

## **BROILER HOUSE PERFORMANCE ANALYSIS AND CHICK ENTRANCE PLANNING FOR A BROILER CHICKEN INTEGRATION**

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### *ABSTRACT*

*Increasing rates of chicken meat production has a substantial impact on the Turkish economy. Per capita production in Turkey is close to the average production of the top 20 countries in the world. The production is done by integrated organizations, incorporating an integrator chicken company affiliated with all aspects of the production in an organization. There are many complicated processes, stages and parties involved in a typical integration, bringing with them many interesting problems. In this paper, we focus on tactical and operational decisions required during the broiler growing stage, motivated by the requirements of a moderate-sized integrator in the Marmara Region of Turkey. We use analysis of variance to compare the performances of available broiler houses, and to find the factors affecting the performance measures. Then, we present a mixed-integer linear program which uses the results of our performance analysis to find the least costly set of broiler houses with which the integrator should work, and the chick entrance times into these houses over a planning period.*

**Keywords:** Analysis of variance, Broiler chicken production, Mixed-integer linear program, Performance analysis.

### **1. INTRODUCTION**

Indigenous chicken meat production (CMP) is a growing industry in the world. According to the United Nations Food and Agriculture Organization, the chicken meat production in 2011 is 89,553,298 tons, up 53% from its value in 2000 (see FAOSTAT, 2012). According to the same source, Turkey ranks ninth in the world with 1,618,350 tons in 2011, up 155% from its value in 2000. Considering the production quantity with respect to population (see Haupt et al., 2011), Turkey produced approximately 21.87 kilograms indigenous chicken meat per capita in 2011. This quantity is not statistically different from the per capita average of the top 20 producer countries at a significance level of 0.05.

Worldwide, the poultry industry has gradually become vertically integrated starting in 1940s. All aspects of the production are nowadays operated by or affiliated with a chicken company. The chicken company is called an "integrator" and is affiliated with all of the following: feed mill (producing the chicken feed), breeding farms (producing fertile eggs), hatchery (producing chicks), broiler house (or growing farm, which grows the broilers, i.e., young chicken grown for its meat), processing plant (or slaughterhouse, which produces and markets the chicken meat), warehouse, distribution center, and veterinary services. In this respect, the broiler production is a complicated system.

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Similar to common practice worldwide, indigenous chicken meat production is mostly done by integrated private broiler organizations in Turkey. Due to the large scope of the system, almost none of the existing broiler chicken integrators operates the broiler houses by themselves. In general, broiler house owners are private entrepreneurs who grow the broilers on behalf of the integrator, and supply broilers to the processing plant. In return, the integrator pays the broiler house owner, depending on three performance measures: mean live weight, feed consumption, and the death rate of the broilers supplied. These performance measures, and hence, the resulting payment are important drivers in generating profits for the overall integrated system. Therefore, identifying factors effective on these performance measures may help the integrator make better tactical decisions.

There exist several studies in the literature, which consider the performance analysis and profitability in the poultry industry. Sariözkan et al. (2009) investigate the effects of some input factors (stocking density, dietary energy level and carnitine supplementation) on some output factors (performance, total costs, egg selling prices and profit) for a specific race of laying hens using analysis of variance (ANOVA). Çiçek et al. (2009) build a regression model with profit as the dependent variable and several independent variables for developing a decision support tool in commercial egg production. These studies focus on the performance analysis of the egg production rather than the meat production system. Tesseraud et al. (2003) study the effects of dietary protein supply on muscle development in chickens, and find that higher protein diet causes faster growth, less feed consumption, lower feed conversion ratio, and higher muscle weights. In our study, we take the death rate of broilers into account (along with the mean live weight and feed consumption) that contribute to the overall performance of a broiler house.

In addition to determining the factors that may be effective on the performance measures of a broiler house, the integrator has to decide on which broiler houses to work with, and plan chick entrance to these houses over a planning horizon. "Chick entrance" is a term used to indicate acceptance of a particular flock of chicks into the broiler house. These decisions depend on the payment amount as well as other factors related with the nature and requirements of growing broilers. On the average, a broiler is slaughtered at the age of 42 days (six weeks). Considering the additional time required to clean the broiler house and prepare it for the next chick entrance, the broiler house is occupied for about eight weeks by a particular flock. Slaughtering is an ongoing process that has to be continuously supplied with the broilers. Therefore, the total number of chicks that the integrator has in the broiler houses is about 50 times of the daily slaughtering quantity, which makes some millions for a moderate-sized integration. In this respect, an integrator needs an operational model that can aid in choosing an efficient set of broiler houses, and planning entrance of the chicks into these broiler houses (see Taube-Netto (1996) for operations research and management science applications in the poultry industry).

In the following section, we describe our related methodology and mathematical models. In Section 3, we present the results of our experiments, and finally conclude with a summary and possible future work in Section 4.

## 2. MATHEMATICAL MODELS

In our study, we focus on tactical and operational decisions required during the broiler growing stage, based on the operations and requirements of a moderate-sized integrated

broiler organization located in the Marmara Region of Turkey. This particular broiler integration has a processing capacity of 50,000 chickens per day, and serves markets throughout Turkey.

In the first subsection we consider a two-way analysis of variance (two-way ANOVA) for the performance analysis of several broiler houses, which may supply the integrator. In the second subsection, we provide a mixed-integer linear programming model that uses the results of the ANOVA along with other inputs to select the least costly set of broiler houses and to determine the chick entrance schedules into the selected houses over a planning period.

**2.1 Two-Way ANOVA for Unbalanced Data with All Cells Filled**

We consider a two-way ANOVA model with factors (geographical) region and chick type, as proposed by the integrator, in order to check for the existence of their effects on a broiler house's performance measures. We consider three performance measures for a broiler house: (i) mean live weight (MLW), (ii) food conversion ratio (FCR), and (iii) death percentage rate (DPR). These performance measures not only determine the payment amount of the integrator to the broiler house, but also affect the overall efficiency of the integrated organization.

There exist 14 regions from which the integrator may select broiler houses for the chicks. In other words, the first factor has 14 levels. Chicks can be either domestically produced (in-house production) within the integration or acquired from the chick market, defining the two levels of the second factor in the analysis. As a result, there are 28 treatments. A total of 1,907 records for chick entrances into broiler houses are summarized in Table 1.

**Table 1. Sample sizes for treatments in two-way ANOVA (number of past entrances into broiler houses according to region and chick type)**

Region	Acquired	In-House	Region totals
Adapazarı	58	113	171
Akçakoca	76	129	205
Akyazı	123	199	322
Düzce	112	180	292
Ferizli	16	19	35
Hendek	17	16	33
İstanbul	4	3	7
Kandıra	24	31	55
Karapürçek	19	19	38
Karasu	70	162	232
Kaynarca	103	174	277
Kocaeli	19	61	80
Söğütlü	13	20	33
Yığılca	57	70	127
Chick type totals	711	1,196	1,907

However, each treatment has unequal number of observations,  $n_{ij} > 0$ . In other words, we have unbalanced data with all cells filled (see the preliminary work of Yates (1934), a survey of two-way unbalanced models by Fujikoshi (1993), Shaw and Mitchell-Olds (1993), and book of Searle (1997) for more information). We apply a two-way ANOVA

using general linear models for unbalanced data sets. The model we use is given in equation (1).

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk} \quad (1)$$

In equation (1),  $y_{ijk}$  represents the performance measure of a broiler house  $k$  at treatment cell  $(i, j)$ ,  $i = 1, \dots, 14, j = 1, 2$ . We note that the model in equation (1) is assumed for each of the three dependent variables (performance measures: MLW, FCR, and DPR). In this model,  $\mu$  denotes the overall mean,  $\tau_i$  is the effect due to region  $i$ ,  $\beta_j$  is the effect due to chick type  $j$ , and  $\gamma_{ij}$  corresponds to the interaction effect of the two factors. Here, the interaction effects are included in the model since it may be possible to have an interaction between a chick type and the region in which this particular type of chick is grown. Finally,  $\varepsilon_{ijk}$  is the error term having a normal distribution with zero mean and variance  $\sigma^2$ .

The results of ANOVA will help the integrator understand the effects of region and chick type on the performance measures (i.e., MLW, FCR, and DPR) for each broiler house. The integrator will be able to make better decisions about which types of chicks to grow, which broiler houses to work with, or at least, in which regions the agreement should continue or terminate.

## 2.2 A Mathematical Programming Model for Chick Entrance Planning

Once the effects of the region and chick type on the performance measures of a broiler house are understood, and it becomes clear whether there are differences in the performances of the broiler houses, the integrator will need to plan the chick entrances. In particular, the integrator has to decide on which particular set of broiler houses to work with in order to meet a pre-specified demand quantity of broilers over a planning period.

Assuming that the ANOVA results yield a plausible set of broiler houses ( $H$  indexed by  $i$ ), and a planning period ( $T$  indexed by  $t$ ), (estimated as eight weeks due to occupation time of a broiler house by a certain flock), the integrator needs an operational decision support tool to plan a cost-effective chick entrance into a set of broiler houses that will be selected from the set,  $H$ , and at certain time periods during  $T$ . We index the performance measure parameters, MLW, FCR, and DPR, for each broiler house  $i \in H$ . We formulate a mathematical programming model, which minimizes the total cost of chick entrance into the broiler houses as follows:

$$\min z = \sum_{t \in T} \sum_{i \in H} S y_{it} + C x_{it} + (MLW_i \times FCR_i \times F) x_{it} + p_{it} \quad (2)$$

s.t.

$$p_{it} = (MLW_i \times W) x_{it} + MLW_i (2 - FCR_i) O x_{it} + (0.06 - DPR_i) M x_{it} \quad \forall i, t \quad (3)$$

$$\sum_{t \in T} y_{it} \leq 1 \quad \forall i \quad (4)$$

$$\sum_{i \in H} x_{it} \geq D_t \quad \forall t \quad (5)$$



$$x_{it} \leq K_i y_{it} \quad \forall i, t \quad (6)$$

$$y_{it} \leq \sum_{k=1}^t A_{ik} \quad \forall i, t \quad (7)$$

$$p_{it} \geq 0, x_{it} \geq 0, y_{it} \in \{0, 1\} \quad \forall i, t \quad (8)$$

The integrator's decision variables are:  $x_{it}$ , the number of chicks to enter in a broiler house  $i$  at week  $t$ ;  $y_{it}$ , the binary variable that takes a value of one if chick entrance occurs into broiler house  $i$  at week  $t$ , and zero otherwise. The mathematical program is a mixed-integer linear program due to the nature of the decision variables given in constraint sets (8).

Once the integrator decides to work with a particular broiler house,  $i$ , initiating the chick entrance into this particular broiler house at time  $t$ , the integrator incurs a fixed cost,  $S$ , for administrative and other similar setup related items. The integrator has to pay for the chicks that will be entering the broiler house, at a unit cost of  $C$  per chick, as well as their feed. The latter cost is calculated based on the quantity of broilers weighted by the product of the mean live weight and the food conversion ratio that estimates the required feed, multiplied by the unit feed cost,  $F$ .

In addition, the integrator has to pay the broiler house for the broilers received from them. As we noted in Section1, the integrator's payment to a broiler house depends on three (realized) performance measures: mean live weights, feed consumption, and death rate of the broilers. The integrator has to pay a unit price of  $W$  per kilogram of live weight per broiler, a premium/penalty of  $O$  per kilogram feed per broiler for deviation from a pre-specified FCR of two (a commonly accepted level of feed conversion ratio), and a premium/penalty of  $M$  per kilogram chick for deviation from a pre-specified DRP of six percent (a commonly accepted death percentage rate). The total payment of the integrator to a particular broiler house at a specific entrance week,  $p_{it}$ , is given by the sum of the payments based on these performance measures as in equation (3). The cost of working with a particular broiler house for a chick entrance,  $c_{it}$ , is given in equation (2), and is the sum of all of the costs we mentioned. Summed over all broiler houses and all chick entrance weeks, the integrator's total cost is the objective function given in equation (1).

The constraint set in (4) ensures that each broiler house can be used at most once during the planning period, i.e., at most one chick entrance can be made into that particular broiler house during the planning period. The quantity of broilers required at each week,  $D_t$ , must be satisfied as provided in constraint set (5) while the weekly capacity of each broiler house,  $K_i$  must not be exceeded as enforced in constraint set (6). The constraint set in (7) is to ensure availability of the broiler houses over time.  $A_{it}$  is an indicator parameter that takes a value of one if broiler house  $i$  is available in week  $t$ , and zero otherwise. Broiler houses are considered to be available for all weeks after the first available week. In reality, if a particular broiler house is unoccupied for a period, the broiler house owner may sign a contract with another integration. However, this is very uncommon in the industry due to small planning period length of eight weeks, and hence, this constraint can be assumed to hold.

Our mathematical programming model for the chick entrance planning is a relaxation of the integer knapsack problem, i.e., we relax the integrality of the decision variables. The

knapsack problem is described as follows: given a set of items of different values and volumes, find the most valuable set of items that fit in a knapsack of fixed volume. In our problem, chicks can be treated as items; values of items are the costs of entrances, which depend on the performance measures. In this respect, we fill available broiler houses with the chicks while minimizing the total cost. A larger-scale of this model is provided by Satır (2003).

### 3. NUMERICAL RESULTS

In this section, we first present our experiments and their results for the performance analysis of the broiler houses. Next, we will discuss our experiment for the chick entrance planning.

We used Minitab (version 15) to conduct the statistical analysis. We summarize the results of the two-way ANOVA in Tables 2, 3 and 4. Note that we transformed the performance variable, FCR, by using Box-Cox transformation with  $\lambda = -3$ , and, DPR, by using the natural logarithm to ensure that assumptions are satisfied.

**Table 2. ANOVA results for MLW**

Source	DF	Seq SS	Adj SS	Adj MS	F	p
Region	13	2.831	2.599	0.218	4.32	0.000
Chick Type	1	0.081	0.013	0.081	0.28	0.186
Region*Chick Type	13	0.596	0.596	0.046	0.99	0.458
Error	1,879	86.972	86.972	0.046		
Total	1,906	90.480				

**Table 3. ANOVA results for FCR<sup>-3</sup>**

Source	DF	Seq SS	Adj SS	Adj MS	F	p
Region	13	0.054	0.044	0.004	2.86	0.000
Chick Type	1	0.000	0.003	0.000	0.25	0.619
Region*Chick Type	13	0.016	0.016	0.001	0.86	0.601
Error	1,879	2.729	2.729	0.001		
Total	1,906	2.800				

**Table 4. ANOVA results for ln DPR**

Source	DF	Seq SS	Adj SS	Adj MS	F	p
Region	13	57.164	50.551	4.397	12.07	0.000
Chick Type	1	0.006	1.921	0.006	0.02	0.898
Region*Chick Type	13	5.387	5.387	0.414	1.14	0.322
Error	1,879	684.703	684.703	0.364		
Total	1,906	747.260				

Based on a significance level of 0.01, we fail to reject the null hypothesis for the interaction and the main effect due to chick type, whereas we reject the null hypothesis for the main effect due to region at each performance measure. Therefore, it is clear that the region has a non-negligible effect on the performance measures. We present the estimates of the region effects and the overall means in Table 5.

**Table 5. Estimates of the region effects and overall means**

Region	$\tau_i$ for MLW	$\tau_i$ for FCR <sup>3</sup>	$\tau_i$ for ln DPR
Adapazarı	0.03	-0.05	-2.33
Akçakoca	-0.03	0.01	0.05
Akyazı	-0.01	0.01	-0.38
Düzce	0.06	-0.01	0.58
Ferizli	0.00	-0.11	-2.64
Hendek	0.04	0.02	0.12
İstanbul	0.17	0.18	7.35
Kandıra	-0.02	0.01	-0.36
Karapürçek	-0.05	-0.04	0.77
Karasu	0.03	-0.02	-1.25
Kaynarca	-0.05	0.02	1.76
Kocaeli	-0.04	0.06	-0.30
Söğütü	-0.01	-0.01	-0.41
Yığılca	0.00	0.01	1.64
Overall means	1.91	1.99	8.08

We note that a positive  $\tau_i$  affects the performance measures FCR and DPR in a negative way, i.e., reduces the performance, whereas it improves MLW. Table 6 lists the signs of the region effect that lead to better performance for each of the measures.

**Table 6. Signs of the region effect estimate leading to better performance**

Region	$\tau_i$ for MLW	$\tau_i$ for FCR <sup>3</sup>	$\tau_i$ for ln DPR
Adapazarı	+	+	+
Akçakoca	-	-	-
Akyazı	-	-	+
Düzce	+	+	-
Ferizli	-	+	+
Hendek	+	-	-
İstanbul	+	-	-
Kandıra	-	-	+
Karapürçek	-	+	-
Karasu	+	+	+
Kaynarca	-	-	-
Kocaeli	-	-	+
Söğütü	-	+	+
Yığılca	+	-	-

From Tables 5 and 6, it can be seen that the integration is better off working with the broiler houses in regions Karasu and Adapazarı, and worse off working with those in regions Akçakoca and Kaynarca. Decisions about other regions need expert opinion. As a result, we remove broiler houses in Akçakoca and Kaynarca regions from the available set of 14 broiler houses. The remaining regions are used in planning the chick entrance to broiler houses in our mathematical programming model. Figures 1 and 2 show that the residuals are approximately normally distributed, and Figures 3(a)-(c) display no pattern or structure supporting the constant variance assumption.

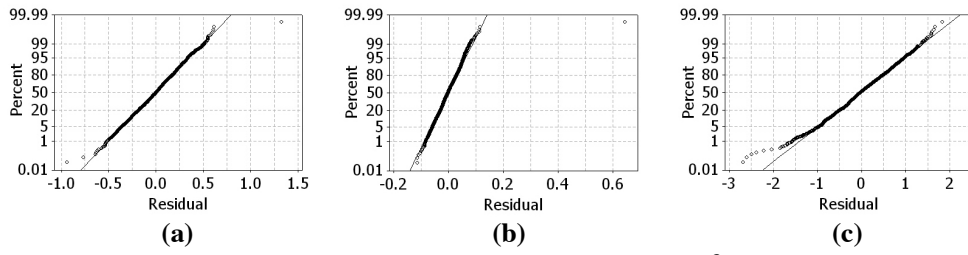


Figure 1. Normal plot of residuals for (a) MLW, (b)  $FCR^{-3}$ , (c)  $\ln DPR$

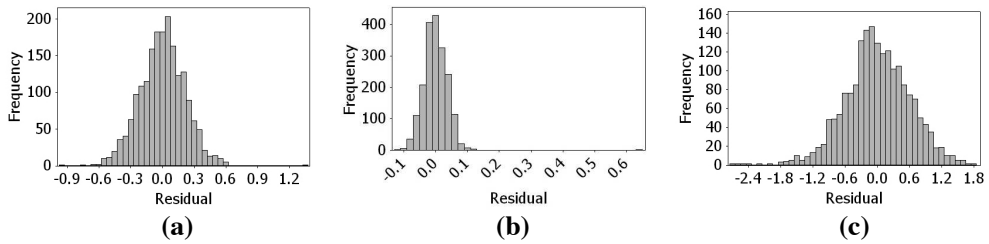


Figure 2. Histogram of residuals for (a) MLW, (b)  $FCR^{-3}$ , (c)  $\ln DPR$

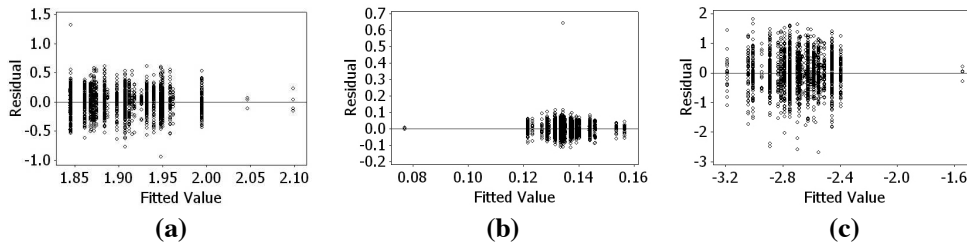


Figure 3. Residuals versus fitted values for (a) MLW, (b)  $FCR^{-3}$ , (c)  $\ln DPR$

Since the mathematical model given in Subsection 2.2 is a relaxation of the knapsack problem, the broiler houses are filled from the “best” to the “worst”. The term “best” defines the production of broilers from chicks by the broiler house owner at minimum cost according to the integration’s perspective. The term “worst” has the opposite meaning. As a result, at most one broiler house may be used fractionally, and the rest of the broiler houses will be filled to their maximum capacity. Our mathematical model includes 5,977 variables, and 6,134 equations based on the data provided by the integration. We solved the mathematical model using GAMS (version 2.01) modeling system, and the solution yielded an objective function value, i.e., total cost of more than 5 million Turkish Liras with a plan for the chick entrances to a set of broiler houses. Due to space limitation and confidentiality requirements of the integrator, we will not be presenting the outputs of the solution.

#### 4. CONCLUSION

The poultry industry, in general, is vertically integrated, i.e., an integrator firm is affiliated with every aspect of the production. This is also the case in Turkey. Due to



complicated processes, many stages and parties involved in a typical integration, there is a need for analytical models to identify opportunities to improve the integration. In this study, we focused on the broiler chicken production, in which an integrator works with one or more broiler houses for the production of broilers over a planning horizon. The performance, and hence the payment, of the integrator for the services of a broiler house depends on three criteria: mean live weight, food conversion ratio, and death percentage rate. The overall performance and profitability of the integration also depends on these criteria. In this study, we focused on the selection of which broiler houses to work with based on performance analysis by using ANOVA, and planning the entrance of chicks to the selected broiler houses in the most economical way by using mathematical programming. We worked with an integrator in the Marmara Region of Turkey, which supplied data for this study.

As a result of ANOVA, we were able to identify that the region of a broiler house has a significant effect on the performance measures. The veterinarians that serve the broiler houses in the selected regions also agreed with the ANOVA results, confirming the results of our study. In the second part of our study, we provided an operational model that uses the plausible set of broiler houses obtained from the former part of this study to help the integrator select the least costly set of broiler houses for chick entrance over a finite planning period.

We discussed the solution obtained from our mixed-integer linear program with the relevant parties in the integration. Even though other omitted factors may play a role in the selection of the broiler houses to work with (for instance, historical business relationships between the integrator and particular broiler houses, unplanned workforce problems, etc.), our model produced results that are agreed on by the integrator as viable and efficient.

For possible future work, more factors can be included in a larger-sized ANOVA study. For instance, races of chicks, qualifications of the labors working in the broiler house, quality of the feed, quality of the veterinary services due to hiring and firing, availability of vaccines, etc., are other factors that we can consider, which will enrich and improve the credibility of the analysis, as well as providing a better input for our mathematical program.

## 5. REFERENCES

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## BİR BROYLER TAVUK ENTEGRASYONU İÇİN KÜMES PERFORMANS ANALİZİ VE CİVCİV GİRİŞİ PLANLAMASI

### ÖZET

*Artan broiler tavuk üretimi Türkiye ekonomisi üzerinde önemli bir etkiye sahiptir. Türkiye'deki kişi başına düşen tavuk eti üretimi dünya sıralamasındaki ilk 20 ülkenin ortalama üretimine çok yakındır. Üretim birleşik organizasyonlar tarafından yapılmakta olup, entegrasyonunun başında üretimin her safhasıyla bağlantılı olan bir tavuk işletmecisi bulunmaktadır. Bir entegrasyonda birçok karmaşık süreç, evre ve grup bulunmakta olup, bunlar beraberlerinde çok sayıda ilginç problem getirmektedir. Bu çalışmada, Türkiye'nin Marmara Bölgesinde yer alan orta büyüklükteki bir tavuk işletmecisinin gereksinimlerinden yola çıkarak, piliç yetiştirme evresindeki taktik ve işletme planları ele alındı. Varyans analizini kullanarak, mevcut kümeslerin performansları karşılaştırıldı ve performans ölçütlerini etkileyen faktörleri belirlendi. Daha sonra, bu performans analizi çalışmasının sonuçlarını kullanan bir tamsayılı doğrusal programlama modeli ile planlama çevreninde tavuk işletmecisinin en ucuz maliyetle birlikte çalışabileceği kümes işletmeleri belirlendi ve bu kümeslere cıvciv girişlerinin planlaması yapıldı.*

**Anahtar Kelimeler:** Varyans analizi, Broiler tavuk üretimi, Tamsayılı doğrusal programlama, Performans analizi.