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New predictor-corrector type iterative methods for solving nonlinear equations

Çiğdem Dinçkal[1*]

ABSTRACT

This paper proposes two new predictor-corrector type iterative methods for finding roots of nonlinear equations. These methods are generated by based on the combination of the two well known Bisection method and Newton-Raphson method. Various numerical examples serve to verify the main purpose of these methods and compare the numerical results. Numerical results are also presented to test the convergence rate of these new proposed methods in terms of number of iterations achieved to reach the exact root of any nonlinear equation. These numerical results obtained also indicate that new proposed methods perform better than both well known methods Bisection and Newton-Raphson and also the other methods in literature.

Keywords: Bisection method, Newton-Raphson method, predictor-corrector type iterative methods, nonlinear equations, numerical examples

Nonlineer denklemleri çözmek için yeni öngörme-düzeltme tipi yineli yöntemler

ÖZ

Bu makale, nonlineer denklemleri çözmek için, iki yeni öngörme-düzeltme tipi yineli yöntem önerir. Bu yöntemler, iyi bilinen ikiye bölme yöntemi ve Newton-Raphson yönteminin kombinasyonuna dayalı bir şekilde oluşturulmuştur. Çeşitli nümerik örnekler, bu yöntemlerin ana amaçlarını doğrulamaya ve nümerik sonuçlarını karşılaştırmaya hizmet etmektedir. Nümerik sonuçlar, herhangi nonlineer bir denklemin tam köküne ulaşmak için elde edilecek yineleme sayısı cinsinden bu yeni önerilen yöntemlerin yakınsama hızlarını test etmek için de sunulmuştur. Elde edilen bu nümerik sonuçlar, önerilen yeni yöntemlerin iyi bilinen her iki yöntemlerden biri olan ikiye bölme ve Newton-Raphson'dan ve ayrıca literatürdeki diğer yöntemlerden de daha iyi performans gösterdiğine de, işaret etmektedir.

Anahtar Kelimeler: İkiye bölme yöntemi, Newton-Raphson yöntemi, örgörme-düzeltme tipi yineli yöntemler, nonlineer denklemler, nümerik örnekler

[*] Çiğdem Dinçkal
1 Çankaya Üniversitesi, Mühendislik Fakültesi, İnşaat Mühendisliği Bölümü, Ankara - cdinckal@cankaya.edu.tr

1. INTRODUCTION (GİRİŞ)

Many problems in applied mathematics or engineering are solved to find the all values of x which satisfy the following equation:

$$f(x) = 0 \tag{1}$$

Analytical methods for solving this equation in (1) are difficult or almost non-existent. Some numerical methods namely; Bisection, Regula Falsi and Newton-Raphson have been employed to find the root of a nonlinear equation [1-3].

In literature, Taylor interpolating polynomials and quadrature formulas have been used to obtain some iterative methods for solving nonlinear equations [2-9]. Ujevic [7] has proposed a predictor-corrector type method such that Newton-Raphson method acts as a predictor and suggested method as a corrector. Similarly Noor and Ahmad [10] have suggested some predictor-corrector type methods by use of the combination of Newton-Raphson and Regula Falsi method (NRF) and Regula Falsi and Newton-Raphson method (RFN).

By inspiring and motivating from these studies [7,10], in this paper, two new predictor-corrector type methods are considered and suggested by combining Bisection and Newton-Raphson method (BNM) and Newton-Raphson and Bisection method (NBM).

Some examples are given to illustrate the efficiency of these new methods and compare with BM and NM. Numerical results of this study demonstrate that new predictor-corrector type iterative methods BNM and NBM give better results than the well known methods BM and NM, correspondingly.

BNM and NBM are also more proficient in most of the examples than the other methods [9-12].

2. MATERIALS AND METHODS (MALZEMELER VE YÖNTEMLER)

BM, NM and proposed methods; BNM and NBM are presented in the following subsections.

2.1. Bisection Method (İkiye Bölme Yöntemi)

1) For a given interval $[a, b]$, f(x) should be continous and the condition $f(a) \cdot f(b) < 0$ should be satisfied then $x_1, x_2, \ldots,$ and calculated such that

$$x_k = \frac{b+a}{2} \tag{2}$$

Convergence Test:

2) If $|x_{k+1}-x_k| < \varepsilon$, then stop iteration.
3) If $f(a)f(x_{k+1}) < 0$, then set $b = x_{k+1}$ else set $a = x_{k+1}$
4) Set k = k + 1 and go to first step.

2.2. Newton-Raphson Method (Newton-Raphson Yöntemi)

NM is the most popular and commonly used numerical method which is used to solve the nonlinear equations with an initial estimate and weighting factor $\alpha \in (0,1]$. The initial estimate should lie in the neighborhood of the root of the equation (1). The algorithm of the NM with is shown as below.

1) For given initial estimate x_0; $x_1, x_2, \ldots,$ calculated such that

$$x_{i+1} = x_i - \alpha \frac{f(x_i)}{f'(x_i)} \tag{3}$$

Convergence Test:

2) If $|x_{i+1}-x_i| < \varepsilon$, then stop the iteration.
3) Set i= i + 1 and go to first step.
α has a significant effect on finding root by NM. This case may also be explained as follows:

When nonlinear equation of the form $f(x) = 0$ is solved numerically, this equation is transformed into the form $x = \varphi(x)$. If the condition $|\varphi'(x)| < 1$ is satisfied in the neighborhood of the solution, then the array determined by the assumption $x_k = \varphi(x_{k-1})$ will be converged and the limit of this array is equal to the solution of $f(x) = 0$. In this method, solution is always satisfied in the neighborhood of the condition $|\varphi'(x)| \leq A < 1$ since $\varphi(x)$ is expressed as the form $\varphi(x) = x - \frac{f(x)}{f'(x)}$. The convergence rate of the method where $\tilde{x}$ is the exact solution of the equation can be viewed in the form of the inequality $|x_k - \tilde{x}| \leq A^k|x_0 - \tilde{x}|$. For this reason, when NM is used, equation (3) yields the best solution in case of α=1 for solving nonlinear equations. Because the value of $|\varphi'(x)| = \left|1 - \alpha \left[\frac{f(x)}{f'(x)}\right]'\right|$ is minumum when α=1. So this case has an impact on number of iterations and convergence rate of the related nonlinear equation.

2.3. Bisection and Newton-Raphson Method (İkiye Bölme ve Newton-Raphson Yöntemi)

In this method, the interval is used to solve the nonlinear equation using BM in each step and the value obtained is employed as the initial estimate for the NM with the weighting factor $\alpha \in (0,1]$.

Steps in BNM are summarized as;

1) For a given interval $[a$, b] $x_1, x_2, \ldots$ calculated such that

$$c_k = \frac{b+a}{2} \tag{4}$$

$$x_{k+1} = c_k - \alpha \frac{f(c_k)}{f'(c_k)} \tag{5}$$

Convergence Test:

2) If $|x_{k+1}-x_k| < \varepsilon$, then stop iteration.
3) If $f(a)f(x_{k+1}) < 0$, then set $b = x_{k+1}$ else set $a = x_{k+1}$
4) Set k = k + 1 and go to first step.

The convergence of this method is quadratic and the rate of convergence p is 2.

BNM is more efficient than well known BM in terms of number iterations.

2.4. Newton-Raphson and Bisection Method (Newton-Raphson ve İkiye Bölme Yöntemi)

In this method NM $\alpha \in (0,1]$ with the weighting factor and BM have been used together. This method is performed by moving the fixed endpoint of the interval using NM. It is significant to note that in NBM, interval must be same as in BM and the derivative of the function should not be zero at the given interval [10].

The algorithm is presented as follows.

1) For a given interval $[a, b]$, $x_1, x_2, \ldots$, calculated such that

$$x_{i+1} = \frac{b+a}{2} \tag{6}$$

Convergence Test:

2) If $|x_{i+1}-x_i| < \varepsilon$, then stop iteration.
3) If $f(a)f(x_{i+1}) < 0$, then set $b = x_{i+1}$ and $a = a - \alpha \frac{f(a)}{f'(a)}$
else set $= x_{i+1}$ and $b = b - \alpha \frac{f(b)}{f'(b)}$
4) Set i = i + 1 and go to first step.

The convergence of NBM is linear and the rate of convergence p is 1.

NBM is more efficient than well known NM in terms of number iterations.

3. NUMERICAL EXAMPLES (SAYISAL ÖRNEKLER)

Some examples are presented to see the number of iterations of the new developed methods for 10 different α values and compared with BM and NM. In all numerical examples ε is taken as 10^{-10} for the accuracy.

Example 1 [11]: Let $f(x) = xe^x - 1$ with initial estimate $x_0=1$ on the interval [-1,1] .

Example 2 [11]: Let $f(x) = 11x^{11} - 1$ with $x_0=0.9$ on the interval [0.1,0.9].

Example 3 [10]: Let $f(x) = x^3 - 2x^2 - 5$ and $x_0=2$ the interval is [2,3].

Example 4 [9,10]: Let $f(x) = e^{x^2+7x-30} - 1$ with $x_0=2.8$ on the interval [2.8,3.2].

Example 5 [10]: Let $f(x) = \frac{1}{x} - \sin(x) + 1$ with $x_0=-1.3$ on the interval [-1.3,-0.5].

Example 6 [10]: Let $f(x) = x - cos(x)$ with $x_0=0$ on the interval [0,π/2].

Example 7 [12]: Let $f(x) = cos(x) - xe^x$ with $x_0=0$ on the interval [0,1].

4. RESULTS (SONUÇLAR)

The number of iterations required to reach the solution are presented in the following tables for all examples. xr is one of the exact roots of the corresponding example. The calculations were conducted in MATLAB 7.1. To these Tables, it is observed that the accuracy obtained in both BNM and NBM requires less or approximately equal number of iteration than BM and NM, respectively.

Table 1. The number of iterations of all methods for example 1 (Tablo 1. Örnek 1. için bütün yöntemlerin iterasyon sayıları)

α	BM	BNM	NBM	NM	xr
1	20	2	5	6	0.5671 433083
0.9	20	3	7	12	0.5671 433083
0.8	20	3	7	16	0.5671 433083

0.7	20	3	8	21	0.5671 433083
0.6	20	4	11	27	0.5671 433083
0.5	20	4	14	34	0.5671 433083
0.4	20	5	16	45	0.5671 433083
0.3	20	6	21	63	0.5671 433083
0.2	20	6	32	98	0.5671 433083
0.1	20	7	66	199	0.5671 433083

Table 2. The number of iterations of all methods for example 2 (Tablo 2. Örnek 2. için bütün yöntemlerin iterasyon sayıları)

α	BM	BNM	NBM	NM	xr
1	15	2	5	6	0.804133
0.9	15	3	7	12	0.804133
0.8	15	4	8	16	0.804133
0.7	15	4	10	20	0.804133
0.6	15	5	12	25	0.804133
0.5	15	6	16	32	0.804133
0.4	15	7	20	43	0.804133
0.3	15	7	26	59	0.804133
0.2	15	8	44	91	0.804133
0.1	15	9	90	185	0.804133

Table 3. The number of iterations of all methods for example 3 (Tablo 3. Örnek 3. için bütün yöntemlerin iterasyon sayıları)

α	BM	BNM	NBM	NM	xr
1	20	3	4	7	2.6906
0.9	20	4	5	12	2.6906
0.8	20	4	5	16	2.6906
0.7	20	4	6	19	2.6906
0.6	20	5	6	23	2.6906
0.5	20	6	7	29	2.6906
0.4	20	7	8	40	2.6906
0.3	20	8	8	57	2.6906
0.2	20	10	8	89	2.6906
0.1	20	14	9	181	2.6906

Table 4. The number of iterations of all methods for example 4 (Tablo 4. Örnek 4. için bütün yöntemlerin iterasyon sayıları)

α	BM	BNM	NBM	NM	xr
1	13	2	3	17	3
0.9	13	3	4	21	3
0.8	13	3	5	24	3
0.7	13	4	6	27	3
0.6	13	4	7	31	3
0.5	13	5	8	37	3
0.4	13	6	9	44	3
0.3	13	7	10	56	3
0.2	13	8	10	73	3
0.1	13	8	11	161	3

Table 5. The number of iterations of all methods for example 5 (Tablo 5. Örnek 5. için bütün yöntemlerin iterasyon sayıları))

α	BM	BNM	NBM	NM	xr
1	13	4	4	26	-0.6294
0.9	13	5	5	17	-0.6294
0.8	13	5	6	19	-0.6294
0.7	13	6	8	22	-0.6294
0.6	13	7	10	26	-0.6294
0.5	13	7	12	30	-0.6294
0.4	13	8	16	42	-0.6294
0.3	13	7	22	60	-0.6294
0.2	13	9	35	94	-0.6294
0.1	13	10	51	192	-0.6294

Table 6. The number of iterations of all methods for example 6 (Tablo 6. Örnek 6. için bütün yöntemlerin iterasyon sayıları)

α	BM	BNM	NBM	NM	xr
1	21	2	4	5	0.7391
0.9	21	3	7	12	0.7391
0.8	21	3	9	15	0.7391
0.7	21	3	13	19	0.7391
0.6	21	4	18	25	0.7391
0.5	21	5	18	33	0.7391
0.4	21	6	22	44	0.7391
0.3	21	6	28	62	0.7391
0.2	21	6	29	96	0.7391
0.1	21	10	31	196	0.7391

Table 7. The number of iterations of all methods for example 7 (Tablo 7. Örnek 7. için bütün yöntemlerin iterasyon sayıları)

α	BM	BNM	NBM	NM	xr
1	20	2	4	7	0.517757
0.9	20	2	6	13	0.517757
0.8	20	3	7	17	0.517757
0.7	20	3	9	21	0.517757
0.6	20	3	11	25	0.517757
0.5	20	3	14	30	0.517757
0.4	20	4	18	43	0.517757

0.3	20	4	25	60	0.517757
0.2	20	7	39	95	0.517757
0.1	20	6	80	193	0.517757

5. CONCLUSION AND DISCUSSION (SONUÇ VE TARTIŞMA)

In this paper two new predictor-corrector type iterative methods were proposed and compared with BM and NM. Results obtained from each example are stated as follows:

For Example 1:

As it is seen in Table 1, BNM gives the best accuracy for all values of α among the other methods. NBM performs better than NM. When the value of α is less than 0.4, BM is more accurate than NBM for this example. While α<0.3, NM gives the greatest number of iterations.

For Example 2:

To Table 2, BNM for all values of α is the best method among the others in terms of least number of iterations. For this example, when α<0.6, number of iterations in NBM increases and BM is going to be the second method after BNM which performs better than NBM. While α<0.3, NM gives the greatest number of iterations.

For Example 3:

The numerical results for example 3 are presented in Table 3 for 10 different values of α.
BNM is the best method in accuracy until α<0.3 since number of iterations are slightly increasing while NBM has the least number of iterations. When α<0.8, NM gives the greatest number of iterations for this example.

For Example 4:

Table 4 gives the numerical results for example 4. BNM for all values of α is the best method among the others in terms of least number of iterations for this example. Number of iterations in NBM are slightly greater than BNM. NM is not the suitable method for this example on account of the greatest number of iterations for all values of α.

For Example 5:

It is obvious that BNM gives the best accuracy for all values of α among the other methods for example 5 in Table 5. When the value of α<0.5, BM gives more accurate results than NBM. NM gives the greatest number of iterations when α<0.4.

For Example 6:
Table 6 which presents the numerical results of example 6, shows that When the value of α<0.6, NM is not the suitable method because of the number of iterations. For all values of α, BNM performs best among the other methods.

For Example 7:

The numerical results of example 7 in Table 7 indicate that BNM is still the best method for all values of α among the other methods in terms of number of iterations. When the value of α<0.3, NM gives the greatest number of iterations for this example.

Briefly, four methods in this study converge to the correct root of the nonlinear equations in different number of iterations for all examples. In other words, the results are not divergent as it is the case in example 4 in Noor et al. (2006).

Furthermore, it is observed that decrease in value of α leads to an increase in the number of iterations in all methods for all examples. NM is the most affected method that the number of iterations are increasing extremely when α values are decreasing in all examples.

On the contrary, the change in value of α does not affect the number of iterations much more in BNM for all examples. Since it is still the most efficient and accurate method in least number of iterations among the other methods; NBM, BM and NM. When α is 1 in all examples, BNM is also more efficient method in terms of less or approximately equal number of iterations than other methods (Noor et al. 2006, Noor and Ahmad 2006, Shaw and Mukhopadhay 2015, Dauhoo and Soobhug 2003) in literature.

REFERENCES (KAYNAKÇA)

[1] S. C. Chapra and R. P. Canale, *Numerical Methods for Engineers*. 6th, edition, McGraw Hill, New York, 2010, pp. 116-145.

[2] R. L. Burden and J. D. Faires, *Numerical Analysis. 6th, edition*, PWS Publishing Company, Boston, 2002, pp. 48-86.

[3] D. Kahaner, C. Moler, and S. Nash, *Numerical Methods and Software*. Prentice Hall, Englewood Cliffs, New Jersey, 1989, pp. 240-247.

[4] S. Amat, S. Busquier, and J. M. Gutierrez, “Geometric construction of iterative functions to

solve nonlinear equations”, J. Comput. Appl. Math., vol. 157, no. 1, pp. 197-205, 2003.

[5] M. Frontini and E. Sormani, “Third-order methods from quadrature formulae for solving systems of nonlinear equations”, Appl. Math. Comput., vol. 149, no. 3, pp. 771-782, 2004.

[6] D. G. Moursund, C. S. Duris, *Elementary Theory and Application of Numerical Analysis*. Dover Publications, New York, 1967, pp. 19-32.

[7] N. Ujevic, “A method for solving nonlinear equations”, Appl. Math. Comput., vol. 174, pp. 1416-1426, 2006.

[8] M. A. Noor and F. Ahmad, “On a predictor-corrector method for solving nonlinear equations”, J. Appl. Math. Comput., vol. 183, pp. 128-133, 2006.

[9] M. A. Noor, F. Ahmad, and S. Javeed, “Two-step iterative methods for nonlinear equations”, Appl. Math. Comput., vol. 181, no. 2, pp. 1068-1075, 2006.

[10] M. A. Noor and F. Ahmad, “Numerical comparison of iterative methods for solving nonlinear equations”, Appl. Math. Comput., vol. 180, no. 1, pp. 167-172, 2006.

[11] S. Shaw and B. Mukhopadhay, “An improved Regula Falsi method for finding simple roots of nonlinear equations”, Appl. Math. Comput., vol. 254, pp. 370-374, 2015.

[12] M. Z. Dauhoo and M. Soobhug, “An adaptive weighted Bisection method for finding roots of non-linear equations”, Int. J. Comput. Math., vol. 80, no. 7, pp. 897-906, 2003.