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Initial value problems spreadsheet solver using VBA for engineering education

Çiğdem Dinçkal[a*]

[a]*Çankaya University, Ankara, Turkey*
[*]*Corresponding author E-mail: cdinckal@cankaya.edu.tr*

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Abstract

Spreadsheet solver using VBA programming has been designed for solving initial value problems (IVPs), analytically and numerically by all Runge-Kutta (RK) methods including also fifth order with calculation of true percent relative error for corresponding RK method. This solver is user-friendly especially for beginner users of Excel and VBA.

1. Introduction

IVPs arise in any field of science and engineering education such as mechanics, geotechnics, dynamics, chemical kinetics, optimization and stability, et cetera. There are computing approaches; exact solution method and numerical methods for solving these IVPs. Numerical methods are both applicable and practical in solving IVPs in many engineering problems because of the existence of complicated problems in engineering and limitations of exact solution method [1, 2]. Numerical methods yield approximate the solutions of the IVPs, particularly for the nonlinear IVPs.

This study mainly has focussed on numerical solutions followed by Euler and various Runge-Kutta methods for solving single IVPs. These methods progress the solution over step starting from some given initial condition at the initial starting point. To simplify the steps in solving IVPs by RK methods, a tool is used. This tool is a prevalent spreadsheet application, fundamentally called as Excel, also commonly used by professionals for diverse applications in business [3], engineering and science [4]-[6].

Numerical methods in science and engineering may also be implemented in by use of Excel and also VBA. Use of VBA in explicit form Visual Basic for Applications programming capability lurks in the background behind Excel handled in the texts like Lilley and Chapra [2, 7]. In addition to this, a series of studies in literature employed spreadsheet as a calculator or solver to focus on design of solver and calculator for polynomial interpolation [8, 9], solution for systems of linear and nonlinear equations [10, 11], computation of eigenvalues [12, 13], design of spreadsheet calculator for numerical differentiation [14]-[16], spreadsheet solver for solution of partial differential equations [17], a spreadsheet solution of system of initial value problems using fourth-order RK method [18], and fourth-order RK method by spreadsheet [19]. Only the works of Tay et al. [20, 21] include design of spreadsheet calculator for solving system of IVPs using fourth-order RK method and also solving IVPs using fourth-order RK method with use of VBA programming.

In this study, a spreadsheet solver is designed to solve both IVPs by all RK methods and also exact solution method in the spreadsheet environment based on VBA programming. Microsoft Excel 2010 and Microsoft Visual Basics for Applications 7.0 are used during this study. The generation of VBA programming includes three steps. The first step is to develop an user interface input form is designed to acquire the needed information such as initial conditions of independent and dependent variables for each RK method, step size and number of steps. Then a general VBA code for any IVPs is created behind the Solve button in user interface input form. The third step is to generate function files depending on the related IVP and its analytical solution. Once the SOLVE button in user interface input form is clicked, the complete numerical and analytical solutions of the IVP and corresponding true percent relative error will be computed automatically for each order of RK method.

Email addresses: **cdinckal@cankaya.edu.tr (Ç. Dinçkal)**

Examples are presented from various fields of engineering to demonstrate the merits of this unconventional solver design which shields the tedious algorithmic implementation details from the user (such as students and educators) and greatly simplifies solving an IVP using RKSOLVER.

This spreadsheet solver is user-friendly such that users only require to enter initial conditions of independent and dependent variables for each RK method, step size and number of steps at the first step to compute the complete solution of the IVPs automatically without typing any commands in the spreadsheet cells. Here, complete solution of the IVPs means solutions from each order of RK method, exact solutions and also true percent relative errors in terms of comparison with each RK method and exact solutions. So users as educators have an oppurtunity to elucidate students the differences and similarities that exist between each order of RK method and also exact solutions at the same time and be able to comment on the solution of any engineering problem including IVPs correctly. There is no need to know the various derivations of RK methods and memorize the complicated formulations of RK methods. The solver is general and standard for any engineering problem. The main aim of this paper is to design a tool in other words spreadsheet solver which employes both numerical methods: RK methods with fifth order and also analytical methods giving exact solutions with automatically calculated true percent relative errors in solving IVPs at the same time. Therefore this solver is called as IVP spreadsheet solver.

2. Runge Kutta (RK) methods

This section is devoted to solving IVPs of the form given below:

$$\frac{dy}{dx} = f(x,y) \tag{2.1}$$

with the initial value $y(x_0) = y_0$ for the number of points n within the interval $x_0 \leq x \leq x_n$. Here x is the independent variable, y is the dependent variable, f is the function of derivation (in other words slope) and h is the fixed step size. n, the number of steps can be found as $(x_n - x_0)/h$ [1].

1) First-Order RK Method

Euler's Method:

$$y_{i+1} = y_i + hk_1 \tag{2.2}$$

where $k_1 = f(x,y)$

2) Second-Order RK Methods

a) Heun's Method:

$$y_{i+1} = y_i + h(\frac{k_1 + k_2}{2}) \tag{2.3}$$

where $k_2 = f(x_i + h, y_i + hk_1)$

b) Midpoint (Improved Polygon) Method:

$$y_{i+1} = y_i + hk_2 \tag{2.4}$$

where $k_2 = f(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2})$

c) Ralston's Method:

$$y_{i+1} = y_i + (\frac{k_1 + 2k_2}{3})h \tag{2.5}$$

where $k_2 = f(x_i + \frac{3h}{4}, y_i + \frac{3hk_1}{4})$

3) Third-Order RK Method

$$y_{i+1} = y_i + (\frac{k_1 + 4k_2 + k_3}{6})h \tag{2.6}$$

where $k_2 = f(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2}), \qquad k_3 = f(x_i + h, y_i - k_1 h + 2k_2 h)$

4) Fourth-Order RK Method

$$y_{i+1} = y_i + (\frac{k_1 + 2k_2 + 2k_3 + k_4}{6})h \tag{2.7}$$

Function f(x, y0, h)
f = y0 / 0.2254
End Function

Table 1: Function module for stress-strain relationship IVP

Function fexact(x, y0, h, i)
fexact = Exp((h * i) / 0.2254)
End Function

Table 2: Function module for exact solution of stress-strain relationship

where $k_2 = f(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2})$, $\quad k_3 = f(x_i + \frac{h}{2}, y_i + \frac{k_2 h}{2})$, $\quad k_4 = f(x_i + h, y_i + k_3 h)$

5) Fifth-Order RK Method

$$y_{i+1} = y_i + (\frac{7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6}{90})h \tag{2.8}$$

where $k_2 = f(x_i + \frac{h}{4}, y_i + \frac{k_1 h}{4})$, $\quad k_3 = f(x_i + \frac{h}{4}, y_i + \frac{k_1 h}{8} + \frac{k_2 h}{8})$, $\quad k_4 = f(x_i + \frac{h}{2}, y_i - \frac{k_2 h}{2} + k_3 h)$, $\quad k_5 = f(x_i + \frac{3h}{4}, y_i + \frac{3k_1 h}{16} + \frac{9k_4 h}{16})$,
and $k_6 = f(x_i + h, y_i - \frac{3k_1 h}{7} + \frac{2k_2 h}{7} + \frac{12k_3 h}{7} - \frac{12k_4 h}{7} + \frac{8k_5 h}{7})$
It should be noted that k's are recurrence relationships. In other words, k_1 appears in the equation for k_2 which appears in the equation for k_3 and so on. Since each k is a functional evaluation, this recurrence makes RK methods efficient for computations [1].
In this work, fifth-order RK method yields the superior results in terms of less error than the other order of RK methods. As the order of RK method increases, convergence to the exact results also increases in terms of less errors.

3. Numerical examples

Numerical examples are presented from various engineering applications.
1) Geotechnical Engineering
To mIVPl the the behavior of soil under the effect of load, it is required to formulate the stress and strain relationship and this is achieved by the following IVP:

$$\frac{d\sigma}{d\varepsilon} = \frac{\sigma}{c_C} \tag{3.1}$$

The exact solution for equation (3.1) is

$$\sigma = e^{\frac{\varepsilon}{c_C}} \tag{3.2}$$

where σ is the stress, ε is the strain of soil and c_C is the compression index and it is 0.2254 for this soil type. Initial conditions are, ε_0 is 0 for independent variable and σ_0 is 1 kPa for dependent variable. Final ε is 1.2 and step size (h) is 0.1. This means that number of steps (n) is 12.
At first, for each numerical example, function modules are prepared for both IVP and exact solution of it respectively. These modules change from example to example. The functions for IVP and exact solution are illustrated in the following tables.
Here x is the independent variable, y0 is the initial dependent variable, i is the counter of steps.
Then equations (2.2) to (2.8) are applied to obtain the solutions by each order of RK method respectively. Besides exact solution of the IVP with true percent relative error for each RK method are also incorporated in the computations.
Finally IVP spreadsheet solver is applied which is discussed in the next section to obtain the complete solutions.
2) Mechanical Engineering
To determine the change in velocity in other words acceleration of a free-falling body to the forces acting on it with considering the air resistance, the following IVP is used:

$$\frac{dv}{dt} = g - \frac{c}{m}v \tag{3.3}$$

The exact solution for equation (3.3), which also gives velocity of the object, is

$$v(t) = \frac{gm}{c}(1 - e^{(-\frac{c}{m})t}) \tag{3.4}$$

where v is the velocity (dependent variable y), t is the time in seconds (indepedent variable x), g is the gravitational constant, 9.8 m/s^2, m is the mass of the object, 68.1 kg and c is the drag coefficient, 12.5 kg/s. Initial conditions are, t_0 is 0 s and v_0 is 0 m/s [1]. Final value of time is 5 s and step size (h) is 0.5. This means that number of steps (n) for computation is 10.
At first, for this example, function modules are written for both IVP and exact solution of it respectively. These functions are illustrated in Table 3 and Table 4 respectively.
Here x is the independent variable corresponding to time, y0 is the initial dependent variable corresponding to velocity.

```
Function f(x, y0, h)
f = 9.8 - ((12.5 / 68.1) * y0)
End Function
```

Table 3: Function module for exact solution yielding velocity

```
Function fexact(x, y0, h,i)
fexact = ((9.8 * 68.1) / 12.5) * (1 - Exp((-12.5 / 68.1) * (h * i)))
End Function
```

Table 4: Function module for exact solution yielding velocity

Like geotechnical engineering example, equations (2.2) to (2.8) are employed to find the solutions by each order of RK method respectively. Besides exact solution of the IVP with true percent relative error for each RK method are also inserted in the computations.
Finally IVP spreadsheet solver is used which is mentioned in the next section to obtain the complete solutions.
3) Chemical Engineering: Mixture Problem
The mixture problem related to a tank containing 1000 L of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same time. In this problem, it is required to determine the amount of salt after t minutes in this tank. For this reason, the following IVP is employed:

$$\frac{dA}{dt} = \frac{-A}{100} \tag{3.5}$$

$A(t)$ is the amount of salt after t minutes in tank, also the dependent variable is obtained by the following exact solution:

$$A(t) = 15e^{\left(\frac{-t}{100}\right)} \tag{3.6}$$

Initial conditions are, t_0 is 0 min and A_0 is 15 kg. Final value of time is 0.96 min and step size (h) is 0.02. Number of steps (n) for computation is 49.
At first, function modules are formed for both IVP and exact solution of the problem respectively. These functions are displayed in Table 5 and Table 6 respectively.
Here x is the independent variable corresponding to time, y0 is the initial dependent variable corresponding to amount of salt after t minutes in the tank.
Then, equations (2.2) to (2.8) are used to determine the solutions by writing codes for each order of RK method respectively. These codes are standard and valid for any scince and engineering problem including IVP. So there is no need to write cIVP for various problems. Besides exact solution of the IVP with true percent relative error for each RK method are also included in the computations. True percent relative error is in the following form:

$$\varepsilon_T = \left| \frac{ExactResult - ApproximateResult}{ExactResult} \right| \times 100 \tag{3.7}$$

Where Exact Result in other words true result represents the solution obtained by analytically. Approximate Result corresponds with the corresponding solution obtained by numerical methods, any order of RK methods.
Finally IVP spreadsheet solver is employed which is argued in the next section to obtain the complete solutions.

4. IVP spreadsheet solver

Using this IVP spreadsheet solver leads to a macro named RKSOLVER which solves the whole IVP at once completely.
The general procedure for obtaining complete solution of an IVP is composed of some steps. These steps are standard and applicable for any type of IVP.
The first step is to design an user interface input form (userform) called as UserForm4 to enable users to enter required data for solving an IVP completely. The standard form of UserForm4 for any problem is illustrated in Figure 4.1.
The second step is to generate a new tab name as IVP Solver with RKSOLVER macro including codes for solving IVP by both numerically (by each order of RK method) and analytically (gives exact solution). RKSOLVER also provides user to compute true percent relative error for each RK method.
Figure 4.2 illustrates the standard IVP Solver tab with RKSOLVER button. One more variation is to add a button assigned RKSOLVER macro in the spreadsheet. So user is able to run the macro simply by clicking this button. It is sufficient to start the complete solution procedure of IVPs.

```
Function f(x, y0, h)
f = -y0 / 100
End Function
```

Table 5: Function module for IVP of the problem

```
Function fexact(x, y0, h, i)
fexact = 15 * Exp(-(h * i) / 100)
End Function
```

Table 6: Function module for exact solution of the problem

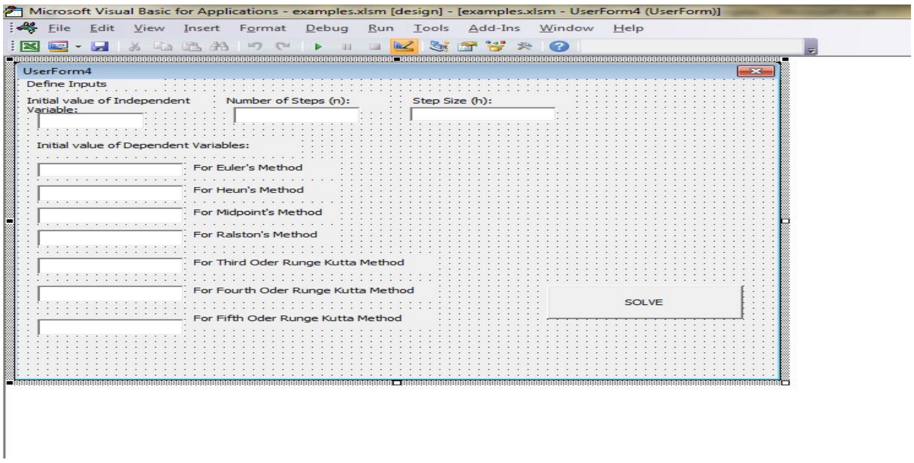


Figure 4.1: The standard userform for all examples

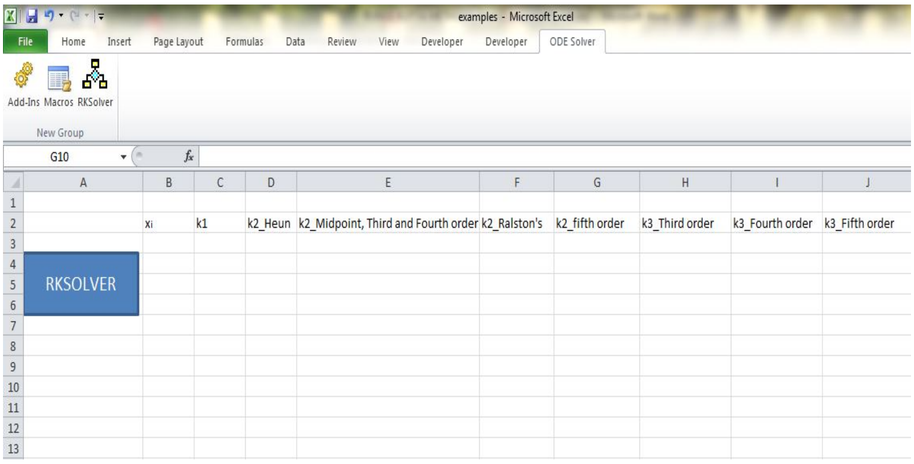


Figure 4.2: The standard IVP Solver tab with RKSOLVER button

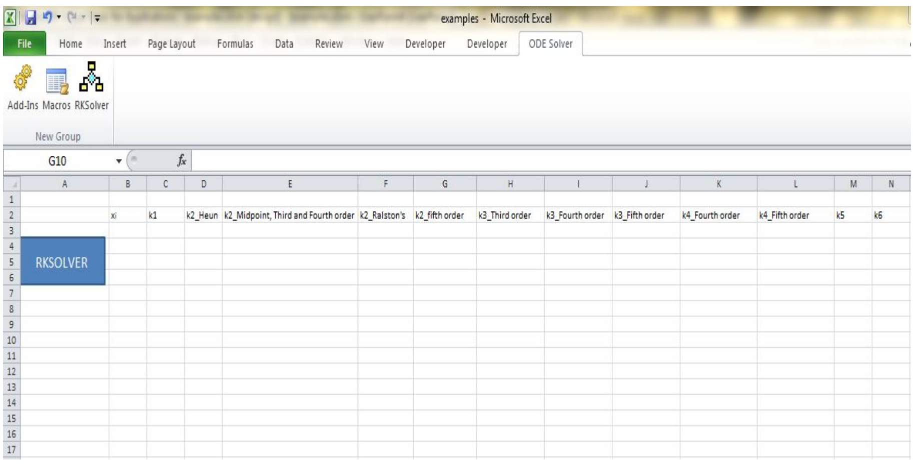


Figure 4.3: The standard blank spreadsheet image with k's (recurrence relationships) titles

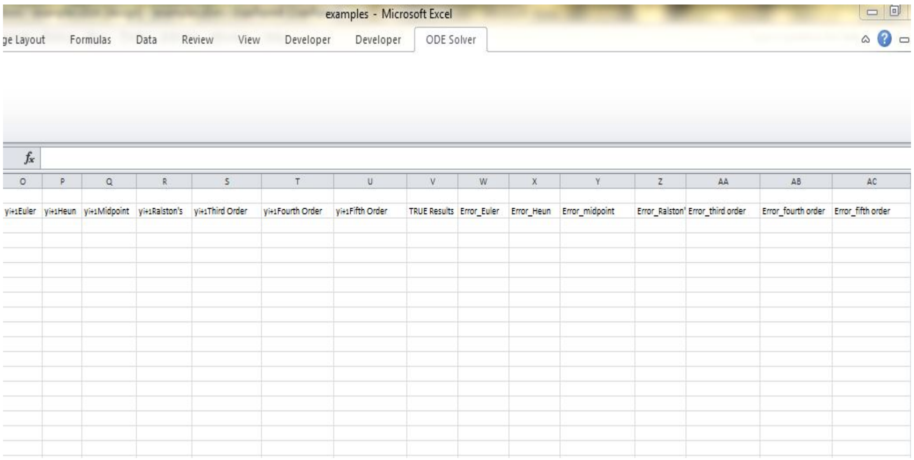


Figure 4.4: The standard blank spreadsheet image with RK results, exact results and error titles

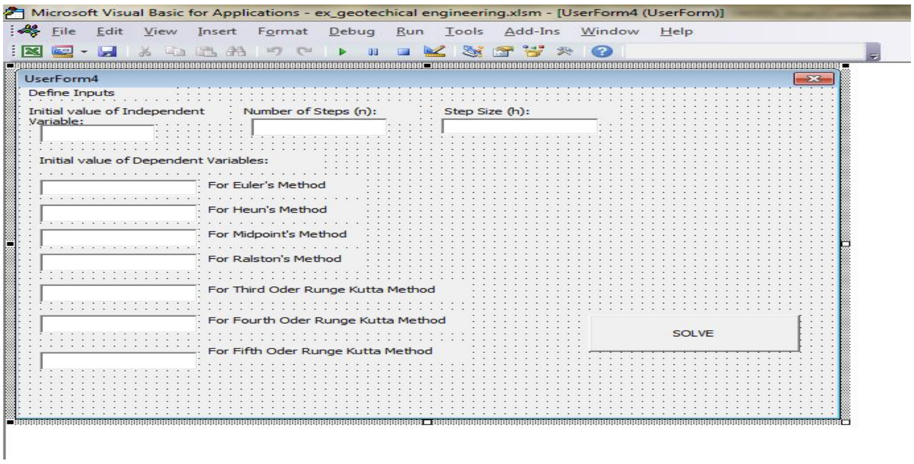


Figure 4.5: Userform for geotechnical engineering example

Then the only thing is to specify sufficient place in spreadsheet cells to make macro fill them with solutions for any IVP examples. For this reason, the titles for k's, RK results, exact results and error titles are written as is the case with Figure 4.3 and Figure 4.4 respectively.
The working procedure for IVP solver namely RKSOLVER is described for each numerical examples (geotechnical engineering, mechanical engineering and chemical engineering). The steps for geotechnical engineering example are illustrated in the Figure 4.5- Figure 4.11.
The first step is to call userform by clicking run in the toolbar or simply clicking RKSOLVER button. The image of this userform for geotechnical engineering example is given in Figure 4.5. This userform is standard for any IVP example.
Due to the fact that initial conditions are different for all IVPs, the filled userform is distinctive for all problems. As is the case with geotechnical engineering example. Userform is filled with initial conditions of the problem in Figure 4.6. Then by clicking SOLVE button in UserForm4; k's, numerical solutions obtained form all RK methods, exact solutions (true solutions) and true percent relative errors can be obtained and displayed as the spreadsheet images in Figure 4.7 to Figure 4.11 respectively.
To Figure 4.10 and Figure 4.11, fifth-order RK method gives the best solution in terms of the least error and best convergence to exact solutions.
Similarly for mechanical engineering, userform is invoked by clicking RKSOLVER in Figure 4.12. Then this form is filled with necessary data as it is shown in Figure 4.13.
By clicking the SOLVE button in userform, computations are performed and given in the spreadsheet images of Figure 4.14 to Figure 4.18.
To Figure 4.17 and Figure 4.18, the worst solution is obtained by Euler's method while fifth-order RK method is the best one with the least error and best convergence to the exact solution.
For mixture problem, userform is called by clicking RKSOLVER button in spreadsheet. Figure 4.19 illustrates this process.
Then this userform is filled by entering initial conditions as given in Figure 4.20. Clicking the SOLVE button in userform leads to complete solution of the problem. These solutions are displayed in Figure 4.21 to Figure 4.25.
To Figure 4.24 and Figure 4.25, all RK methods give quite well solutions with convergence to exact results in terms of less errors.

5. Conclusion

An IVP solver with use of RK methods including also the highest order; fifth order has been generated by VBA for the first time in literature. Emphasis was on all types of RK methods usable simultaneously and the solver generated applicable to IVPs for science and engineering problems.

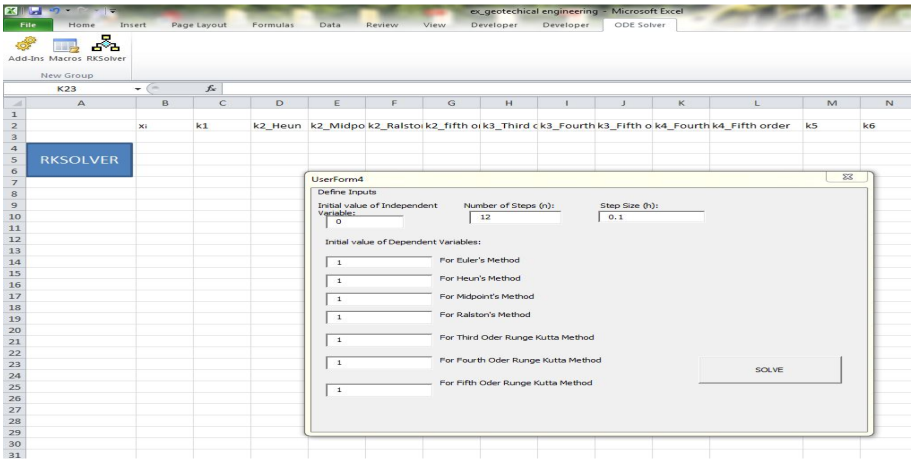
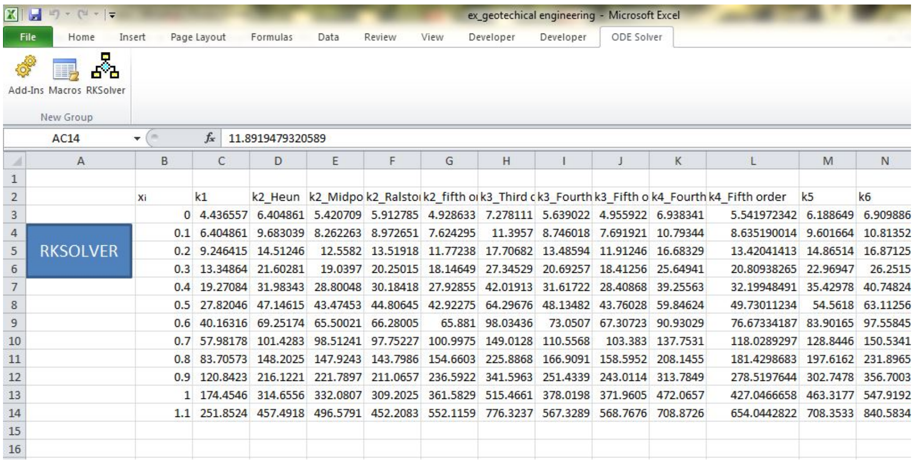
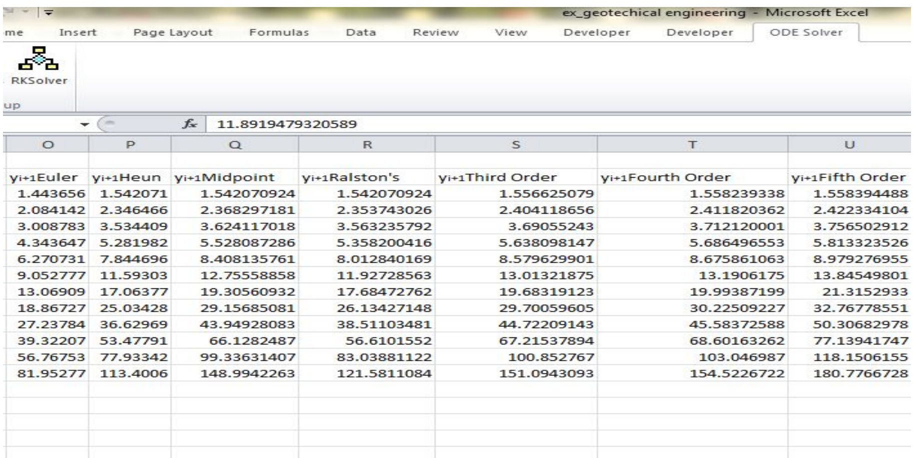


Figure 4.6: Filled userform for geotechnical engineering example



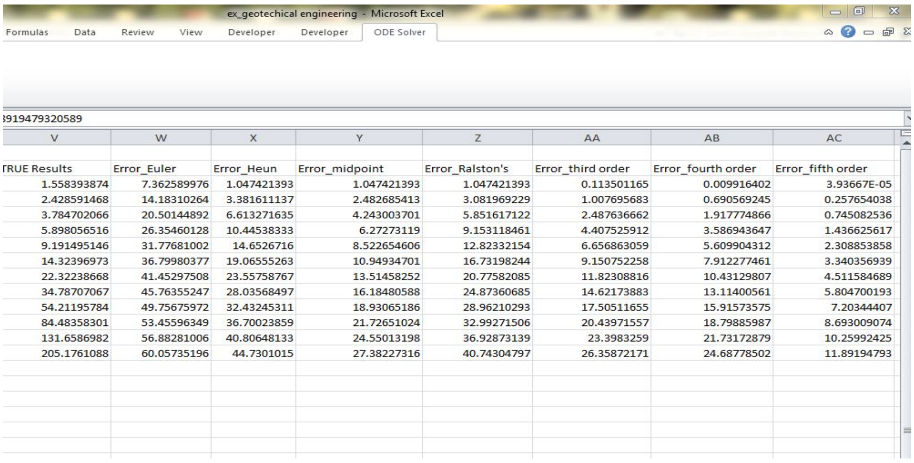
xi	k1	k2_Heun	k2_Midpo	k2_Ralstoı	k2_fifth oı	k3_Third c	k3_Fourth	k3_Fifth o	k4_Fourth	k4_Fifth order	k5	k6
0	4.436557	6.404861	5.420709	5.912785	4.928633	7.278111	5.639022	4.955922	6.938341	5.541972342	6.188649	6.909886
0.1	6.404861	9.683039	8.262263	8.972651	7.624295	11.3957	8.746018	7.691921	10.79344	8.635190014	9.601664	10.81352
0.2	9.246415	14.51246	12.5582	13.51918	11.77238	17.70682	13.48594	11.91246	16.68329	13.42041413	14.86514	16.87125
0.3	13.34864	21.60281	19.0397	20.25015	18.14649	27.34529	20.69257	18.41256	25.64941	20.80938265	22.96947	26.2515
0.4	19.27084	31.98343	28.80048	30.18418	27.92855	42.01913	31.61722	28.40868	39.25563	32.19948491	35.42978	40.74824
0.5	27.82046	47.14615	43.47453	44.80645	42.92275	64.29676	48.13482	43.76028	59.84624	49.73011234	54.5618	63.11256
0.6	40.16316	69.25174	65.50021	66.28005	65.881	98.03436	73.0507	67.30723	90.93029	76.67334187	83.90165	97.55845
0.7	57.98178	101.4283	98.51241	97.75227	100.9975	149.0128	110.5568	103.383	137.7531	118.0289297	128.8446	150.5341
0.8	83.70573	148.2025	147.9243	143.7986	154.6603	225.8868	166.9091	158.5952	208.1455	181.4298683	197.6162	231.8965
0.9	120.8423	216.1221	221.7897	211.0657	236.5922	341.5963	251.4339	243.0114	313.7849	278.5197644	302.7478	356.7003
1	174.4546	314.6556	332.0807	309.2025	361.5829	515.4661	378.0198	371.9605	472.0657	427.0466658	463.3177	547.9192
1.1	251.8524	457.4918	496.5791	452.2083	552.1159	776.3237	567.3289	568.7676	708.8726	654.0442822	708.3533	840.5834

Figure 4.7: Computation results for k's for geotechnical engineering example



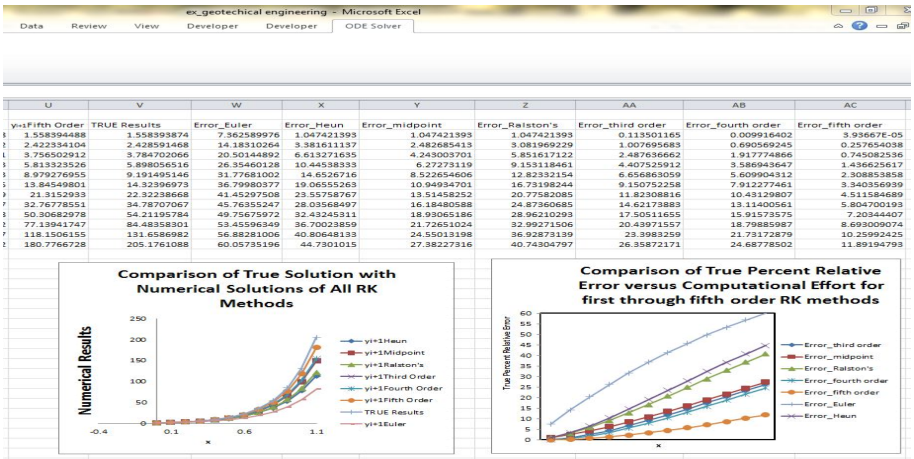
yi+1Euler	yi+1Heun	yi+1Midpoint	yi+1Ralston's	yi+1Third Order	yi+1Fourth Order	yi+1Fifth Order
1.443656	1.542071	1.542070924	1.542070924	1.556625079	1.558239338	1.558394488
2.084142	2.346466	2.368297181	2.353743026	2.404118656	2.411820362	2.422334104
3.008783	3.534409	3.624117018	3.563235792	3.69055243	3.712120001	3.756502912
4.343647	5.281982	5.528087286	5.358200416	5.638098147	5.686496553	5.813323526
6.270731	7.844696	8.408135761	8.012840169	8.579629901	8.675861063	8.979276955
9.052777	11.59303	12.75558858	11.92728563	13.01321875	13.1906175	13.84549801
13.06909	17.06377	19.30560932	17.68472762	19.68319123	19.99387199	21.3152933
18.86727	25.03428	29.15685081	26.13427148	29.70059605	30.22509227	32.76778551
27.23784	36.62969	43.94928083	38.51103481	44.72209143	45.58372588	50.30682978
39.32207	53.47791	66.1282487	56.6101552	67.21537894	68.60163262	77.13941747
56.76753	77.93342	99.33631407	83.03881122	100.852767	103.046987	118.1506155
81.95277	113.4006	148.9942263	121.5811084	151.0943093	154.5226722	180.7766728

Figure 4.8: Computation results for each RK method for geotechnical engineering example



TRUE Results	Error_Euler	Error_Heun	Error_midpoint	Error_Ralston's	Error_third order	Error_fourth order	Error_fifth order
1.558393874	7.362589976	1.047421393	1.047421393	1.047421393	0.113501165	0.009916402	3.93667E-05
2.428591468	14.18310264	3.381611137	2.482685413	3.081969229	1.007695683	0.690569245	0.257654038
3.784702066	20.50144892	6.613271635	4.243003701	5.851617122	2.487636662	1.917774866	0.745082536
5.898056516	26.35460128	10.44538333	6.27273119	9.153118461	4.407525912	3.586943647	1.436625617
9.191495146	31.77681002	14.6526716	8.522654606	12.82332154	6.656863059	5.609904312	2.308853858
14.32396973	36.79980377	19.06555263	10.94934701	16.73198244	9.150752258	7.912277461	3.340356939
22.32238668	41.45297508	23.55758767	13.51458252	20.77582085	11.82308816	10.43129807	4.511584689
34.78707067	45.76355247	28.03568497	16.18480588	24.87360685	14.62173883	13.11400561	5.804700193
54.21195784	49.75675972	32.43245311	18.93065186	28.96210293	17.50511655	15.91573575	7.20344407
84.48358301	53.45596349	36.70023859	21.72651024	32.99271506	20.43971557	18.79885987	8.693009074
131.6586982	56.88281006	40.80648133	24.55013198	36.92873139	23.3983259	21.73172879	10.25992425
205.1761088	60.05735196	44.7301015	27.38227316	40.74304797	26.35872171	24.68778502	11.89194793

Figure 4.9: Computation results for exact results (true results) and true percent relative errors of each RK method for geotechnical engineering example



yi+1Fifth Order	TRUE Results	Error_Euler	Error_Heun	Error_midpoint	Error_Ralston's	Error_third order	Error_fourth order	Error_fifth order
1.558394488	1.558393874	7.362589976	1.047421393	1.047421393	1.047421393	0.113501165	0.009916402	3.93667E-05
2.422334104	2.428591468	14.18310264	3.381611137	2.482685413	3.081969229	1.007695683	0.690569245	0.257654038
3.756502912	3.784702066	20.50144892	6.613271635	4.243003701	5.851617122	2.487636662	1.917774866	0.745082536
5.813323526	5.898056516	26.35460128	10.44538333	6.27273119	9.153118461	4.407525912	3.586943647	1.436625617
8.979276955	9.191495146	31.77681002	14.6526716	8.522654606	12.82332154	6.656863059	5.609904312	2.308853858
13.84549801	14.32396973	36.79980377	19.06555263	10.94934701	16.73198244	9.150752258	7.912277461	3.340356939
21.3152933	22.32238668	41.45297508	23.55758767	13.51458252	20.77582085	11.82308816	10.43129807	4.511584689
32.76778551	34.78707067	45.76355247	28.03568497	16.18480588	24.87360685	14.62173883	13.11400561	5.804700193
50.30682978	54.21195784	49.75675972	32.43245311	18.93065186	28.96210293	17.50511655	15.91573575	7.20344407
77.13941747	84.48358301	53.45596349	36.70023859	21.72651024	32.99271506	20.43971557	18.79885987	8.693009074
118.1506155	131.6586982	56.88281006	40.80648133	24.55013198	36.92873139	23.3983259	21.73172879	10.25992425
180.7766728	205.1761088	60.05735196	44.7301015	27.38227316	40.74304797	26.35872171	24.68778502	11.89194793

Figure 4.10: Graphical display of the computation results for geotechnical engineering example

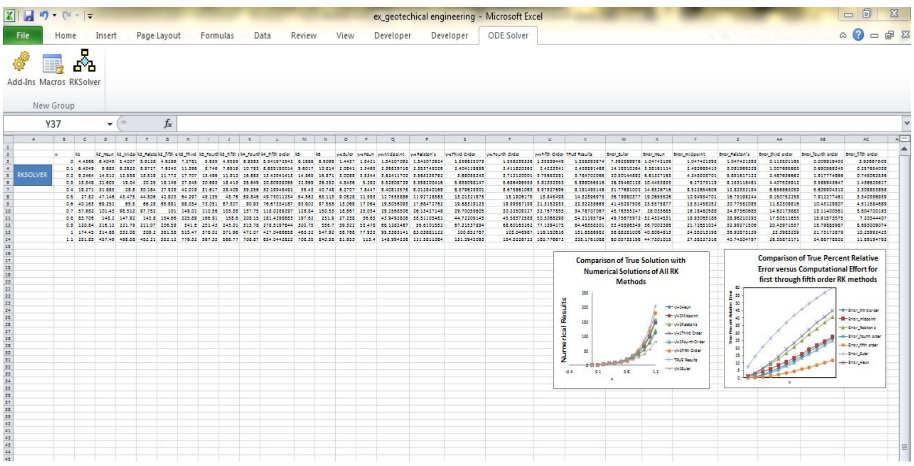


Figure 4.11: The spreadsheet image of full computation results for geotechnical engineering

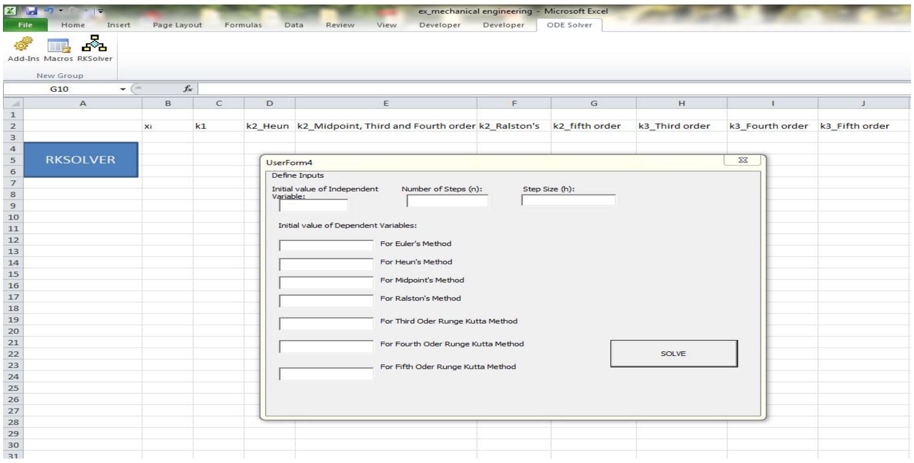


Figure 4.12: Userform in spreadsheet for mechanical engineering example

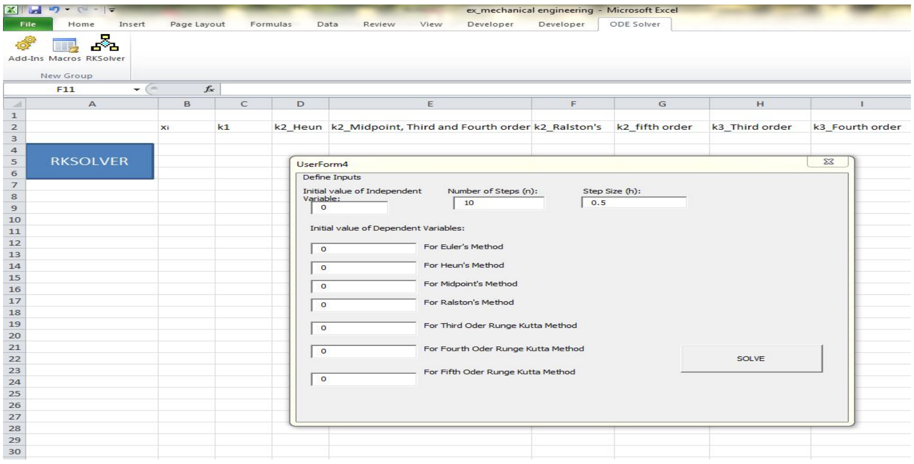
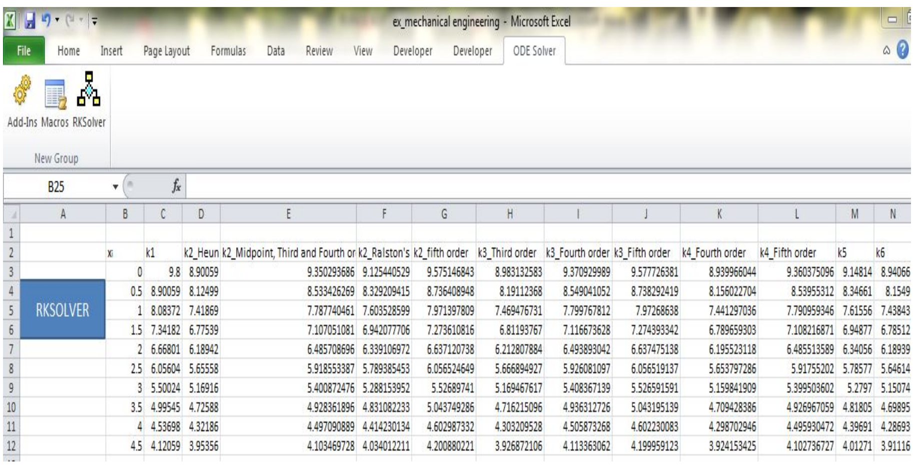
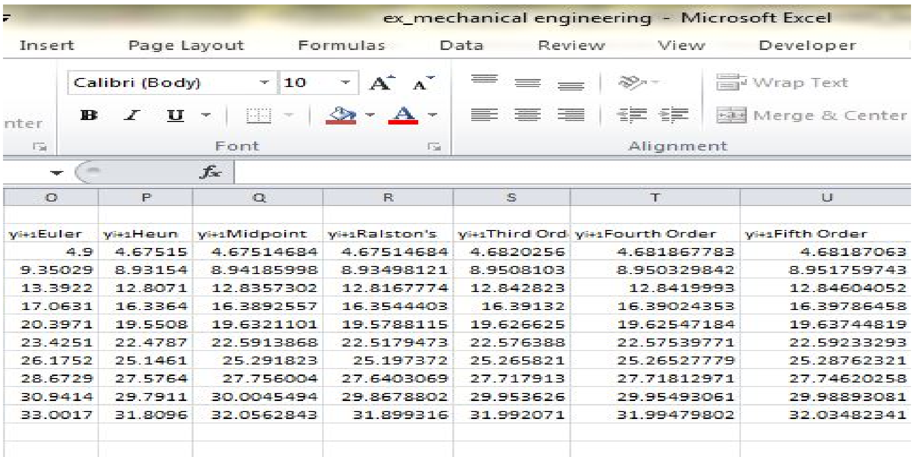


Figure 4.13: Filled userform for mechanical engineering example



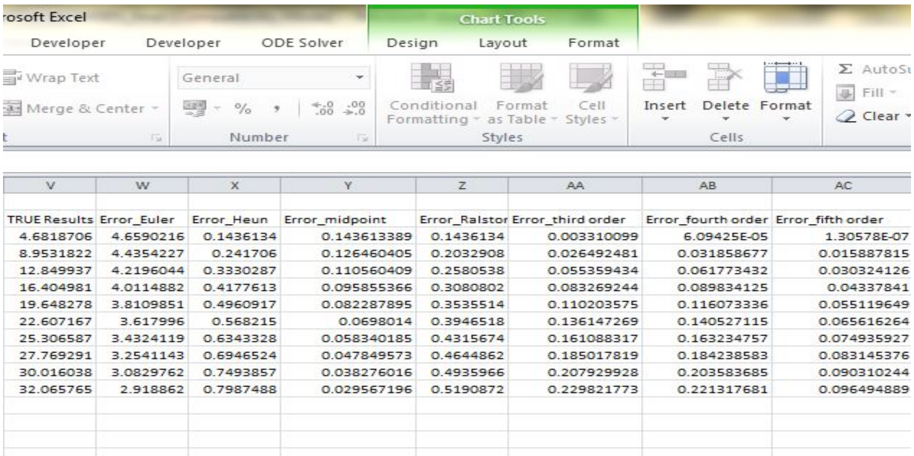
xi	k1	k2_Heun	k2_Midpoint, Third and Fourth or	k2_Ralston's	k2_fifth order	k3_Third order	k3_Fourth order	k3_Fifth order	k4_Fourth order	k4_Fifth order	k5	k6
0	9.8	8.90059	9.350293686	9.125440529	9.575146843	8.983132583	9.370929989	9.577726381	8.939966044	9.360375096	9.14814	8.94066
0.5	8.90059	8.12499	8.533426269	8.329209415	8.736408948	8.19112368	8.549041052	8.738292419	8.156022704	8.53955312	8.34661	8.1549
1	8.08372	7.41869	7.787740461	7.603528599	7.971397809	7.469476731	7.799767812	7.97268638	7.441297036	7.790959346	7.61556	7.43843
1.5	7.34182	6.77539	7.107051081	6.942077706	7.273610816	6.81193767	7.116673628	7.274393342	6.789659303	7.108216871	6.94877	6.78512
2	6.66801	6.18942	6.485708696	6.339106972	6.637120738	6.212807884	6.493893042	6.637475138	6.195523118	6.485513589	6.34056	6.18939
2.5	6.05604	5.65558	5.918553387	5.789385453	6.056524649	5.666894927	5.926081097	6.056519137	5.653797286	5.91755202	5.78577	5.64614
3	5.50024	5.16916	5.400872476	5.288153952	5.52689741	5.169467617	5.408367139	5.526591591	5.159841909	5.399503602	5.2797	5.15074
3.5	4.99545	4.72588	4.928361896	4.831082233	5.043749286	4.716215096	4.936312726	5.043195139	4.709428386	4.926967059	4.81805	4.69895
4	4.53698	4.32186	4.497090889	4.414230134	4.602987332	4.303209528	4.505873268	4.602230083	4.298702946	4.495930472	4.39691	4.28693
4.5	4.12059	3.95356	4.103469728	4.034012211	4.200880221	3.926872106	4.113363062	4.199959123	3.924153425	4.102736727	4.01271	3.91116

Figure 4.14: Computation results for k's for mechanical engineering example



y_{i+1}Euler	y_{i+1}Heun	y_{i+1}Midpoint	y_{i+1}Ralston's	y_{i+1}Third Ord	y_{i+1}Fourth Order	y_{i+1}Fifth Order
4.9	4.67515	4.67514684	4.67514684	4.6820256	4.681867783	4.68187063
9.35029	8.93154	8.94185998	8.93498121	8.9508103	8.950329842	8.951759743
13.3922	12.8071	12.8357302	12.8167774	12.842823	12.8419993	12.84604052
17.0631	16.3364	16.3892557	16.3544403	16.39132	16.39024353	16.39786458
20.3971	19.5508	19.6321101	19.5788115	19.626625	19.62547184	19.63744819
23.4251	22.4787	22.5913868	22.5179473	22.576388	22.57539771	22.59233293
26.1752	25.1461	25.291823	25.197372	25.265821	25.26527779	25.28762321
28.6729	27.5764	27.756004	27.6403069	27.717913	27.71812971	27.74620258
30.9414	29.7911	30.0045494	29.8678802	29.953626	29.95493061	29.98893081
33.0017	31.8096	32.0562843	31.899316	31.992071	31.99479802	32.03482341

Figure 4.15: Computation results for each RK method for mechanical engineering example



TRUE Results	Error_Euler	Error_Heun	Error_midpoint	Error_Ralstor	Error_third order	Error_fourth order	Error_fifth order
4.6818706	4.6590216	0.1436134	0.143613389	0.1436134	0.003310099	6.09425E-05	1.30578E-07
8.9531822	4.4354227	0.241706	0.126460405	0.2032908	0.026492481	0.031858677	0.015887815
12.849937	4.2196044	0.3330287	0.110560409	0.2580538	0.055359434	0.061773432	0.030324126
16.404981	4.0114882	0.4177613	0.095855366	0.3080802	0.083269244	0.089834125	0.04337841
19.648278	3.8109851	0.4960917	0.082287895	0.3535514	0.110203575	0.116073336	0.055119649
22.607167	3.617996	0.568215	0.0698014	0.3946518	0.136147269	0.140527115	0.065616264
25.306587	3.4324119	0.6343328	0.058340185	0.4315674	0.161088317	0.163234757	0.074935927
27.769291	3.2541143	0.6946524	0.047849573	0.4644862	0.185017819	0.184238583	0.083145376
30.016038	3.0829762	0.7493857	0.038276016	0.4935966	0.207929928	0.203583685	0.090310244
32.065765	2.918862	0.7987488	0.029567196	0.5190872	0.229821773	0.221317681	0.096494889

Figure 4.16: Computation results for exact results (true results) and true percent relative errors of each RK method for mechanical engineering example

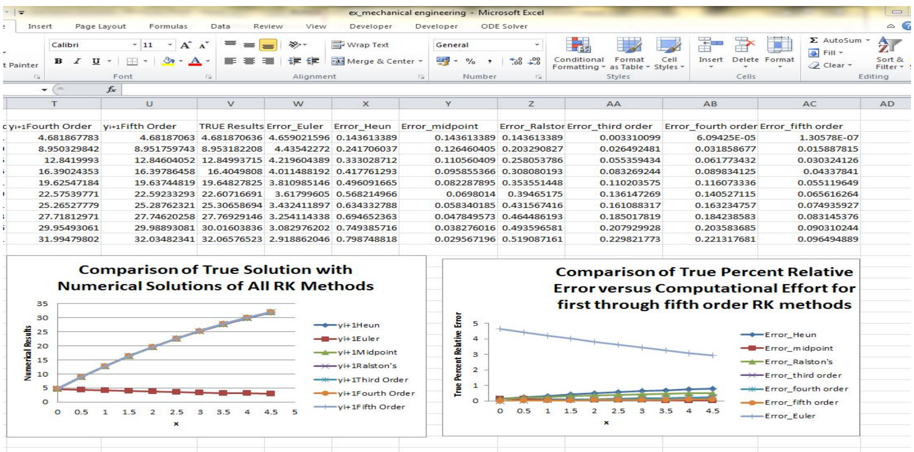


Figure 4.17: Graphical display of the computation results for mechanical engineering example

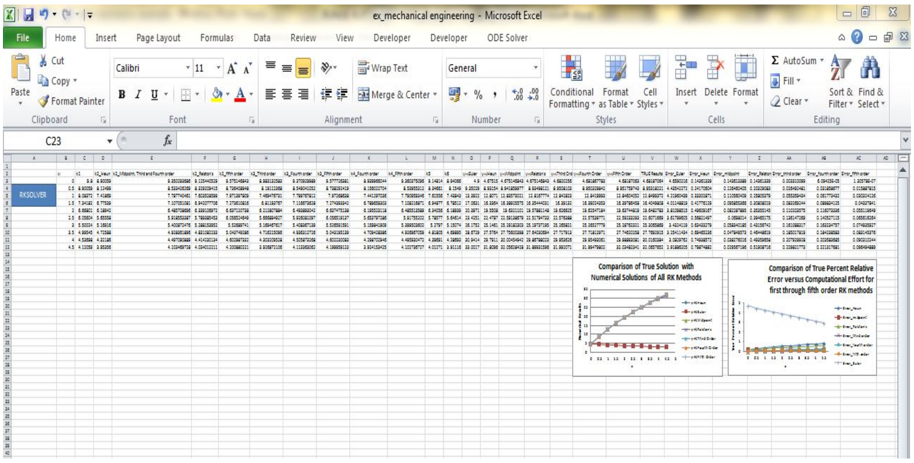


Figure 4.18: The spreadsheet image of full computation results for mechanical engineering example

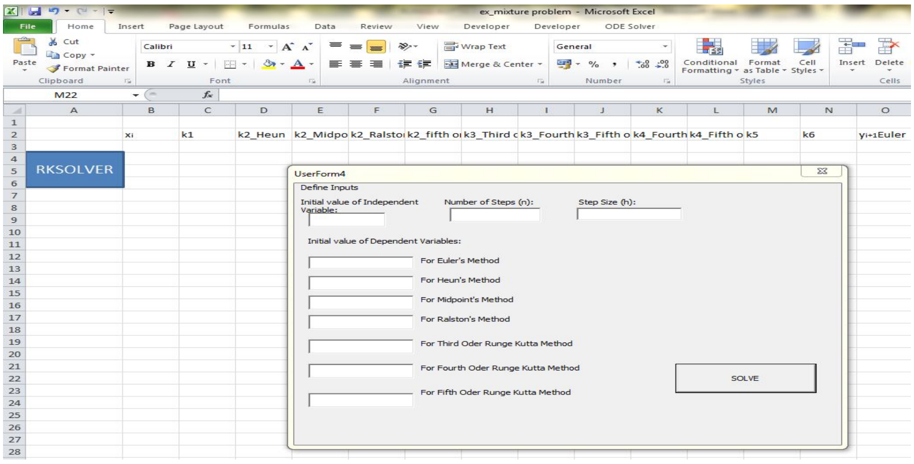


Figure 4.19: Userform in spreadsheet for mixture problem

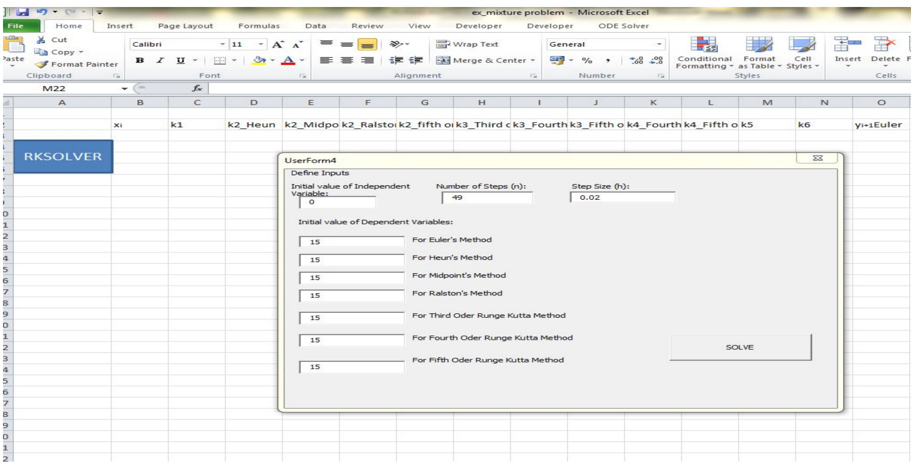


Figure 4.20: Filled userform for mixture problem

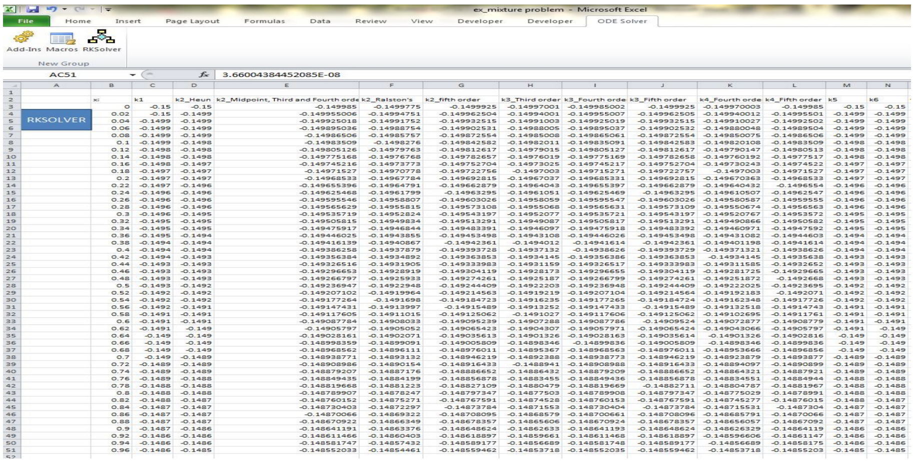


Figure 4.21: Computation results for k's for mixture problem

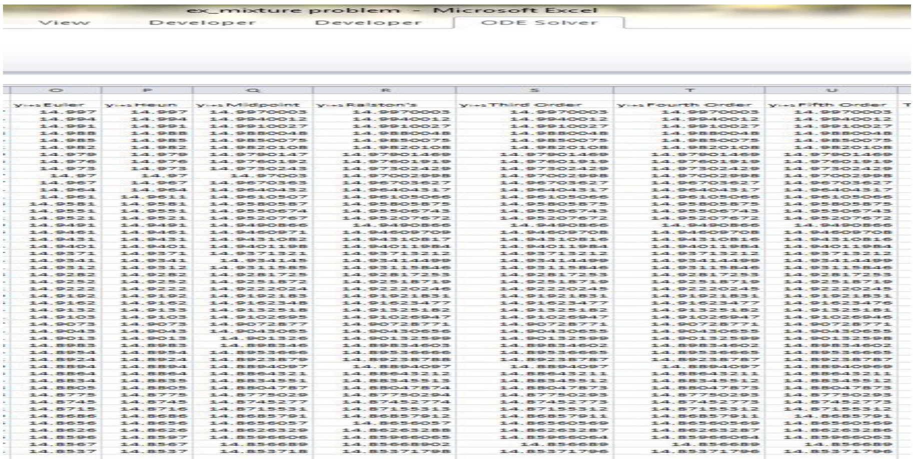


Figure 4.22: Computation results for each RK method for mixture problem

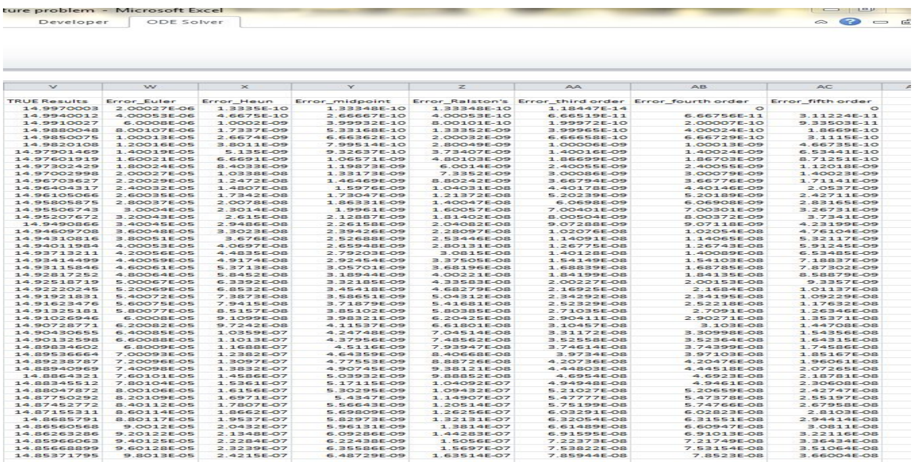


Figure 4.23: Computation results for exact results (true results) and true percent relative errors of each RK method for mixture problem

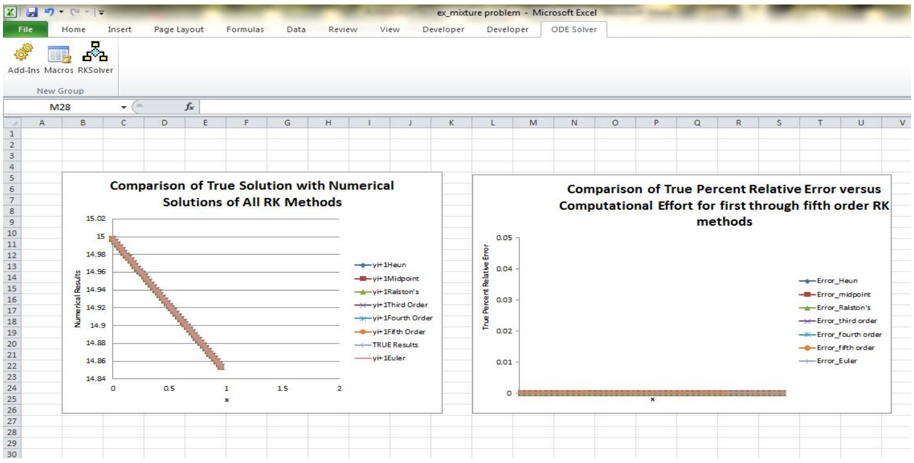


Figure 4.24: Graphical display of the computation results for mixture problem

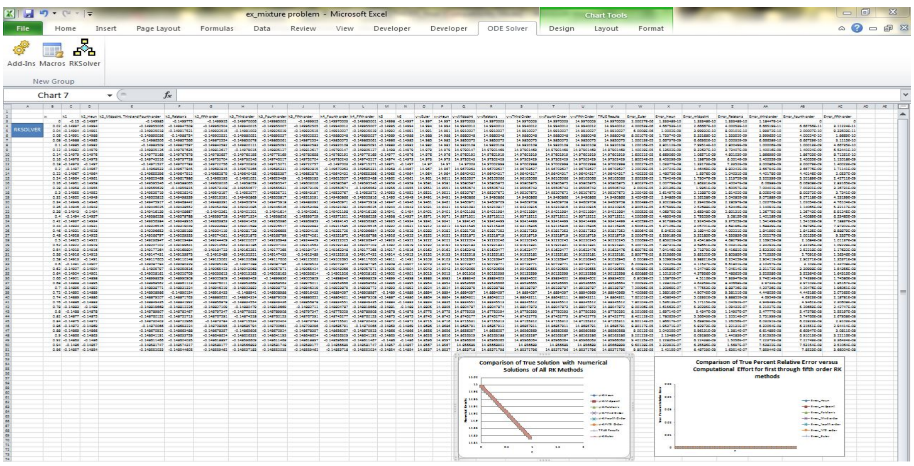


Figure 4.25: The spreadsheet image of full computation results for mixture problem

This spreadsheet solver is so user-friendly that users (students, educators and also beginner users of Excel and VBA) only require to click RKSOLVER button and enter relevant information in userform to perform all computations for the complete solution of IVPs efficiently without typing any commands in the spreadsheet.
It is hoped that this spreadsheet solver can be used as a marking scheme for users who need the complete solutions of IVPs numerically and analytically with comparison of them in terms of error at the same time. Lastly, it is hoped that this spreadsheet solver could serve as not only a numerical IVP tool but also an analytical IVP tool with a comparison of them that is convenient for the community of engineering educators and students.

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