STUDY OF ELECTRO-OSMOTIC NANOFLUID TRANSPORT FOR SCRAPED SURFACE HEAT EXCHANGER WITH HEAT TRANSFER PHENOMENON

by

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In this study a novel mathematical model for electroosmotic flow for Cu-water based nanofluid with heat transfer phenomenon is reported for scraped-surface heat exchanger. The flow is initiated due to motion of lower wall of the channel and axial pressure gradient. The flow is modelled with aid of low Reynolds number and lubrication approximation theory. Exact analytical expressions are gathered for axial velocity, and stream functions for various stations of scraped-surface heat exchanger. Physical phenomenon of electro osmotic parameter are investigated on velocity profile, velocity distribution and pressure rise at edge of the blades. It is reported that electro-osmotic parameter mainly works as dragging force, it can be used to control the flow. This controlling mechanism may be helpful in mixing different materials in scraped-surface heat exchanger. Pressure rise at edge of the blades mainly rises below the blades with electro-osmotic, whereas, this profiles is suppressed for region above the blades and between the blades.

Key words: *scraped-surface heat exchanger, electric field, Cu-water nanofluids*

Introduction

Scraped-surface heat exchangers (SSHE) are abundantly used in various industries. They are used in pharmaceutical, chemical, and food industries. But they are vibrantly used in food industry to carry out different food processing operations. Due to complex structure of SSHE, large capital investment is required to carry out experimental research, also food industrialists are naturally inclined to minimize the cost and to optimize the profit. Due to these and other reason large research has been done to study flow behavior inside SSHE.

Duffy *et al.* [1] studied mathematically flow of Newtonian and rheological power law fluid, and explored analytical expressions for the velocities, stream functions and flow rates, and suggested the equilibrium position of the blades. Fitt *et al.* [2] provide the understanding about channeling process for Newtonian fluid in a simplest model of SSHE. Pascual *et al.* [3] studied the flow in laboratory construed SSHE and gathered great similarity with result which

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already established through numerical computations. Smith *et al.* [4] extended the work of Duffy *et al.* [1], they studied the flow by taking temperature dependent viscosity. Siddiqui *et al.* [5] explored the exact analytical solution and provided many important flow phenomena. To optimize the heat transfer process a precise construction of SSHE with rotating blades has been constructed by Blasiak *et al.* [6]. Imran *et al.* [7, 8] carried out research to study the rheological fluid transport using Adomian decomposition method through SSHE. Exact analytical solution for narrow gap SSHE for the second-grade fluid has been studied by [9]. Magnetic properties of Newtonian fluid transport with heat phenomenon has been studied by Imran *et al.* [10, 11]. Acosta *et al.* [12, 13] explored the flow by using dynamic heat exchanger for homogeneous viscous fluid with heat transfer.

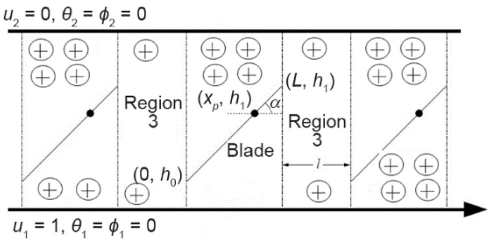

Figure 1. Geometrical scheme of Cu-water nanofluid surface heat exchanger

Problem formulation

Steady heat transfer of aqueous ionic Cu-nanofluid flow in a slender gap scraped-surface heat exchanger is taken, fig. 1. In this problem water is taken as base fluid and Cu as nanofluid particles. Flow for SSHE is investigated by dividing geometry into three regions of channel. Region below the blades $(0 \le x \le L)$ is termed as Region 1, above the blade $(0 \le x \le L)$ is Region 2, and between the blade $(l \le x \le L + l)$ is Region 3. It is assumed that lower boundary of the channel is moving with velocity, U, and upper boundary is at rest, the flow is developed by motion of the lower boundary and due to axial pressure gradient. Also, electric field, E_x, is applied in the axial direction. The SSHE blade occupies the position $(0 \le x \le L)$ and their pivot is situated at (x_p, h_p).

If α signifies angle of inclination of the blade along the x-axis like $y = h(x)$:

$$h = h_\mathrm{p} + \alpha(x - x_\mathrm{p})$$

In order to non-dimensionalise, incorporating the following scaling variables:

$$\hat{x} = \frac{x}{L},\quad \hat{y} = \frac{y}{h_p},\quad \hat{x}_p = \frac{x_p}{L},\quad \hat{l} = \frac{l}{L},\quad \hat{\alpha} = \frac{L\alpha}{h_p}$$

$$\hat{h} = \frac{h}{h_p},\quad \hat{H} = \frac{H}{h_p},\quad \hat{h}_0 = \frac{h_0}{h_p},\quad \hat{h}_1 = \frac{h_1}{h_p},\quad \mathrm{Re} = \frac{\rho_f UL}{\mu_f}$$

$$\mathrm{Pr} = \frac{\mu_f c_f}{k_f},\quad \hat{u}_k = \frac{u_k}{U},\quad \hat{p}_k = \frac{p_k h_p^2}{\mu UL},\quad \hat{\phi} = \frac{z_\mathrm{e}\phi}{\kappa_B T},\quad \theta = \frac{T - T_0}{T_0}$$

$$\mathrm{Pe} = \frac{UL}{D},\quad Uhs = -\frac{E_x \varepsilon_0 \varepsilon T_{av} K_B}{\mathrm{e} z_v c \mu_f},\quad \mathrm{Gr} = \frac{L^2 T_0 \rho \beta_f}{c\mu_f} \tag{1}$$

Using the previous scaling variables and using the lubrication approximation theory, and Debye-Huckel linearization [14] the equations of motion the take the form:

$$d_2 \frac{\partial^2 u_k}{\partial y^2} + d_3 \mathrm{Gr}\theta + Uhsm_\mathrm{e}^2 \phi_k = p_{kx} \tag{2}$$

$$\frac{\partial^2 \phi_k}{\partial y^2} = m_\mathrm{e}^2 \phi_k \tag{3}$$

$$d_1 \frac{\partial^2 \theta_k}{\partial y^2} + d_2 B = 0 \tag{4}$$

where

$$d_1 = \frac{\alpha_\mathrm{nf}}{\alpha_f}, \quad d_2 = \frac{(\rho c_p)_\mathrm{nf}}{(\rho c_p)_f}, \quad d_3 = \frac{\mu_\mathrm{nf}}{\mu_f}, \quad d_4 = \frac{(\rho\beta)_\mathrm{nf}}{(\rho\beta)_f}, \quad d_5 = \frac{-(-2d_1 - Bd_2)}{2d_1}$$

For no-slip condition k = 1, 2, 3:

$$u_1 = u_3 = 1, \quad \theta_1 = \phi_1 = 0 \quad \text{at} \quad y = 0 \tag{5}$$

$$u_1 = u_2 = 0, \quad \theta_2 = \phi_2 = 0, \quad \phi_1 = \phi_2 \quad \text{and} \quad \frac{\partial \phi_1}{\partial y} = \frac{\partial \phi_2}{\partial y}$$
$$\frac{\partial \theta_1}{\partial y} = \frac{\partial \theta_2}{\partial y} \quad \text{at} \quad y = h = 1 + \alpha(x - x_\mathrm{p}) \tag{6}$$

$$u_1 = u_2 = u_3 = 0, \quad \theta_3 = \phi_3 = 1 \quad \text{at} \quad y = H \tag{7}$$

Exact solution for physical problem

Solving eqs. (2)-(4) along with boundary conditions one may gather the exact solution:

$$\phi_k = Csch(\kappa)Sinh(y\kappa) \tag{8}$$

$$\theta_k = \frac{-(Bd_2 y^2)}{2d_1} + yd_5 \tag{9}$$

$$u_k = \frac{p_{1x} y^2}{2d_3} - \frac{d_5 d_4 \mathrm{Gr} y^3}{6d_3} + \frac{Bd_2 d_4 \mathrm{Gr} y^4}{24 d_1 d_3} + c_{1k} + yc_{2k} - \frac{UhsCsch(m_\mathrm{e})Sinh(m_\mathrm{e} y)}{d_3} \tag{10}$$

$$\psi_k = \frac{p_{kx} y^3}{6d_3} - \frac{d_5 d_4 \mathrm{Gr} y^4}{24 d_3} + \frac{Bd_2 d_4 \mathrm{Gr} y^5}{120 d_1 d_3} + yc_{1k} + \frac{1}{2} y^2 c_{3k} + c_{4k} - \frac{UhsCosh(m_\mathrm{e} y)Csch(m_\mathrm{e})}{d_3 m_\mathrm{e}} \tag{11}$$

for k = 1, 2, 3.

where c_{1k}, c_{2k}, c_{3k}, and c_{4k} are variables terms used for simplification, their values are not included here for brevity.

Results and discussions

Flow of electro-osmotic Cu-nanofluid with heat transfer in SSHE has been investigated. The flow is modelled by using lubrication approximation theory. Maxwell-Garnelt model thermal model [15] has been capitalized. In this section effect of electro osmotic parameter is investigated on the velocity profile, velocity distribution and pressure rise at the edge of blades. It is observed from fig. 2 that velocity profile in Regions 1 and 3 rise at start and then tend to decline in the middle of the flow with rise in electro-osmotic, m_e, whereas, velocity in Region 2 continuously declines, here electro-osmotic parameter mainly working as dragging force, it mean that this can be used to control the flow. This controlling mechanism may be helpful in mixing different materials in SSHE. Stream lines pattern are exhibited in fig. 3, it is seen that there are some back flow region under the blades, and it is observed that stream lines in Region 2 possesses parabolic profile. On other-hand rectilinear flow behaviour in Region 3 is recorded. From fig. 4 it is quite obvious that in region 1 pressure rise at edge of the blades mainly rises with electro-osmotic m_e, whereas, pressure rise profiles are suppressed for Regions 2 and 3.

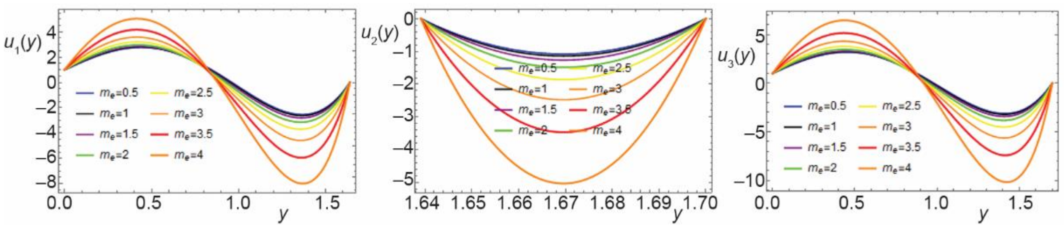

Figure 2. Analysis of electro-osmotic parameter m_e in different station of SSHE by fixing Gr = 0.01, H = 1.7, l = 1, B = 0.6, x_p = 0.49, α = 1.25322, x = 1, Uhs = –1, χ = 0.2
(*for color image see journal web site*)

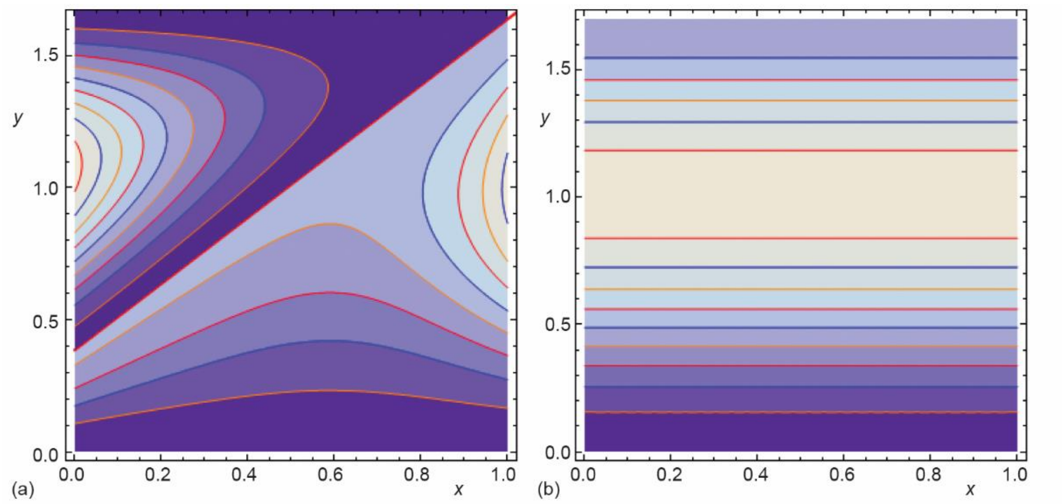

Figure 3. Stream line patterns in SSHE by fixing Gr = 0.01, H = 1.7, m_e = 2, l = 1, B = 0.6, x_p = 0.49, α = 1.25322, x = 1, Uhs = –2, χ = 0.2 (*for color image see journal web site*)

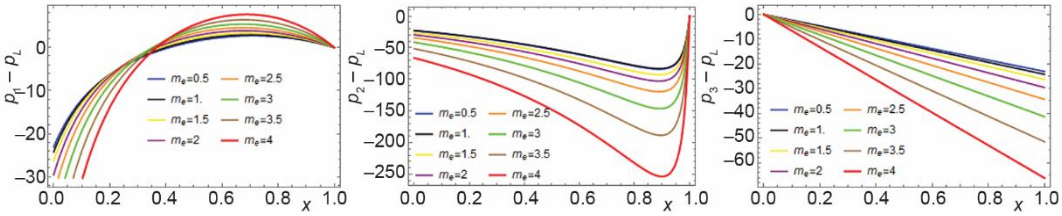

Figure 4. Analysis of pressure rise at the edge of the blades as function of electro-osmotic parameter m_e by fixing Gr = 0.01, H = 1.7, l = 1, B = 0.9, x_p = 0.49, α = 1.25322, x = 1, Uhs = –2, χ = 0.2
(*for color image see journal web site*)

Conclusions

Flow of electro-osmotic Cu-nanofluid with heat transfer in SSHE has been investigated by using lubrication approximation theory. Some key findings of current investigation are the following.

- Electro-osmotic parameter mainly retards the flow, it mean that this can be used to control the flow. This mechanism may be helpful in mixing different materials in SSHE.
- It is seen that there are some back flow region under the blades, and it is observed that stream lines in Region 2 possesses parabolic profile, and has rectilinear flow behaviour in Region 3.
- Pressure rise at edge of the blades mainly rises with electro-osmotic, m_e, parameter in Region 1, whereas, pressure rise profiles are suppressed in Regions 2 and 3.

References

[1] Duffy, B. R. *et al.*, A Mathematical Model of Fluid Flow in a Scraped-Surface Heat Exchanger, *Journal of Engineering Mathematics, 57* (2007), 4, pp. 381-405

[2] Fitt, A. D., *et al.*, Analysis of Heat Flow and "Channelling"• in a Scraped-Surface Heat Exchanger, *Journal of Engineering Mathematics, 57* (2007), 4, pp. 407-422

[3] Pascual, *et al.*, Flow and Particle Motion in Scraped Heat Exchanger Crystallizers, *Chemical engineering science, 64* (2009), 24, pp. 5153-5161

[4] Smith, A. A. T., *et al.*, Heat and Fluid Flow in a Scraped-Surface Heat Exchanger Containing a Fluid with Temperature-Dependent Viscosity, *Journal of Engineering Mathematics, 68* (2010), 3-4, pp. 301-325

[5] Siddiqui, *et al.*, Magnetohydrodynamic Flow of Newtonian Fluid in a Scraped-surface Heat Exchanger, *Canadian Journal of Physics, 93* (2015), 10, pp. 1088-1099

[6] Blasiak, P., Pietrowicz, S., Towards a Better Understanding of 2D Thermal-Flow Processes in a Scraped-surface Heat Exchanger, *International Journal of Heat and Mass Transfer, 98* (2016), July, pp. 240-256

[7] Imran, A., *et al.*, Flow of Oldroyd 8-Constant Fluid in a Scraped-surface Heat Exchanger, T*he European Physical Journal Plus, 131*(2016), 12, pp. 1-19

[8] Imran, A., *et al.*, Study of a Eyring-Powell Fluid in a Scraped-surface Heat Exchanger, *International Journal of Applied and Computational Mathematics, 4* (2018), 1, pp. 1-20

[9] Imran, A., *et al.*, Flow of Second Grade Fluid in a Scraped-surface Heat Exchanger, *Journal of Food Process Engineering, 40* (2017), 2, e12393

[10] Imran, A., *et al.*, MHD and Heat Transfer Analyses of a Fluid Flow Through Scraped-surface Heat Exchanger by Analytical Solver, *AIP Advances, 9* (2019), 7, 075201

[11] Imran, A., *et al.*, Mathematical moDel of Eyring Fluid in a Scraped-surface Heat Exchanger, *Journal of the National Science Foundation of Sri Lanka, 48* (2020), 1, pp. 3-14

[12] Acosta, C. A., *et al.*, Numerical and Experimental Study of the Glass-Transition Temperature of a Non-Newtonian Fluid in a Dynamic Scraped-surface Heat Exchanger, *International Journal of Heat and Mass Transfer, 152* (2020), May, 119525

[13] Acosta, C. A., *et al.*, Empirical and Numerical Determination of the Freezing Point Depression of an Unsteady Flow in a Scraped-surface Crystallizer, *Applied Thermal Engineering, 179* (2020), Oct., 115734

[14] Prakash, J., *et al*., *A Model for Electro-Osmotic Flow of Pseudoplastic Nanofluids in Presence of Peristaltic Pumping: An Application to Smart Pumping in Energy Systems, Nanotechnology for Energy and Environmental Engineering*, Springer Cham, Denmark, 2020. pp. 185-213
[15] Hayat, T., *et al*., A Model for an Application to Biomedical Engineering Through Nanoparticles, *International Journal of Heat and Mass Transfer*, *101* (2016), Oct., pp.112-120

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