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RESEARCH ARTICLE

Hermite–Hadamard Type Inequalities for Interval-Valued Preinvex Functions via Fractional Integral Operators

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Abstract
In this article, the notion of interval-valued preinvex functions involving the Riemann–Liouville fractional integral is described. By applying this, some new refinements of the Hermite–Hadamard inequality for the fractional integral operator are presented. Some novel special cases of the presented results are discussed as well. Also, some examples are presented to validate our results. The established outcomes of our article may open another direction for different types of integral inequalities for fractional interval-valued functions, fuzzy interval-valued functions, and their associated optimization problems.

Keywords H–H inequalities · Preinvex functions · Interval-valued functions · R–L fractional integral · Fractional integral inequalities

Mathematics Subject Classification 26A51 · 26A33 · 26D07 · 26D10 · 26D15

Abbreviations
H–H Hermite–Hadamard
R–L Riemann–Liouville

1 Introduction

The idea of convexity has a lot of recognition in the theory of inequality, and assumes an essential part in probability theory, operations research, finance, decision-making, and numerical analysis. As of late, several generalizations related to convex functions have been set up. The idea of integral inequality is a fascinating area for research within mathematical analysis. Some fundamental integral inequalities are being utilized as a tool for fostering the subjective properties of convexity. Moreover, fractional calculus deals with derivatives and integrations of any fractional order. Recently, studies on integral inequalities using fractional operators have become an interesting topic for several mathematicians. Sarikaya et al. [1] started the trend of applying Riemann–Liouville (R–L) fractional integral operator for the classical convex function to refine the well-known Hermite–Hadamard (H–H) inequality. It is seen that fractional calculus aims at establishing mathematical models.

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