Hindawi
Journal of Mathematics
Volume 2022, Article ID 9240772, 16 pages
https://doi.org/10.1155/2022/9240772

Research Article

A Nonsingular Fractional Derivative Approach for Heat and Mass Transfer Flow with Hybrid Nanoparticles

Muhammad Imran Asjad,[1] **Rabia Naz**,[1] **Muhammad Danish Ikram**,[1] **Azhar Iqbal**,[2] **and Fahd Jarad**[3,4,5]

[1]*Department of Mathematics, University of Management and Technology, Lahore 54770, Pakistan*
[2]*Department of Mathematics, Dawood University of Engineering and Technology, Karachi 74800, Pakistan*
[3]*Department of Mathematics, Cankaya University, Etimesgut 06790, Ankara, Turkey*
[4]*Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia*
[5]*Department of Medical Research, China Medical University, Taichung 40402, Taiwan*

Correspondence should be addressed to Fahd Jarad; fahd@cankaya.edu.tr

Received 29 December 2021; Accepted 23 February 2022; Published 22 April 2022

Academic Editor: Melike Kaplan

This paper deals with the study of MHD Brinkman type fluid flow containing hybrid titanium (TiO2) and silver (Ag) nanoparticles with nonlocal noninteger type Atangana-Baleanu (ABC) fractional differential operator. The problem is designed for the convective flow restrained in a microchannel. With the Mittag–Leffler kernel, the conventional governing equations are converted into dimensionless form and then generalised with noninteger order fractional operators. The solutions for temperature and velocity fields obtained via Laplace transform method and expressed in the series form. The effect of related parameters is dignified graphically with the help of Mathcad and presented in the graphical section. Finally, the results show that the AB fractional operator exhibited improved memory effect as compared to CF fractional operator. Furthermore, due to increasing the values volume fractional temperature can be enhanced and velocity decreases. In comparison between nanoparticles for different types of based fluid, velocity and temperature of water based (TiO2) and silver (Ag) is higher than other base fluids.

1. Introduction

Fractional calculus is a field of mathematics that deals with the integration and derivation of real and complex numbers. Despite the fact that calculus is an old subject, in recent decades, it has regained popularity due to its numerous applications. Researchers have recently investigated mathematical models of heat transfer fluids with considerable applications, which play an essential role in manufacturing. These simulations are usually represented as integer-order partial differential equations. Its important to note that traditional PDEs cannot account for the dynamical behavior of physical flow parameters or retention effects. To cater and eliminate the above mentioned flaws researchers concentrated on fractional dynamic systems of heat transmission in simple and complex fluid models. Nanoparticles are tiny particles with a diameter of 1 to 100 nanometers. Nanoparticles, which are invisible to the naked eye, can have drastically varying physical and chemical characteristics than their bigger material counterparts. Only a few hundred atoms make up most nanoparticles. Nanofluids are often used as coolants in heat transfer equipment such as heat exchangers, electronic cooling systems (such as solid surfaces), and radiators due to their better cooling characteristics [1]. Nanofluids have unique features that make them potentially helpful in a variety of heat transfer applications, such as microelectronics, battery storage, pharmaceutical processes, hybrid-powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, superheater, and crusher [2]. Das et al. [3] conducted a research nanofuids and their application in research and technology. Heat transmission procedures play a significant role in general energy intake, that is why improvement of its effectiveness is a significant research area [4]. Nonrenewable assets such as natural gas, coal and petroleum are the core energy resources for international economy motion [5]. First of all, Caputo established the fractional derivative with Laplace transform technique. The (RL) fractional derivative issue was solved using this operator. In the literature, (RL) and (C) fractional operators were usually rejected in numerous fields of science therefore, a new fractional derivative (ABC) was introduced, which provided more accurate and promising results [6]. Caputo and Fabrizio introduced (CF) fractional derivative with a nonsingular kernel [7]. It has been effectively applied in many fields of science.

Atangana andBaleanu [8] developed a new operator in 2016 which was better than Caputo and Fabrizio's model in describing the physical behaviour of natural phenomena. Saqib et al. [9] described MHD movement of Casson fluid in a microchannel by (ABC) fractional operator. Atangana and Koca [10] discussed (ABC) fractional operator in nonlinear baggs and freedman model. Siddique et al. [11] explained unsteady free convection movements of an incompressible fluid over an infinite vertical plate via (ABC) and (CF) approaches. Imran et al. [12] provided a full description of fractional (ABC) and (CF) MHD viscous fluid convective flow. Mohammad et al. [13] used fractional derivatives to examine the outcomes of the uncertain fractional backward difference equations. (CF) and (ABC) fractional operators were used to discover the influences of carbon nanotubes on magnetohydrodynamic movement of methanol based nanofluid in [14]. Hammouch and Makkaoui [15] described the dynamical behavior of a novel 3D fractional-ordered chaotic system. Ikram et al. [16] analyzed the heat transfer of viscous nanofluid over an exponentially moving vertical plate via (ABC) approach. The thermal conductivity of Newtonian fluid with C and CF fractional operator was discussed in [17, 18].

In [19] nonsingular kernel inside the Atangana-Baleanu fractional operator, the Painlev as well as BagleyTorvik calculations are answered according to the initial conditions. In [20] Imran et al. discussed the thermophoresis properties on simultaneous mass as well as heat transfer in MHD fluid over a porous medium by even heat flux with the help of (AB) fractional operator. In [21] paper emphases on conditions that a fresh high-order system for the arithmetic results of fractional instruction Volterra integro-DE's with the help of AB operator. In [22] Khan discussed the novel idea of ABC fractional operator on human blood pour in Nanofluids. Sania and Yusuf [23] discussed the modeling and results of chickenpox infection with different fractional operators such as C, CF and ABC operators. Raza and Ullah [24] presented a comparison of C and CF fractional operators with heat transfer analysis of fractional Maxwell fluid. They used Tzous and Stehfests algorithms to find the solutions numerically. Imran et al. [25] studied the unsteady natural convection flow of Maxwell fluid with fractional derivative over an exponentially accelerated infinite vertical plate using Stehfests and Tzous algorithms. Chu et al. [26] explained a nonlocal fractional model of hybrid nanofluid with MHD free convection flow in microchannel. Imran et al. [27] used constant proportional Caputo (CPC) fractional operator to study the heat transfer flow of caly-water base nanofluids over an infinite vertical surface moving with constant velocity. Ikram et al. [28] calculated the heat transfer flow of hybrid nanofluid between parallel plates via (CPC) fractional approach and offered a comparison with CF fractional derivative. Imran et al. [29] discussed the heat and mass transfer flow for adhesive fluid between two upright plates through fractional approach. Dawar et al. [30] analyzed the heat transfer of 3D magnetohydrodynamic and sodium alginate based on iron oxide nanofluid moving through a spinning disk with homotopy analysis method. Vaidya et al. [31] calculated the influence of homogeneous and heterogeneous responses on the peristaltic flow of MHD Jeffrey fluid in a nonuniform vertical channel with slip conditions. Sabu et al. [32] presented the analysis of hydromagnetic alumina-water nanoliquid flow due to a rotating rigid disk numerically. They considered different nanoparticle shapes and the thermo-hydrodynamic slip conditions. The following references show a wide range of fractional models and their applications in applied sciences [33–36].

Freshly, In [37] the fractional derivative of (BTF) along with hybrid nanofluid is explained with the help of (CF) fractional operator. We applied the LT technique to acquire the logical results for velocity as well as temperature fields and is indicated as M-function. After that a few figures were plotted to show the impression of parameters on velocity and temperature. The physical properties of hybrid nanofluids are presented in Table 1 and relation of the physical values of nano and hybrid nanofluids can be seen in Tables 2 and 3.

The significance of the present work is that a nonsingular fractional operator is applied in the heat and mass transfer phenomena containing nanoparticles of the type Ag and TiO_2. As the nanoparticles used in the base fluid to improve the physical quantities of base fluid, this paper contains some new comparisons between fractional operators as well as different kinds of nanoparticles which can be useful in the heat transfer process where we can make a choice with different base fluids according to desired targets. These objectives have already been justified physically in this paper.

2. Mathematical Formulation

Suppose that magnetohydrodynamic natural convection movement of Brinkman fluid happens in microchannel of a comprehensive, electrically conductive $(\mathrm{Ag} - \mathrm{TiO_2} - \mathrm{H_2O})$ hybrid nanofluid.

The hypotheses forgoes as shown in Figure 1:

(a) Width and length of microchannel is l and infinite, respectively

(b) Channel is beside horizontal line and normal to vertical line

(c) T_0 is the temperature of framework, when $t \leq 0$

(d) Subsequently $t = 0^+$, temperature rises T_0 into T_W

(e) C_0 is the concentration of framework, when $t \leq 0$

Table 1: Nanoparticles and base fluid thermophysical characteristics [37].

Material	Base fluid	Nanoparticles	Nanoparticles
	TiO_2	Ag	H_2O
ρ	425	10500	997.1
k	8.9538	429	0.613
C_p	6862	235	4179
σ	1×10^{-12}	3.6×10^{7}	0.05
$\beta_T\times 10^{-5}$	0.9	1.89	21
Pr	—	—	6.2

Table 2: Expressions of nanofluid [37].

Properties	Nanofluid
Density	$\rho_{nf}=\phi\rho_s+(1-\phi)\rho_f$
Dynamic viscosity	$\mu_{nf}=(\mu_f/(1-\phi)^{2.5})$
Thermal expansion	$(\rho\beta_T)_{nf}=\phi(\rho\beta_T)_s+(1-\phi)(\rho\beta_T)_f$
Heat capacitance	$(\rho C_p)_{nf}=\phi(\rho C_p)_s+(1-\phi)(\rho C_p)_f$
Electric conductivity	$(\sigma_{nf}/\sigma_f)=1+(3((\sigma_s/\sigma_f)-1)\phi/((\sigma_s/\sigma_f)+2)-((\sigma_s/\sigma_f)-1)\phi)$
Thermal conductivity	$(K_{nf}/K_f)=(2k_f+k_s-2\phi(k_s-k_f)/2k_f+k_s+\phi(k_s-k_f))$

(f) Subsequently $t=0^+$, concentration rises C_0 into C_W

(g) Motion is along x-axis

(h) MF of strength B_0 works diagonally into the direction of flow

Flow of electrically conductive $Ag-TiO_2-H_2O$ hybrid nanofluid undergoes electromotive energy, which generate current. The induced MF is overlooked for the assumption of reduced Reynolds number. The electromagnetic energy revolves on the power of electric instability [18].

Velocity component of hybrid nanofluid is of the form $(v(y,t)\hat{i},0,0)$. The electromagnetic force F_{em} is merged into the momentum equation of the natural convection flow of an incompressible $Ag-TiO_2-H_2O$ hybrid nanofluid in the absence of pressure gradient and an inclined magnetic field is imposed.

The governing equations are [37]

$$\rho_{hnf}\left(v_t(y,t)+\beta_{b^*}v(y,t)\right)=\mu_{hnf}v_{yy}(y,t)-\sigma_{hnf}B_0^2v(y,t)+g(\rho\beta_T)_{hnf}(T(y,t)-T_0)+g(\rho\beta_c)_{hnf}(C(y,t)-C_0), \tag{1}$$

$$\left(\rho C_p\right)_{hnf}T_t(y,t)=K_{hnf}T_{yy}(y,t)+Q_0(T(y,t)-T_0), \tag{2}$$

$$C_t(y,t)=DC_{yy}(y,t), \tag{3}$$

with the following initial and boundary conditions

$$\begin{aligned}v(y,0)&=0,\\ T(y,0)&=T_0,\\ C(y,0)&=C_0,\\ y\geq 0,&\ t<0,\end{aligned} \tag{4}$$

$$\begin{aligned}v(l,t)&=0,\\ T(l,t)&=T_w,\\ C(l,t)&=C_w,\\ t&>0.\end{aligned} \tag{5}$$

By presenting the accompanying dimensionless variables,

$$\begin{aligned}\tau_1&=\frac{\nu}{l^2}t,\\ U&=\frac{l}{\nu}v,\\ C&=\frac{C-C_0}{C_w-C_0},\\ \varpi&=\frac{y}{l},\\ \varphi&=\frac{T-T_0}{T_w-T_0},\end{aligned} \tag{6}$$

into equations (1)–(5), we have the subsequent dimensionless problem:

Table 3: Expressions of hybrid nanofluid [37].

Properties	Hybrid nanofluid
Density	$\rho_{hnf} = \phi_{\mathrm{Ag}}\rho_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}\rho_{\mathrm{TiO_2}} + (1-\phi_{hnf})\rho_f$
Dynamic viscosity	$\mu_{hnf} = (\mu_f/[1-(\phi_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}})]^{2.5})$
Thermal expansion	$(\rho\beta_T)_{hnf} = \phi_{\mathrm{Ag}}(\rho\beta_T)_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}(\rho\beta_T)_{\mathrm{TiO_2}} + (1-\phi_{hnf})(\rho\beta_T)_f$
Heat capacitance	$(\rho C_p)_{hnf} = \phi_{\mathrm{Ag}}(\rho C_p)_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}(\rho C_p)_{\mathrm{TiO_2}} + (1-\phi_{hnf})(\rho C_p)_f$
Electric conductivity	$(\sigma_{hnf}/\sigma_f) = 1 + (3((\phi_{\mathrm{Ag}}\sigma_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}\sigma_{\mathrm{TiO_2}}/\sigma_f) - \phi)/((\phi_{\mathrm{Ag}}\sigma_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}\sigma_{\mathrm{TiO_2}}/\phi\sigma_f) + 2) - ((\phi_{\mathrm{Ag}}\sigma_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}\sigma_{\mathrm{TiO_2}}/\sigma_f) - \phi))$
Thermal conductivity	$(K_{hnf}/K_f) = ((\phi_{\mathrm{Ag}}k_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}k_{\mathrm{TiO_2}}/\phi_{hnf}) + 2k_f + 2(\phi_{\mathrm{Ag}}k_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}k_{\mathrm{TiO_2}}) - 2\phi_{hnf}k_f/(\phi_{\mathrm{Ag}}k_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}k_{\mathrm{TiO_2}}/\phi_{hnf}) + 2k_f + (\phi_{\mathrm{Ag}}k_{\mathrm{Ag}} + \phi_{\mathrm{TiO_2}}k_{\mathrm{TiO_2}}) - \phi_{hnf}k_f)$

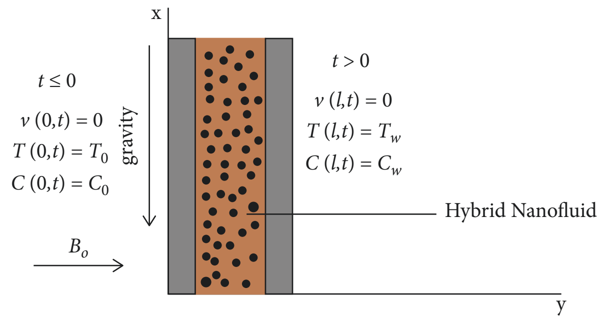

FIGURE 1: Formation of microchannel and coordinate system.

$$C_0 U_{\tau_1}(\varpi,\tau_1) + \beta_b U(\varpi,\tau_1) = C_1 U_{\varpi\varpi}(\varpi,\tau_1) - C_2 M U(\varpi,\tau_1) + C_3 Gr \varphi(\varpi,\tau_1) + C_4 G_m C(w,\tau_1), \tag{7}$$

$$C_5 \Pr \varphi_{\tau_1}(\varpi,\tau_1) = \lambda_{hnf}\varphi_{\varpi\varpi}(\varpi,\tau_1) + Q\varphi(\varpi,\tau_1), \tag{8}$$

$$C_{\tau_1}(\varpi,\tau_1) = \frac{1}{S_c} C_{\varpi\varpi}(\varpi,\tau_1), \tag{9}$$

where Sc is Schmidt number (dimensionless).

Initial and boundary conditions which are dimensionless,

$$\begin{aligned} U(\varpi,0) &= 0,\\ \varphi(\varpi,0) &= 0,\\ C(\varpi,0) &= 0,\\ \varpi \geq 0,\ \tau_1 &< 0,\\ U(l,\tau_1) &= 0,\\ \varphi(l,\tau_1) &= 1,\\ C(l,\tau_1) &= 1,\\ \tau_1 &> 0, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \beta_b &= \frac{l^2 \beta_{b^*}\rho_{hnf}}{\mu},\\ M &= \frac{l^2 \sigma_f B_0^2}{\mu},\\ \lambda_{hnf} &= \frac{k_{hnf}}{k_f},\\ Q &= \frac{Q_0 l^2}{k_f},\\ \Pr &= \frac{(\mu C_p)}{k_f},\\ Gr &= \frac{l^3 g (\beta_T)_f (T_w - T_0)}{\nu^2},\\ Sc &= \frac{\nu}{D},\\ C_0 &= 1 - \phi_{hnf} + \frac{\phi_{Ag}\rho_{Ag} + \phi_{TiO_2}\rho_{TiO_2}}{\rho_f},\\ C_1 &= \frac{1}{\left[1 - \left(\phi_{Ag} + \phi_{TiO_2}\right)\right]^{2.5}},\\ C_2 &= \frac{\sigma_{hnf}}{\sigma_f},\\ C_3 &= 1 - \phi_{hnf} + \frac{\phi_{Ag}(\rho\beta_T)_{Ag} + \phi_{TiO_2}(\rho\beta_T)_{TiO_2}}{(\rho\beta_T)_f},\\ C_4 &= 1 - \phi_{hnf} + \frac{\phi_{Ag}(\rho\beta_p)_{Ag} + \phi_{TiO_2}(\rho\beta_p)_{TiO_2}}{(\rho\beta_p)_f},\\ C_5 &= 1 - \phi_{hnf} + \frac{\phi_{Ag}(\rho C_p)_{Ag} + \phi_{TiO_2}(\rho C_p)_{TiO_2}}{(\rho C_p)_f}. \end{aligned} \tag{11}$$

3. Result of the Problem

Fractional model obtained by equations (7)–(9) using the definition of ABC defined in [8].

$$U_{\varpi\varpi}(\varpi,\tau_1) - D_1^{ABC} D_{\tau_1}^{\alpha} U(\varpi,\tau_1) - D_2 U(\varpi,\tau_1) + D_3\varphi(\varpi,\tau_1) + D_4 C(\varpi,\tau_1) = 0, \tag{12}$$

$$\varphi_{\varpi\varpi}(\varpi,\tau_1) - D_5^{ABC} D_{\tau_1}^{\alpha}\varphi(\varpi,\tau_1) + D_6\varphi(\varpi,\tau_1) = 0, \tag{13}$$

$$S_c^{ABCD_{\tau_1}^{\alpha}} C(\varpi,\tau_1) = C_{\varpi\varpi}(\varpi,\tau_1), \tag{14}$$

where as $D_1 = (C_0/C_1)$, $D_2 = (C_2M + \beta_b/C_1)$, $D_3 = (C_3Gr/C_1)$, $D_4 = (C_4G_m/C_1)$, $D_5 = (C_4\Pr/\lambda_{hnf})$, and $D_6 = (Q/\lambda_{hnf})$.

3.1. Formulation of Concentration. Implementing LT on equation (14) and using initial and boundary conditions, we have

$$\left[D^2 - S_c \cdot \frac{q^\alpha}{(1-\alpha)q^\alpha + \alpha}\right]\bar{C}(\varpi, q) = 0, \tag{15}$$

satisfying

$$\bar{C}(0,q) = 0, \qquad \bar{C}(1,q) = \frac{1}{q}. \tag{16}$$

The general solution of equation (15) subject to equation (16)

$$\bar{C}(\varpi, q) = \frac{\operatorname{Sinh}\left(\varpi\sqrt{(S_c q^\alpha/(1-\alpha)q^\alpha + \alpha)}\right)}{q\operatorname{Sinh}\left(\sqrt{(S_c q^\alpha/(1-\alpha)q^\alpha + \alpha)}\right)}. \tag{17}$$

Equation (17) can be written in suitable form by using formula [28].

$$\frac{Sinh(ab)}{Sinha} = \sum_{n=0}^{\infty}\left[e^{-(2n+1-b)a} - e^{-(2n+1+b)a}\right].$$

as (18)

$$\bar{C}(\varpi, q) = \frac{1}{q}\sum_{i=0}^{\infty}\left[e^{-(2i+1-\varpi)\cdot\sqrt{S_c q^\alpha/(1-\alpha)q^\alpha+\alpha}} - e^{-(2i+1+\varpi)\cdot\sqrt{(S_c q^\alpha/(1-\alpha)q^\alpha+\alpha)}}\right],$$

By converting into series form as in [28],

$$\bar{C}(\varpi, q) = \sum_{i=0}^{\infty}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\frac{(-(2i+1-\varpi))^{i_1}\left(\sqrt{S_c}\right)^{i_2}(\alpha)^{i_2}\Gamma((i_1/2)+1)}{i_1!i_2!\left(1-\alpha^{(i_1/2)+i_2}\right)q^{\alpha i_2+1}\Gamma((i_1/2)+1-i_2)} - \sum_{i=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{i_4=0}^{\infty}\frac{(-(2i+1-\varpi))^{i_3}\left(\sqrt{S_c}\right)^{i_3}(\alpha)^{i_4}\Gamma((i_3/2)+1)}{i_3!i_4!\left(1-\alpha^{(i_3/2)+i_4}\right)q^{\alpha i_4+1}\Gamma((i_3/2)+1-i_4)}, \tag{19}$$

Taking inverse LT

$$\begin{aligned}C(\varpi, \tau_1) = &\sum_{i=0}^{\infty}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\frac{(-(2i+1-\varpi))^{i_1}\left(\sqrt{S_c}\right)^{i_2}(\alpha)^{i_2}\left(\tau_1^{\alpha_{i_2}}\right)\Gamma((i_1/2)+1)}{i_1!i_2!\left(1-\alpha^{(i_1/2)+i_2}\right)\Gamma((i_1/2)+1-i_2)}\\ &-\sum_{i=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{i_4=0}^{\infty}\frac{(-(2i+1-\varpi))^{i_3}\left(\sqrt{S_c}\right)^{i_3}(\alpha)^{i_4}\left(\tau_1^{\alpha i_4}\right)\Gamma((i_3/2)+1)}{i_3!i_4!\left(1-\alpha^{(i_3/2)+i_4}\right)\Gamma((i_3/2)+1-i_4)}.\end{aligned} \tag{20}$$

3.2. Formulation of Temperature Field. By implementing LT on equation (13) and using initial condition, we obtain

$$\left[D^2 - D_5\frac{q^\alpha}{(1-\alpha)q^\alpha + \alpha} + D_6\right]\bar{\varphi}(\varpi, q) = 0, \tag{21}$$

which satisfies

$$\bar{\varphi}(0,q) = 0, \qquad \bar{\varphi}(1,q) = \frac{1}{q}. \tag{22}$$

The general solution of equation (21) subject to equation (22)

$$\bar{\varphi}(\varpi, q) = \frac{1}{q}\left[\frac{\sinh\left(\varpi\sqrt{\left(D_5 q^\alpha/(1-\alpha)q^\alpha + \alpha\right) - D_6}\right)}{\sinh\left(\sqrt{\left(D_5 q^\alpha/(1-\alpha)q^\alpha + \alpha\right) - D_6}\right)}\right], \quad (23)$$

Suitably written in equivalent form

$$\bar{\varphi}(\varpi, q) = \frac{1}{q}\sum_{\xi=0}^{\infty}\left[e^{-(2\xi+1-\varpi)\sqrt{\left(D_5 q^\alpha/(1-\alpha)q^\alpha+\alpha\right)-D_6}} - e^{-(2\xi+1+\varpi)\sqrt{\left(D_5 q^\alpha/(1-\alpha)q^\alpha+\alpha\right)-D_6}}\right]. \quad (24)$$

By applying LT to equation (24) after expressed in series form,

$$\begin{aligned}\bar{\varphi}(\varpi, q) = \frac{1}{q} + \sum_{\xi=0}^{\infty}\left[\sum_{\xi_1=1}^{\infty}\sum_{\xi_2=1}^{\infty}\sum_{\xi_3=1}^{\infty}\frac{(\varpi-2\xi-1)^{\xi_1}(-1)^{(\xi_1/2)+\xi_2+\xi_3}(D_5)^{\xi_2}(\alpha)^{\xi_3}}{\xi_1!\xi_2!\xi_3!(D_6)^{\xi_2-(\xi_1/2)}(1-\alpha)^{\xi_2+\xi_3}q^{\alpha\xi_3+1}}\frac{\Gamma((\xi_1/2)+1)\Gamma(\xi_2+\xi_3)}{\Gamma((\xi_1/2)+1-\xi_2)\Gamma(\xi_2)}\right] \\ - \sum_{\xi=0}^{\infty}\left[\sum_{\xi_4=0}^{\infty}\sum_{\xi_5=0}^{\infty}\sum_{\xi_6=0}^{\infty}\frac{(-(2\xi+1+\varpi))^{\xi_4}(-1)^{(\xi_4/2)+\xi_5+\xi_6}(D_5)^{\xi_5}(\alpha)^{\xi_6}}{\xi_4!\xi_5!\xi_6!(D_6)^{\xi_5-(\xi_4/2)}(1-\alpha)^{\xi_5+\xi_6}q^{\alpha\xi_6+1}}\frac{\Gamma((\xi_4/2)+1)\Gamma(\xi_5+\xi_6)}{\Gamma((\xi_4/2)+1-\xi_5)\Gamma(\xi_5)}\right].\end{aligned} \quad (25)$$

Taking inverse LT on equation (25) and expressing in M-function defined in [7], we have

$$\begin{aligned}\varphi(\varpi, \tau_1) = 1 + \sum_{\xi=0}^{\infty}\sum_{\xi_1=1}^{\infty}\sum_{\xi_2=1}^{\infty}\frac{(\varpi-2\xi-1)^{\xi_1}(-1)^{(\xi_1/2)+\xi_2}(D_5)^{\xi_2}}{\xi_1!\xi_2!(D_6)^{\xi_2-(\xi_1/2)}(1-\alpha)^{\xi_2}}M_3^2\left[\frac{-\alpha\tau_1^\alpha}{1-\alpha}\Big|_{((\xi_1/2)+1-\xi_2,0),(\xi_2,0),(1,\alpha)}^{((\xi_1/2)+1,0),(\xi_2,1)}\right] \\ - \sum_{\xi=0}^{\infty}\sum_{\xi_4=0}^{\infty}\sum_{\xi_5=0}^{\infty}\frac{(-(2\xi+1+\varpi))^{\xi_4}(-1)^{(\xi_4/2)+\xi_5}(D_5)^{\xi_5}}{\xi_4!\xi_5!(D_6)^{\xi_5-(\xi_4/2)}(1-\alpha)^{\xi_5}}M_3^2\left[\frac{-\alpha\tau_1^\alpha}{1-\alpha}\Big|_{((\xi_4/2)+1-\xi_5,0),(\xi_5,0),(1,\alpha)}^{((\xi_4/2)+1,0),(\xi_5,1)}\right].\end{aligned} \quad (26)$$

3.3. Nusselt Number

3.3. Nusselt Number. We have computed the heat transfer rate in terms of Nusselt number through the following relation and presented in Table 4.

$$Nu = -\frac{\partial\varphi(0, \tau_1)}{\partial\varpi}. \quad (27)$$

3.4. Formulation of Velocity Field. By utilizing the LT on equation (12) and using initial condition, we obtain

$$\left[D^2 - D_1\frac{q^\alpha}{(1-\alpha)q^\alpha+\alpha} - D_2\right]\bar{U}(\varpi, q) = -D_3\bar{\varphi}(\varpi, q) - D_4\bar{c}(w, q), \quad (28)$$

which satisfies

Table 4: Numerically analysis of Nusselt number for the effect of fractional parameter with comparison of (ABC) and (CF).

α	Nu (ABC)	Nu (CF)	Nu (ABC)	Nu (CF)
	$t=0.5$	$t=0.5$	$t=2$	$t=2$
0.1	2.057	2.111	2.041	1.936
0.2	2.065	2.169	2.000	1.798
0.3	2.080	2.225	1.930	1.642
0.4	2.100	2.277	1.830	1.469
0.5	2.124	2.317	1.697	1.288
0.6	2.144	2.330	1.531	1.111
0.7	2.144	2.288	1.333	0.956
0.8	2.085	2.138	1.112	0.838
0.9	1.891	1.838	0.892	0.755

$$\begin{aligned}\bar{U}(0,q) &= 0,\\ \bar{U}(1,q) &= 0.\end{aligned} \tag{29}$$

The general solution of equation (28) subject to equation (29) is

$$\begin{aligned}
\bar{U}(\varpi,q) = {} & \frac{D_3}{q}\sum_{\xi=0}^{\infty}\left[\frac{e^{-2\xi\sqrt{\left(D_5q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_6}}-e^{-(2\xi+2)\sqrt{\left(D_5q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_6}}}{\left(D_7q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_8}\right]\\
& \times\sum_{\chi=0}^{\infty}\left[e^{-(2\chi+1-\varpi)\sqrt{\left(D_1q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)+D_2}}-e^{-(2\chi+1+\varpi)\sqrt{\left(D_1q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)+D_2}}\right]\\
& -\frac{D_3}{q}\sum_{\xi=0}^{\infty}\left[\frac{e^{-(2\xi+1-\varpi)\sqrt{\left(D_5q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_6}}-e^{-(2\xi+1+\varpi)\sqrt{\left(D_5q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_6}}}{\left(D_7q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_8}\right]\\
& +\frac{D_4}{q}\sum_{\imath=0}^{\infty}\left[\frac{e^{-2\imath\sqrt{\left(S_cq^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)}}-e^{-(2\imath+2)\sqrt{\left(S_cq^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_6}}}{\left(D_7q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_8}\right]\\
& \times\sum_{\chi=0}^{\infty}\left[e^{-(2\chi+1-\varpi)\sqrt{\left(D_1q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)+D_2}}-e^{-(2\chi+1+\varpi)\sqrt{\left(D_1q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)+D_2}}\right]\\
& -\frac{D_4}{q}\sum_{\imath=0}^{\infty}\left[\frac{e^{-(2\imath+1-\varpi)\sqrt{\left(S_cq^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)}}-e^{-(2\xi+1+\varpi)\sqrt{\left(S_cq^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)}}}{\left(D_7q^{\alpha}/(1-\alpha)q^{\alpha}+\alpha\right)-D_8}\right].
\end{aligned} \tag{30}$$

where $D_7 = D_5 - D_1, D_8 = D_2 + D_6$.

Equation (30) is too much complicated, so that we cannot apply inverse LT. Therefore solutions obtained numerically.

4. Results and Discussion

The current study of MHD Brinkman type fluid flow is the focus of this article, which contain sliver (Ag) and hybrid titanium (TiO_2) nanoparticles with nonlocal noninteger type fractional differential operators the problem is created for the convective flow restrained in a microchannel. With the help of LT temperature and velocity solutions are obtained and then converted to the series form. In order insight the physical significance of flow parameters some graphs are plotted.

The fractional operator was used to measure the hybrid nanofluid ($TiO_2 - H_2O$). The (BTF) model with conflict along with affected by magnetic field was distinguished with the help of PDEs. Appraisal between the (ABC) fractional derivative was applied to determine the performance of hybrid nanofluid with the help of LT procedure. The results of the temperature and velocity fields are exposed as M-function. The parameters influences of α, β_b, ϕ_{hnf}, Gr, Q and M on temperature as well as velocity fields are also calculated and shown graphically with their physical impact.

Figures 2-5 will compare the (ABC) derivative with the (CF) derivative, which is argued in [37], to analyze the impact of α on temperature and velocity fields. The velocity as well as temperature is decreasing functions of α as a result of reduction in momentum and heat boundary layers. It is due to that the fractional parameter controls the phenomena

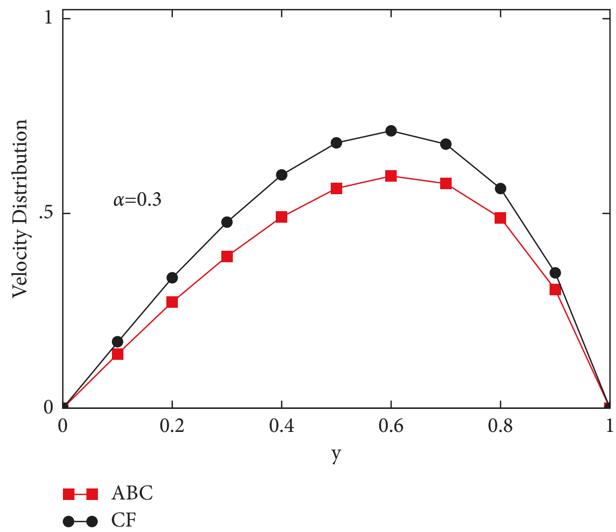

FIGURE 2: Correlation between the velocities through CF [37] and present, when Pr = 6.2, $t = 3$, $M = 0.2$, Gr = 20, $Q = 0.5$, $\phi_{hnf} = 0.08$ and $\beta = 0.8$.

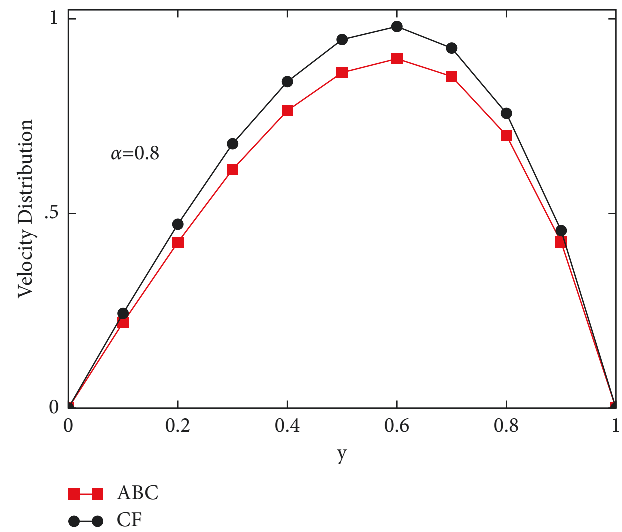

FIGURE 4: Correlation between the velocities through CF [37] and ABC, Pr = 6.2, when $t = 2$, $Q = 0.5$, Gr = 23, $M = 0.2$, $\beta = 0.8$ and $\phi_{hnf} = 0.08$.

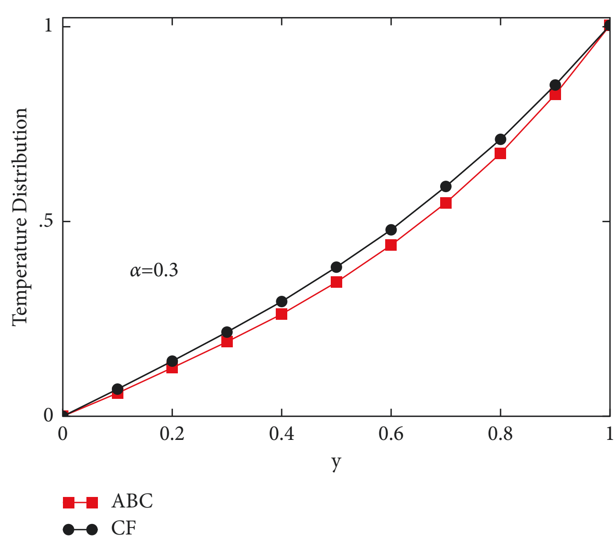

FIGURE 3: Correlation between the temperatures through CF [37] and present, Pr = 6.2, when $t = 2$, $Q = 1.2$ and $\phi_{hnf} = 0.2$.

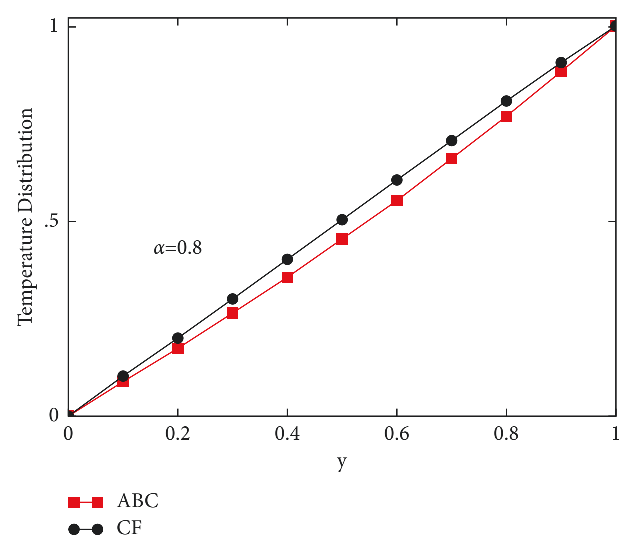

FIGURE 5: Correlation between the temperatures through CF [37] and ABC, Pr = 6.2, when $t = 3$, $Q = 0.5$ and $\phi_{hnf} = 0.04$.

of fluid flow physically. The rate of heat transfer is also calculated numerically in the form of Nusselt number by comparing (ABC) and (CF) fractional operators as shown in Table 4. It is observed that (ABC) derivatives show better retention than (CF) derivatives. Figures 6 and 7 signify the appraisal of velocity and temperature fields for nanofluid particles $(Ag - H_2O)$ and $(TiO_2 - H_2O)$. Meanwhile TiO_2 is semiconductor and Ag is good conductor, consequently $(Ag - H_2O)$ had high temperature profile than $(TiO_2 - H_2O)$. However the nanofluid concentration has more significance in the velocity field. Because of antiparticles adding in base fluid, the resultant nanofluid creates much denser which decrease the velocity. $(Ag - H_2O)$ decreases velocity more than $(TiO_2 - H_2O)$ relatively.

Figures 8 and 9 are designed to see the impact of Q which is very predictable. By increasing values of Q, both velocity and temperature increased. This is owing to heat producing by system through rising Q. Figures 10 and 11 display the influence of ϕ_{hnf} on the velocity and temperature fields. The velocity is a decreasing function of ϕ_{hnf} whereas temperature is increasing function. This is because of ϕ_{hnf} increases the viscosity and density of fluid from Table 2. The similar trend for ϕ_{hnf} was concluded by [34].

Figures 12 and 13 are made to show the evaluation of changed base fluids (engine oil, water, kerosene) for the temperature and velocity arenas. Because of the changed thermal conductivity of water and base fluids by hybrid nanoparticles has unusual temperature due to greater heat

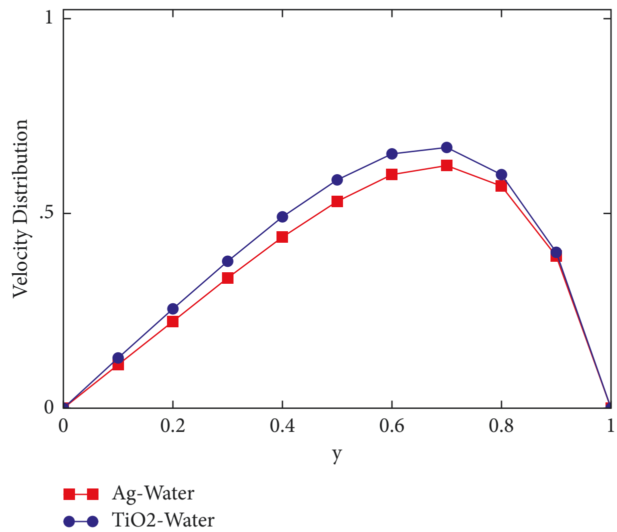

FIGURE 6: Correlation between nanofluid for velocity, when Pr = 6.2, $t = 0.4$, Gr = 40, $Q = 0.5$, $M = 10$, $\beta = 2$ and $\alpha = 0.2$.

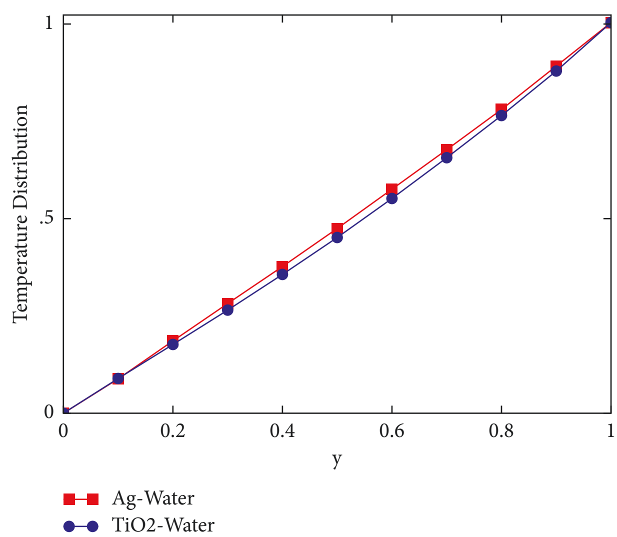

FIGURE 7: Correlation among nanofluids for temperature, when $t = 3$, $\alpha = 0.2$, $Q = 3$, and Pr = 6.2.

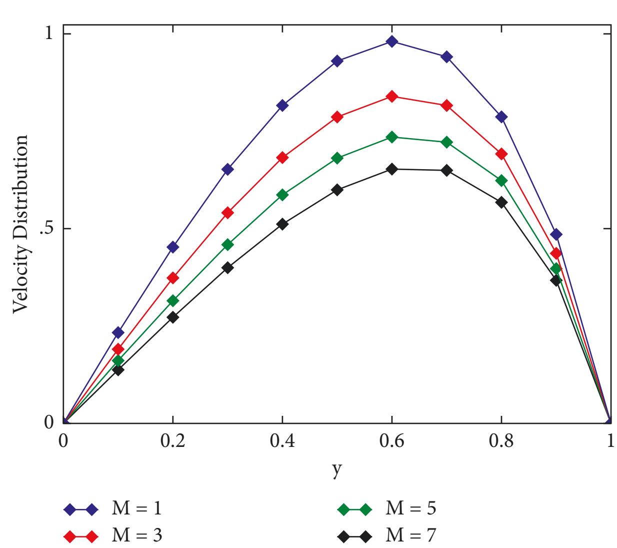

FIGURE 8: Heat source Q Impact on velocity when Pr = 6.2, $\phi_{hnf} = 0.04$, $t = 0.2$, $\beta = 0.5$, Gr = 30, $M = 3$, and $\alpha = 0.5$.

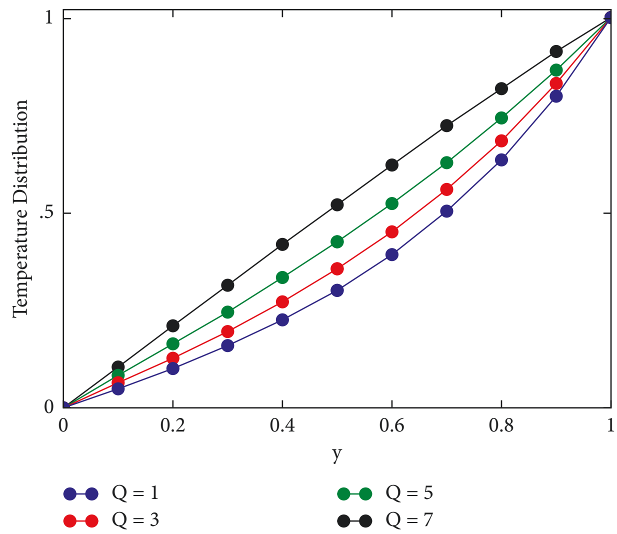

FIGURE 9: Effect of heat source Q on temperature when Pr = 6.2, $t = 0.4$, $\phi_{hnf} = 0.2$ and $\alpha = 0.5$.

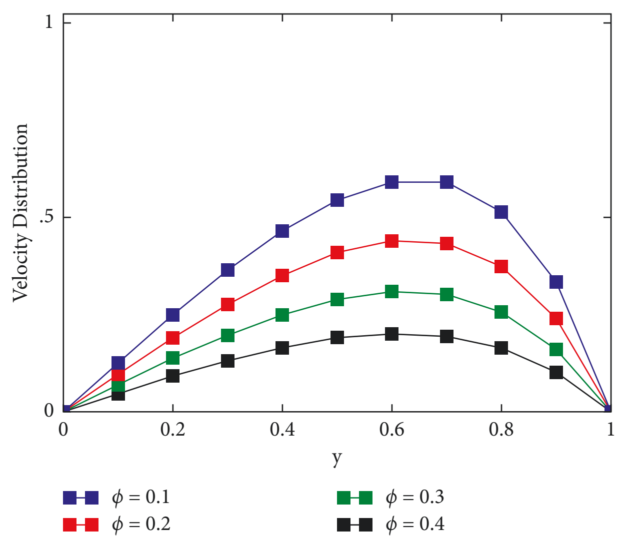

FIGURE 10: Impact of ϕ_{hnf} on velocity when Pr = 6.2, $t = 0.1$, $M = 3$, $\beta = 0.5$, $Q = 2$, Gr = 30, and $\alpha = 0.5$.

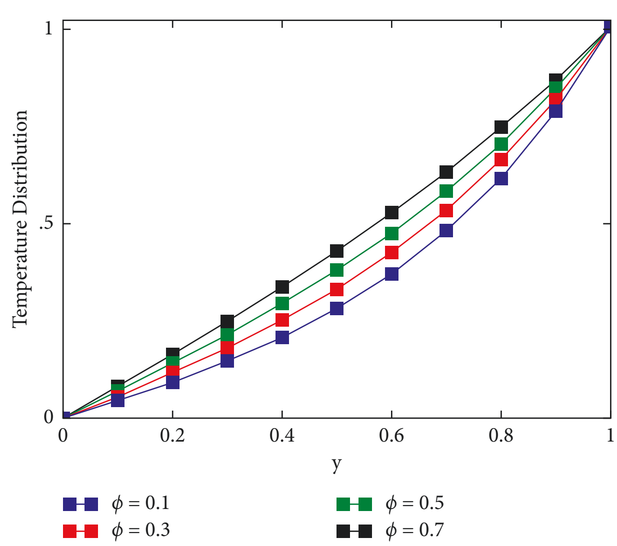

FIGURE 11: Effect of ϕ_{hnf} on temperature after Pr = 6.2, $t = 0.4$, $Q = 1.2$ and $\alpha = 0.5$.

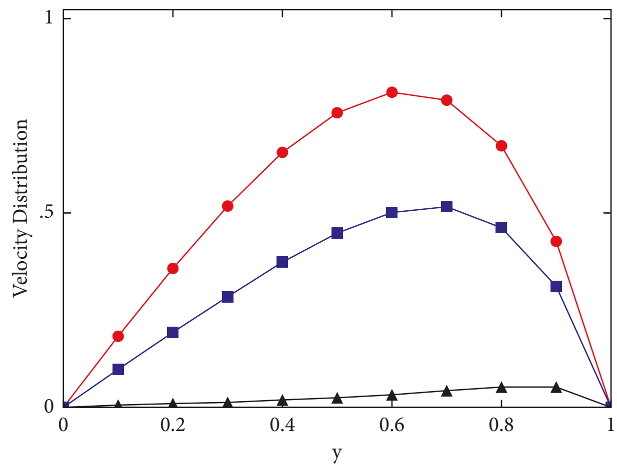

FIGURE 12: Comparison among base fluids (engine oil, water and kerosene) for velocity after $Q = 0.5$, $t = 4$, $\mathrm{Gr} = 30$, $M = 4$, $\beta = 0.2$ $\alpha = 0.5$ and $\phi_{hnf} = 0.08$.

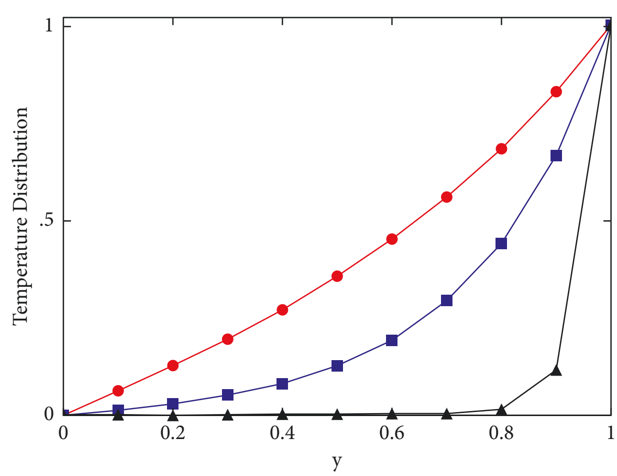

FIGURE 13: Comparison among unlike base fluids (engine oil, water and kerosene) for Temperature after $\alpha = 0.5$, $t = 0.4$, $Q = 3$ and $\phi_{hnf} = 0.2$.

conductivity than further base fluids, it is observed that water-based hybrid nanoparticles has higher velocity as it moves away from the plate, whereas the velocity of engine oil-based fluid decrees near the plate. Figure 14 represent the effect of β_b on the velocity field. After increasing β_b the fluid velocity shrinks. This is due to higher values of β_b make the drag forces stronger physically, so velocity decreases. The impression of M is verified in Figure 15. It guides that by rising M, velocity diminishes. Meanwhile values of M transmit to restive type forces, called Lorentz forces. Thus the velocity was declined. The effect of Gr on the velocity field represented in Figure 16. It is comprehended that by

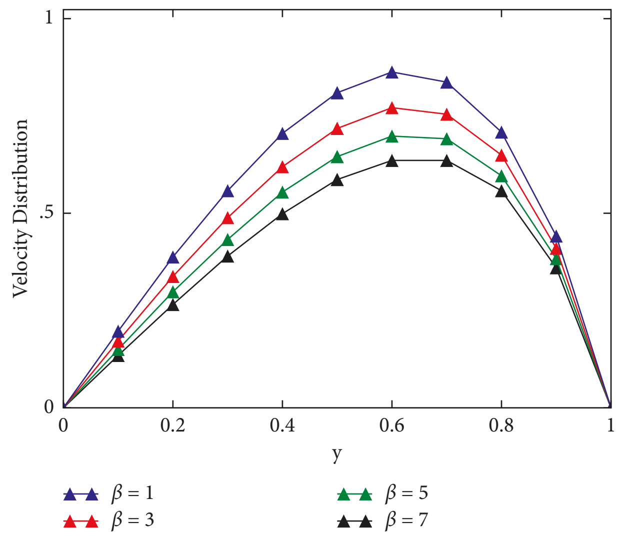

FIGURE 14: The velocity effect of the BT fluid *beta*, $\mathrm{Pr} = 6.2$, $\mathrm{Gr} = 30$, $\phi_{hnf} = 0.08$ when $t = 3$, $Q = 0.5$, and $\alpha = 0.5$.

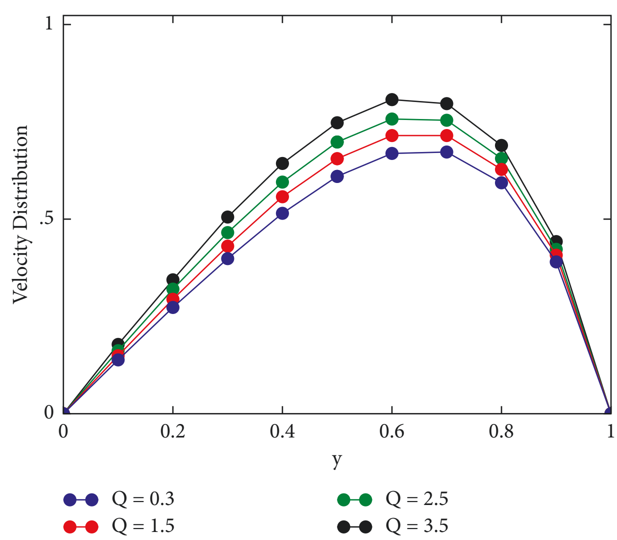

FIGURE 15: Effect of Magnetic field M on velocity after $\mathrm{Pr} = 6.2$, $t = 3$, $\beta = 0.2$, $Q = 0.5$, $\alpha = 0.5$, $\phi_{hnf} = 0.08$ and $\mathrm{Gr} = 30$.

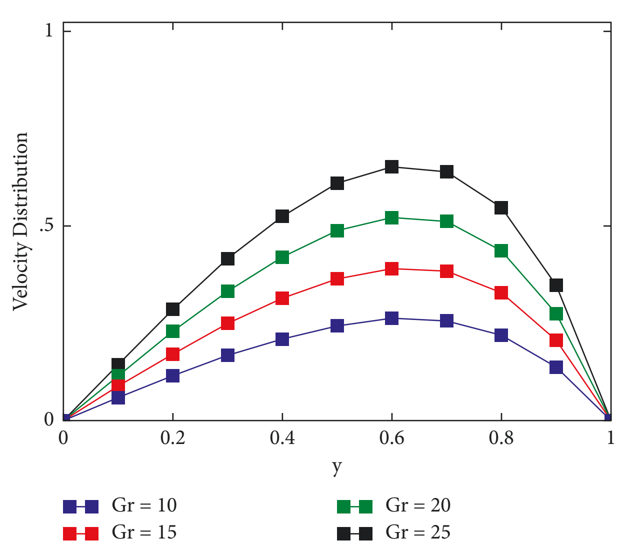

FIGURE 16: The influence of Gron velocity when $\beta = 0, t = 3, \mathrm{Pr} = 6.2$, $Q = 0.5$, $M = 4$, $\phi_{hnf} = 0.08$, $\beta = 0.2$ and $\alpha = 0.5$.

rising Gr, the velocity way quicker. As a result, Gr is linked to buoyancy forces, which elevate conventional convection and hence increase velocity.

5. Conclusions

In present research work, the characteristics of $(Ag - TiO_2 - H_2O)$ hybrid nanofluid has been studied. In microchannel the flow of the hybrid nanofluid was calculated. The (MHD) effects and heat source Q were measure likewise. After converting into fractional model, (ABC) fractional operator used to solve the problem. The exact solutions were attained with the help of LT method and listed in the term of M-function, velocity fields.

The significance of the present work is the nonsingular fractional operator is applied in the heat and mass transfer phenomena containing nanoparticles of the type Ag and TiO_2. As the nanoparticles used in the base fluid to improve the physical quantities of base fluid. This paper contains some new comparisons between fractional operators as well as different kinds of nanoparticles which can be useful in the heat transfer process where we can make a choice with different base fluids according to desired targets. These objectives have already been justified physically in this study.

Major outcomes of present paper are as per the following:

(i) The results show that the Atangana–Baleanu fractional operator exhibited improved memory effect as compared to Caputo–Fabrizio fractional operator.

(ii) Further, due to increasing the values volume fractional temperature can be enhanced and velocity decreases.

(iii) In comparison between nanoparticles for different types of based fluid, velocity and temperature of water based (TiO_2) and silver (Ag) is higher than other base fluids.

(iv) Fractional parameter used to control the thermal and momentum boundary layer thickness.

(v) For larger values of concentration of nanoparticles, thermal as well as momentum boundary layer decrease.

Abbreviations

RL: Riemann–Liouville
C: Caputo
AB: Atangana–Baleanu
CF: Caputo–Fabrizio
B^*: Magnetic fluidity force
μ_0: Magnetic absorptivity
E^*: Electrical field strength
J: Current density
σ_0: Electrical conductivity
U: Velocity field (ms^{-1})
ρ_{hnf}: Density of hybrid nanofluid (kgm^{-3})
v: Fluid velocity
F_{em}: Electromagnetic force
β_{b^*}: Brinkman parameter
μ_{hnf}: Dynamic viscosity $(kgm^{-1}s^{-1})$
σ_{hnf}: Electrical conductivity
β_{hnf}: Thermal expansion
B_0: Magnetic field
$(C_p)_{hnf}$: Specific heat $(jkg^{-1}K^{-1})$
T: Temperature (K)
k_{hnf}: Thermal conductivity $(Wm^{-2}K^{-1})$
T_w: Fluid temperature at the plate (K)
Gr: Thermal Grashof number (dimensionless)
Pr: Prandtl number (dimensionless)
M: Magnetic parameter (dimensionless)
t: Time (s)
α: Fractional parameter
Q: Heat generation parameter
q: Laplace transform parameter
BTF: Brinkman type fluid
CCE: Clasius–Clapeyron equation
LHE: Latent heat evaporation
MF: Magnetic field
LT: Laplace transform.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest in submission of this paper.

Authors' Contributions

All of the authors substantially contributed to this work.

Acknowledgments

The authors extend a sense of gratitude to the University of Management and Technology in Lahore, Pakistan, for facilitating and supporting our research.

References

[1] D. Singh, J. Toutbort, and G. Chen, "Heavy vehicle systems optimization merit review and peer evaluation," Annual Report, Argonne National Laboratory, Lemont, IL, USA, 2006.

[2] A. Kamyar, R. Saidur, and M. Hasanuzzaman, "Application of computational fluid dynamics (CFD) for nanofluids," *International Journal of Heat and Mass Transfer*, vol. 55, no. I 1516, pp. 4104–4115, 2012.

[3] S. K. Das, S. U. Choi, W. Yu, and T. Pradeep, *Nanofluids, Science and Technology*, Wiley, Hoboken, NJ, USA, 2007.

[4] J. P. Vallejo, G. Żyła, J. Fernández-Seara, and L. Lugo, "Rheological behaviour of functionalized graphene nanoplatelet nanofluids based on water and propylene glycol:water mixtures," *International Communications in Heat and Mass Transfer*, vol. 99, pp. 43–53, 2018.

[5] J. P. Vallejo, G. Żyła, J. Fernández-Seara, and L. Lugo, "Influence of six carbon-based nanomaterials on the rheological properties of nanofluids," *Nanomaterials*, vol. 9, no. 146, 2019.
[6] M. Caputo, "Linear models of dissipation whose Q is almost frequency independent--II," *Geophysical Journal International*, vol. 13, no. 5, pp. 529–539, 1967.
[7] M. Caputo and M. Fabrizio, "A new definition of fractional derivative without singular kernel, Progress in fractional differentiation and applications," *International Journal*, vol. 2, pp. 73–85, 2015.
[8] A. Atangana and D. Baleanu, "New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model," *Thermal Science*, vol. 20, 2016.
[9] M. Saqib, I. Khan, and S. Shafie, "New direction of atangana-baleanu fractional derivative with mittag-leffler kernel for non-Newtonian channel flow," in *Fractional Derivatives with Mittag-Leffler Kernel*, pp. 253–268, Springer, Basel, Switzerland, 2019.
[10] A. Atangana and I. Koca, "On the new fractional derivative and application to nonlinear baggs and freedman model," *The Journal of Nonlinear Science and Applications*, vol. 9, pp. 2476–2480, 2016.
[11] I. Siddique, I. Tlili, S. M. Bukhari, and Y. Mahsud, "Heat transfer analysis in convective flows of fractional second grade fluids with CaputoFabrizio and AtanganaBaleanu derivative subject to newtonion heating," *Mechanics of Time-dependent Materials*, vol. 25, pp. 291–311, 2021.
[12] M. A. Imran, M. Aleem, M. B. Riaz, R. Ali, and I. Khan, "A comprehensive report on convective flow of fractional (ABC) and (CF) MHD viscous fluid subject to generalized boundary conditions," *Chaos, Solitons & Fractals*, vol. 118, pp. 274–289, 2019.
[13] P. O. Mohammed, T. Abdeljawad, F. Jarad, and Y. M. Chu, "Existence and uniqueness of uncertain fractional backward difference equations of RiemannLiouville Type," *Mathematical Problems in Engineering*, vol. 2020, Article ID 6598682, 8 pages, 2020.
[14] K. A. Abro, I. Khan, N. K. Soopy, and A. A. Sulaiman, "Effects of carbon nanotubes on magnetohydrodynamic flow of methanol based nanofluids via Atangana-Baleanu and Caputo-Fabrizio fractional derivatives," *Thermal Science*, vol. 23, no. 2B, pp. 883–893, 2019.
[15] Z. Hammouch and T. Mekkaoui, "Circuit design and simulation for the fractional-order chaotic behavior in a new dynamical system," *Complex & Intelligent Systems*, vol. 4, no. 4, pp. 251–260, 2018.
[16] M. D. Ikram, M. I. Asjad, A. Ahmadian, and M. Ferrara, "A new fractional mathematical model of extraction nanofluids using clay nanoparticles for different based fluids," *Mathematical Methods in the Applied Sciences*, pp. 1–14, 2020.
[17] A. Atangana and B. Alkahtani, "Analysis of the keller-segel model with a fractional derivative without singular kernel," *Entropy*, vol. 17, no. 12, pp. 4439–4453, 2015.
[18] A. Atangana and B. Dumitru, "Caputo-Fabrizio derivative applied to groundwater flow within confined aquifer," *Journal of Engineering Mechanics*, vol. 143, no. 5, pp. 1943–7889, 2016.
[19] O. Abu Arqub and M. Al-Smadi, "Atangana-Baleanu fractional approach to the solutions of Bagley-Torvik and Painlevé equations in Hilbert space," *Chaos, Solitons & Fractals*, vol. 117, pp. 161–167, 2018.
[20] M. A. Imran, F. Miraj, and I. Khan, "Soret effects on simultaneous heat and mass transfer in MHD viscous fluid through a porous medium with uniform heat flux and Atangana-Baleanu fractional derivative approach," *European Physical Journal Plus*, vol. 133, no. 224, 2018.
[21] O. A. Arqub and B. Maayah, "Fitted fractional reproducing kernel algorithm for the numerical solutions of ABCFractional Volterra integro-differential equations," *Chaos, Solitons & Fractals*, vol. 126, Article ID 394402, 2019.
[22] I. Khan, "New idea of Atangana and Baleanu fractional derivatives to human blood flow in nanofluids," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 29, no. 1, Article ID 013121, 2019.
[23] Q. Sania and A. Yusuf, "Modeling chickenpox disease with fractional derivatives: from caputo to atangana-baleanu," *Chaos, Solitons & Fractals*, vol. 122, pp. 111–118, 2019.
[24] N. Raza and M. A. Ullah, "A comparative study of heat transfer analysis of fractional Maxwell fluid by using Caputo and Caputo-Fabrizio derivatives," *Canadian Journal of Physics*, vol. 98, no. 1, pp. 89–101, 2020.
[25] M. A. Imran, M. B. Riaz, N. A. Shah, and A. A. Zafar, "Boundary layer flow of MHD generalized Maxwell fluid over an exponentially accelerated infinite vertical surface with slip and Newtonian heating at the boundary," *Results in Physics*, vol. 8, pp. 1061–1067, 2018.
[26] Y. M. Chu, M. D. Ikram, M. I. Asjad, A. Ahmadian, and F. Ghaemi, "Influence of hybrid nanofluids and heat generation on coupled heat and mass transfer flow of a viscous fluid with novel fractional derivative," *Journal of Thermal Analysis and Calorimetry*, vol. 144, 2021.
[27] M. A. Imran, R. Ali, M. D. Ikram, and D. Baleanu, "New analytical solutions of heat transfer flow of clay-water base nanoparticles with the application of novel hybrid fractional derivative," *Thermal Science*, vol. 24, no. 1, pp. S343–S350, 2020.
[28] M. D. Ikram, M. I. Asjad, A. Akgul, and D. Baleanu, "Effects of hybrid nanofluid on novel fractional model of heat transfer flow between two parallel plates," *Alexandria Engineering Journal*, vol. 60, Article ID 35933604, 2021.
[29] M. A. Imran, M. A. Ullah, A. Raza, and M. D. Ikram, "Mathematical fractional modeling of transpot phenomena of viscous fluid-flow between two plates," *Thermal Science*, vol. 25, no. 2, pp. 1–5, 2021.
[30] A. Dawar, A. Wakif, T. Thumma, and N. A. Shah, "Towards a new MHD non-homogeneous convective nanofluid flow model for simulating a rotating inclined thin layer of sodium alginate-based Iron oxide exposed to incident solar energy," *International Communications in Heat and Mass Transfer*, vol. 130, Article ID 105800, 2022.
[31] H. Vaidya, C. Rajashekhar, G. Manjunatha et al., "Analysis of entropy generation and biomechanical investigation of MHD Jeffery fluid through a vertical non-uniform channel," *Case Studies in Thermal Engineering*, vol. 28, 2021.
[32] A. S. Sabu, A. Wakif, S. Areekara, A. Mathew, and N. Ali Shah, "Significance of nanoparticles' shape and thermo-hydrodynamic slip constraints on MHD alumina-water nanoliquid flows over a rotating heated disk: the passive control approach," *International Communications in Heat and Mass Transfer*, vol. 129, Article ID 105711, 2021.
[33] M. D. Ikram, M. A. Imran, Y. M. Chu, and A. Akgül, "MHD flow of a Newtonian fluid in symmetric channel with ABC fractional model containing hybrid nanoparticles," *Combinatorial Chemistry & High Throughput Screening*, vol. 24, 2021.
[34] N. A. Shah, M. I. Asjad, and F. Miraj, "Exact solutions of time fractional free convection flows of viscous fluid over an isothermal vertical plate with Caputo and Caputo-fabrizio

derivatives," *Journal of Prime Research and Mathematics*, vol. 13, p. 5674, 2017.

[35] S. Salahshour, A. Ahmadian, N. Senu, D. Baleanu, and P. Agarwal, "On analytical solutions of the fractional differential equation with uncertainty: application to the basset problem," *Entropy*, vol. 17, no. 2, pp. 885–902, 2015.

[36] M. Ahmad, M. A. Imran, M. Aleem, and I. Khan, "A comparative study and analysis of natural convection flow of MHD non-Newtonian fluid in the presence of heat source and first-order chemical reaction," *Journal of Thermal Analysis and Calorimetry*, vol. 137, no. 5, pp. 1783–1796, 2019.

[37] M. Saqib, S. Shafie, I. Khan, Y. M. Chu, and K. S. Nisar, "Symmetric MHD channel flow of nonlocal fractional model of BTF containing hybrid nanoparticles," *Symmetry*, vol. 12, no. 663, 2020.