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Impact of activation energy and MHD on Williamson fluid flow in the presence of bioconvection

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Stretching Sheet

Abstract The main purpose of the current study is to invetigate the influence of Brownian motion and thermophoresis diffusion in non-Newtonian Williamson fluid flow through exponentially stretching sheet with the effects of thermal radiation and the bioconvection of microorganisms. For this purpose, similarity functions are involved to transmute partial differential equations to corresponding ordinary differential equations. Then Runge–Kutta method with shooting technique is hired to evaluate the desired findings with utilization of MATLAB script. The fluid velocity becomes slow against strength of magnetic parameter and it boosts with mixed convection. The temperature rises with parameter of Brownian motion and thermophoresis. The bioconvection Lewis number diminishes the velocity field. Compared with the existing literature, the results show satisfactory congruence's.

1. Introduction

The convoluted and quick process in massive machinery and little gadgets have produced a significant problem of thermal imbalance. Varied extraneous techniques like fins and fans are used however their utility is restricted because of giant size. In 1995 the scientist Choi [1]introduced the nano-sized

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Nomenclature

$\mathrm{M} = \frac{2\sigma B_0^2 \ell}{\rho \widetilde{U}_w}$ Magne

tic field parameter

$\mathrm{s} = V_0\sqrt{\frac{2l}{a_0 v}}$ Suction/injection parameter

$\mathrm{We} = \Gamma\sqrt{\frac{\widetilde{U}_w^3}{lv}}$ Williamson fluid parameter

$\lambda = \frac{\left(1-\widetilde{C}_\infty\right)\beta g\left(\widetilde{T}_w-\widetilde{T}_\infty\right)2\ell}{\widetilde{U}_w^2}$ Mixed convection parameter

$\mathrm{Nr} = \frac{\left(\rho_p-\rho_f\right)\left(\widetilde{C}_w-\widetilde{C}_\infty\right)}{\beta\left(1-\widetilde{C}_\infty\right)\rho\left(\widetilde{T}_w-\widetilde{T}_\infty\right)}$ Buoyancy ratio number

$\mathrm{Rb} = \frac{\left(\rho_m-\rho_f\right)\gamma\left(\widetilde{N}_w-\widetilde{N}_\infty\right)}{\rho\left(1-\widetilde{C}_\infty\right)\beta\left(\widetilde{T}_w-\widetilde{T}_\infty\right)}$ Bioconvection Rayleigh number

$\mathrm{Nt} = \frac{\tau D_T\left(\widetilde{T}_w-\widetilde{T}_\infty\right)}{v\widetilde{T}_\infty}$ Thermopherosis diffusion

$\mathrm{Nb} = \frac{\tau D_B\left(\widetilde{C}_w-\widetilde{C}_\infty\right)}{v}$ Brownian motion factor

$\sigma_m = \frac{2K_r^2\ell}{\widetilde{U}_w}$ Dimensionless reaction rate

$\mathrm{E} = \frac{Ea}{k\widetilde{T}_\infty}$ Dimensionless activation energy

$\mathrm{K} = \frac{4\sigma^* \widetilde{T}_\infty^3}{k^* K}$ Radiation parameter

$\sigma_1 = \frac{\widetilde{N}_\infty}{\widetilde{N}_w-\widetilde{N}_\infty}$ Bioconvection difference parameter

$\delta = \frac{\left(\widetilde{T}_w-\widetilde{T}_\infty\right)}{\widetilde{T}_\infty}$ Temperature distinction parameter

$\mathrm{Sc} = \frac{v}{D}$ Schmidt number

$\mathrm{Lb} = \frac{\alpha}{D_n}$ Bio-convection Lewis number

$\mathrm{Pe} = \frac{dW_c}{D_n}$ Peclet number

Re	Reynold's number
$\widetilde{T}$	Fluid temperature
$\widetilde{T}_w$	Wall temperature
$\widetilde{T}_\infty$	Temperature far away from the plate
$\tilde{u}, \tilde{v}$	Velocity components along (x, y)-axes
μ	Dynamic viscosity
v	Kinematic viscosity

particles mixed in the fluid called nanofluid have the more capacity of heat transfer as compared to fluid without nano-sized particles. Das et al.[2] explained the recent and future applications of fluid involving nano-sized particles. Khan et al. [3] using shooting method analyzed flow features of Williamson nanofluid influenced by variable viscosity depends on temperature and Lorentz force past an inclined nonlinear extending surface. Koo et al. [4] described influences of convection, conduction,viscous dissipation and thermal transportation on nanofluid flow in micro channel. Sui et al. [5] studied double diffusion to analyze the significance of slip velocity, Brownian motion, thermophoresis, mass and thermal transportation, and variable viscosity on the stream of Maxwell upper convected nanofluid across extending surface. Imran et al. [6] determined unsteady stream of Maxwell fluid through an accelerated exponentially vertical surface with influences of radiation, Newtonian heating, MHD, and slip condition taken into account. Khan et al. [7] investigated flow of micro polar base nanofluid through stretching sheet with thermal radiation and magnetic dipole. Sheikholeslami et al. [8] scrutinized magnetic field impacts on the thermal transport rate in nanofluid. Stretching sheet because of its wide applications like plastic film, metal drawing and spinning, glass fiber, paper processing, and heat moving has become an important topic in the past decades. Recently some researchers investigated the magneto hydrodynamic flow with the different effects such as viscous dissipation, and chemical reaction using stretching sheet Ismail et al. [9], Rajput et al. [10], Swain et al. [11], Reddy et al. [12], Jat et al. [13], Sajid et al. [14], Ishak et al. [15], Abd El-Aziz [16], Makinde [17], and Goud et al. [18].

Bioconvection described the phenomena in which living microorganisms denser than water swims upward in suspensions. These microorganisms pile up in the layer of upper surface and because of this pile up lower surface becomes less dense than upper surface and distribution of density becomes unstable due to which microorganisms fall down and phenomena of bioconvection occur. Bioconvection have applications in biological systems and biotechnology such as purify cultures, enzyme biosensors and to separate dead and living cells [19]. Raees et al. [20] examined the unsteady stream of bioconvection mixed nanofluid having gyroitactic motile microorganisms through horizontal channel. Siddiqa et al. [21] numerically studied the bioconvection flow of nano fluid having mass and thermal transportation along gyrotactic microorganisms through curved vertical cone. Abbasi et al. [22] introduced the bioconvection stream of viscoelastic nano fluid because of gyrotactic microorganisms past a rotating extending disk having zero mass flux and convective boundary condition, also described the relatable parameters influences on velocity, temperature, local density, Sherwood number, and Nusselt number in detail. Chu et al. [23] analyzed the stream of bioconvection MHD fluid through extending sheet with significance of motile microorganisms, activation energy, thermophoresis diffusion, Brownian motion, and chemical reaction are taken into account. Henda et al. [24] examined the magnetized bioconvection flow of fluid past an extending cylinder with thermal radiation, activation energy, and heat source. Makinde et al. [25] discussed the influences of magnetic field and nonlinear thermal radiation on bioconvection MHD nanofluid flow through upper surface of a paraboloid of revolution. Makinde et al. [26] studied the bioconvection of nanofluid over an upper horizontal surface of a paraboloid of revolution in the presence of Brownian motion, thermophoresis and quartic autocatalytic kind of chemical reaction. Puneeth et al. [27] analyzed bioconvection of a radiating hybrid nanofluid through a thin needle in the presence of heterogeneous-homogeneous chemical reaction. Mutuku et al.[28] scrutinized hydromagnetic bioconvection flow of nanofluid due to motile microorganisms through a permeable vertical surface with Brownian motion and thermophoresis effects taken into account. Khan and Makinde [29] investigated the boundary layer flow of a nanofluid due to gyrotactic microorganisms across vertical plate containing navier slip and magnetic field.

Non-Newtonian fluid urges the researchers to analyzed the phenomena of mass and heat transportation due to its role in engineering and industrial process. Class of non-Newtonian fluids such as shampoos, jelly, sugar, honey, human blood, pulps, etc. Williamson fluid is also category of fluid model that is pseudo-plastic. Pseudo-plastic fluids have applications in the engineering and industrial field such as food processing, blood cells, photographic films, and inkjet printing etc. Bhatti et al. [30] described the MHD Williamson nanofluid flow through rotating circular plates influenced by Gyrotactic Microorganism. Bhatti et al. [31]scrutinized non-Newtonian Eyring-Powell nanofluid flow through stretching sheet in the presence of nonlinear thermal radiation and bioconvection. Rana et al. [32] explained the oblique Oldroyd-B fluid flow with the influences of gyrotactic microorganisms, suction/injection and zero mass flux. Kumar et al.[33] investigated the effects of thermal radiation and chemical reaction on MHD Williamson fluid flow due to curved sheet. Raju et al. [34]determined the Williamson and Casson fluid flow past a stretching sheet with heat and mass transfer in the presence of temperature dependent heat source. Kumaran et al. [35] analyzed the influences of viscous dissipation and thermal transfer in radiative MHD Williamson fluid flow across upper paraboloid of revolution. Tlili et al. [36] described the thermal transport of Casson and normal fluid flow due to stretching sheet with cross-diffusion, Brownian motion, thermophoresis and joule heating taken into account. Sulochana et al. [37] numerically discussed the effects of thermal radiation and chemical reaction on MHD Casson fluid flow through wedge with joule heating. Kumar et al.[38] introduced viscous dissipation, thermal radiation, joule heating and magnetic field to analyzed the stream of Williamson nano-fluid past an extending sheet influenced by chemical reactions. Shateyi et al. [39] analyzed thorough and detailed study of incompressible conductive Williamson nano-fluid on the extending permeable sheet. Ali et al. [40] by using FEM scrutinized the significance of thermal diffusion, thermal radiation, and MHD on time dependent flow of Maxwell nano-fluid past an extending geometry.

Motivated by ongoing research on thermal transportation of fluid flow through different regimes, we discussed Brownian motion and thermophoresis diffusion in Williamson fluid flow across exponentially stretching sheet. The principal concern of current work is to examined the characteristics of thermal radiation and bioconvection influence on Williamson MHD fluid flow with the insertion of gyrotactic, auto-motile organisms to avoid possible sedimentation. The connotation of such meaningful attributes can be useful extension and the results can be utilized for desired effective thermal transportation in the heat exchanger of various technological process.

2. Problem Formulation

Here we considered, steady incompressible magneto hydrodynamic non-Newtonian Williamson fluids flow through exponentially stretching sheet along x-axis and y-axis taken to be normal with velocity $\widetilde{U}_w = a_0 e^{\frac{x}{l}}$ as shown in Fig. 1. Magnetic field applied to flow region and acts in the y-direction. A mild diffusion of microorganisms and nano-particles are set in the non-Newtonian base fluid. Thermal radiation is considered and bioconvection take place because of microorganism's movement. The fluid velocity for two dimensional flow is $\tilde{u}, \tilde{v}$. Under above conditions, the governing equations are as follows [12,18,40].

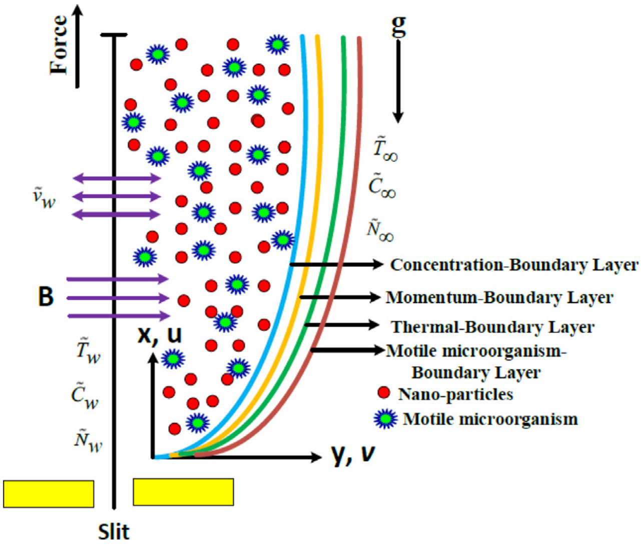

Fig. 1 Geometry of the problem.

Continuity equation

$$\tilde{u}_x + \tilde{v}_y = 0, \tag{1}$$

momentum equation

$$\begin{aligned}\tilde{u}\tilde{u_x} + \tilde{v}\tilde{u_y} = {} & v\tilde{u_{yy}} + \sqrt{2}\Gamma \tilde{u_y}\tilde{u_{yy}} - \frac{\sigma}{\rho}\left(B_0^2\tilde{u}\right) \\ & + \frac{1}{\rho}\Big[g\beta\rho\left(1 - \widetilde{C}_\infty\right)\left(\widetilde{T} - \widetilde{T}_\infty\right) \\ & - g\left(\rho_p - \rho_f\right)\left(\widetilde{C} - \widetilde{C}_\infty\right) - g\gamma\left(\rho_m - \rho_f\right)\left(\widetilde{N} - \widetilde{N}_\infty\right)\Big],\end{aligned} \tag{2}$$

energy equation

$$\tilde{u}\tilde{T}_x + \tilde{v}\tilde{T}_y = \alpha\tilde{T}_{yy} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \tau\left(D_B\tilde{T}_y\tilde{C}_y + \frac{D_T}{\tilde{T}_\infty}\left(\tilde{T}_y\right)^2\right), \tag{3}$$

concentration equation

$$\tilde{u}\tilde{C}_x + \tilde{v}\tilde{C}_y = D\tilde{C}_{yy} - K_r^2\left(\widetilde{C} - \widetilde{C}_\infty\right)\left(\frac{\widetilde{T}}{\widetilde{T}_\infty}\right)^n exp\left(\frac{-Ea}{k\widetilde{T}}\right) + \frac{D_T}{\tilde{T}_\infty}\tilde{T}_{yy}, \tag{4}$$

bioconvection equation

$$\tilde{u}\tilde{N}_x + \tilde{v}\tilde{N}_y + dW_c\frac{\partial}{\partial y}\left(\frac{\widetilde{N}}{\Delta\widetilde{C}}\tilde{C}_y\right) = \tilde{N}_{yy}D_n. \tag{5}$$

With constraints

$$\tilde{u} = \widetilde{U}_w,\ \tilde{v} = -\gamma(x),\ \widetilde{T} = \widetilde{T}_w,\ \widetilde{C} = \widetilde{C}_w,\ \widetilde{N} = \widetilde{N}_w\ at\ y \to 0,$$

$$\tilde{u} \to 0,\ \widetilde{T} \to \widetilde{T}_\infty,\ \widetilde{C} \to \widetilde{C}_\infty,\ \widetilde{N} \to \widetilde{N}_\infty\ as\ y \to \infty. \tag{6}$$

Now introducing:

$$\widetilde{U}_w = a_0 e^{\frac{x}{\ell}},\ \gamma(x) = -V_0 e^{\frac{x}{2\ell}},\ \widetilde{T}_w = \widetilde{T}_\infty + \widetilde{T}_0 e^{\frac{x}{2\ell}},$$
$$\widetilde{C}_w = \widetilde{C}_\infty + \widetilde{C}_0 e^{\frac{x}{2\ell}},\ \widetilde{N}_w = \widetilde{N}_\infty + \widetilde{N}_0 e^{\frac{x}{2\ell}}. \tag{7}$$

Under the Rosseland approximation q_r [18], Eq. (3) can be written as

$$\tilde{u}\tilde{T}_x + \tilde{v}\tilde{T}_y = \tilde{T}_{yy}\left(\alpha + \frac{16\sigma^* T_\infty^3}{\rho C_p 3k_1}\right) + \tau\left(D_B \tilde{T}_y \tilde{C}_y + \frac{D_T}{\tilde{T}_\infty}\left(\tilde{T}_y\right)^2\right). \tag{8}$$

Introducing, similarity transformation,

$$\eta = y\sqrt{\frac{a_0}{2v\ell}}e^{\frac{x}{2\ell}},\ \tilde{u} = a_0 e^{\frac{x}{\ell}} f'(\eta),\ \tilde{v} = -\sqrt{\frac{a_0 v}{2\ell}}e^{\frac{x}{2\ell}}[f(\eta) + \eta f'(\eta)],$$
$$\psi = \sqrt{2vLa_0}f(\eta)e^{\frac{x}{2\ell}}, \widetilde{T} = \widetilde{T}_\infty + \widetilde{T}_0 e^{\frac{x}{2\ell}}\theta(\eta),\ \widetilde{C} = \widetilde{C}_\infty + \widetilde{C}_0 e^{\frac{x}{2\ell}}\phi(\eta),\ \widetilde{N} = \widetilde{N}_\infty + \widetilde{N}_0 e^{\frac{x}{2\ell}}\chi(\eta). \tag{9}$$

In view of the above appropriate relations, Eq. (1) is satisfied identically and Eqs. (2)–(5) respectively becomes

$$f''' - Mf' - 2f'^2 + ff'' + Wef''f''' + \lambda(\theta - Nr\phi - Rb\chi) = 0, \tag{10}$$

$$\left(1 + \frac{4}{3}K\right)\theta'' + Prf\theta' - Pr\theta f' + Pr\theta'(Nb\phi' + Nt\theta') = 0, \tag{11}$$

$$\phi'' + \left[f\phi' - \phi f' - \sigma_m \phi(1 + \delta\theta)^n exp\left(\frac{-E}{1 + \delta\theta}\right)\right]Sc + \frac{Nt}{Nb}\theta'' = 0, \tag{12}$$

$$\chi''(\xi) + LbPrf(\xi)\chi'(\xi) - LbPrf'(\xi)\chi(\xi)$$
$$- Pe(\sigma_1\phi''(\xi) + \chi(\xi)\phi''(\xi) + \chi'(\xi)\phi'(\xi)) = 0, \tag{13}$$

the constraints reduces to

$$f'(0) = 1,\ f(0) = -s,\ \phi(0) = 1,\ \theta(0) = 1,\ \chi(0) = 1,\ at\quad \eta = 0,$$

$$f'(\infty) \to 0,\ \phi(\infty) \to 0,\ \theta(\infty) \to 0,\ \chi(\infty) \to 0.\ as\ \eta \to \infty. \tag{14}$$

The associated parameters are appearing in the paper

$$M = \frac{2\sigma B_0^2 \ell}{\rho \widetilde{U}_w},\ Pr = \frac{v}{\alpha},\ We = \Gamma\sqrt{\frac{\widetilde{U}_w^3}{lv}},\ s = V_0\sqrt{\frac{2l}{a_0 v}},\ \lambda = \frac{\left(1-\widetilde{C}_\infty\right)\beta g\left(\widetilde{T}_w - \widetilde{T}_\infty\right)2\ell}{\widetilde{U}_w^2},\ Nr = \frac{\left(\rho_p - \rho_f\right)\left(\widetilde{C}_w - \widetilde{C}_\infty\right)}{\beta\left(1-\widetilde{C}_\infty\right)\rho\left(\widetilde{T}_w - \widetilde{T}_\infty\right)},$$
$$Rb = \frac{\left(\rho_m - \rho_f\right)\gamma\left(\widetilde{N}_w - \widetilde{N}_\infty\right)}{\rho\left(1-\widetilde{C}_\infty\right)\beta\left(\widetilde{T}_w - \widetilde{T}_\infty\right)},\ Nt = \frac{\tau D_T\left(\widetilde{T}_w - \widetilde{T}_\infty\right)}{v\widetilde{T}_\infty},\ Nb = \frac{\tau D_B\left(\widetilde{C}_w - \widetilde{C}_\infty\right)}{v},\ \sigma_m = \frac{2K_r^2\ell}{\widetilde{U}_w},\ E = \frac{Ea}{k\widetilde{T}_\infty},\ K = \frac{4\sigma^* \widetilde{T}_\infty^3}{k^* K},$$
$$\sigma_1 = \frac{\widetilde{N}_\infty}{\widetilde{N}_w - \widetilde{N}_\infty},\ \delta = \frac{\left(\widetilde{T}_w - \widetilde{T}_\infty\right)}{\widetilde{T}_\infty},\quad Sc = \frac{v}{D},\ Lb = \frac{\alpha}{D_n},\ Pe = \frac{dW_c}{D_n}.$$

Where M is magnetic field parameter, Pr is Prandtl number, We is the dimensionless Williamson fluid parameter, $(s < 0)$ is the suction and $(s > 0)$ is the injection parameter, λ is mixed convection parameter, Nr is the buoyancy ratio number, Rb is bioconvection Rayleigh number, Nt is the thermopherosis diffusion factor, Nb is the Brownian factor, σ_m denotes dimensionless reaction rate, E means non-dimensional energy activation, K is radiation parameter, σ_1 is bioconvective difference parameter, δ is used as temperature distinction parameter, Sc is Schmidt number, Lb Bio-convection Lewis number and Pe is peclet number.

The wall shear stress, thermal flux, and mass flux respectively given The local skin friction C_{f_x}, Nusselt number Nu_x, sherwood number Sh_x and motile density number Nn_x given below

$$C_{f_x} = \frac{\tau_w}{\rho \widetilde{U}_w^2},\ Nu_x = \frac{xq_w}{k\left(\widetilde{T}_w - \widetilde{T}_\infty\right)},$$
$$Sh_x = \frac{xq_m}{D_B\left(\widetilde{C}_w - \widetilde{C}_\infty\right)},\ Nn_x = \frac{xq_n}{D_N\left(\widetilde{N}_w - \widetilde{N}_\infty\right)}, \tag{15}$$

in the above equation τ_w represents the shear stress along stretching surface, q_w denotes the thermal flux, q_m indicates the mass flux and q_n stands for motile microorganisms flux which are stated as

$$\tau_w = \mu\left(\frac{\partial\tilde{u}}{\partial y} + \frac{\Gamma}{\sqrt{2}}\left(\frac{\partial\tilde{u}}{\partial y}\right)^2\right)_{y=0},\ q_w = -\left(K + \frac{4\sigma^* \widetilde{T}_\infty^3}{k^*}\left(\frac{\partial\widetilde{T}}{\partial y}\right)\right)_{y=0},$$

$$q_m = -D_B\left(\frac{\partial\widetilde{C}}{\partial y}\right)_{y=0}\quad and\quad q_n = -D_N\left(\frac{\partial\widetilde{N}}{\partial y}\right)_{y=0}. \tag{16}$$

Skin friction coefficient in non-dimensional form is

$$\sqrt{Re_x}C_{f_x} = \left(f''(0) + \frac{\lambda}{2}f''(0)^2\right), \tag{17}$$

by using Eqs. (7),(9) and (14) we get the following

$$\frac{Nu_x}{\sqrt{Re_x}} = -\left(1 + \frac{4}{3}K\right)\theta'(0),\ \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0),\ \frac{Nn_x}{\sqrt{Re_x}} = -\chi'(0). \tag{18}$$

Here $R_{e_x} = \frac{x\widetilde{U}_w(x)}{v}$

3. Results and discussion

Physical meanings of the final non-dimensional formulation of time independent MHD flow of Williamson nanofluid due to stretch of an exponentially sheet in the presence of bioconvection along the boundary constraints is solved numerically. Table 1 represents the comparative output for skin friction factor $\left(f''(0) + \frac{\lambda}{2}f''(0)^2\right)$. Table 2 contains results for -$\theta'(0)$ (Nusselt number). Comparison of the results indicates acceptable agreement to validate this numeric procedure. In Fig.2 the velocity of the flow seems to be reduced significantly when magnetic parameter M $(0.0 \leqslant M \leqslant 2.5)$ is increased. The increasing value of magnetic parameter M caused to produce Lorentz force. The improvement of mixed convection parameter λ causes to boost the flow velocity $f'(\eta)$ as shown in Fig.3. The mixed convection provides an stronger buoyancy effect to the flow with the temperature difference and density variation. This phenomenon enhanced the fluid flow. A notable rise in the temperature field $\theta(\eta)$ due to progressive value of radiation parameter K is delineated in Fig.4. The greater K means strong radiation mod of heat transfer which helps to rise the temperature. From Fig. 5 and 6, significant rising behavior of $\theta(\eta)$ are noticed with enhanced value of Brownian motion parameter Nb and thermophorosis parameter Nt. The fast random motion of nano-particles characterized by larger Nb is responsible for enhanced heat transfer to raise $\theta(\eta)$. Similarly, the higher Nt means greater thermophoretic effect which move the nano-particles hoter regime to the colder one and

Table 1 Comparison of $\sqrt{Re_x}Cf$ with changed values of λ, s and M.

λ	s	M	Ahmed et al. [41]	Our Results $\left(f''(0)+\frac{\lambda}{2}f''(0)^2\right)$
0.1	0.2	2.0	1.754213	1.7542131
0.2			1.678675	1.6786760
0.3			1.579827	1.5798270
	0.1		1.799249	1.7992499
	0.2		1.754213	1.7542131
	0.3		1.710489	1.7104894
		0.1	1.201556	1.2015510
		0.2	1.237223	1.2372243
		0.3	1.271816	1.2718166

Table 2 Comparison of $-\theta'(0)$ with changed values of K,M and Pr.

K	M	Pr	Ishak et al. [15]	Goud et al. [18]	Bidin et al. [42]	Our Results
0.0	0	1.0	0.9548	0.954784	0.9547	0.9548106
		2.0	1.4715	1.471462	1.4714	1.4714540
		3.0	1.8691	1.869073	1.8691	1.8690688
		5.0	2.5001	2.500111		2.5001280
		10.0	3.6604	3.660346		3.6603693
	1.0	1.0	0.8611	0.861097		0.8615086
1.0	0.0		0.5312	0.53117	0	0.5313112
	1.0		0.4505	0.450687	0	0.4506955

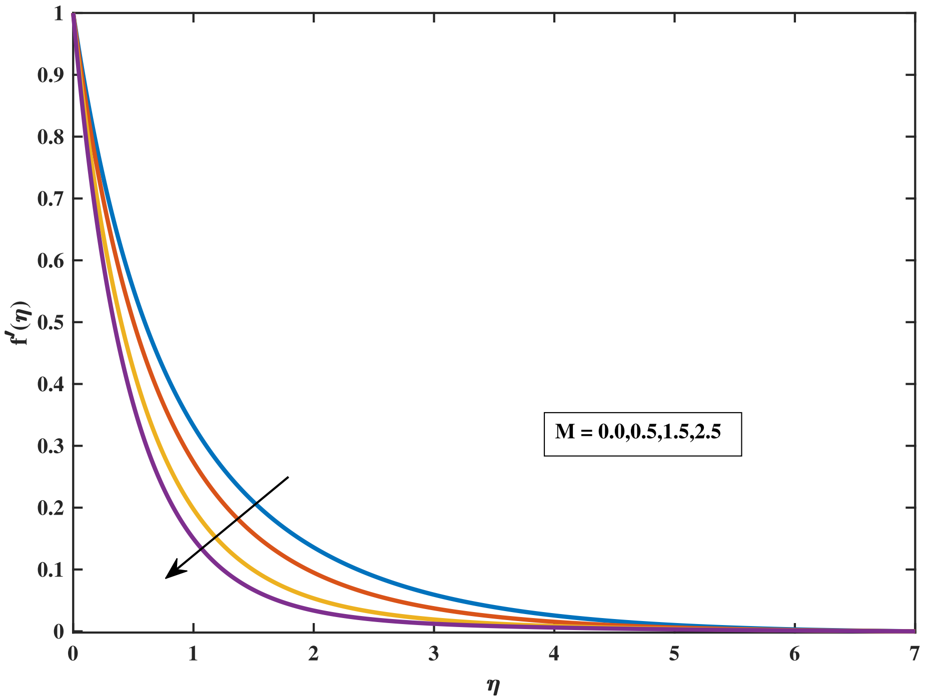

Fig. 2 Influence of M on $f\prime$.

increase the thermal distribution. Fig.7 display the decrement in $\phi(\eta)$ due to larger value of Sc (Schmidt number), the larger Schmidt number mean less mass diffusivity to decrease $\phi(\eta)$. However buoyancy ratio parameter provides strength to $\phi(\eta)$ as depicted in Fig.8. The larger value of σ_m causes the decrement in $\phi(\eta)$ as shown in Fig.9. Similarly larger value

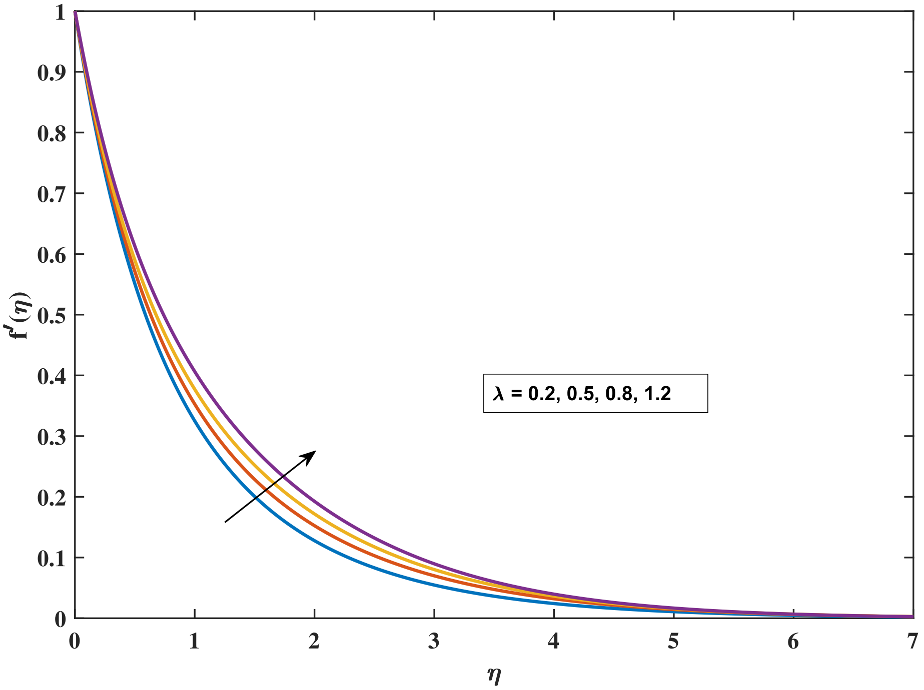

Fig. 3 Influence of λ on $f\prime$.

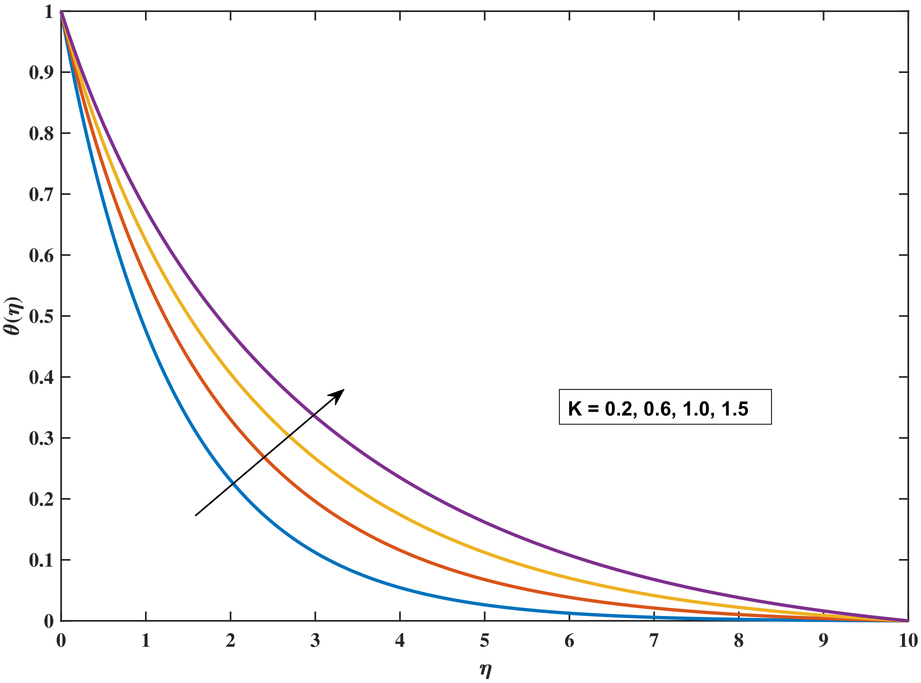

Fig. 4 Influence of K on θ.

of E provides strength to $\phi(\eta)$ as depicted in Fig.10. The bioconvection Rayleigh number Rb is responsible to given direct increment to $\chi(\eta)$ as demonstrated in Fig.11. The density of the motile microorganisms $\chi(\eta)$ increases as Rb increases. From Fig.12 the significant reduction of microorganisms distribution function $\chi(\eta)$ is attained against improved inputs of bioconvection Lewis number Lb which is reciprocal to mass diffusivity of microorganisms. The larger values of Picklet number Pe causes to decline microorganism's distribution as seems from Fig.13. Table 3–5.

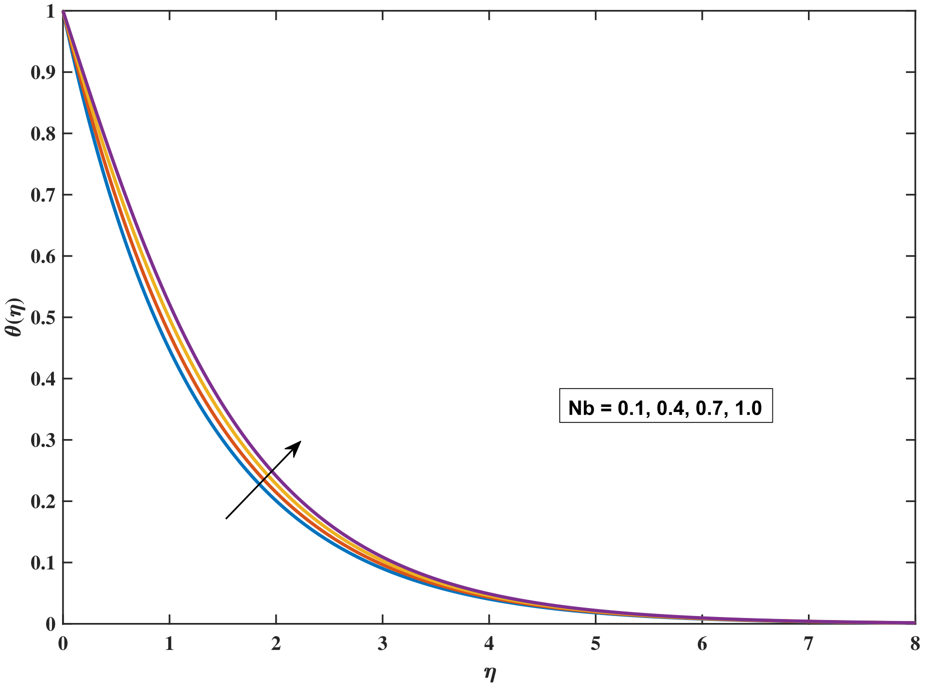

Fig. 5 Influence of Nb on θ.

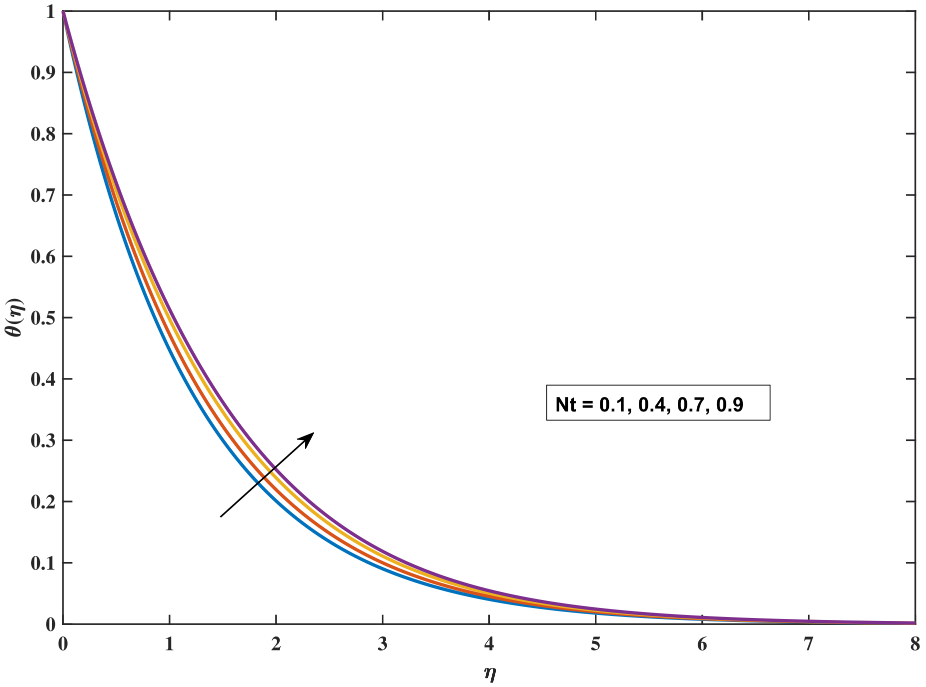

Fig. 6 Influence of Nt on θ.

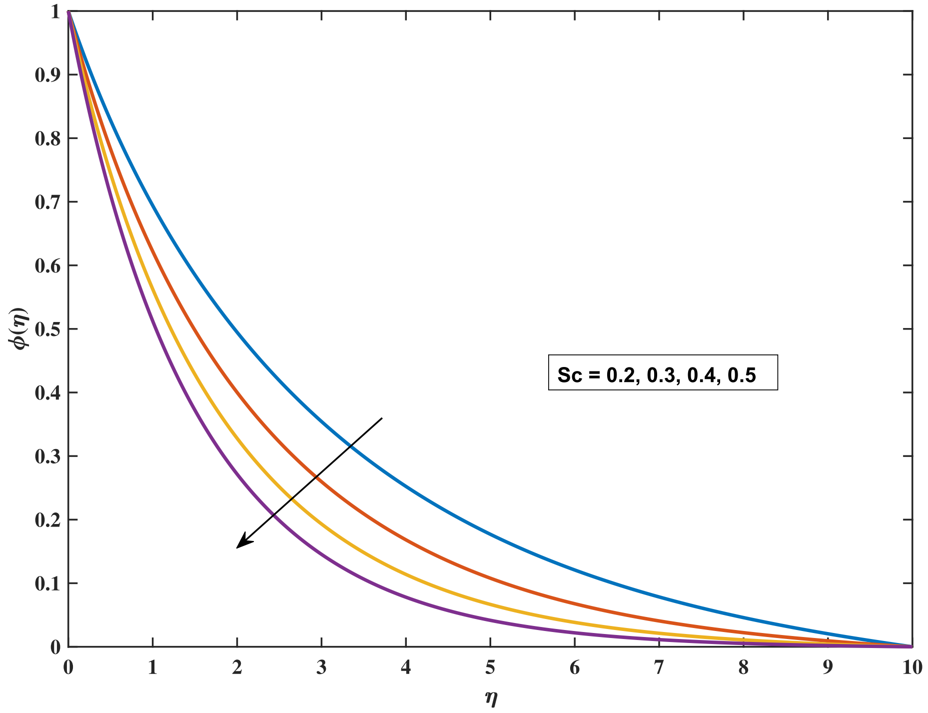

Fig. 7 Influence of Sc on ϕ.

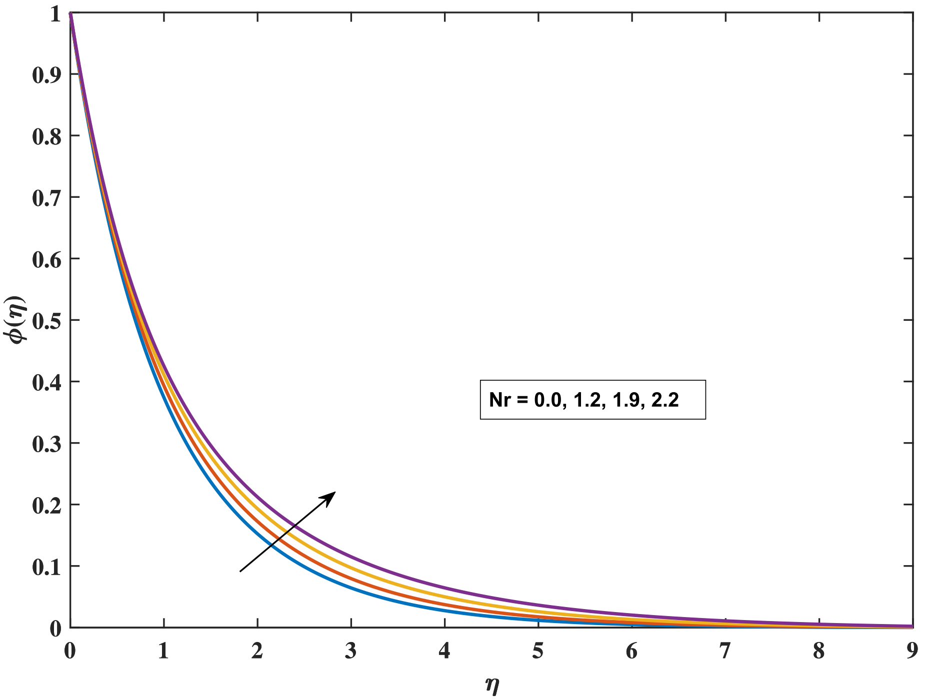

Fig. 8 Influence of Nr on ϕ.

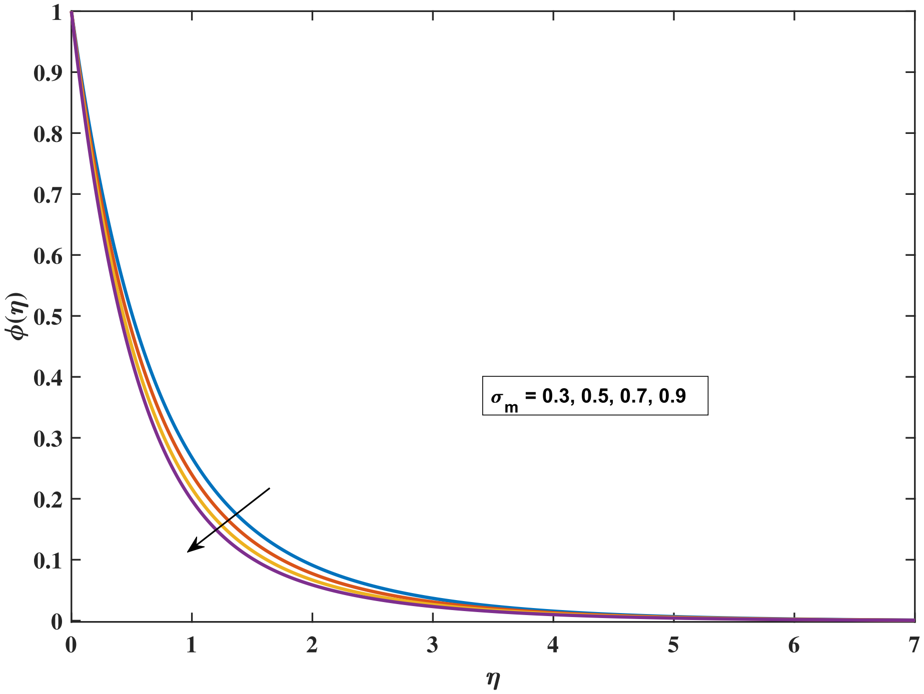

Fig. 9 Influence of σ_m on ϕ.

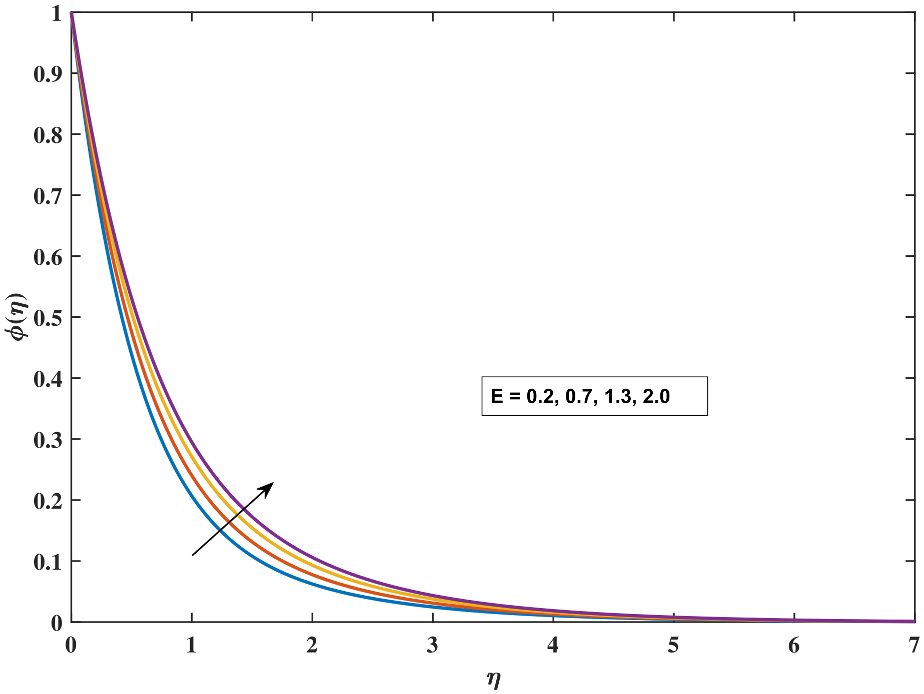

Fig. 10 Influence of E on ϕ.

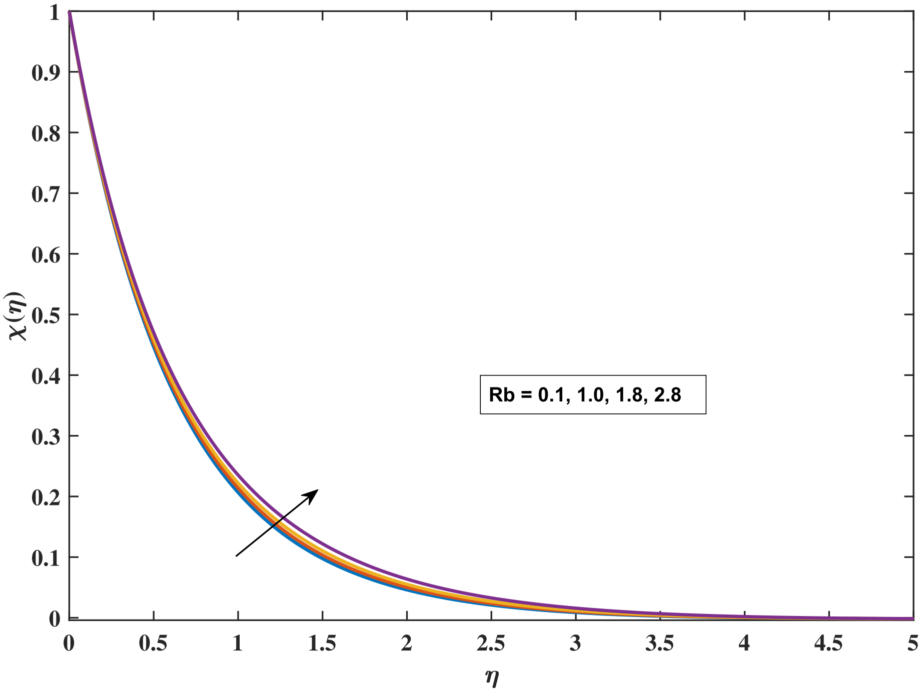

Fig. 11 Influence of Rb on χ.

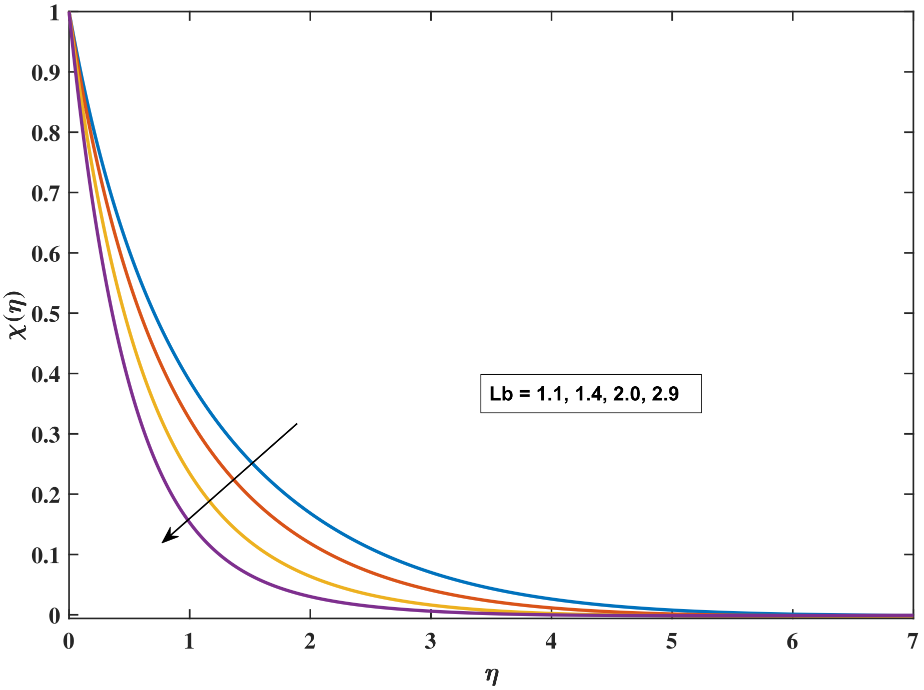

Fig. 12 Influence of Lb on χ.

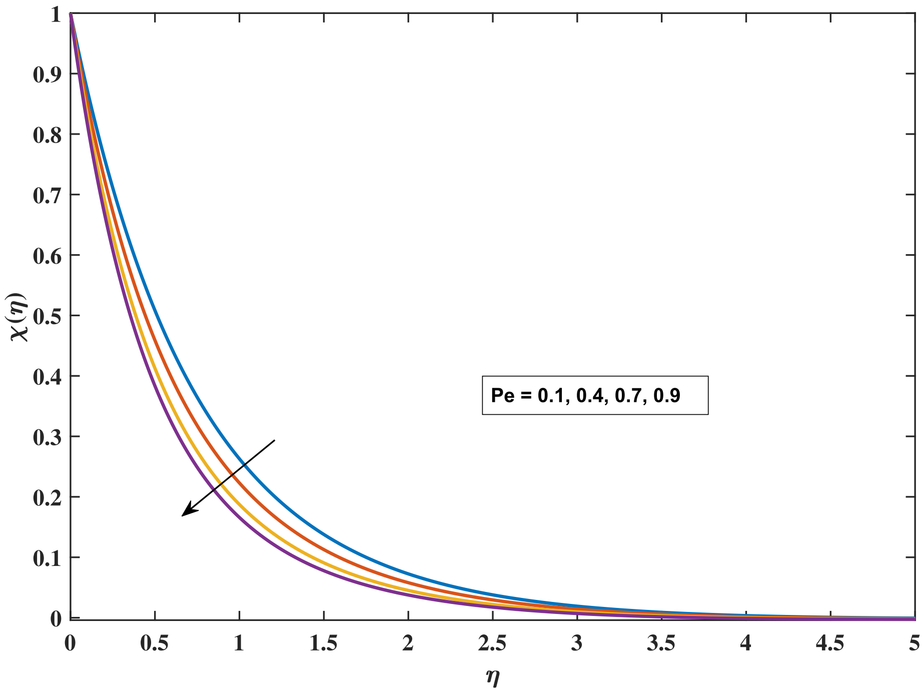

Fig. 13 Influence of pe on χ.

Table 3 Effects of different parameters on Nusselt number.

Pr	K	Nt	Nb	$Nu_x = -\left(1+\frac{4}{3}K\right)\theta'(0).$
1.3	0.2	0.1	0.1	0.9455
1.4				0.9849
1.5				1.0224
	0.2			0.9455
	0.6			1.1063
	1.0			1.2368
		0.1		0.9455
		0.4		0.8729
		0.7		0.8115
			0.1	0.9455
			0.4	0.8268
			0.7	0.7226

Table 4 Effects of different parameters on Sherwood number.

σ_m	E	Nb	Nt	$Sh_x = -\phi'(0).$
0.3	0.2	0.1	0.1	1.0906
0.5				1.2341
0.7				1.3607
	0.2			1.0906
	0.7			0.9991
	1.3			0.9254
		0.1		1.0906
		0.4		1.3399
		0.7		1.3750
			0.1	1.0906
			0.2	0.8038
			0.3	0.5354

Table 5 Effects of different parameters on profile of motile microorganism.

Pe	Lb	σ_1	$Nn_x = -\chi'(0)$.
0.1	1.1	0.1	0.8122
0.4			1.0677
0.7			1.3322
	1.1		0.8122
	1.4		0.9800
	2.0		1.2506
		0.1	0.8122
		0.3	0.8259
		0.5	0.8396

4. Conclusions

Theoretical and numeric analysis for MHD Williamson fluid flow owing to sudden stretched in an exponentially sheet has been made in this communication. Effects of the emerging parameters are enumerated on physical field namely velocity, temperature and microorganisms distribution. Significant outcomes are summarized as:

- The velocity profile reduces with magnetic parameter M and boost with mixed convection parameter λ.
- Temperature profile increased for radiation parameter K, Brownian motion parameters Nb and thermophoresis Nt.
- Concentration recede with Schmidt number Sc, reaction rate parameter σ_m and enhanced with buoyancy ratio number Nr and activation energy parameter E.
- The motile microorganism profile increased with bioconvection Rayleigh number Rb and recced with bioconvection Lewis number Lb and Picklet number Pe.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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