}{\tan\left(\frac{\theta}{2}\right)} . \tag{4.58}$$

In the small circle Γ_i centered at Q_1, Fig. 4.7, the element dl can be written as

$$dl = \sigma_i \, d\phi \; . \tag{4.59}$$

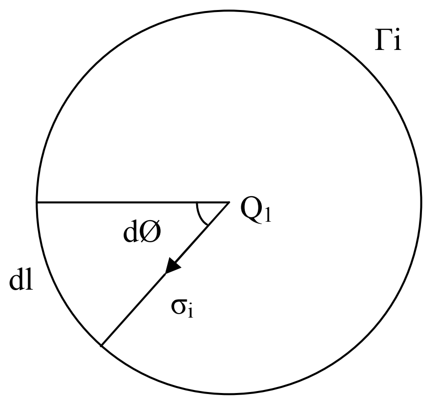

Figure 4.7: Illustrating the small circle Γi

The position vector $\vec{r}_1$ of the point Q_1, Fig. 4.8, can be expressed as:

$$\vec{r}_1 = \vec{r} + \vec{s} \tag{4.60}$$

where, $\vec{r}$ denotes the position vector of the observation point P and $\vec{s}$ is the vector between P and Q_1.

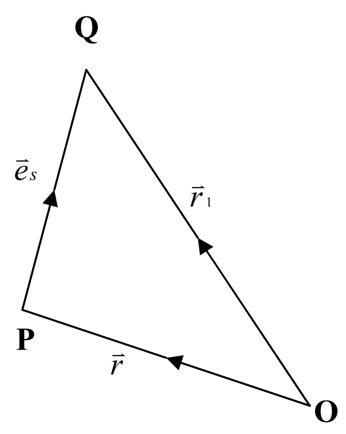

Figure 4.8: Showing the relation between the position vectors of P and Q.

By substituting from Eqs. (4.58),(4.59) and (4.60) into Eq. (4.57), where ϕ is the azimuthal angle, the disturbance from the point Q_1 at p will be

$$F_1(p) = \lim_{\sigma_i \to 0} \frac{A}{4\pi} e^{ik\vec{m}.\vec{r}} \int_0^{2\pi} \frac{e^{ik\vec{m}.\vec{s}} e^{iks}}{s} \frac{\sigma_i}{\tan\left(\theta/2\right)} d\phi \tag{4.61}$$

since $\vec{m}$ and $\vec{e}_s$ are approximately applied to each other, then the angle θ is considered to be very small, then this expression can be written

$$\tan\left(\theta/2\right) = \left(\theta/2\right). \tag{4.62}$$

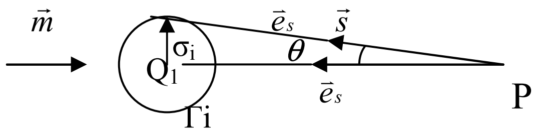

Figure 4.9: Illustrating the relation between σ_i and $\vec{s}$

Hence the circle Γi is very small circle, the distance s is very large with respect to the radius σi of Γi , so the angle θ can be written as $\sigma i/s$, Fig. 4.9 and the tangent of the angle θ can be expressed as:

$$\tan\left(\theta/2\right) = \left(\theta/2\right) = \frac{\sigma_i}{2s}. \tag{4.63}$$

By substituting from Eq. (4.63) into Eq. (4.61)

$$F_1(p) = \lim_{\sigma_i \to 0} \frac{A}{4\pi} e^{ik\vec{m}.\vec{r}} \int_0^{2\pi} \frac{e^{ik\left(\vec{m}.s\vec{e}_s + \vec{s}\vec{e}_s\right)}}{s} \frac{\sigma_i \times 2s}{\sigma_i} d\phi. \tag{4.64}$$

From Fig. 4.9, the angle between the incident wave unit vector $\vec{m}$ and $\vec{e}s$ the unit vector of the distance s is very small and the directions of them are opposite to each other, the exponent of e in Eq. (4.64) can be written as:

$$\vec{m}\,.\,s\vec{e}_s + s\vec{e}_s \approx s\cos(\pi) + s = 0\,. \tag{4.65}$$

From Eq. (4.65), Eq. (4.64) can be written as:

$$F_1(p) = \lim_{\sigma_i \to 0} \frac{A}{4\pi} e^{ik\vec{m}.\vec{r}} \int_0^{2\pi} 2 d\phi \tag{4.66}$$

by solving the integral in Eq. (4.66), the contribution from the aperture at the observation point (P) will be

$$F_1(p) = \frac{A}{4\pi} e^{ik\vec{m}.\vec{r}}\, 2(2\pi - 0) = A e^{ik\vec{m}.\vec{r}} \quad . \tag{4.67}$$

The total contribution from the aperture and the boundary of the aperture can be evaluated by summation the contribution of the boundary Eq. (4.40) and the contribution from the aperture Eq. (4.67), the summation may be expressed as:

$$\text{The total contribution at (P)} = U^{(B)}(P) + F_1(p) \tag{4.68}$$

CHAPTER 5

OUR SOLUTION VERSUS TO GANCI'S SOLUTION OF TWO SPECIAL CASES OF INCIDENT WAVE AND THE EXACT SOLUTION

5.1. Introduction

The solution of the half-plane diffraction problem by GANCI is examined in a case of an oblique incident and a case of a normal incident of a unit amplitude plane monochromatic wave [4, 5].

In this chapter we compare our solution, by the boundary diffraction wave method, with these two cases for a half plane problem.

5.2. The Comparison of The Solution of The Boundary Diffraction Wave Method With The Case of Normal Incidence of GANCI Solution for a Half Plane Problem

The aim of this section is to compare our solution, by the boundary diffraction wave method, with the solution of GANCI when the incident ray is normal to the plane of the aperture for the same problem (a half-plane problem) regard that the two methods consider that the diffracted wave field U(P) at the observation point (P) can be expressed as the summation of the incidence wave $U_G(P)$ that represents the unperturbed wave transmitted through the aperture and the boundary wave $U_B(P)$ which is arising at the points Qi of the boundary line of the aperture, but the formulas used to find the second part $U_B(P)$ of the diffracted field are a bit different and it is expressed in GANCI solution as[4]

$$U_{\mathbf{B}}(P) = \frac{1}{4\pi} \int_{\Gamma} e^{ikr} \frac{e^{iks}}{s} \frac{\cos(\vec{n},\vec{s})}{1+\cos(\vec{s},\vec{m})} \sin(\vec{m}, d\vec{l})dl \tag{5.1}$$

where the unit vector $\vec{n}$ is orthogonal to both the unit vector $\vec{m}$ of the incident wave and the line element $d\vec{l}$ of the boundary line Γ Fig. 5.1.

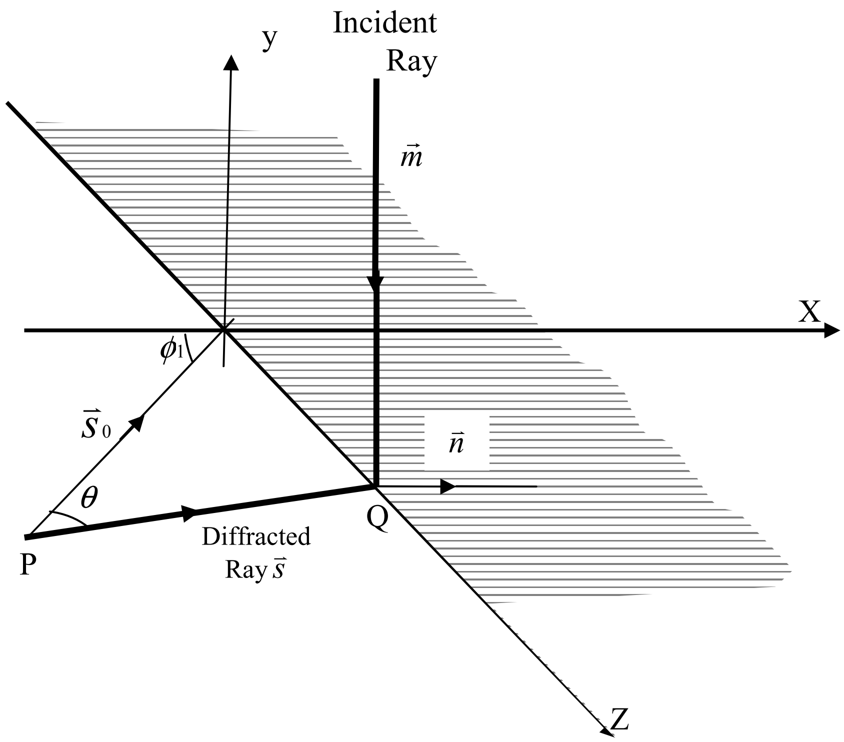

Figure 5.1: Illustrating the normal incident ray

The solution of Eq. (5.1) according to GANCI [4]

$$U^{(B)}(P) = \frac{1}{2\sqrt{2\pi}} \frac{\cos(\phi_1)}{1 - \sin(\phi_1)} \frac{e^{i\left(ks_0 + \frac{\pi}{4}\right)}}{\sqrt{ks_0}}. \tag{5.2}$$

In the solution by the boundary diffraction wave method, the formula used to evaluate the boundary diffraction is

$$U^{(B)}(P) = \vec{e}z \int_{\Gamma} A \frac{e^{iks}}{s} \frac{1}{4\pi} \frac{\sin(\phi_0 - \phi)\vec{e}z}{1 + \cos(\phi - \phi_0)} dz' \tag{5.3}$$

the solution of Eq. (5.3) according to the boundary diffraction wave method is

$$U^{(B)}(P) = -\frac{A}{2\sqrt{2\pi}} \frac{\sin(\phi_0 - \phi)}{1 + \cos(\phi - \phi_0)} \frac{e^{\left(ikR_1 - \frac{i\pi}{4}\right)}}{\sqrt{kR_1}} \tag{5.4}$$

Since in the solution by the boundary diffraction wave method, the incident ray $\vec{m}$ lies at the origin and as a special case when the incident angle ($\phi_0 = \pi/2$) Fig. 5.2, and the incident and the diffracted rays lie on the same plane (x-y), the valve of R_1 is equal to $\sqrt{x_1^2 + y_1^2}$.

$$\vec{s}_0 = \vec{R}_1. \tag{5.5}$$

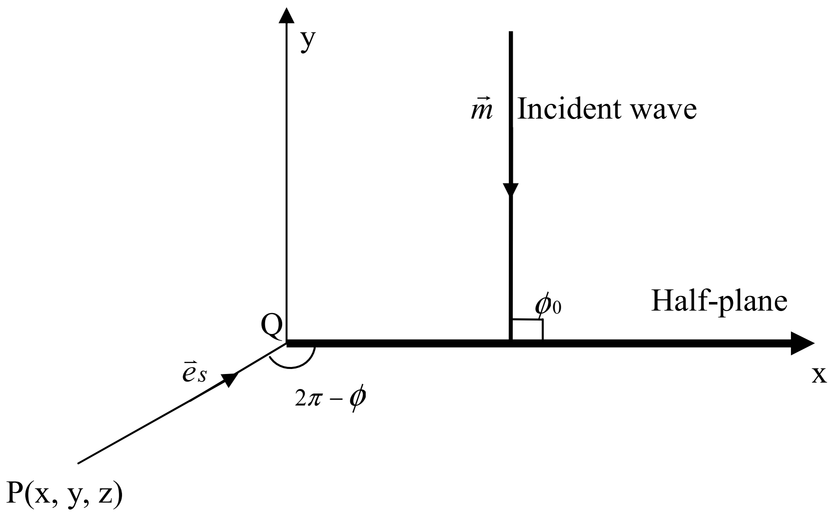

Figure 5.2: Illustrates the incidence angle $\phi_0 = \pi/2$

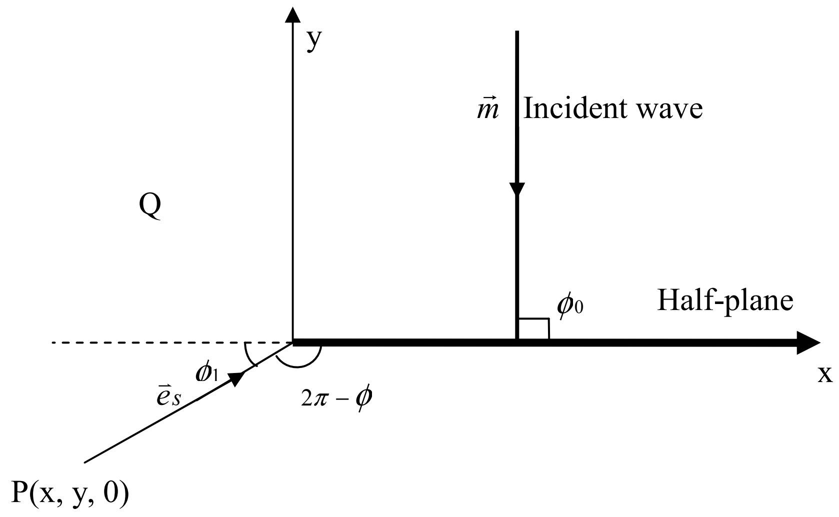

Figure 5.3: Illustrates the relation between $\vec{s}_0$ and $\vec{R}_1$

In addition Fig. 5.3 can be drawn, and from it the relation between the diffraction angle ϕ and the angle ϕ_1 between $\vec{s}_0$ and the negative side of x-axis can be expressed as

$$\phi = \pi + \phi_1 \tag{5.6}$$

for $\phi_0 = \pi/2$ the following relations can be written

$$-\sin\left(\frac{\pi}{2} - \phi\right) = \cos(\phi_1) \tag{5.7.1}$$

$$\cos\left(\phi - \frac{\pi}{2}\right) = -\sin(\phi_1). \tag{5.7.2}$$

From Eq. (5.4), we can get the contribution from the boundary of the aperture by using the boundary diffraction wave method By substituting from Eqs. (5.5), (5.7.1) and (5.7.2) into Eq. (5.4) we will get

$$U^{(B)}(P) = \frac{A}{2\sqrt{2\pi}} \frac{\cos(\phi_1)}{1 - \sin(\phi_1)} \frac{e^{\left(iks_0 - \frac{i\pi}{4}\right)}}{\sqrt{ks_0}}. \tag{5.8}$$

The Eq. (5.8) is equal to Eq. (5.2) which represents the contribution from the boundary of the aperture at the observation point P by using the GANCI solution.

5.3. The Comparison of The Solution of The Boundary Diffraction Wave Method With The Solution of The Oblique Case In GANCI Method for a Half-Plane Problem.

In this case the direction of the incident unit amplitude plane monochromatic wave lies on y-z plane; and the half plane lies in the x-z plane on the half plane x>0, θ_i is the angle between the incident ray and the edge of the half-plane, Fig. 5.4.

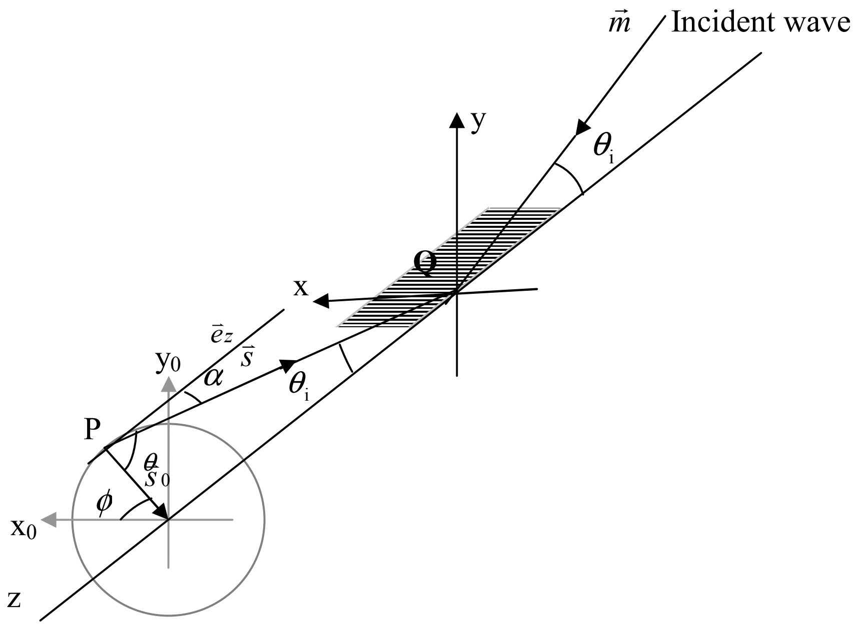

Figure 5.4: Illustrating the positions of the incident ray $\vec{m}$ and the diffracted ray $\vec{s}$

In the oblique incidence case the diffracted field $U_{\mathbf{B}}(P)$ from the boundary of the aperture has the following expression [5]:

$$U_{\mathbf{B}}(P) = \frac{1}{4\pi}\int_{\Gamma} e^{ikr}\,\frac{e^{iks}}{s}\,\frac{\cos(\vec{n},\vec{s})}{1+\cos(\vec{s},\vec{m})}\sin\left(\vec{m},d\vec{l}\right)dl \tag{5.9}$$

where the unit vector $\vec{n}$ is orthogonal to both the unit vector $\vec{m}$ of the incident wave and the line element $d\vec{l}$ of the boundary line Γ. According to GANCI solution for the oblique incidence case, the solution of Eq. (5.9) is expressed as [5]:

$$\mathrm{U}^{(\mathrm{B})}(\mathrm{P}) = \frac{1}{2\sqrt{2\pi}}\,\frac{\cos\phi}{1-\sin\phi}\,\frac{e^{i\left(ks\sin\theta_i + \frac{\pi}{4}\right)}}{\sqrt{ks\sin\theta_i}}. \tag{5.10}$$

In the solution of the boundary diffraction wave method the formula used to evaluate the boundary diffraction is

$$U^{(B)}(P) = \vec{e}z \int_{\Gamma} A \frac{e^{iks}}{s} \frac{1}{4\pi} \frac{\sin(\phi_0 - \phi)\vec{e}z}{1 + \cos(\phi - \phi_0)} dz' \tag{5.11}$$

since the solution of Eq. (5.11) according to the boundary diffraction wave method is

$$U^{(B)}(P) = -\frac{A}{2\sqrt{2\pi}} \frac{\sin(\phi_0 - \phi)}{1 + \cos(\phi - \phi_0)} \frac{e^{\left(ikR_1 - \frac{i\pi}{4}\right)}}{\sqrt{kR_1}} \tag{5.12}$$

According to Fig. 5.4 the vector of the diffracted ray $\vec{s}$ can be expressed as:

$$\vec{s} = -s \sin\theta_i \cos\phi \, \vec{e}_x - s \sin\theta_i \sin\phi \, \vec{e}_y + s \cos\theta_i \cos\alpha \vec{e}_z \tag{5.13}$$

so the magnitude of the vector $\vec{s}$ is

$$s = \sqrt{(-s \sin\theta_i \cos\phi)^2 + (-s \sin\theta_i \sin\phi)^2 + (s \cos\theta_i \cos\alpha)^2} \tag{5.14}$$

since in the solution by the boundary diffraction wave method the valve of R_1 is equal to $\sqrt{x_1^2 + y_1^2}$, so from Eq. (5.14), R_1 can be written as:

$$R_1 = \sqrt{(-s \sin\theta_i \cos\phi)^2 + (-s \sin\theta_i \sin\phi)^2} \tag{5.15}$$

Eq. (5.15) can be rewritten as:

$$R_1 = s \sin\theta_i . \tag{5.16}$$

Since the incident ray lies on the y-z plane, so if the incident angle ϕ_0 is measured with respect to x-axis it will be equal to $\pi/2$.

From Fig. 5.4 the value $\frac{\sin(\phi_0 - \phi)}{1 + \cos(\phi - \phi_0)}$ in Eq. (5.13) can be written as:

$$\frac{\sin(\phi_0 - \phi)}{1 + \cos(\phi - \phi_0)} = \frac{\cos\phi}{1 - \sin\phi} \tag{5.17}$$

substitute from Eqs. (5.16) and (5.17) into Eq.(5.12) to get

$$U^{(B)}(P) = -\frac{A}{2\sqrt{2\pi}} \frac{\cos\phi}{1-\sin\phi} \frac{e^{\left(iks\sin\theta_i - \frac{i\pi}{4}\right)}}{\sqrt{ks\sin\theta_i}} \tag{5.18}$$

This Eq. (5.18) is equal to Eq. (5.10) which represents the contribution from the boundary of the aperture by using the GANCI solution. Inaddition, It can be concluded from Fig. (5.1) and Fig (5.4) that the limits of the integral in the solution of the boundary diffraction wave method (from z equals to -∞ to z equals to ∞) are corresponding to the limits of θ (from $-\pi/2$ to $\pi/2$) in GANCI solution.

It's concluded from the above analysis, that the contribution from the boundary of the aperture by using the GANCI method is equal to the contribution from the boundary of the aperture by using the boundary diffraction wave method.

5.4. The Comparison Between The Exact Solution And BDWM Solution

This section illustrates a comparison between our solution to evaluate the diffracted field and the exact solution to evaluate the same field which originated from a perfectly conducting half plane. The solution for this problem by the Boundary Diffraction Wave Method is an approximate solution. Both solutions is multiplied by the transition function to cancel the defect approaching infinity at the transition regions. When multiply the transition function which is expressed in Eq (4.41) by our solution we will get

$$U^{(B)}(P) = -\frac{A}{2\sqrt{2\pi}} \frac{\cos\phi}{1-\sin\phi} \frac{e^{\left(iks\sin\theta_i - \frac{i\pi}{4}\right)}}{\sqrt{ks\sin\theta_i}} \times p(\phi, \pi \mp \phi_0) \times \left[1 - \exp\left(-\sqrt{2\pi kR_1}\left|\cos\frac{\phi \pm \phi_0}{2}\right|\right)\right] \tag{519}$$

The Following graphs illustrate the comparison between our solution and the exact solution, since both of them is multiplied by the transition function:

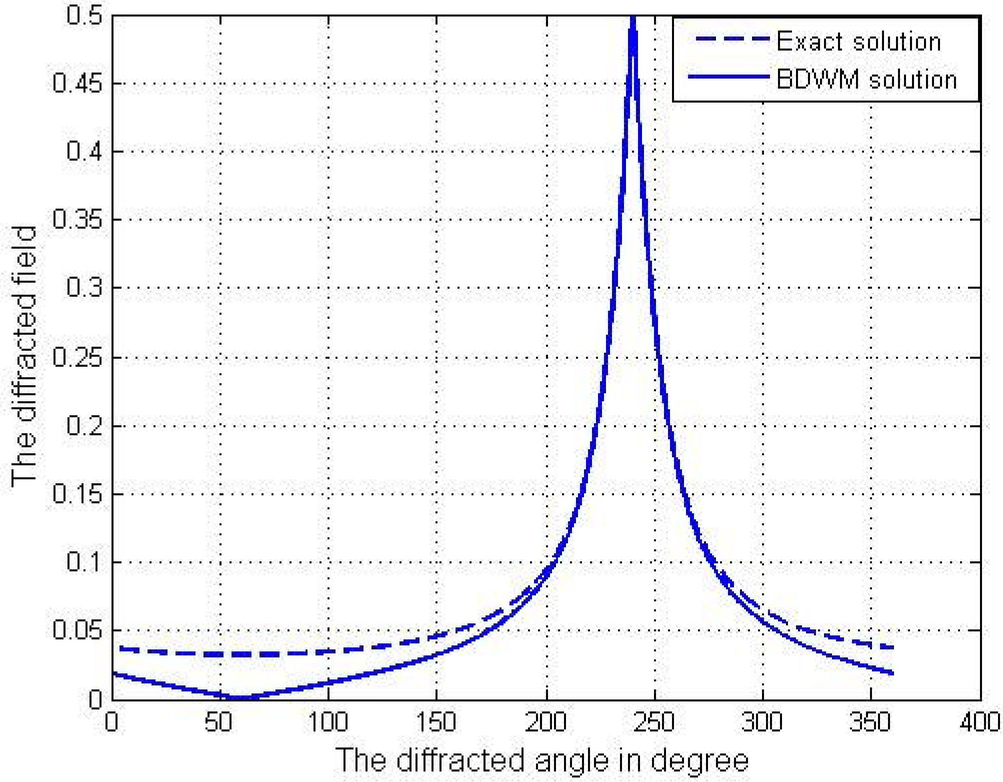

Figure 5.5a: Diffracted fields from perfectly conducting half plane

(BDWM and exact solution, $\phi_0 = \pi/3$)

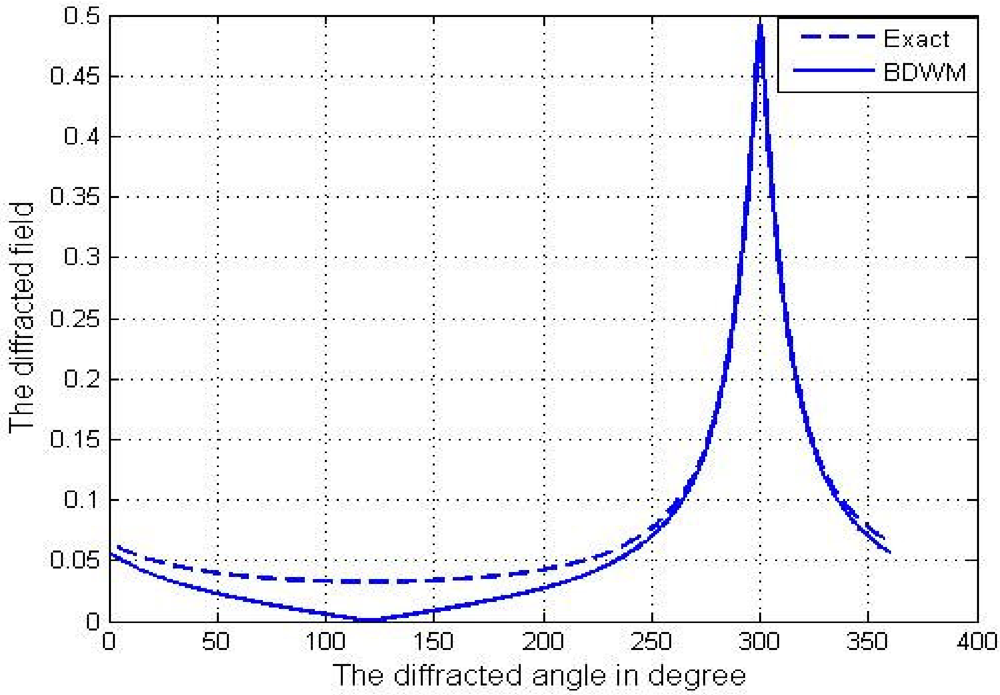

Figure 5.5b: Diffracted fields from perfectly conducting half plane

(BDWM and exact solution, $\phi_0 = 2\pi/3$)

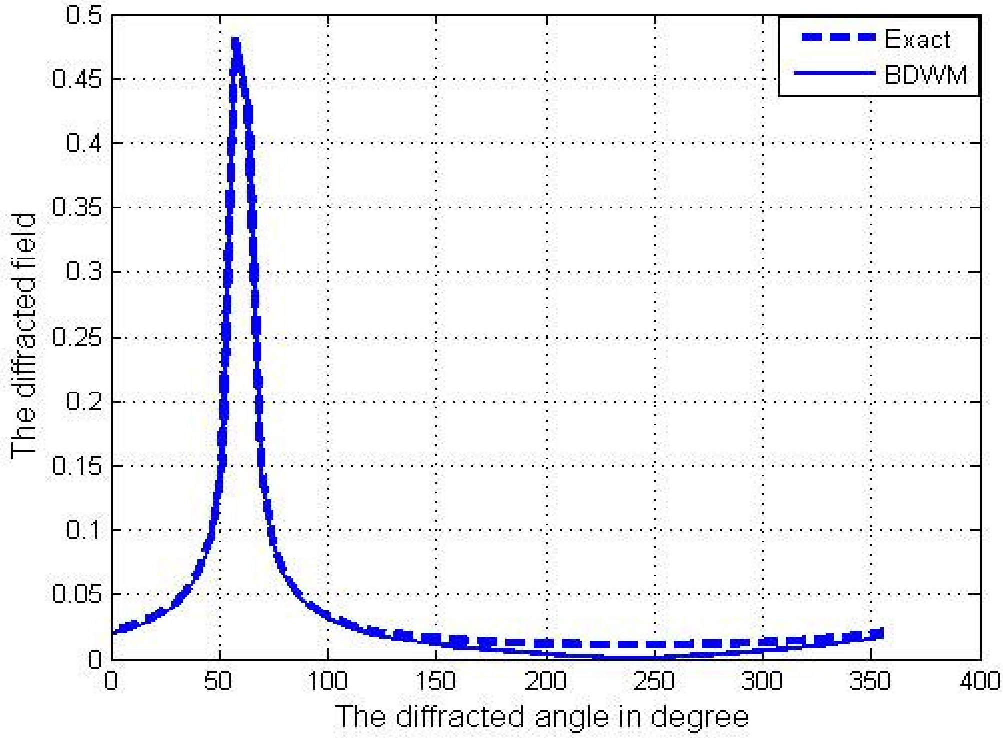

Figure 5.5c: Diffracted fields from perfectly conducting half plane

(BDWM and exact solution, $\phi_0 = 4\pi/3$)

It is concluded from the comparison between the exact solution and the BDWM solution and according to the curves plotted in Fig. 5.5 that the curve of the diffracted field which is plotted according to BDWM goes to zero when the diffracted angle is equal to the incident angle. In addition, for a certain value of the diffracted angle and when the incident angle is equal to $\pi/3$ the difference between the two diffracted fields which are originated by the two methods is the biggest compared with the differences between the two diffracted fields at same diffracted angle in the cases of $\phi_0 = 2\pi/3$ and $\phi_0 = 4\pi/3$. Moreover, the curve of the diffracted field which is plotted according to the exact solution has always a lowest value when the diffracted angle is equal to the incident angle. Furthermore, In the solutions of the two methods, the highest value of the field occur always at $\phi = \pi + \phi_0$, since the angle between the highest and the lowest values of the diffracted field is equal to π .

CHAPTER 6

CONCLUSIONS

The diffraction is the bending of waves around the edge of an obstacle, when light strikes an opaque body, for instance, a shadow forms on the side of the body that is shielded from the light source. Ordinarily, light travels in straight lines through a uniform, transparent medium. But these light waves that just pass the edges of the opaque body are bent.

According to Huygens and Fresnel principle each point of unobstructed part of a primary wave is assumed to be a center of the secondary disturbance and the diffracted field is considered to arise from the superposition of this secondary disturbance.

In addition, each monochromatic scalar wavefield has a vector potential which is associated with that scalar wavefield. This vector potential has property that the normal component of its curl, taken with respect to the coordinates of any point on a closed surface surrounding an observation point is equal to the integrand of the Helmholtz-Kirchhoff integral. This property is exploit to evaluate the diffracted field is originated by the diffraction of the scalar wave by an obstacle whose linear dimensions are large compared to the wavelength. Moreover, there are singularities for the vector potential at some points Qi on the surface S, the total field can be evaluated by sum of the disturbances of these points at a certain observation point.

The generalization of the Maggi-Rubinowics theory of the boundary diffraction wave is the defining of the new vector potential which is associated with any scalar wavefield. The first step of our thesis is the expression of the Helmholtz-Kirchhoff formula in terms of the vector potential to exploit the new formula of Helmholtz- Kirchhoff integral to evaluate the disturbance from certain points Qi on a surface S at a typical observation point, the vector potential must have singularities on the surface S otherwise the disturbance at the observation point will be zero. For any homogeneous plane wave incident on an obstacle with an aperture, the value of the field generated from point Q at the obstacle and measured at an observation point P after the aperture is equal to the origin homogeneous plane wave field that incident on the aperture, since the incident plane wave on the aperture is transmitted unpertureped wave and the wave incident along the line from the main source point to the observation point. Furthermore, there is a general expression for the vector potential associated with any given wave field, the thesis evaluated this expression for the vector potential, since the residual contribution of the vector potential is equal to zero. The general expression for the vector potential associated with a diverging spherical wave is same as the expression of the vector potential associated with the plane wave, which is studied in the application of the boundary diffraction wave method in our thesis as incident wave, also when the incident wave is a diverging spherical wave the disturbance at the observation point, which is associated with any point Q which is located at the aperture is equal to the incident wave field it self. Moreover, the vector potential with the converging spherical wave field is evaluated on the thesis.

When a monochromatic scalar plane wave incident on a plane opaque screen with an aperture, the scattered field (Kirchhoff field), which is originated by this plane opaque screen with aperture and measured at the observation point, is equal to the summation of the disturbance from the boundary of the aperture and the contribution from the aperture itself.. The disturbance from the aperture is depending on the location of the observation point and the vector potential associated with the incident plane wave has only one singularity point in the plane of the aperture, this singularity point located at the first intersection point which is located between the incident ray and the plane of the aperture, at this point the angle between the incident and transmitted rays is equal to π, also the location of the singularity point is depending on the location of the observation point whether the observation on the direct beam or in the geometrical shadow. As a result the value of the of the transmitted field is depending on the location of the observation point, since if the observation point located in the geometrical shadow, the transmitted field is equal to zero whereas if the observation point in the direct beam, the transmitted field is equal to the incident plane wave field.

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APPENDICESY

The Expressions for the Residual Contribution $\vec{W}_\infty$ [2]

$$\vec{W}_0^+(R\vec{u},\vec{e}_s) \approx -\frac{2\pi i\mu_z}{k}\left(\frac{\vec{u}}{1+\vec{e}_s.\vec{u}}\right)A_0^+(\mu_x,\mu_y)\frac{e^{ikR}}{R}, \qquad \text{when } \mu_z > 0, \qquad \text{(A1)}$$

$$\vec{W}_0^+(R\vec{u},\vec{e}_s) \approx -\frac{2\pi i\mu_z}{k}\left(\frac{-\vec{u}}{1-\vec{e}_s.\vec{u}}\right)A_0^+(-\mu_x,-\mu_y)\frac{e^{-ikR}}{R}, \qquad \text{when } \mu_z < 0, \qquad \text{(A2)}$$

$$\vec{W}_0^-(R\vec{u},\vec{e}_s) \approx \frac{2\pi i\mu_z}{k}\left(\frac{-\vec{u}}{1-\vec{e}_s.\vec{u}}\right)A_0^-(-\mu_x,-\mu_y)\frac{e^{-ikR}}{R}, \qquad \text{when } \mu_z > 0, \qquad \text{(A3)}$$

$$\vec{W}_0^-(R\vec{u},\vec{e}_s) \approx \frac{2\pi i\mu_z}{k}\left(\frac{\vec{u}}{1+\vec{e}_s.\vec{u}}\right)A_0^-(\mu_x,\mu_y)\frac{e^{ikR}}{R}, \qquad \text{when } \mu_z < 0, \qquad \text{(A4)}$$