INVESTIGATION OF ELECTROOSMOSIS FLOW OF COPPER NANOPARTICLES WITH HEAT TRANSFER DUE TO METACHRONAL RHYTHM

by

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A mathematical model is explored to establish the electroosmotic flow for Cu-water nanoliquids within a ciliated symmetric micro-channel, the flow is established with aid of ciliary motion and axial pressure gradient. Nanofluid comprise of Cu as a nanofluid particles and water as base fluid. Maxwell-Garnelt model is exploited for viscosity and thermal conductivity of nanoliquid. Magnetic field is applied in the transverse direction and external electric field is enforced in the axial direction. Equations of motion are simplified for nanofluid flow in the micro-channel by employing low Reynolds number and long wavelength approximation theory. Crucial exact analytical expression are gathered for electric potential, temperature profile, axial velocity, volume flux, pressure gradient, stream function, and result for pressure rise per wavelength explored numerically. The influence of crucial flow parameters on, flow behaviour, pumping phenomena, and temperature profile are thoroughly investigated.

Key words: *ciliated micro-channel, electric field, magnetic field, Cu-water nanofluids*

Introduction

Cilia are minute hairlike projections which exists over free surface of specific cell. They are found nearly all animal kingdom. Their motion play significant part in very crucial physiological phenomena like respiration, circulation, alimentation, reproduction, *etc.* [1]. Cilia are classified into two major branches *i.e.*, motile cilia, and non-motile cilia. Emphasis in this study is on motile cilia which beat in well co-ordinated scheme.

Ilio and Hess [2] narrated that the composition of ciliated and non-ciliated cells in various animals normally fluctuates between 1:3 to 1:8. Maiti and Pandey [3] and Haroon *et al.* [4] developed a theoretical model for power-law rheological fluid transport in an axisymmetric ciliated tube. Akbar *et al.* [5] and Bhatti *et al.* [6] revealed the ciliary hydrodynamics of rheological fluid. Nadeem and Sadaf [7] presented the ciliary flow of Newtonian nanofluid in

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a curved ciliated channel by capitalizing wall phenomenon. Theoretical study of a Newtonian fluid due cilia induce flow because of metachronal waves through the curved permeable channel has been done by Asghar *et al.* [8]. Mann *et al.* [9] gathered analytical solution for fractional Burgers fluid through inclined ciliated tube by fractional Adomian decomposition methodology. Imran *et al.* [10, 11] explored temperature variant viscosity for the MHD Newtonian fluid in the ductus efferentes of human male reproductive tract. Electroosmosis and ciliary induced metachronal wave has great significance in various biological and engineering applications, namely biomimetic pumping and in microfluidic appliances. Electro-kinetic phenomena are generated in a scenario when the ionized fluid in transported with respect to the static charged surface underneath the externally functional electric field, which is termed as electroosmosis. The electroosmosis initially investigated with the aid of experiments for peristalsis [12].

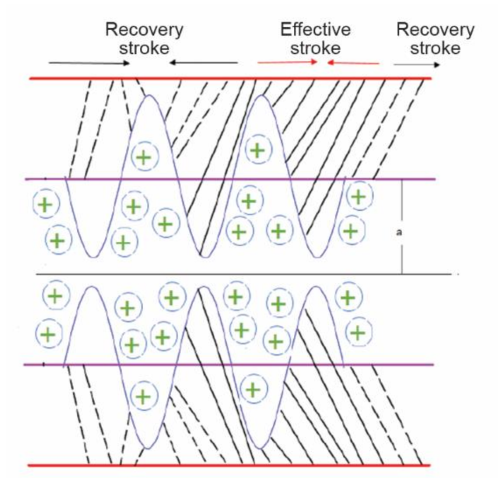

Figure 1. Geometry for Cu-water cilia induce electro osmotic MHD flow through micro-channel

Mathematical modelling of the problem

Consider the 2-D electro-osmotic of unsteady incompressible electrically conducting Cu nanofluid model in symmetric ciliated micro-channel. Geometry of physical problem is exhibited in fig. 1. It is assumed that ciliated micro-channel is filled with pure water and copper nanoparticles. Further it is considered that Cu-water nanofluid mixture along with heat transfer phenomenon is possessing thermal equilibrium state, and moved with metachronal wave velocity, c. The wall of micro-channel which is moving with metachronal rhythm is possessing temperature, T_0. The wave moves along the $\hat{X}$-direction and $\hat{Y}$ is transverse to it, and it is assumed that Cu-water nanofluid is exposed to uniform magnetic filed, B_0, which is applied transversely and electric field is considered in $\hat{X}$-direction.

Dimensionless investigation

In order to non-dimensionalize the equations of motion, one may invoke the following non-dimensional quantities for equations of motion:

$$h=\frac{H}{a},\quad t=\frac{ct'}{\lambda},\quad m_\mathrm{e}=\frac{a}{\lambda_D},\quad n=\frac{\hat{n}}{n_0},\quad x=\frac{X}{\lambda},\quad y=\frac{Y}{a},\quad \delta=\frac{a}{\lambda},\quad u=\frac{U}{c},\quad v=\frac{V}{c}$$

$$B=\frac{a^2Q}{T_0k_f},\quad p=\frac{a^2p}{c\lambda\mu_f},\quad \mathrm{Re}=\frac{\rho_f ca}{\mu_f},\quad \mathrm{Pr}=\frac{\mu_f c_f}{k_f},\quad \theta=\frac{T-T_0}{T_0},\quad \mathrm{Pe}=\frac{ca}{D} \tag{1}$$

$$\mathrm{Hr}=\frac{a^2B_0^2\sigma_e}{\mu_f},\quad Uhs=-\frac{E_x\varepsilon_0\varepsilon T_{av}K_B}{ez_v c\mu_f},\quad \phi=\hat{\phi}\frac{ez_v}{T_{av}K_B},\quad \mathrm{Gr}=\frac{a^2T_0(\rho\beta)_f}{c\mu_f}$$

where m_e is the electroosmotic parameter, λ_p – the Debye length, Uhs – the Helmholtz-Smoluchowski velocity, p – the pressure, θ – the temperature, Re – the Reynolds number, Pe – the ionic Peclet number, Pr – the Prandtl number, Gr – the Grashof number, Hr – the Hartmann number, and B – the constant heat generation. Invoking Debye Hckel linearization, the renowned Poisson Boltzmann equation is expressed:

$$\phi'' - m_e^2\phi = 0 \tag{2}$$

Further, by capitalizing the wave frame transformation and using non-dimensional parameter, and lastly low Reynolds and long wave length approximations, the governing eqs. of motion [13] takes the form:

$$\frac{dp}{dx} = a_3\frac{\partial^2 u}{\partial y^2} - \mathrm{Hr}^2 u + a_4 \mathrm{Gr}\theta + Uhs m_e^2\phi \tag{3}$$

$$a_1\frac{\partial^2\theta}{\partial y^2} + a_2 B = 0 \tag{4}$$

where

$$a_1 = \frac{\alpha_{\mathrm{nf}}}{\alpha_f}, \quad a_2 = \frac{(\rho c_p)_{\mathrm{nf}}}{(\rho c_p)_f}, \quad a_3 = \frac{\mu_{\mathrm{nf}}}{\mu_f}, \quad a_4 = \frac{(\rho\beta)_{\mathrm{nf}}}{(\rho\beta)_f}$$

using these simplifications, the x-component of momentum equation takes the form:

$$u = -1 - 2\pi \in \alpha\delta\cos 2\pi x, \quad v = 2\pi \in \sin 2\pi x + 2\pi^2 \in^2 \alpha\delta\sin 4\pi x$$

$$\phi = 1, \quad \theta = 1 \quad \text{at} \quad y = h = 1 + \in\cos 2\pi x \tag{5}$$

$$\frac{\partial u}{\partial y} = 0, \quad \phi = 0, \quad \theta = 0, \quad \text{at} \quad y = 0 \tag{6}$$

Exact analytical solution

Solving eqs. (2) and (3) w.r.t to boundary conditions (5)-(6):

$$\phi = Csch(hm_e)\sin h(m_e y) \tag{7}$$

$$\theta = \frac{y}{h} + \frac{a_2 B(h-y)y}{2a_1} \tag{8}$$

Now invoking eq. (7) in eq. (8) along with boundary conditions to get exact expression for axial component of velocity:

$$u = \frac{-p + \frac{a_4 \mathrm{Gr}y}{h}}{\mathrm{Hr}^2} + \frac{a_2 a_4 B\mathrm{Gr}[-2a_3 + \mathrm{Hr}^2(h-y)y]}{2a_1\mathrm{Hr}^4} + e^{\frac{\mathrm{Hr}y}{\sqrt{a_3}}}c_1 + e^{-\frac{\mathrm{Hr}y}{\sqrt{a_3}}}c_2 +$$

$$+\frac{m_e^2 UhsCsch(hm_e)\sin h(m_e y)}{\mathrm{Hr}^2 - a_3 m_e^2} \tag{9}$$

Mathematical expressions for the volume flux and pressure gradient may be extracted for ciliary are:

$$F=-\frac{a_2a_3a_4B\mathrm{Gr}h}{a_1\mathrm{Hr}^4}+\frac{a_4\mathrm{Gr}h}{2\mathrm{Hr}^2}+\frac{a_2a_4B\mathrm{Gr}h^3}{12a_1\mathrm{Hr}^2}-\frac{h}{\mathrm{Hr}^2}\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{\sqrt{a_3}\left(-1+\mathrm{e}^{\frac{h\mathrm{Hr}}{\sqrt{a_3}}}\right)c_1}{\mathrm{Hr}}+$$
$$+\frac{\sqrt{a_3}\left(1-\mathrm{e}^{-\frac{h\mathrm{Hr}}{\sqrt{a_3}}}\right)c_2}{\mathrm{Hr}}+\frac{m_eUhs\tan h\left(\frac{hm_\mathrm{e}}{2}\right)}{\mathrm{Hr}^2-a_3m_\mathrm{e}^2} \tag{10}$$

$$\frac{\mathrm{d}p}{\mathrm{d}x}=\frac{\mathrm{Hr}^2}{h}\left[F+\frac{a_2a_3a_4B\mathrm{Gr}h}{a_1\mathrm{Hr}^4}-\frac{a_4\mathrm{Gr}h}{2\mathrm{Hr}^2}-\frac{a_2a_4B\mathrm{Gr}h^3}{12a_1\mathrm{Hr}^2}-\frac{\sqrt{a_3}\left(-1+\mathrm{e}^{\frac{h\mathrm{Hr}}{\sqrt{a_3}}}\right)c_1}{\mathrm{Hr}}-\right.$$
$$\left.-\frac{\sqrt{a_3}\left(1-\mathrm{e}^{-\frac{h\mathrm{Hr}}{\sqrt{a_3}}}\right)c_2}{\mathrm{Hr}}-\frac{m_eUhs\tan h\left(\frac{hm_\mathrm{e}}{2}\right)}{\mathrm{Hr}^2-a_3m_e^2}\right] \tag{11}$$

Stream function may be extracted using the relation $u=\partial\psi/\partial y$ in eq. (9):

$$\psi=\frac{a_4\mathrm{Gr}y^2(3b_2-b_3y)+6a_3\mathrm{e}^{-\frac{\mathrm{Hr}y}{\sqrt{a_3}}}\left(\mathrm{e}^{\frac{2\mathrm{Hr}y}{\sqrt{a_3}}}c_3+c_4\right)}{6\mathrm{Hr}^2}+c_5+yc_6+\frac{b_1Uhs\sin h(m_\mathrm{e}y)}{\mathrm{Hr}^2-a_3m_\mathrm{e}^2} \tag{12}$$

where $c_1, c_2, \ldots c_6$ are variables terms with large expression their value are not included in the manuscript for brevity.

Results and discussions

In this section physical impact of crucial flow parameter are exhibited. In fig. 2 impact of electroosmotic parameter, m_e, and Helmholtz-Smoluchowski velocity, *Uhs*, on axial component of velocity are investigated, it is seen that axial velocity profile initially rise with the inclination of electroosmotic parameter then opposite trend is recorded in the middle of the

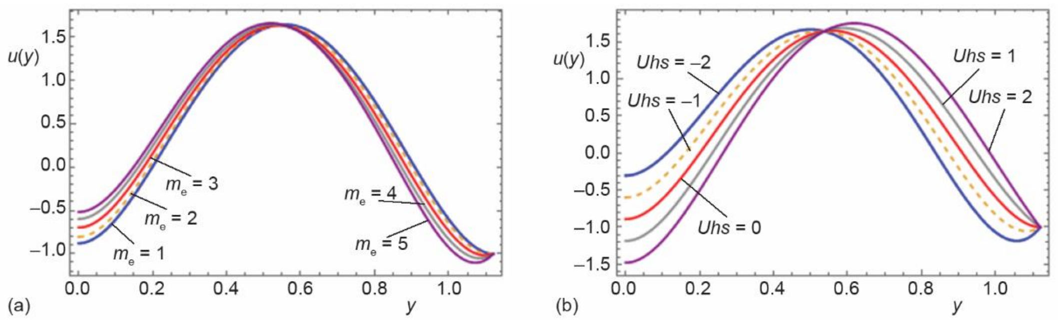

Figure 2. Analysis of electro-osmotic parameter and Helmholtz-Smoluchowski velocity on axial component of velocity with $\alpha = 0.25$, $\epsilon = 0.12$, $F = 0.5$, $\delta = 0.01$, $B = 2$, $\chi = 0.02$, *Uhs* = –1; (a) Hr = 1 and (b) $m_e = 2$

flow. Impact of the *Uhs* velocity profile are revealed in fig. 2(b). Decline in the velocity profile is seen at centre of the flow and then significant rise is recorded. It is seen form fig. 3 that stream lines surrounded by the trapped bolus are decreased and size of the bolus is inflated with enhancement in m_e. Temperature profile is investigated is in fig. 4 with variations in heat generation and volume fraction parameter of nanofluid, significant enhancement in the temperature profile is recorded with increasing B and decline is observed with χ. Pressure gradient profile is comprehensively investigated as function of electro-osmotic parameter, Helmholtz-Smoluchowski velocity in figs. 5. In pressure gradient small decline is recorded with m_e and sharp decline with *Uhs* is exhibited.

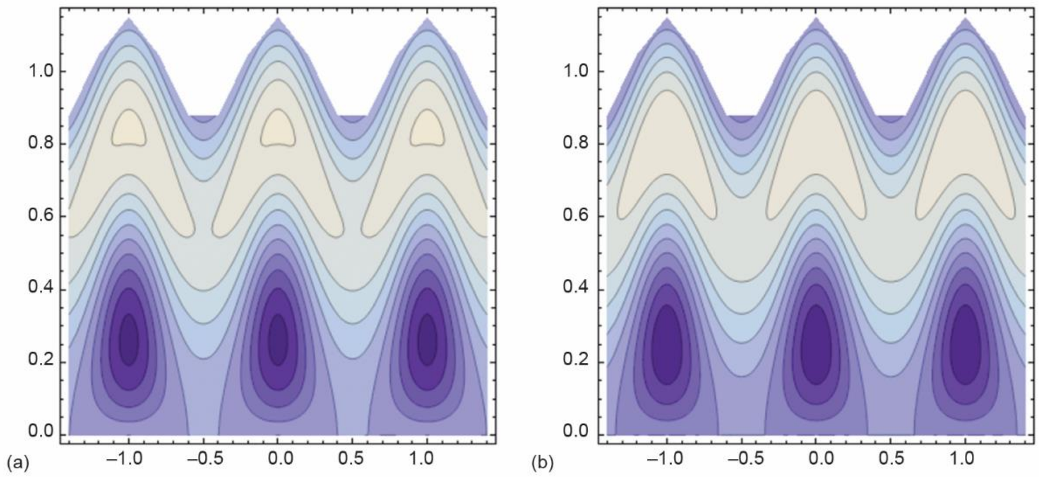

Figure 3. Stream function plot for various value of with m_e: α = 0.2, ϵ = 0.15, F = 0.0.02, Uhs = –1, δ = 0.01, Hr = 1, B = 2, χ = 0.02; (a) m_e = 5 and (b) m_e = 7

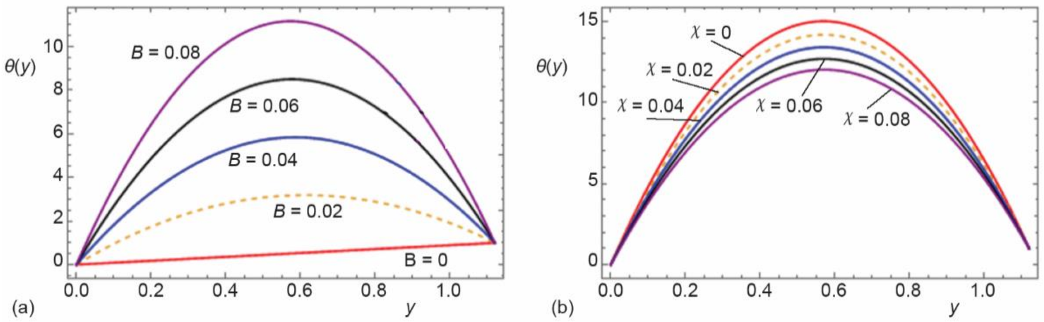

Figure 4. Plots of temperature profile for various values of heat generation and volume fraction parameters of nanofluid with α = 1, ϵ = 0.12, F = 0.5, Uhs = –5, δ = 0.01; (a) χ = 0.06 and (b) B = 0.02

Conclusion

In this investigation we have developed mathematical for nanofluid in a ciliated channel with magnetic field effect which is applied in the transverse direction and external electric field applied in the axial direction. The flow is established with aid of ciliary motion

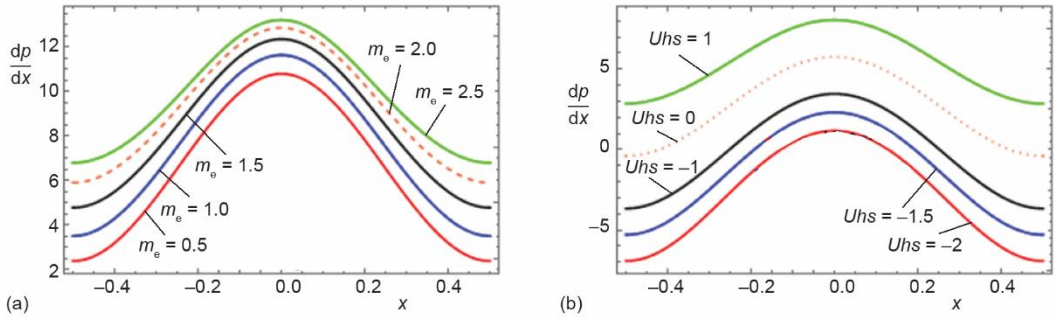

Figure 5. Pressure gradient plot for various value of m_e and *Uhs* with $\alpha = 1$, $\epsilon = 0.2$, $F = 0.1$, $\delta = 0.01$, $B = 0,05$, $\chi = 0.02$, Hr = 1; (a) *Uhs* = 2 and (b) $m_e = 5$

and axial pressure gradient. Equations of motion were simplified for nanofluid flow in the micro-channel by employing low Reynolds number and long wavelength simplification theory.

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