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Numerical analysis of Atangana-Baleanu fractional model to understand the propagation of a novel corona virus pandemic

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Abstract In this manuscript, we formulated a new nonlinear SEIQR fractional order pandemic model for the Corona virus disease (COVID-19) with Atangana-Baleanu derivative. Two main equilibrium points F_0^* , F_1^* of the proposed model are stated. Threshold parameter \mathcal{R}_0 for the model using next generation technique is computed to investigate the future dynamics of the disease. The existence and uniqueness of solution is proved using a fixed point theorem. For the numerical solution of fractional model, we implemented a newly proposed Toufik-Atangana numerical scheme to validate the importance of arbitrary order derivative ρ and our obtained theoretical results. It is worth mentioning that fractional order derivative provides much deeper information about the complex dynamics of Corona model. Results obtained through the proposed scheme are dynamically consistent and good in agreement with the analytical results. To draw our conclusions, we explore a complete quantitative analysis of the given model for different quarantine levels. It is claimed through numerical simulations that pandemic could be eradicated faster if a human community selfishly adopts mandatory quarantine measures at various coverage levels with proper awareness. Finally, we have executed the joint variability of all classes to understand the effectiveness of quarantine policy on human population.

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1. Introduction

Corona virus disease (COVID-19) is a severe pandemic with an extremely high fatality rate caused by an infectious respiratory virus called SARS-CoV-2. The virus spreads uniformly among individuals and affecting people worldwide through direct and

indirect physical contact [1]. The first and overall third outbreak of novel Corona virus disease was experienced at the end of December 2019 in the Wuhan City of China. It spreads rapidly in different parts of China and then worldwide in almost 223 countries of Asia, Australia, America and Europe [2,4]. World Health Organization (WHO) recorded the global situation and reported that there are more than 232,636,622 confirmed cases with 4,762,089 deaths by September 29, 2021. It is remarkable that WHO reported 408,990 new cases to date while the number of confirmed cases are continuing to increase. However, almost 210,951,891 people have been recovered from COVID-19. A total of 6,136,962,861 vaccine doses have been administered by 29 September, 2021. Currently, the number of highest positive cases encountered in United States of America followed by India, Brazil and Spain respectively.

The current evidence suggests that the main source of Corona virus transmission among individuals is the respiratory droplets through sneezing, coughing and spitting of an infected person [3,4]. During the treatment of Corona patients, health care staff can also get infected. Incubation period for the Corona virus disease normally varies from 2 to 14 days and on average 5-6 days. Every individual can get Corona infection at any age and time instant and become seriously ill or die. It is observed that people around 60 years and older and all those who develop severe diseases like heart disease, obesity or cancer, diabetes or lung disease are at higher risk of getting seriously sick with Corona virus and can be infectious for longer. On the other hand, people with a variety of symptoms like fever, dry cough, sneezing, red eyes, shortness of breath, muscle pain, fatigue, severe headache, new loss of smell or taste, persistent pain in the chest, sore throat, runny nose, sleep disorders, vomiting, and diarrhea, may have Corona virus disease [5]. It may develop to acute multi-organ failure, respiratory distress syndrome, blood clots, and septic shock, among other conditions. Without needing hospital treatment, about 5% among those who develop these symptoms become critically ill and need intensive care, about 80% may recover from the viral disease and about 15% become seriously ill and require oxygen [6,7]. They may face a rapid progression to death due to their ill condition. The virus may spread on a large scale if the pandemic is not restricted. It will threaten the entire human population. Therefore, to take care of infected patients, it is inevitable to take comprehensive preventive measures [8].

Public health experts worldwide are continuously learning and monitoring the dynamics of novel Corona virus these days. They always look for multiple solutions to control the pandemic. The first important step is to develop appropriate community engagement, awareness campaign and protection measures among individuals. For example, wearing proper clothes and gloves, and washing hands after visiting the patients should be taken care of. To protect yourself and other people, we should wear a surgical mask that especially covers our nose and mouth. Keep a distance of at least 6 feet from other peoples who do not live with you, especially in crowded areas. Avoid indoor spaces as much as possible, particularly ones that are not well ventilated. Use hand sanitizer with at least 60% alcohol content. Wash your hands with soap and water for 20 s. Everyone takes these actions to help slow down COVID-19. The other rigorous measures are the use of vaccination, public quarantine facilities, lock down, isolation or treatment of infected humans [9,10]. This also includes raising

the awareness like public health education, laboratory testing, risk communication, start of antiviral drugs, designed special hospitals and health screening at borders. It is remarkable that antibiotics do not work against COVID-19. Moreover, there is no licensed medicine which is used to cure pandemic.

Ever since COVID-19 came, many researchers [10–24] have been formulating and using different mathematical models as a way to explore the dynamical behavior of a Corona virus disease. In all of the research articles cited therein, mathematical models were based on integer-order derivatives involving some restrictions on the order of ODE's. The aim was to gain an insight into the mode of transmission, spread, impact, prevention and control of the pandemic. Performing a reliable and competitive mathematical analysis of these models plays a key role in the field of epidemiology. This actually provides a deeper understanding about the patterns of disease pandemic. It always helps to develop a mechanism to control the future spread of the disease.

Fractional calculus is a rapidly growing field of mathematics which has various applications in widespread and diverse fields of science and engineering such as viscoelasticity, fluid mechanics, electromagnetics, electrochemistry, optics, signals processing and biological population models. It has been used to model engineering and physical processes that are found to be best described by fractional differential equations. Fractional order derivatives [26,27] are useful in demonstrating many natural facts and phenomena having non-local complex dynamical behavior. These operators are helpful to overcome all the restrictions on the order of differential equations while solving them. The fractional order mathematical models are more versatile than classical integer-order models due to the hereditary features and description of memory [28–39]. Such models yield more accurate outcomes about the complex behavior of epidemic diseases as compared to the integer-order models. Indeed, the usual integer order models do not enjoy subsequent memory effects occurring in biological models.

The Caputo fractional derivatives and integrals are the most well known operators, historically been used for modeling many real world problems. Moreover, the Riemann-Liouville derivative operator has a strong relation with the Caputo fractional derivative. When these operators are used to investigate the structure of different models, they may block obtaining better results. The main issue with the Riemann-Liouville and Caputo operators is the singularity property of their kernels [54,55]. Due to this reason, researchers have felt the need of fractional operators with nonsingular kernels to better understand the dynamics of models. To this extent, many scientists succeeded in presenting fractional operators with nonsingular kernels, for example, Caputo-Fabrizio fractional operator [25,54,56]. In the recent past, Atangana and Baleanu [57] proposed a new fractional derivative operator (ABC) with a Mittag-Leffer kernel with one parameter. The main advantage of this operator is that it has a nonlocal and nonsingular kernel. It provides an advantage for those people who work in numerical modeling of real world problems. More recently, new advances and studies in fractional differential equations with ABC derivative operator has been published [28–30].

Among non-pharmaceutical interventions, execution of various quarantine strategies in human communities is a significant public health measure, the most effective and a safe way that historically been utilized to control the spread of all com-

municable diseases. Physically weak populations from the whole world especially in developing countries are at the highest priority for quarantine process. World Health Organization works closely with researchers around the world to assess the developed global standards, safety norms and the effectiveness of the implemented quarantine policy during the quarantine period. To control the spread of epidemic diseases, many researchers are attracted to develop mathematical models [10–16] dealing with quarantine strategies. A several number of articles [17–24, 59–61, 70] have been published recently on the effectiveness of implemented quarantine program. It is worth mentioning that the most effective and a safe way to control the spread of Corona infection is the prospective quarantine of exposed humans. Recently, a research article [11] has been published to understand the effect of quarantine policy on the propagation of Corona virus epidemic. Authors analyzed Corona virus disease through an integer order SEIQR epidemic model. To the best of our knowledge, no one explored a complete mathematical analysis of this model for the fractional order using Atangana-Baleanu derivative operator. We will move in this direction.

In this paper, our aim is to promote the applications of Atangana-Baleanu operator to a SEIQR model [11] of a Corona virus disease. We analyzed the developed fractional order initial value problem with Atangana-Baleanu derivative operator to observe class wise transmission dynamics of the infection in a human population. Mathematical modeling of COVID-19 shall work confidentially to predict and comprehend how virus spread. These predictions will also help to understand how the Corona infection might decrease or increase in the future. The proposed model [11] incorporates a class of those humans who have quarantined through appropriate quarantine programme at their level of exposedness. We assumed that the quarantine is perfect; that is, the humans will not get infected during quarantine period and will be healthy at the end of this period. The impact of the quarantine programme on individuals with the help of fractional model will be assessed. Using some known techniques, we shall perform a complete mathematical analysis of this model with an aim of controlling the spread of COVID-19.

In Chaos theory, an important area of research is to study various mathematical models with different descriptions and requirements for their numerical stability. Recently, the stability of various epidemic models [12, 62–64] of Ebola and Corona viruses with respect to the involved parameters was obtained. Some appropriate reliable numerical techniques [68, 67] to solve the governing equations were employed. The dynamics of both the diseases was studied with the help of graphic illustrations. We tend to expect that our projected SEIQR fractional model also contains all vital self-propelling properties such as: well-posedness of the model along with the boundedness and positivity of obtained solutions. It is ascertained that most of the traditional standard methods sometimes become unbounded divergent when implemented to a nonlinear system. Implementation of these numerical methods could cause major problems such as generating oscillations, bifurcations, chaos, negative solutions, or solutions converging to false equilibrium states as time grid size increased [62, 65]. Therefore, the best approach to solve our fractional model herein is to develop Toufik-Atangana finite difference scheme [45, 46]. We will prove the dynamic consistency of the developed scheme by performing detailed numerical analysis of the proposed model.

Furthermore, we will simulate computationally the dynamics of the spread of Corona virus disease over time t . The numerical analysis highlights the effect of quarantine strategy on the dynamics of COVID-19 at different quarantine levels. The effect of quarantine on susceptible and exposed class is executed reveals that number of susceptible and the exposed humans are decreased continuously by increasing the levels of quarantine. A rapid regress in the population of infected humans is observed when quarantine rates are high. It is noticed that infectivity will approach to zero as time t goes to infinity. However, the number of recovered humans are quickly increased by increasing the coverage levels of proper quarantine. An increment in number of recovered and the decrease in number of infected humans will significantly reduce the risk of Corona virus disease. Consequently, the host population will become healthy if the coverage levels and effectiveness of implemented quarantine strategy are high. Additionally, we will discuss statistically the joint variability among all populations.

The paper has been organized into ten sections. The derivation of proposed compartmental model is outlined in section 2. The fractional form of the proposed Corona model is given in section 3. In section 4, we explore in details some mathematical properties of the fractional model. Existence and uniqueness of solutions is proved using some fixed point theorem. The proofs of the positivity and boundedness of solutions are done in section 5. Disease free and endemic equilibrium points of the fractional model are obtained in the section 6. Threshold parameter \mathcal{R}_0 is calculated analytically in section 7. We proved the local and global stability of equilibrium points in section 8. Numerical analysis of the fractional model along with graphical illustrations and necessary discussions is performed in section 9. We have developed a Toufik-Atangana numerical scheme for the fractional model to approximate its solution. Moreover, the computational advantages of proposed Toufik-Atangana scheme are discussed in details. We conclude the present work in section 10.

2. Formulation of the integer order model

Nonlinear models [30–37] describing the transmission dynamics of deadly Corona virus play a significant role in the disciplines of epidemiology. These models assist the public health planners and policy makers in many ways. There is a series of various mathematical models in the existing literature [38–53] with different assumptions depending on the propagation mechanism of Corona virus disease. In this section, we have considered a new real world integer order SEIQR Corona epidemic model [11]. All of our efforts will be to study the impact of proposed quarantine strategy on the dynamics of COVID-19 through the fractional form of this model. Detail of variables used to model the flow pattern of Corona pandemic within the human population is given below: The total human population $N(t)$ is separated into five classes denoted by $S(t)$, $E(t)$, $I(t)$, $Q(t)$ and $R(t)$, all remain disjoint over the continuous time evolution. The first class denoted by $S(t)$ includes the number of susceptible humans at time instant t . The second class which represents the number of exposed humans at time instant t is denoted by $E(t)$. The third epidemic class characterizes number of infected humans at time instant t who are able to spread the infection and is denoted by $I(t)$. It is not possible for

any regime to quarantine the entire population, hence not everyone will be able to be quarantined right away. The fourth class which is denoted by $Q(t)$ gives the number of exposed humans who have been quarantined at time instant t . Finally, the fifth class denoted by $R(t)$ represents the number of recovered humans who have developed complete immunity from the Corona virus disease at time instant t . We assume that the recovered humans will remain in the class $R(t)$ throughout their life. Thus S, E, I, Q , and R are the variables for the Corona model (1). All of these variables are assumed to be real valued continuously differentiable functions defined on the interval $[0, +\infty)$ of t values. Fig. 1 describes a schematic which represents the involvement of Corona infection in the human population. The description of employed parameters and their physical relevance are provided in Table 1.

The SEIQR mathematical model representing the flow pattern of Corona virus in a community is given by the following system of ordinary differential equations:

$$\frac{dS}{dt} = \lambda - \beta_1 SI - \beta_2 SE - \mu S, \tag{1a}$$

$$\frac{dE}{dt} = \beta_1 SI + \beta_2 SE - (q_1 + \mu + \alpha + k)E, \tag{1b}$$

$$\frac{dI}{dt} = \alpha E - (\mu + d_1 + r)I, \tag{1c}$$

$$\frac{dQ}{dt} = q_1 E - (q + \mu + d_2)Q, \tag{1d}$$

$$\frac{dR}{dt} = kE + rI + qQ - \mu R, \tag{1e}$$

We have assumed that all the initial conditions and involved parameters of the system are non-negative.

3. Fractional model

We first recall some fundamental notions related to Caputo [54] and Atangana-Baleanu fractional derivatives [57].

Definition 1. Let Ω be an open subset of \mathbb{R} and $p \in [1, \infty)$, the Sobolev space $H^p(\Omega)$ is defined by

$$H^p(\Omega) = \{v \in L^2(\Omega) : D^\alpha v \in L^2, \text{ for all } |\alpha| \leq p\}.$$

Table 1 Representation of involved parameters and their estimated values.

Parameter	Interpretations	Value (day ⁻¹)	Source
λ	Natality or recruitment rate	0.5000	[11,12]
μ	Natural mortality rate of each class	0.5000	[11,12]
k	Rate of transfer of exposed humans into recovered class	0.00398	[11,12]
r	Rate of transfer of infected humans into recovered class	0.09871	[11,12]
α	Rate of transfer of exposed humans into infected class	0.085432	[11,12]
d_1	Mortality rate of infected humans due to COVID-19	0.0047876	[11,12]
d_2	Mortality rate of quarantined humans due to COVID-19	0.000001231	[11,12]
q_1	Rate of transfer of exposed humans into quarantine class	0.001	[11]
q	Rate of transfer of quarantined humans into recovered class	1.05	[11]
β_1	Rate of transfer of susceptible humans into infected class	1.05	[11]
β_2	Rate of transfer of susceptible humans into exposed class	0.005 (F_0^*)	[11]
		1.05 (F_1^*)	[11]

Definition 2. The Caputo fractional derivative of order $\rho \in (n - 1, n), n \in \mathbb{N}$, for a given function $v : [a, b] \rightarrow \mathbb{R}$, is stated [47,54] as:

$${}_a^C D_t^\rho v(t) = \frac{1}{\Gamma(n - \rho)} \int_a^t v^n(\xi)(t - \xi)^{n-\rho-1} d\xi. \tag{2}$$

Clearly, ${}_a^C D_t^\rho v(t)$ tends to $\dot{v}(t) = \frac{dv}{dt}$ as $\rho \rightarrow 1$.

Definition 3. If $v : [a, b] \rightarrow \mathbb{N}$ is a differentiable function defined on $[a, b]$ such that $v \in H^1(a, b), b > a$ and $\rho \in (0, 1]$, then the Atangana-Baleanu fractional derivative of v in Caputo sense (ABC) [57] is defined as:

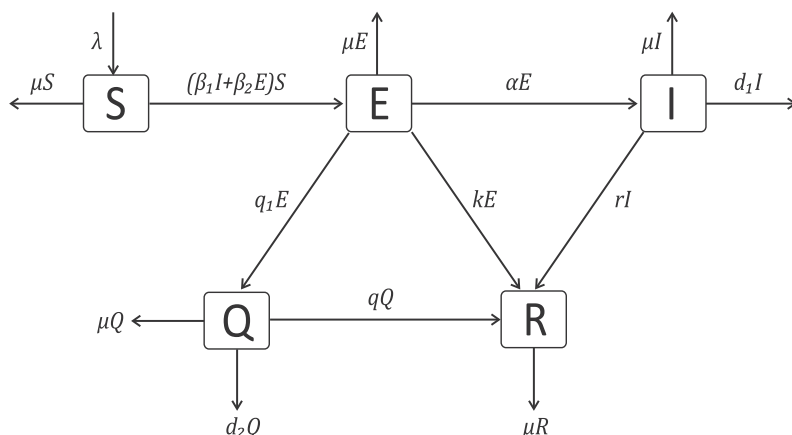


Fig. 1 A schematic representation of the disease dynamics in a compartmental model (1).

$${}_a^{ABC} D_t^\rho v(t) = \frac{B(\rho)}{1-\rho} \int_a^t \dot{v}(\xi) E_\rho \left[-\rho \frac{(t-\xi)^\rho}{1-\rho} \right] d\xi, \tag{1c}$$

where E_ρ is the well-known one parameter Mittag-Leffler function of form:

$$E_\rho(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\rho n + 1)},$$

and $B(\rho) > 0$ is a normalization function satisfying $B(0) = B(1) = 1$.

Definition 4. The Laplace Transform [57,58] of (1c) is defined as follows:

$$\begin{aligned} \mathcal{L} [{}_a^{ABC} D_t^\rho v(t)](s) &= \frac{B(\rho)}{1-\rho} \mathcal{L} \left[\int_a^t \dot{v}(\xi) E_\rho \left\{ -\rho \frac{(t-\xi)^\rho}{1-\rho} \right\} d\xi \right] \\ &= \frac{B(\rho)}{1-\rho} \left[\frac{s^\rho \mathcal{L}\{v(t)\}(s) - s^{\rho-1} v(0)}{s^\rho + \frac{\rho}{1-\rho}} \right]. \end{aligned} \tag{1d}$$

Definition 5. The associated fractional integral [47,57] with non-local kernel is defined by

$${}_a^{ABC} D_t^\rho v(t) = \frac{1-\rho}{B(\rho)} v(t) + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_a^t v(\xi) (t-\xi)^{\rho-1} d\xi. \tag{1e}$$

For more details about the properties of Atangana-Baleanu fractional derivative, we refer the readers to [44–52].

Theorem 1. The fractional order differential equation

$${}_a^{ABC} D_t^\rho v(t) = f(t), \tag{1f}$$

has the following unique solution [28,29,57]

$$v(t) = v(a) + \frac{1-\rho}{B(\rho)} f(t) + \frac{\rho}{B(\rho)\Gamma(\rho)} \int_a^t f(\xi) (t-\xi)^{\rho-1} d\xi.$$

Generally, to analyze the dynamical behavior of an infectious disease, the classical integer order models are neither reliable nor more helpful. However, the fractional order models show cooperatively better fit to the real data. Hence, to generate the system (1) for the present work, we replace the classical integer order time derivative D_t by fractional Atangana-Baleanu derivative ${}_0^{ABC} D_t^\rho$ given in (1c). Through this reformulation, we will be able to observe memory effects and gain more insights about the pandemic. For $t \geq 0$ and $\rho \in (0, 1]$, the proposed nonlinear fractional order Corona virus model in the sense of ABC-fractional operator is given as:

$${}_0^{ABC} D_t^\rho S(t) = \lambda - \beta_1 S(t)I(t) - \beta_2 S(t)E(t) - \mu S(t), \tag{7a}$$

$${}_0^{ABC} D_t^\rho E(t) = \beta_1 S(t)I(t) + \beta_2 S(t)E(t) - (q_1 + \mu + \alpha + k)E(t), \tag{7b}$$

$${}_0^{ABC} D_t^\rho I(t) = \alpha E(t) - (\mu + d_1 + r)I(t), \tag{7c}$$

$${}_0^{ABC} D_t^\rho Q(t) = q_1 E(t) - (q + \mu + d_2)Q(t), \tag{7d}$$

$${}_0^{ABC} D_t^\rho R(t) = kE(t) + rI(t) + qQ(t) - \mu R(t), \tag{7e}$$

and it can be written in the following simple form:

$${}_0^{ABC} D_t^\rho v(t) = F(t, v(t)), \quad 0 < t < T < +\infty \tag{8a}$$

along with

$$v(0) = v_0, \tag{8b}$$

where $v : [0, +\infty) \rightarrow \mathbb{R}^5$ and $F : \mathbb{R}^5 \rightarrow \mathbb{R}^5$ are vector valued functions such that

$$v(t) = \begin{pmatrix} S(t) \\ E(t) \\ I(t) \\ Q(t) \\ R(t) \end{pmatrix}$$

and

$$F(v(t)) = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{pmatrix} = \begin{pmatrix} \lambda - \beta_1 SI - \beta_2 SE - \mu S \\ \beta_1 SI + \beta_2 SE - (q_1 + \mu + \alpha + k)E \\ \alpha E - (\mu + d_1 + r)I \\ q_1 E - (q + \mu + d_2)Q \\ kE + rI + qQ - \mu R \end{pmatrix}$$

respectively. Clearly, $F_i; i = 1, 2, 3, 4, 5$, are functions of t, S, E, I, Q , and R . In the next section we investigate the existence and uniqueness of the solution for system (8) by fixed point theorem.

4. Existence and uniqueness of solution

In this section, we prove existence and uniqueness [45,46] of solution with the help of theorems.

Theorem 2. The function F in (8) is Lipschitz continuous in v .

Proof. On inspection, the components, F_1, F_2, F_3, F_4 , and F_5 of the function F are continuously differentiable functions of t . Since the set $\mathcal{Q}(v_1, v_2, \alpha) = \{v_1 + \alpha(v_2 - v_1) : \alpha \in [0, 1], v_1, v_2 \in \mathbb{R}^5\}$ is a line segment in \mathbb{R}^5 which joins the point v_1 to v_2 as α varies from 0 to 1, and is a compact subset of \mathbb{R}^5 . Thus, we may choose a point $w \in \mathcal{Q}(v_1, v_2, \alpha)$ using Mean Value Theorem, such that

$$\|F(v_2) - F(v_1)\|_\infty = \|\dot{F}(w; v_2 - v_1)\|_\infty, \tag{9}$$

where $\dot{F}(w; v_2 - v_1)$ is the directional derivative of function F at w in the direction of $v_2 - v_1$. However,

$$\begin{aligned} \|\dot{F}(w; v_2 - v_1)\|_\infty &= \left\| \sum_{i=1}^5 (\mathbf{J}F_i(w) \cdot (v_2 - v_1)) \mathbf{e}_i \right\|_\infty \\ &\leq \left\| \sum_{i=1}^5 \mathbf{J}F_i(w) \right\| \|v_2 - v_1\|_\infty, \end{aligned}$$

where \mathbf{e}_i is the i th coordinate unit vector in \mathbb{R}^5 and $\mathbf{J}F_i(w)$ is a bounded linear operator. As all the partial derivatives of function $F_i, i = 1, 2, 3, 4, 5$, are bounded, there exists a number $\mathcal{M} > 0$ such that

$$\left\| \sum_{i=1}^5 \mathbf{J}F_i(w) \right\| \leq \mathcal{M},$$

for all $\mathcal{Q}(v_1, v_2, \alpha) \subseteq \mathbb{R}^5$. Consequently, we have

$$\|F(v_2) - F(v_1)\|_\infty \leq \mathcal{M} \|v_2 - v_1\|_\infty \tag{10}$$

and hence F is Lipschitz continuous. \square

Theorem 3. Suppose that the function $F(t, v(t))$ satisfies the Lipschitz condition

$$\|F(v_2) - F(v_1)\|_\infty \leq \mathcal{M} \|v_2 - v_1\|_\infty,$$

then the problem (8) has a unique solution for the specific initial population if $F(0, v(0)) = 0$ and

$$\mathcal{M} \left(\frac{1 - \rho}{B(\rho)} + \frac{\rho}{B(\rho)\Gamma(\rho)} T^* \right) < 1.$$

Proof. Suppose that $F(0, v(0)) = 0$ for a specific initial data. We will prove that the function $v(t)$ satisfies Eqs. (8a) and (8b) if and only if it satisfies the relation

$$v(t) = {}_0^{ABC}I_t^\rho F(t, v(t)). \tag{11}$$

Let $v(t)$ satisfies Eq. (8a). Applying Atangana-Beleanu fractional integral (1e) to both sides of system (8a), that is

$${}_0^{ABC}I_t^\rho [{}_0^{ABC}D_t^\rho v(t)] = {}_0^{ABC}I_t^\rho F(t, v(t)).$$

On making use of the Proposition 3.4 in [57], we obtain the following non-linear Volterra integral equation

$$v(t) = v(0) + \frac{1 - \rho}{B(\rho)} F(t, v(t)) + \frac{\rho}{B(\rho)\Gamma(\rho)} \times \int_0^t (t - \xi)^{\rho-1} F(\xi, v(\xi)) d\xi. \tag{12}$$

Since $F(0, v(0)) = 0$, and from Eq. (8b), $v(0) = v_0$, then Eq. (11) is satisfied. Conversely, let $v(t)$ satisfies Eq. (11), then by using the fact $F(0, v(0)) = 0$, it is obvious that $v(0) = v_0$. Implies that the solution representation satisfies the initial data.

Secondly, we prove the uniqueness of solution with the condition

$$\mathcal{M} \left(\frac{1 - \rho}{B(\rho)} + \frac{\rho}{B(\rho)\Gamma(\rho)} T^* \right) < 1.$$

Let $\mathcal{J} = (0, T)$ and consider the operator $\psi : C(\mathcal{J}, \mathbb{R}^5) \rightarrow C(\mathcal{J}, \mathbb{R}^5)$ defined by

$$\psi[v(t)] = v(0) + \frac{1 - \rho}{B(\rho)} F(t, v(t)) + \frac{\rho}{B(\rho)\Gamma(\rho)} \times \int_0^t (t - \xi)^{\rho-1} F(\xi, v(\xi)) d\xi.$$

Eq. (12) becomes

$$v(t) = \psi[v(t)]. \tag{13}$$

The supremum norm on $\mathcal{J}, \|\cdot\|_{\mathcal{J}}$ is:

$$\|v(t)\|_{\mathcal{J}} = \sup_{t \in \mathcal{J}} \|v(t)\|.$$

Clearly, $C(\mathcal{J}, \mathbb{R}^5)$ along with norm $\|\cdot\|_{\mathcal{J}}$ construct a Banach space. Furthermore, we can show an important inequality proved in the following:

Let us define an integral operator $\varphi : C[0, T] \rightarrow C[0, T]$ defined by

$$\varphi v = w,$$

where

$$w = w(t) = \int_0^t k(t, \xi) v(\xi) d\xi.$$

Here $k(t, \xi) : \mathcal{J} \times \mathcal{J} \rightarrow \mathbb{R}$ is a given function called the kernel of φ . It is assumed to be a continuous function on the closed square $G = \mathcal{J} \times \mathcal{J}$ in the $t\xi$ -plane, and hence bounded. Then there exist a real number k_0 such that

$$|k(t, \xi)| \leq k_0, \quad (k, \xi) \in \mathcal{J} \times \mathcal{J}.$$

Furthermore,

$$\sup_{t, \xi \in \mathcal{J}} |k(t, \xi)| \leq k_0,$$

implies that

$$\|k(t, \xi)\| \leq k_0, \quad (k, \xi) \in \mathcal{J} \times \mathcal{J}.$$

Our target is to show that φ is bounded. Consider

$$\|\varphi v\|_{\mathcal{J}} = \left\| \int_0^t k(t, \xi) v(\xi) d\xi \right\|_{\mathcal{J}}.$$

Since $k(t, \xi)$ and $v(\xi)$ are continuous, therefore

$$\begin{aligned} \left\| \int_0^t k(t, \xi) v(\xi) d\xi \right\|_{\mathcal{J}} &= \sup_{t \in \mathcal{J}} \left\| \int_0^t k(t, \xi) v(\xi) d\xi \right\|_{\mathcal{J}} \\ &\leq \sup_{t \in \mathcal{J}} \int_0^t \|k(t, \xi)\| \|v(\xi)\| d\xi. \end{aligned}$$

Hence

$$\left\| \int_0^t k(t, \xi) v(\xi) d\xi \right\|_{\mathcal{J}} \leq T^* \|k(t, \xi)\|_{\mathcal{J}} \|v(t)\|_{\mathcal{J}}. \tag{14}$$

with $v(t) \in C(\mathcal{J}, \mathbb{R}^5), k(t, \xi) \in C(\mathcal{J}^2, \mathbb{R})$ such that

$$\|k(t, \xi)\|_{\mathcal{J}} = \sup_{t, \xi \in \mathcal{J}} |k(t, \xi)|.$$

On make use of the definition given in (13), we get

$$\begin{aligned} \|\psi[v_1(t)] - \psi[v_2(t)]\|_{\mathcal{J}} &\leq \left\| \frac{\rho}{B(\rho)\Gamma(\rho)} \int_0^t (t - \xi)^{\rho-1} (F(t, v_1) - F(t, v_2)) d\xi \right. \\ &\quad \left. + \frac{1 - \rho}{B(\rho)} (F(t, v_1) - F(t, v_2)) \right\|_{\mathcal{J}}. \end{aligned}$$

Applying the Lipschitz condition (10) along with the result in (14), we deduce that

$$\|\psi[v_1(t)] - \psi[v_2(t)]\|_{\mathcal{J}} \leq \mathcal{M} \left(\frac{1 - \rho}{B(\rho)} + \frac{\rho}{B(\rho)\Gamma(\rho)} T^* \right) \|v_1(t) - v_2(t)\|_{\mathcal{J}}.$$

Thus, the operator ψ will be a contraction if the constant

$$\mathcal{M} \left(\frac{1 - \rho}{B(\rho)} + \frac{\rho}{B(\rho)\Gamma(\rho)} T^* \right),$$

is less than one. Hence, by the Banach Contraction Principle there exists a unique solution of the model (8). \square

5. Properties

In this section, we discuss some important properties of the model (7) or equivalently system (8) such as boundedness and positivity of the solutions for $t \geq 0$. Thus the obtained solution will be realistic and physically realizable and the problem becomes well-posed. All these computations are done in the following steps.

5.1. Invariant region

In this subsection, we determine the boundary of solutions (S, E, I, Q, R) of the non-linear system (7) with a non-negative initial data. Our main target is to show that the obtained feasible region in \mathbb{R}_+^5 which is positively invariant with respect to the fractional model (7).

Theorem 4. *The epidemiologically feasible region of Atangana-Baleanu fractional model (7) is given by*

$$\Pi = \left\{ (S, E, I, Q, R) \in \mathbb{R}_+^5 : 0 \leq N \leq \frac{\lambda}{\mu}; S, E, I, Q, R \geq 0 \right\}. \quad (8o)$$

The existence and uniqueness of the solution of model (7) are proved in the previous section, it remains to show that the set Π is positively invariant with initial conditions $S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, Q(0) \geq 0$ and $R(0) \geq 0$. The following theorem will be used for the proof of [Theorem 4](#).

Theorem 5. *The solutions of system (7) are bounded.*

Proof. Addition of all the five equations in the fractional model (7) gives

$$\begin{aligned} {}_0^{ABC}D_t^\rho N(t) &= {}_0^{ABC}D_t^\rho S(t) + {}_0^{ABC}D_t^\rho E(t) + {}_0^{ABC}D_t^\rho I(t) + {}_0^{ABC}D_t^\rho Q(t) \\ &+ {}_0^{ABC}D_t^\rho R(t), = \lambda - \mu N(t) - d_1 I(t) - d_2 Q(t), \end{aligned} \quad (8p)$$

where $N(t) = S(t) + E(t) + I(t) + Q(t) + R(t), t \geq 0$, is the total population under consideration. Clearly

$$\lambda - \mu N(t) - d_1 I(t) - d_2 Q(t) \leq \lambda - \mu N(t)$$

Therefore, from Eq. (8p) it follows that

$${}_0^{ABC}D_t^\rho N(t) \leq \lambda - \mu N(t).$$

Applying the Laplace transform on both sides of above inequality, we obtain

$$\mathcal{L}[{}_0^{ABC}D_t^\rho N(t)](s) = \frac{\lambda}{s} - \mu \mathcal{L}[N(t)](s),$$

$$\frac{B(\rho)s^\rho}{\rho + (1-\rho)s^\rho} N(s) + \mu N(s) \leq \lambda s^{\rho-(\rho+1)} + \frac{B(\rho)N(0)s^{\rho-1}}{\rho + (1-\rho)s^\rho}$$

where $N(s) = [N(t)](s)$ and $N(0)$ represents the initial value of the total population. Implies

$$N(s) \leq \frac{\lambda[(1-\rho)s^\rho + \rho]s^{\rho-(\rho+1)} + B(\rho)s^{\rho-1}N(0)}{\mu\rho + \rho(1-\rho)s^\rho + B(\rho)s^\rho}$$

$$\begin{aligned} N(s) &\leq \frac{\lambda\rho}{(1-\rho)\mu + B(\rho)} \frac{s^{\rho-(\rho+1)}}{s^\rho + \frac{\rho\mu}{(1-\rho)\mu + B(\rho)}} \\ &+ \left[\frac{\lambda(1-\rho)}{(1-\rho)\mu + B(\rho)} + \frac{B(\rho)N(0)}{(1-\rho)\mu + B(\rho)} \right] \frac{s^{\rho-1}}{s^\rho + \frac{\rho\mu}{B(\rho) + (1-\rho)\mu}}, \end{aligned}$$

Applying the inverse Laplace, we arrive at

$$\begin{aligned} N(t) &\leq \frac{\lambda\rho t^\rho}{(1-\rho)\mu + B(\rho)} E_{\rho, \rho+1}(-\Omega t^\rho) \\ &+ \left[\frac{\lambda(1-\rho)}{(1-\rho)\mu + B(\rho)} + \frac{B(\rho)N(0)}{B(\rho) + (1-\rho)\mu} \right] E_{\rho, 1}(-\Omega t^\rho), \end{aligned} \quad (8q)$$

where $\Omega = \frac{\rho\mu}{B(\rho) + (1-\rho)\mu}$, and $E_{\alpha, \beta}$ is the Mittag-Leffler function with two parameters $\alpha > 0$ and $\beta > 0$ may be defined by the series

$$E_{\alpha, \beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)},$$

whose Laplace transform is

$$\mathcal{L}[t^{\beta-1} E_{\alpha, \beta}(\pm \Lambda t^\alpha)] = \frac{s^{-\beta}}{s^\alpha \mp \Lambda},$$

provided that $s > |\Lambda|^{1/\alpha}$. For $\alpha, \beta > 0$, the Mittag-Leffler function satisfies

$$E_{\alpha, \beta}(z) = \frac{1}{z} \left[E_{\alpha, \beta-\alpha}(z) - \frac{1}{\Gamma(\beta-\alpha)} \right],$$

and for the case $\alpha = \rho, \beta = \rho + 1$ and $z = -\Omega t^\rho$, we have

$$E_{\rho, \rho+1}(-\Omega t^\rho) = \frac{1}{\Omega t^\rho} [1 - E_{\rho, 1}(-\Omega t^\rho)]. \quad (8r)$$

The Mittag-Leffler function is bounded for all $t > 0$, possess an asymptotic behavior [57], introducing the relation (8r) in inequality (8q), it is obvious that $N(t) \leq \frac{\lambda}{\mu}$ as $t \rightarrow \infty$. Thus $N(t)$ and all other variables of the model (7) are bounded in a region Π .

The solution remains $\mathbf{x} = 0$ for all $t > 0$ if $\mathbf{x}_0 = 0$. Moreover, for any nonnegative set of initial conditions in Π , every solution of model (7) in

$$\mathbb{R}_+^5 = \{ \mathbf{y} \in \mathbb{R}^5 : \mathbf{y} \geq 0 \}, \quad \mathbf{y}(t) = (S(t), E(t), I(t), Q(t), R(t))^T,$$

approach asymptotically in finite time t , enters and remains in the region Π . Implies the region Π attracts all solutions in \mathbb{R}_+^5 . Thus the closed set Π is a positively invariant [12] for our system (7). \square

5.2. Positivity of solutions

We have to prove that the involved state variables S, E, I, Q and R are non-negative for any $t > 0$, that is the solution will remain positive forever corresponding to any non-negative initial data in \mathbb{R}_+^5 . This property is required to show our model physically realizable.

Theorem 6. *The solution space (S, E, I, Q, R) of the system (7) will remain positive forever with any positive initial costs.*

Proof. First equation of the model (7) is rearranged to give

$${}_0^{ABC}D_t^\rho S(t) \geq -[\mu + \beta_1 I(t) + \beta_2 E(t)]S(t).$$

Since all the solutions are bounded, let $E(t)$ and $I(t)$ are bounded by σ_1 and σ_2 respectively. Then,

$${}^0_{ABC}D_t^\rho S(t) \geq -c S(t), \tag{8s}$$

where $c = \mu + \beta_1\sigma_2 + \beta_2\sigma_1$ is a constant. Applying the Laplace transform in (8s), we deduce

$$\frac{B(\rho)s^\rho}{\rho + (1-\rho)s^\rho} \mathcal{L}[S(t)](s) - \frac{B(\rho)s^{\rho-1}}{\rho + (1-\rho)s^\rho} S(0) \geq -c \mathcal{L}[S(t)](s),$$

$$[B(\rho)s^\rho + c\rho + c(1-\rho)s^\rho] \mathcal{L}[S(t)](s) \geq B(\rho)s^{\rho-1} S(0),$$

$$\mathcal{L}[S(t)](s) \geq \frac{B(\rho)s^{\rho-1} S(0)}{B(\rho) + c(1-\rho)} \frac{s^{\rho-1}}{s^\rho + \frac{c\rho}{B(\rho) + c(1-\rho)}}.$$

Introducing inverse Laplace transform in the above inequality, we obtain

$$S(t) \geq \frac{B(\rho)s^{\rho-1} S(0)}{B(\rho) + c(1-\rho)} E_{\rho,1} \left(-\frac{c\rho}{B(\rho) + c(1-\rho)} t^\rho \right). \tag{8t}$$

Since both the quantities on right hand side of (8t) are positive. Implies the solution $S(t)$ is positive for all $t \geq 0$. Similarly, we can easily prove that $E(t) \geq 0, I(t) \geq 0, Q(t) \geq 0$ and $R(t) \geq 0$ for all $t \geq 0$ corresponding to any non-negative initial data. Thus the solutions in \mathbb{R}_+^5 remain positive forever. \square

Proofs of existence and uniqueness, boundedness and the positivity of all solutions show that the problem (8) is mathematically well-posed.

6. Equilibrium points

Equilibrium points of the proposed fractional model (7) are obtained by solving the system

$$\begin{aligned} {}^0_{ABC}D_t^\rho S(t) &= {}^0_{ABC}D_t^\rho E(t) = {}^0_{ABC}D_t^\rho I(t) = {}^0_{ABC}D_t^\rho Q(t) \\ &= {}^0_{ABC}D_t^\rho R(t) = 0. \end{aligned}$$

After simple calculations, the SEIQR model (7) has a unique non-negative Corona free equilibrium at the point given by

$$F_0^* = (S_0^*, E_0^*, I_0^*, Q_0^*, R_0^*) = \left(\frac{\lambda}{\mu}, 0, 0, 0, 0 \right),$$

and a unique non-negative Corona present equilibrium at the point given by

$$F_1^* = (S_1^*, E_1^*, I_1^*, Q_1^*, R_1^*),$$

in the epidemiological region Π , where

$$S_1^* = \frac{(q_1+k+\alpha+\mu)(r+\mu+d_1)}{\beta_1\alpha+\beta_2(r+\mu+d_1)} > 0,$$

$$E_1^* = \frac{(\lambda-\mu S_1^*)(r+\mu+d_1)}{[\beta_1\alpha+\beta_2(r+\mu+d_1)]S_1^*} > 0,$$

$$I_1^* = \frac{\alpha E_1^*}{r+\mu+d_1} > 0,$$

$$Q_1^* = \frac{q_1 E_1^*}{q+\mu+d_2} > 0,$$

$$R_1^* = \frac{1}{\mu} (kE_1^* + rI_1^* + qQ_1^*) > 0.$$

7. Threshold parameter \mathcal{R}_0

We can determine the dynamics of mathematical models by computing the value of threshold parameter, usually denoted by \mathcal{R}_0 . The spread of any infectious disease begins when an infected person enters into a class of completely susceptible humans. Therefore, a threshold parameter gives the total number of new infections produced by an infected human at $t \geq 0$. It works confidentially to predict the dynamical behavior of a model. Implies it provides a better information about the future spread or control of any disease.

The value of threshold parameter for the model (7) is computed by using the standard next generation matrix approach [12,62–64]. If $v = (E, I, Q, S, R)^T \in \mathbb{R}_+^5$, then the model (7) can be written as

$${}^0_{ABC}D_t^\rho v = \mathcal{F}(v) - \mathcal{G}(v)$$

where

$$\mathcal{F}(v) = \begin{pmatrix} \beta_1 SI + \beta_2 SE \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\mathcal{G}(v) = \begin{pmatrix} (q_1 + \mu + \alpha + k)E \\ (\mu + d_1 + r)I - \alpha E \\ (q + \mu + d_2)Q - q_1 E \\ \mu S - \lambda \\ \mu R - kE - rI - qQ \end{pmatrix}.$$

The functions, F and V for the rate of new infection terms entering, and the rate of transfer into and out of the exposed, infected, and the quarantined class, respectively are as follows:

$$F = \begin{pmatrix} \beta_1 SI + \beta_2 SE \\ 0 \\ 0 \end{pmatrix},$$

whereas the matrix G representing the transition rate of existing or transported cases is computed as

$$G = \begin{pmatrix} (q_1 + k + \alpha + \mu)E \\ (r + \mu + d_1)I - \alpha E \\ (q + \mu + d_2)Q - q_1 E \end{pmatrix}.$$

The jacobian of the matrices F and G evaluated at point F_0^* are

$$\begin{aligned} \bar{F} &= \begin{pmatrix} \frac{\partial \mathcal{F}_1}{\partial E} & \frac{\partial \mathcal{F}_1}{\partial I} & \frac{\partial \mathcal{F}_1}{\partial Q} \\ \frac{\partial \mathcal{F}_2}{\partial E} & \frac{\partial \mathcal{F}_2}{\partial I} & \frac{\partial \mathcal{F}_2}{\partial Q} \\ \frac{\partial \mathcal{F}_3}{\partial E} & \frac{\partial \mathcal{F}_3}{\partial I} & \frac{\partial \mathcal{F}_3}{\partial Q} \end{pmatrix} = \begin{pmatrix} \frac{\beta_2 \lambda}{\mu} & \frac{\beta_1 \lambda}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \bar{G} &= \begin{pmatrix} \frac{\partial \mathcal{G}_1}{\partial E} & \frac{\partial \mathcal{G}_1}{\partial I} & \frac{\partial \mathcal{G}_1}{\partial Q} \\ \frac{\partial \mathcal{G}_2}{\partial E} & \frac{\partial \mathcal{G}_2}{\partial I} & \frac{\partial \mathcal{G}_2}{\partial Q} \\ \frac{\partial \mathcal{G}_3}{\partial E} & \frac{\partial \mathcal{G}_3}{\partial I} & \frac{\partial \mathcal{G}_3}{\partial Q} \end{pmatrix} \\ &= \begin{pmatrix} q_1 + k + \alpha + \mu & 0 & 0 \\ -\alpha & r + \mu + d_1 & 0 \\ -q_1 & 0 & q + \mu + d_2 \end{pmatrix}. \end{aligned}$$

The maximum absolute eigenvalue of the positive matrix \overline{FG}^{-1} is the value of \mathcal{R}_0 for the model (7). That is,

$$\mathcal{R}_0 = \frac{\lambda[\beta_1\alpha + \beta_2(r + \mu + d_1)]}{(r + \mu + d_1)(q_1 + k + \alpha + \mu)}.$$

8. Stability analysis

In this part, the local and global stabilities of a Corona SEIQR fractional model (7) at both the equilibria are discussed theoretically. Lyapunov function theory as implemented in [12,62–64,66,67] is employed to prove global stability of the model. So, this section is completed in the following way.

8.0.1. Local behavior of the model

Theorem 7. *A Corona free equilibrium F_0^* is locally asymptotically stable (LAS) when $\mathcal{R}_0 < 1$. If $\mathcal{R}_0 > 1$, it is unstable.*

Proof. The Jacobian J for the system (1) at F_0^* can be written as

$$J(F_0^*) = \begin{pmatrix} -\mu & -\beta_2 \frac{\lambda}{\mu} & -\beta_1 \frac{\lambda}{\mu} & 0 & 0 \\ 0 & \beta_2 \frac{\lambda}{\mu} - (q_1 + k + \alpha + \mu) & \beta_1 \frac{\lambda}{\mu} & 0 & 0 \\ 0 & \alpha & -(\gamma + \mu + d_1) & 0 & 0 \\ 0 & q_1 & 0 & -(q + \mu + d_2) & 0 \\ 0 & k & \gamma & q & -\mu \end{pmatrix},$$

Jacobian matrix $J(F_0^*)$ has the following eigenvalues:

$$\lambda_1 = -\mu, \quad (21a)$$

$$\lambda_2 = -\mu, \quad (21b)$$

$$\lambda_3 = -(q + \mu + d_2) \quad (21c)$$

$$\lambda_4 = \beta_2 \frac{\lambda}{\mu} - (q_1 + k + \alpha + \mu), \quad (21d)$$

$$\lambda_5 = \frac{-\frac{\lambda}{\mu}(\gamma + \mu + d_1)(q_1 + k + \alpha + \mu)(\mathcal{R}_0 - 1)}{\beta_2 \frac{\lambda}{\mu} - (q_1 + k + \alpha + \mu)}. \quad (21e)$$

It is to be noted that the parameters in the proposed model are assumed to be positive. Therefore, the eigenvalues λ_1, λ_2 and λ_3 are clearly negative. Indeed the quantities μ , and $q + \mu + d_2$ are strictly positive. Now consider Eq. (21d), that is $\lambda_4 = \beta_2 \frac{\lambda}{\mu} - (q_1 + k + \alpha + \mu)$. Hence $\lambda_4 < 0 \iff \beta_2 \frac{\lambda}{\mu} < (q_1 + k + \alpha + \mu)$, which is true. Lastly, it has been proved that

$$\begin{aligned} \frac{-\lambda}{\mu}(\gamma + \mu + d_1)(q_1 + k + \alpha + \mu) &< 0, \\ \beta_2 \frac{\lambda}{\mu} - (q_1 + k + \alpha + \mu) &< 0. \end{aligned}$$

Therefore, Eq. (21e) implies, $\lambda_5 < 0$ if and only if $\mathcal{R}_0 < 1$. Thus all the eigenvalues of $J(F_0^*)$ when \mathcal{R}_0 is less than unity, are negative and hence F_0^* is locally asymptotically stable. Once the dynamical system is stable at disease free point F_0^* , then there will be no pandemic threat. \square

Theorem 8. *A Corona present equilibrium F_1^* is locally asymptotically stable (LAS) whenever $\mathcal{R}_0 > 1$. Whenever $\mathcal{R}_0 < 1$, it is unstable.*

Proof. The Jacobian for the system (1) at F_1^* can be written as

$$J(F_1^*) = \begin{pmatrix} -\mu & -\beta_2 S_1^* & -\beta_1 S_1^* & 0 & 0 \\ M_{21} & M_{22} & \beta_1 S_1^* & 0 & 0 \\ 0 & \alpha & -(\gamma + \mu + d_1) & 0 & 0 \\ 0 & q_1 & 0 & -(q + \mu + d_2) & 0 \\ 0 & k & \gamma & q & -\mu \end{pmatrix},$$

where

$$M_{21} = \beta_1 I_1^* + \beta_2 E_1^*,$$

$$M_{22} = \beta_2 S_1^* - (q_1 + k + \alpha + \mu).$$

Then the Jacobian matrix $J(F_1^*)$ has the following eigenvalues:

$$\lambda_1 = -\mu < 0, \quad (22a)$$

$$\lambda_2 = -\mu < 0, \quad (22b)$$

$$\lambda_3 = -(q + \mu + d_2) < 0, \quad (22c)$$

$$\lambda_4 = \beta_2 S_1^* - \left[(q_1 + k + \alpha + \mu) + \frac{1}{\mu} \beta_2 S_1^* (\beta_1 I_1^* + \beta_2 E_1^*) \right], \quad (22d)$$

$$\lambda_5 = \frac{\mu(\gamma + \mu + d_1)M_{22} + \mu\alpha\beta_1 S_1^* - [\beta_2(\gamma + \mu + d_1)M_{21}S_1^* + \alpha\beta_1 M_{21}S_1^*]}{M_{21}\beta_2 S_1^* - \mu M_{22}}. \quad (22e)$$

On inspection, it is verified that

$$M_{21}\beta_2 S_1^* - \mu M_{22} > 0,$$

$$\beta_2 S_1^* < (q_1 + k + \alpha + \mu) + \frac{1}{\mu} \beta_2 S_1^* (\beta_1 I_1^* + \beta_2 E_1^*),$$

$$\begin{aligned} \mu(\gamma + \mu + d_1)M_{22} + \mu\alpha\beta_1 S_1^* &< \beta_2(\gamma + \mu + d_1)M_{21}S_1^* \\ &+ \alpha\beta_1 M_{21}S_1^*. \end{aligned}$$

Implies that λ_4 and λ_5 are negative. Hence F_1^* is locally asymptotically stable. \square

8.0.2. Global behavior of the quarantine model

Theorem 9. *A Corona free equilibrium F_0^* is globally asymptotically stable (GAS) in the domain Π when $\mathcal{R}_0 < 1$.*

Proof. Let $S_0^* = \frac{\lambda}{\mu}$. We construct a candidate [48] Lyapunov function $U_0 : \Pi \rightarrow \mathbb{R}$ such that,

$$U_0 = S - S_0^* - S_0^* \ln \frac{S}{S_0^*} + E + I + Q + R.$$

Differentiating U_0 with respect to time t yields

$$\begin{aligned} {}_0^{ABC} D_t^\rho U_0 &= \left(1 - \frac{S_0^*}{S}\right) {}_0^{ABC} D_t^\rho S + {}_0^{ABC} D_t^\rho E + {}_0^{ABC} D_t^\rho I \\ &+ {}_0^{ABC} D_t^\rho Q + {}_0^{ABC} D_t^\rho R. \end{aligned}$$

Substituting the values of fractional derivatives from system (1), we have

$${}^0_{ABC}D_t^\rho U_0 = \left(1 - \frac{S_0^*}{S}\right)[\lambda - \beta_1 SI - \beta_2 SE - \mu S] + [\beta_1 SI + \beta_2 SE - (q_1 + k + \alpha + \mu)E] + [\alpha E - (\gamma + \mu + d_1)I] + [q_1 E - (q + \mu + d_2)Q] + [kE + \gamma I + qQ - \mu R]$$

Implies that

$${}^0_{ABC}D_t^\rho U_0 = \frac{-\mu}{S}(S - S_0^*)^2 - \mu(E + I + Q + R) + \lambda_n \left(\frac{S_0^*}{S}\right) - (d_1 I + d_3 Q),$$

where

$$\lambda_n = (\beta_1 I + \beta_2 E)S.$$

Clearly, $\lambda_n \geq 0$, therefore

$${}^0_{ABC}D_t^\rho U_0 \leq \frac{-\mu}{S}(S - S_0^*)^2 - \mu(E + I + Q + R) - (d_1 I + d_3 Q),$$

Clearly, ${}^0_{ABC}D_t^\rho U_0 \leq 0$ for all $(S, E, I, Q, R) \in \mathbb{R}_+^5$. It can be noticed that ${}^0_{ABC}D_t^\rho U_0 = 0$ if and only if $S = S_0^*$, $E = E_0^* = 0$, $Q = Q_0^*$, $I = I_0^* = 0$, $R = R_0^*$. Hence by LaSalle’s invariance principle [69], F_0^* is globally asymptotically stable in Π . As a result, Corona virus disease disappears from the human population. □

Theorem 10. A Corona present steady state F_1^* is globally asymptotically stable (GAS) in the feasible region Π if $\mathcal{R}_0 > 1$.

Proof. To show the global stability of a Corona present steady state F_1^* , we construct a proposed [48] Lyapunov function $U_1 : \Pi \rightarrow \mathbb{R}$ such that

$$U_1 = S - S_1^* - S_1^* \ln \frac{S}{S_1^*} + E - E_1^* - E_1^* \ln \frac{E}{E_1^*} + Q - Q_1^* - Q_1^* \ln \frac{Q}{Q_1^*} + I - I_1^* - I_1^* \ln \frac{I}{I_1^*} + R - R_1^* - R_1^* \ln \frac{R}{R_1^*}.$$

The Atangana-Baleanu fractional derivative of U_1 with respect to time t can be written as

$${}^0_{ABC}D_t^\rho U_1 = \left(1 - \frac{S_1^*}{S}\right) {}^0_{ABC}D_t^\rho S + \left(1 - \frac{E_1^*}{E}\right) {}^0_{ABC}D_t^\rho E + \left(1 - \frac{I_1^*}{I}\right) {}^0_{ABC}D_t^\rho I + \left(1 - \frac{Q_1^*}{Q}\right) {}^0_{ABC}D_t^\rho Q + \left(1 - \frac{R_1^*}{R}\right) {}^0_{ABC}D_t^\rho R.$$

Using equations given in the model (1), we obtain that

$${}^0_{ABC}D_t^\rho U_1 = \left(1 - \frac{S_1^*}{S}\right)[\lambda - (\mu + \beta_1 I + \beta_2 E)(S - S_1^*) - (\mu + \beta_1 I + \beta_2 E)S_1^*] + \left(1 - \frac{E_1^*}{E}\right)[\beta_1 SI + \beta_2 SE - (q_1 + k + \alpha + \mu)(E - E_1^*) - (q_1 + k + \alpha + \mu)E_1^*] + \left(1 - \frac{I_1^*}{I}\right)[\alpha E - (\gamma + \mu + d_1)(I - I_1^*) - (\gamma + \mu + d_1)I_1^*] + \left(1 - \frac{Q_1^*}{Q}\right)[q_1 E - (q + \mu + d_2)(Q - Q_1^*) - (q + \mu + d_2)Q_1^*] + \left(1 - \frac{R_1^*}{R}\right)[kE + \gamma I + qQ - \mu(R - R_1^*) - \mu R_1^*].$$

A simple calculation yields

$${}^0_{ABC}D_t^\rho U_1 = \sigma_1 - \sigma_2,$$

where

$$\sigma_1 = \lambda + (\mu + \beta_1 I + \beta_2 E) \frac{(S_1^*)^2}{S} + (\beta_1 I + \beta_2 E)S + (q_1 + k + \alpha + \mu) \frac{(E_1^*)^2}{E} + (\gamma + \mu + d_1) \frac{(I_1^*)^2}{I} + (\alpha + q_1)E + (q + \mu + d_2) \frac{(Q_1^*)^2}{Q} + (kE + \gamma I + qQ) + \mu \frac{(R_1^*)^2}{R},$$

and

$$\sigma_2 = (\mu + \beta_1 I + \beta_2 E) \frac{(S - S_1^*)^2}{S} + (\mu + \beta_1 I + \beta_2 E)S_1^* + (q_1 + k + \alpha + \mu) \frac{(E - E_1^*)^2}{E} + \lambda \frac{S_1^*}{S} + (\beta_1 I + \beta_2 E)S \frac{E_1^*}{E} + (\gamma + \mu + d_1) \frac{(I - I_1^*)^2}{I} + (q + \mu + d_2) \frac{(Q - Q_1^*)^2}{Q} + \mu \frac{(R - R_1^*)^2}{R} + (q_1 + k + \alpha + \mu)E_1^* + (\gamma + \mu + d_1)I_1^* + \alpha E \frac{I_1^*}{I} + (q + \mu + d_2)Q_1^* + q_1 E \frac{Q_1^*}{Q} + \mu R_1^* + (kE + \gamma I + qQ) \frac{R_1^*}{R}.$$

Since all the parameters used in the model (1) are non-negative, we have ${}^0_{ABC}D_t^\rho U_1 \leq 0$ for $\sigma_1 \leq \sigma_2$. The equality ${}^0_{ABC}D_t^\rho U_1 = 0$ holds if and only if $S = S_1^*$, $E = E_1^*$, $I = I_1^*$, $Q = Q_1^*$, $R = R_1^*$. Implies $\{F_1^*\}$ is the greatest invariant set contained in

$$N_1 = \{(S, E, I, Q, R) \in \Pi : {}^0_{ABC}D_t^\rho U_1 = 0\}.$$

Hence by LaSalle’s invariance principle [69], it is concluded that F_1^* is globally asymptotically stable in Π . Global stability of F_1^* enables that COVID-19 will persist in the human population and eventually lead to pandemic. □

9. Numerical study

In this segment, we employ Toufik-Atangana scheme [45,46] to acquire a numerical solution of fractional model for different values of ρ . The numerical values of involved parameters, given in Table 1 are taken from [11,12]. Dynamical behavior of Corona virus disease over time t is simulated for various values of fractional order ρ . Moreover, the effectiveness of quarantine programme is analyzed numerically for different coverage levels. The presented numerical results are discussed in detail.

9.1. Toufik-Atangana scheme and simulations

In this section, we develop a numerical scheme which is based on a recently developed Toufik-Atangana rule to analyze and predict the numerical stability of a Corona fractional model (8). To get an iterative scheme, we firstly describe the method briefly and then apply it to the fractional model (8). By applying the fundamental theorem of fractional calculus, the system (8) can be written as

$$v(t) - v(0) = \frac{1 - \rho}{B(\rho)} F(t, v(t)) + \frac{\rho}{B(\rho)\Gamma(\rho)} \times \int_0^t (t - \xi)^{\rho-1} F(\xi, v(\xi)) d\xi.$$

At $t = t_{n+1}$, $n = 0, 1, 2, \dots, N$ with $h = \frac{T}{N}$, we have

$$v(t_{n+1}) - v(0) = \frac{1 - \rho}{B(\rho)} F(t_n, v(t_n)) + \frac{\rho}{B(\rho)\Gamma(\rho)} \times \int_0^{t_{n+1}} (t_{n+1} - \xi)^{\rho-1} F(\xi, v(\xi)) d\xi,$$

or equivalently,

$$v(t_{n+1}) = v(0) + \frac{1-\rho}{B(\rho)} F(t_n, v(t_n)) + \frac{\rho}{B(\rho)\Gamma(\rho)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} (t_{n+1} - \xi)^{\rho-1} F(\xi, v(\xi)) d\xi. \quad (23)$$

The function $F(\xi, v(\xi))$ can be approximated over $[t_j, t_{j+1}]$, using the interpolation polynomial

$$F(\xi, v(\xi)) = \frac{F(t_j, v(t_j))}{h} (t - t_{j-1}) - \frac{F(t_{j-1}, v(t_{j-1}))}{h} (t - t_j).$$

Putting in Eq. (23) we get

$$v(t_{n+1}) = v(0) + \frac{1-\rho}{B(\rho)} F(t_n, v(t_n)) + \frac{\rho}{B(\rho)\Gamma(\rho)} \sum_{j=0}^n \left[\frac{F(t_j, v(t_j))}{h} \int_{t_j}^{t_{j+1}} (t_{n+1} - t)^{\rho-1} (t - t_{j-1}) dt - \frac{F(t_{j-1}, v(t_{j-1}))}{h} \int_{t_j}^{t_{j+1}} (t_{n+1} - t)^{\rho-1} (t - t_j) dt \right].$$

Calculating these integrals we finally get the approximate solution as:

$$v(t_{n+1}) = v(t_0) + \frac{1-\rho}{B(\rho)} F(t_n, v(t_n)) + \frac{\rho}{B(\rho)} \sum_{j=0}^n \left[\frac{h^\rho F(t_j, v(t_j))}{\Gamma(\rho+2)} \{ (n-j+2+\rho)(n+1-j)^\rho (n-j+2+2\rho)(n-j)^\rho \} - \frac{h^\rho F(t_{j-1}, v(t_{j-1}))}{\Gamma(\rho+2)} \{ (n+1-j)^{\rho+1} - (n-j+1+\rho)(n-j)^\rho \} \right].$$

Hence, we obtain the following recursive formulae for the model equations:

$$\begin{aligned} S(t_{n+1}) &= S(t_0) + \frac{1-\rho}{B(\rho)} F_1(t_n, v(t_n)) + \frac{\rho}{B(\rho)} \sum_{j=0}^n \left[\frac{h^\rho F_1(t_j, v(t_j))}{\Gamma(\rho+2)} \{ (n-j+2+\rho)(n+1-j)^\rho \} \right. \\ &\quad \left. (n-j+2+2\rho)(n-j)^\rho \} - \frac{h^\rho F_1(t_{j-1}, v(t_{j-1}))}{\Gamma(\rho+2)} \{ (n+1-j)^{\rho+1} - (n-j+1+\rho)(n-j)^\rho \} \right], \\ E(t_{n+1}) &= E(t_0) + \frac{1-\rho}{B(\rho)} F_2(t_n, v(t_n)) + \frac{\rho}{B(\rho)} \sum_{j=0}^n \left[\frac{h^\rho F_2(t_j, v(t_j))}{\Gamma(\rho+2)} \{ (n-j+2+\rho)(n+1-j)^\rho \} \right. \\ &\quad \left. (n-j+2+2\rho)(n-j)^\rho \} - \frac{h^\rho F_2(t_{j-1}, v(t_{j-1}))}{\Gamma(\rho+2)} \{ (n+1-j)^{\rho+1} - (n-j+1+\rho)(n-j)^\rho \} \right], \\ I(t_{n+1}) &= I(t_0) + \frac{1-\rho}{B(\rho)} F_3(t_n, v(t_n)) + \frac{\rho}{B(\rho)} \sum_{j=0}^n \left[\frac{h^\rho F_3(t_j, v(t_j))}{\Gamma(\rho+2)} \{ (n-j+2+\rho)(n+1-j)^\rho \} \right. \\ &\quad \left. (n-j+2+2\rho)(n-j)^\rho \} - \frac{h^\rho F_3(t_{j-1}, v(t_{j-1}))}{\Gamma(\rho+2)} \{ (n+1-j)^{\rho+1} - (n-j+1+\rho)(n-j)^\rho \} \right], \\ Q(t_{n+1}) &= Q(t_0) + \frac{1-\rho}{B(\rho)} F_4(t_n, v(t_n)) + \frac{\rho}{B(\rho)} \sum_{j=0}^n \left[\frac{h^\rho F_4(t_j, v(t_j))}{\Gamma(\rho+2)} \{ (n-j+2+\rho)(n+1-j)^\rho \} \right. \\ &\quad \left. (n-j+2+2\rho)(n-j)^\rho \} - \frac{h^\rho F_4(t_{j-1}, v(t_{j-1}))}{\Gamma(\rho+2)} \{ (n+1-j)^{\rho+1} - (n-j+1+\rho)(n-j)^\rho \} \right], \\ R(t_{n+1}) &= R(t_0) + \frac{1-\rho}{B(\rho)} F_5(t_n, v(t_n)) + \frac{\rho}{B(\rho)} \sum_{j=0}^n \left[\frac{h^\rho F_5(t_j, v(t_j))}{\Gamma(\rho+2)} \{ (n-j+2+\rho)(n+1-j)^\rho \} \right. \\ &\quad \left. (n-j+2+2\rho)(n-j)^\rho \} - \frac{h^\rho F_5(t_{j-1}, v(t_{j-1}))}{\Gamma(\rho+2)} \{ (n+1-j)^{\rho+1} - (n-j+1+\rho)(n-j)^\rho \} \right]. \end{aligned} \quad (24)$$

The purpose of this segment is to explain the role of fractional order ρ on the transmission patterns and control of COVID-19 through the model (7). To this end, we present some graphical results of the proposed fractional model (7) using an iterative finite difference scheme (24) developed by Toufik and Atangana [57]. In Fig. 2, the effect of arbitrary fractional order ρ on total number of humans in each class is demonstrated. The dynamics of the model (7) is simulated for $\rho = 0.2, 0.4, 0.6, 0.8, 1$ respectively. It is remarkable that we

were at the endemic equilibrium F_1^* for the integer case $\rho = 1$. A significant decrease in total Corona exposed and the infected cases is observed by decreasing the value of order ρ . However, the total number of humans in susceptible, quarantined and the recovered class increases slowly by reducing the value of ρ from 1. Implies we can reduce the transmission of corona infection in human population by decreasing the value of order ρ of the fractional model. It is noticed that as ρ gets close to zero from the right, the memory effects of the epidemic model decreases and consequently the number of cases in E and I gets close to 0. Moreover, for the case $\rho = 1$, we obtained the same graphical results as given in [11] where NSFD numerical scheme [68], was used.

9.2. Effect of quarantine policy on populations

In this subsection, we have studied the impact of quarantine on the dynamics of Corona virus pandemic quantitatively with the help of proposed fractional model (7). The aim was to handle the spread of COVID-19 in a human population through persistent quarantine program. For this purpose, several numerical simulations under the effect of different quarantine levels are demonstrated by employing a numerical scheme (24), see Figs. 3–8. By taking value of fractional order ρ other than 1, this numerical analysis has given more interesting and biologically feasible results in order to investigate the stability pattern of corona virus disease. The time level in all simulations is considered up to 100 days.

The effect of parameter q_1 (quarantine level) and fractional order ρ versus the total humans in each class is analyzed in Figs. 3–8. The study indicates that there is a significant decrease in the corona exposed and infected humans by increasing the order of fractional derivative and the level of quarantine. It is also noticed that the number of susceptible humans increases as the value of ρ and q_1 increases. Moreover, in Fig. 5, when $\rho = 0.6$, the number of recovered humans gradually increases for the quarantine levels $q_1 = 0, q_1 = 0.1$ and

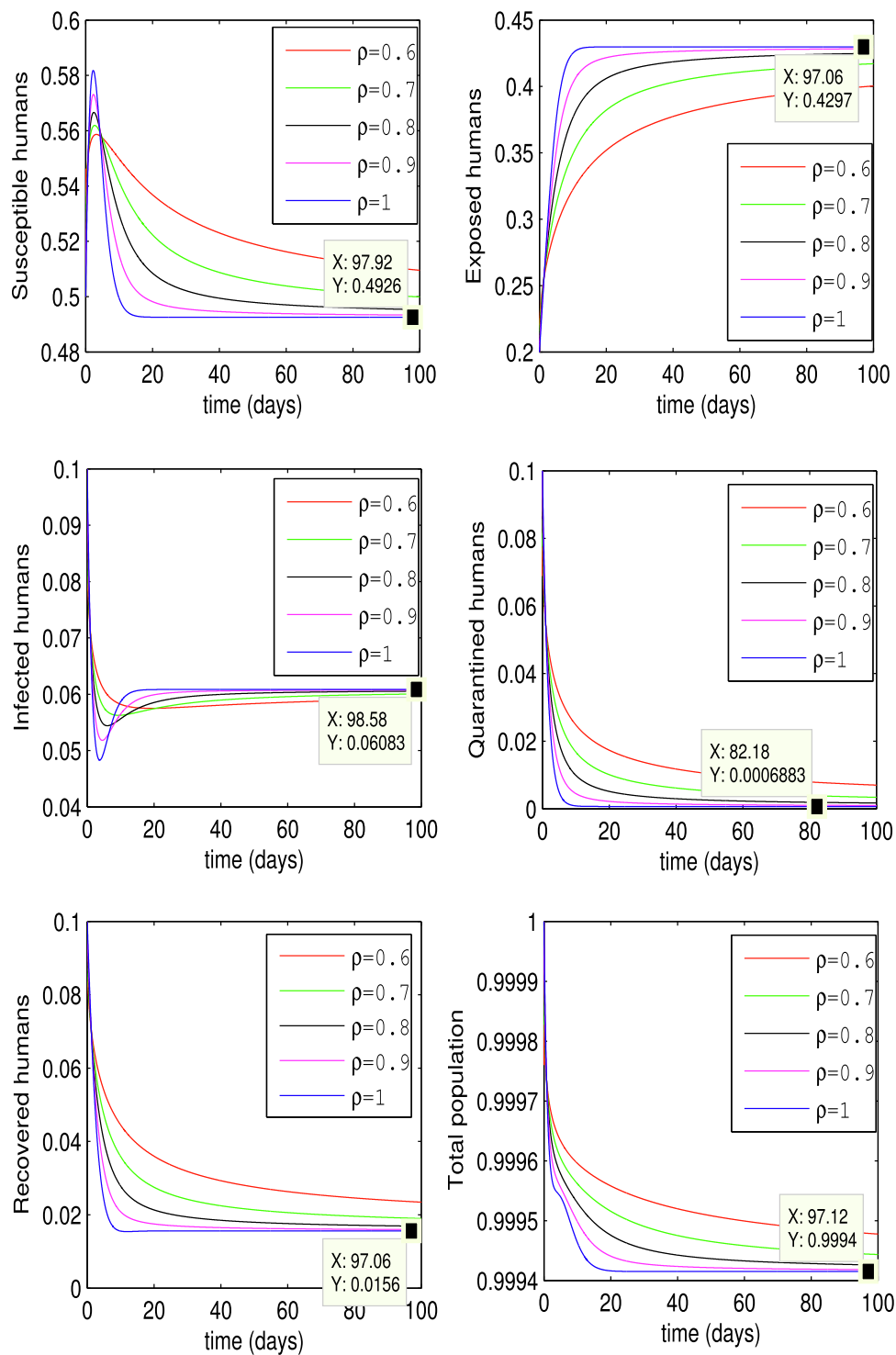


Fig. 2 Dynamical behavior of fractional SEIQR Corona epidemic model for different values of ρ using Toufik-Atangana numerical scheme. It is found that number of susceptible and the recovered humans gradually increases by decreasing the value of order ρ from 1. However, with the decrease in the value of fractional order ρ from 1, the number of exposed and infected humans also decreases. The curves for each of the class S, E, I, Q, R , and N get flattened as ρ reduces from 1 to 0.6. .

$q_1 = 0.3$ respectively. The curves of recovered humans eventually goes down for higher quarantine levels over time t , as needed.

With the continuous increase in the value of fractional order and quarantine level, the number of exposed, infected, and the recovered humans finally goes to zero whereas the number of susceptible humans goes to the value $S_0^* = \lambda/\mu$ for

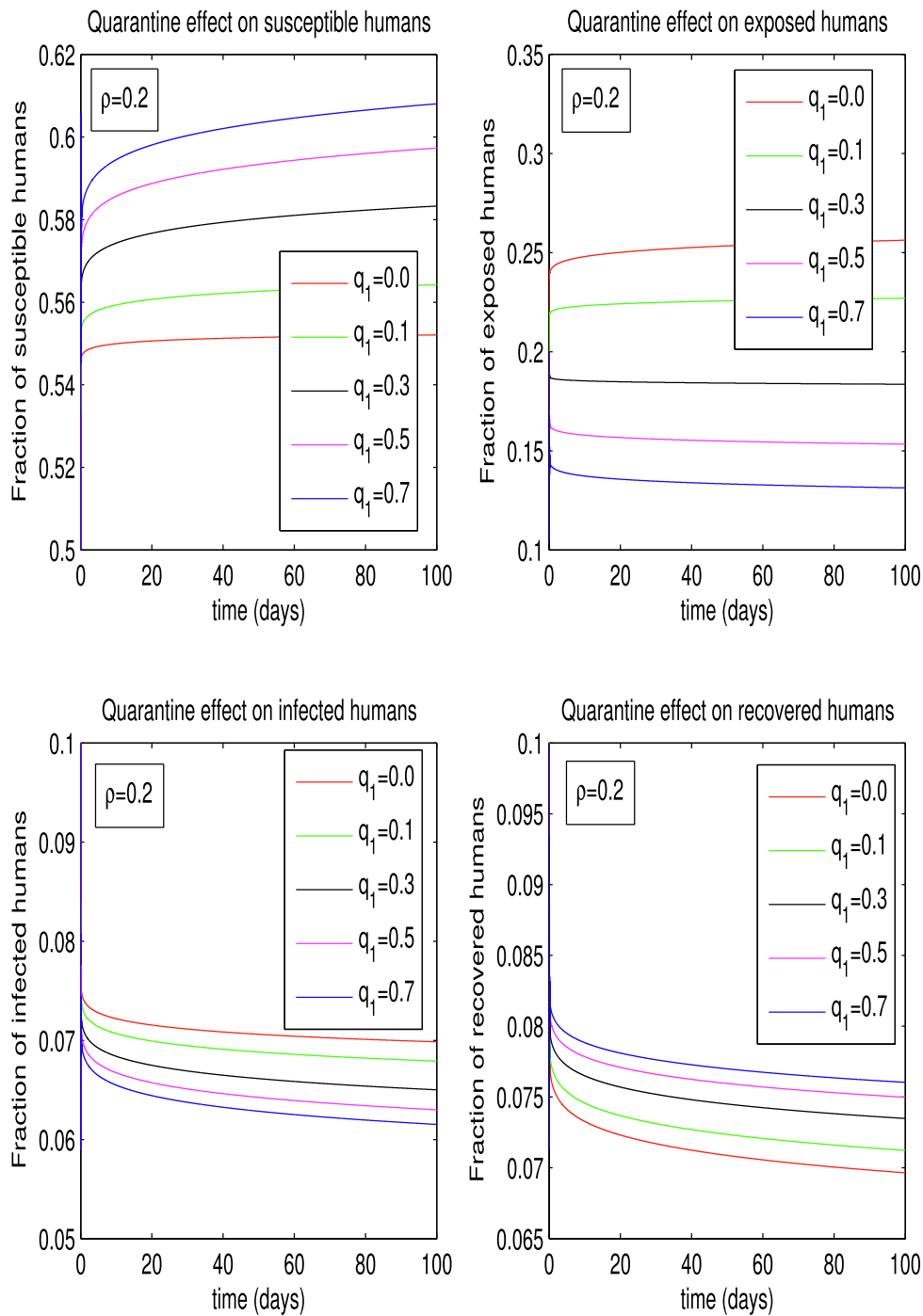


Fig. 3 Quarantine effect on the dynamics of Corona pandemic for the case $\rho = 0.2$ at endemic point F_1^* . Profiles for each class are obtained using Toufik-Atangana numerical scheme with step size $h = 0.01$. It is seen that number of susceptible and recovered humans gradually increases by increasing the level of quarantine. However, the number of exposed and infected humans decreases with the increase in quarantine rate q_1 .

the case $\rho = 1$ and $q_1 = 0.7$. This is a particular state of corona free point F_0^* at which the total population will become healthy. It shows that at all quarantine levels ranging 0% to $\leq 70\%$, the Corona virus disease will persist in the human population. Indeed, from 70% onward, each population approach the equilibrium state of F_0^* automatically as illustrated in Table 2. Hence, our study indicates that Corona virus disease

could be eradicated if we quarantine at least 70% of the human population, which justifies the title of our paper.

9.3. Computational advantages

We preferred Toufik-Atangana scheme over all standard and non-standard methods that solve a given problem only for

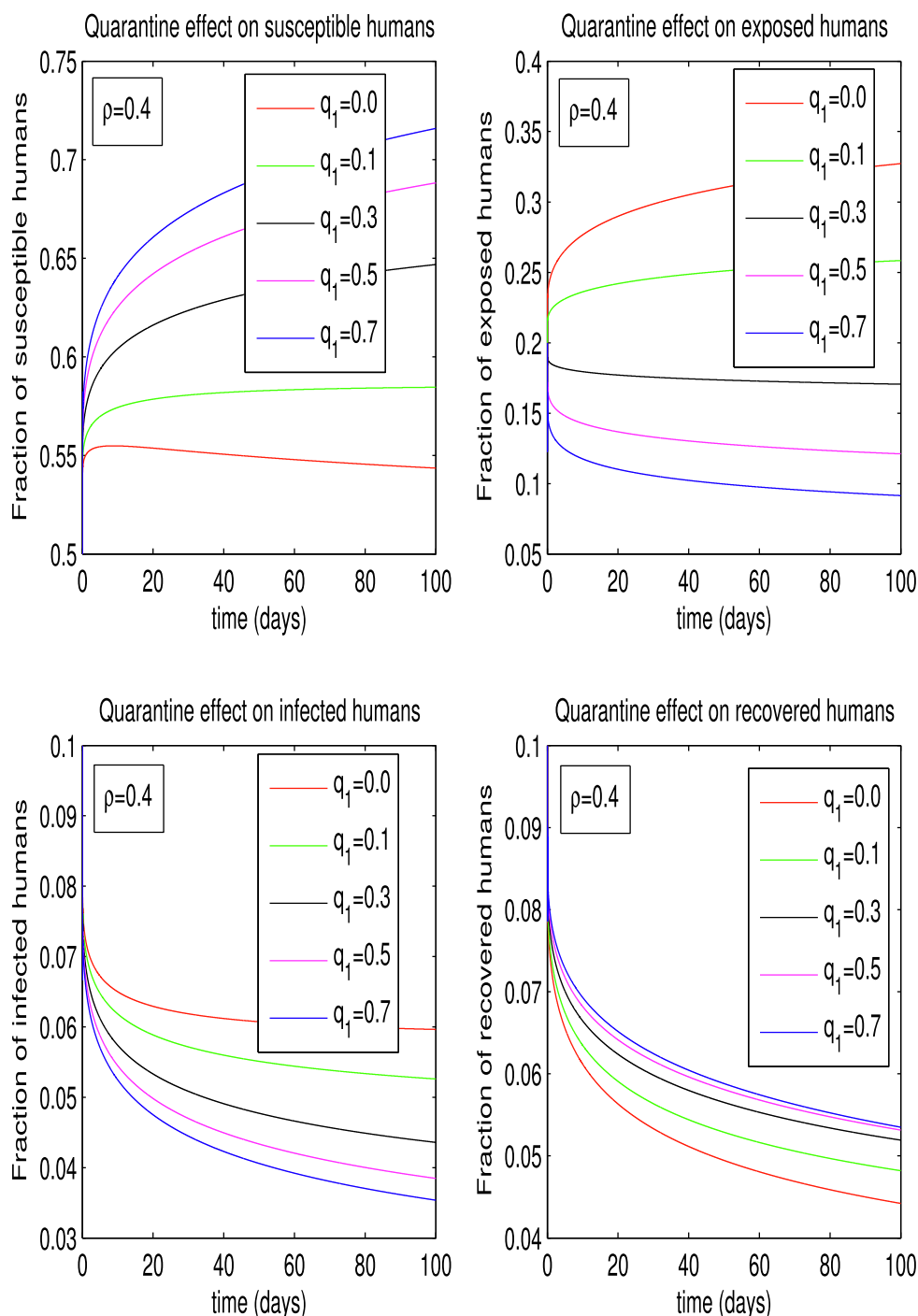


Fig. 4 Quarantine effect on the dynamics of Corona pandemic for the case $\rho = 0.4$ at endemic point F_1^* . Profiles for each class are obtained using Toufik-Atangana numerical scheme with step size $h = 0.01$. It is seen that number of susceptible and recovered humans gradually increases by increasing the level of quarantine. However, the number of exposed and infected humans decreases with the increase in quarantine rate q_1 .

an integer order. The preference is justified taking into account several numerical aspects of the fractional model, as it works gently for non-integer values of order ρ . Indeed, the fractional order ρ affects the global dynamical behavior of COVID-19 pandemic and provides a detailed information about the model at least in this work. We have studied efficiently the effect of fractional order ρ and the proposed

quarantine strategy on disease dynamics over the longer time. Importance of graphic results is attributed to the numerical scheme (24) for the fractional model involving Atangana-Baleanu fractional operator with the hereditary property. Furthermore, it is claimed that all the hidden properties of real world problems in engineering and physical sciences can be revealed better by a mathematical model

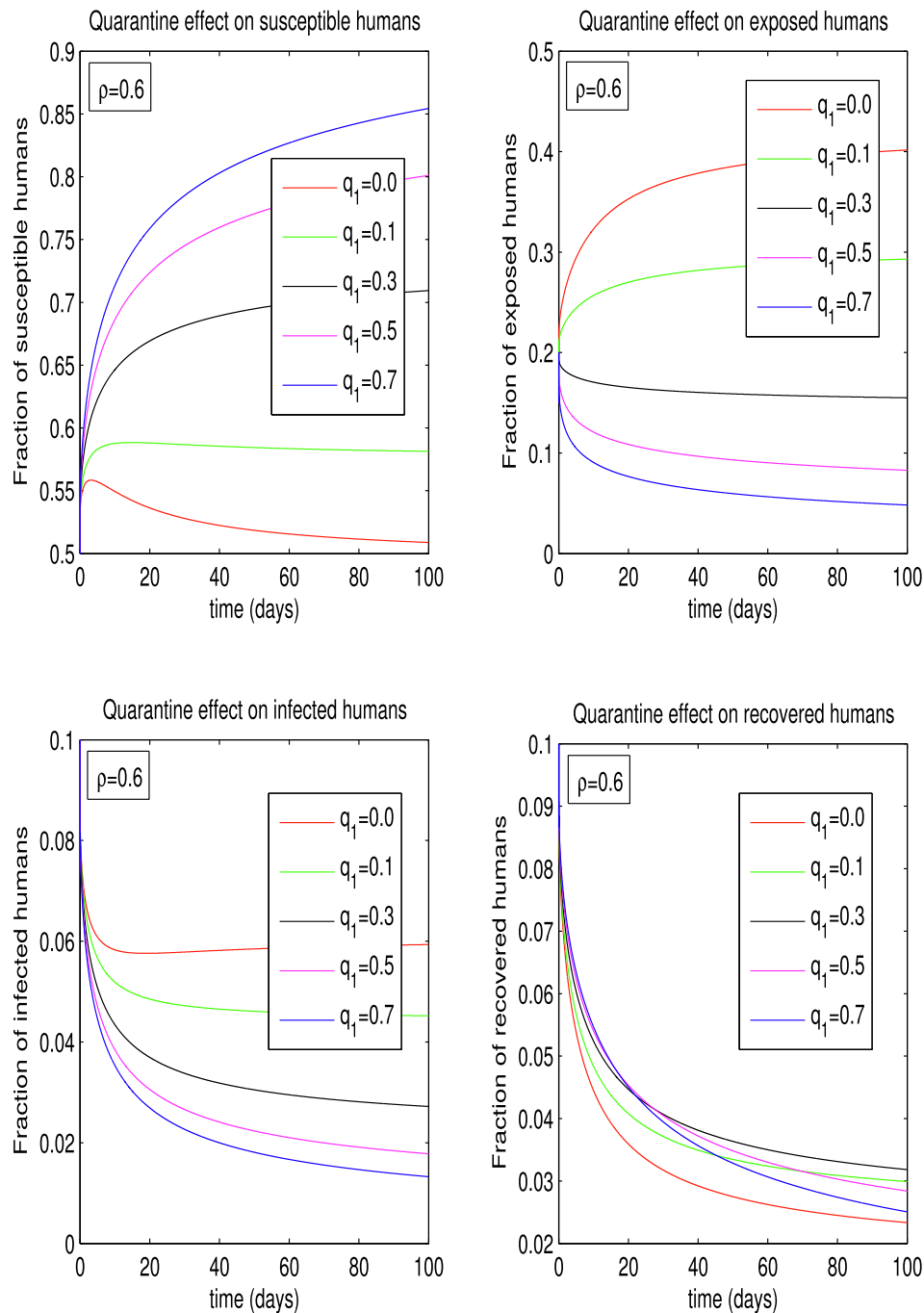


Fig. 5 Quarantine effect on the dynamics of Corona pandemic for the case $\rho = 0.6$ at endemic point F_1^* . Profiles are obtained using Toufik-Atangana numerical scheme with step size $h = 0.01$. It is seen that number of susceptible humans gradually increases by increasing the level of quarantine whereas the number of exposed and infected humans decreases. One important thing is to be noted that number of recovered humans initially increases (see for $q_1 = 0, 0.1, 0.3$) and then reduces (see for $q_1 = 0.5, 0.7$) with the increase in quarantine rate.

involving Atangana-Baleanu fractional operator. To verify this argument, further studies can be conducted on the effect of other fractional derivative operators such as Caputo-Fabrizio operator for the proposed model, where one can compare the results of the same model with the Atangana-Baleanu fractional operator with a Mittag-Leffer kernel includes exponential function as its special case.

9.4. Covariance

In the current subsection, the joint variability [12,62] of all classes is studied numerically. For each pair of populations, we have computed the relationship quantity ρ^* and its consequences as illustrated in Table 3. A numerical study performed in Figs. 3–7 authentically justified the inverse relationship

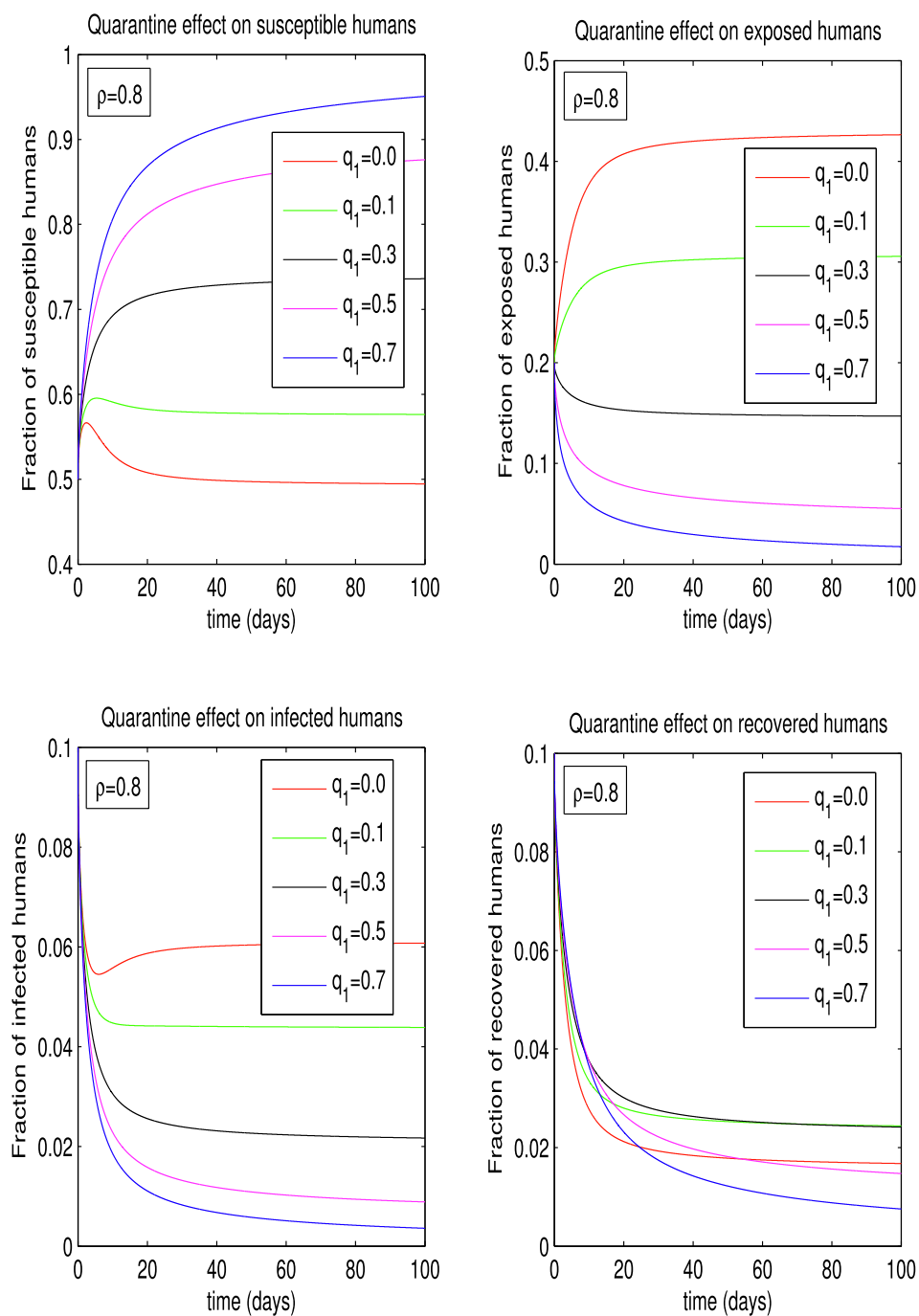


Fig. 6 Quarantine effect on the dynamics of Corona pandemic for the case $\rho = 0.8$ at endemic point F_1^* . Profiles are obtained using Toufik-Atangana numerical scheme with step size $h = 0.01$. It is seen that number of susceptible humans gradually increases by increasing the level of quarantine whereas the number of exposed and infected humans decreases. One important thing is to be noted that number of recovered humans initially increases (see for $q_1 = 0, 0.1, 0.3$) and then reduces (see for $q_1 = 0.5, 0.7$) with the increase in quarantine rate.

among exposed, infected, recovered and the quarantined humans. However, it is noticed that there is a direct correlation between the susceptible and quarantined class as demonstrated in each of the Figs. 3–7. Subsequently, a continuous rapid increment in the susceptible class will lead the entire human population to Corona free state of F_0^* .

9.5. Is it possible to quarantine 70% of the population?

A mathematical analysis of the proposed model (7) suggests that we will have to quarantine at least 70% of the human population to control the spread of COVID-19. It looks difficult for all human communities, especially in developing

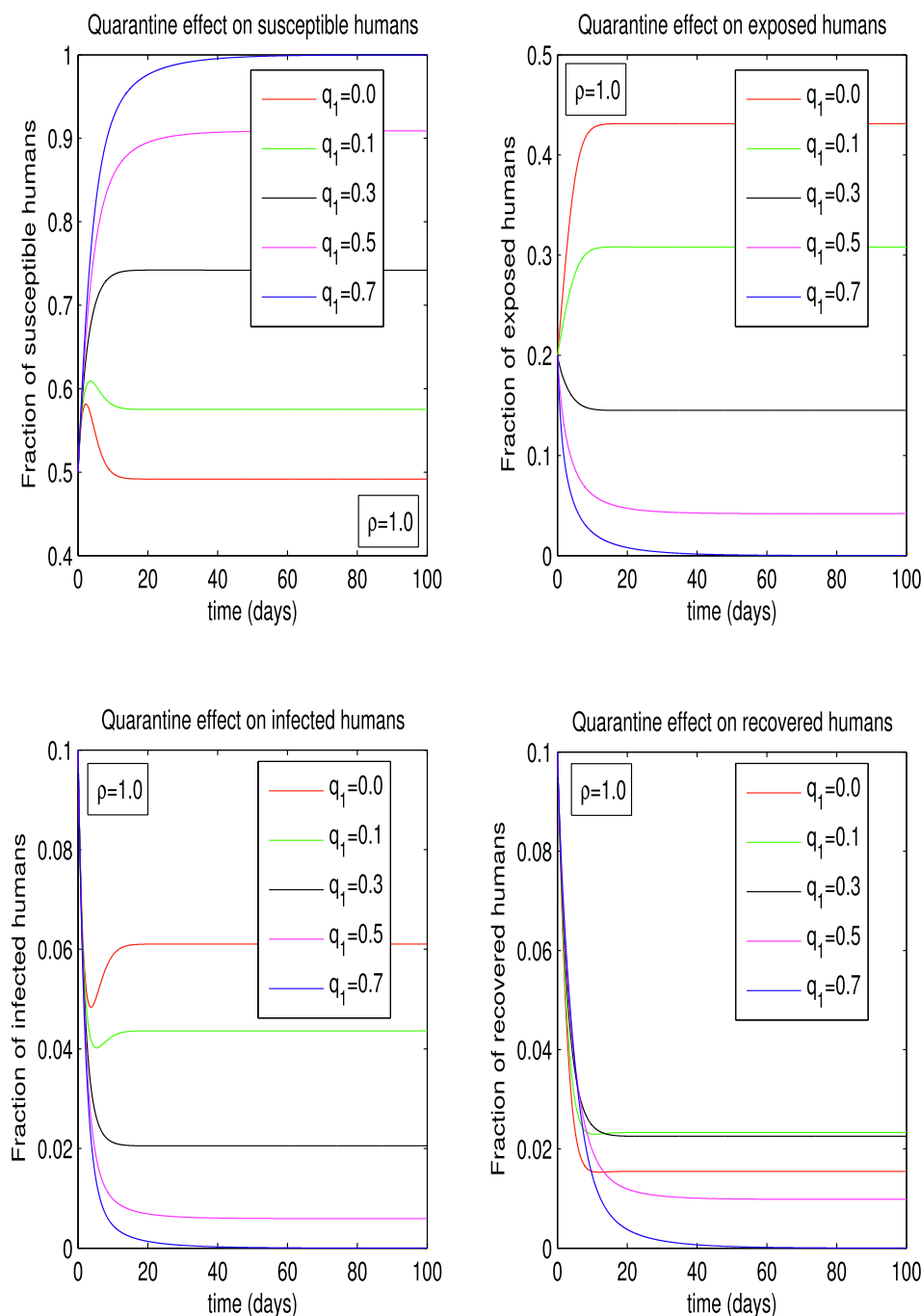


Fig. 7 Quarantine effect on the dynamics of Corona pandemic for the case $\rho = 1.0$ at endemic point F_1^* . Profiles are obtained using Toufik-Atangana numerical scheme with step size $h = 0.01$. It is seen that number of susceptible humans gradually increases by increasing the level of quarantine whereas the number of exposed and infected and the recovered humans decreases. The graph shows that we are at disease free state F_0^* when round about 70% of the human population is quarantined.

countries due to their limited resources. To make the quarantine strategy effective, firstly, we need to prepare priming people who can help others to think about many ways to reduce issues of loss of freedom, boredom, irritability or anxiety during quarantine period. We should encourage people to make plans such as structuring of time for virtual socialising and for exercise, or other shared activities. Every government or agency should plan for what will happen as and after quarantine restrictions are lifted. We need to

provide a reliable internet access to psychological, social and medical support. Confidential telephone hotlines that provide professional counseling should be provided. The state needs to provide a clean and a safe place to quarantine along with basic supplies such as clothes, water, food, and financial support. The basic necessities such as payment for house rent, electricity, child care and other utilities should be provided. We need to provide people the ability to work remotely.

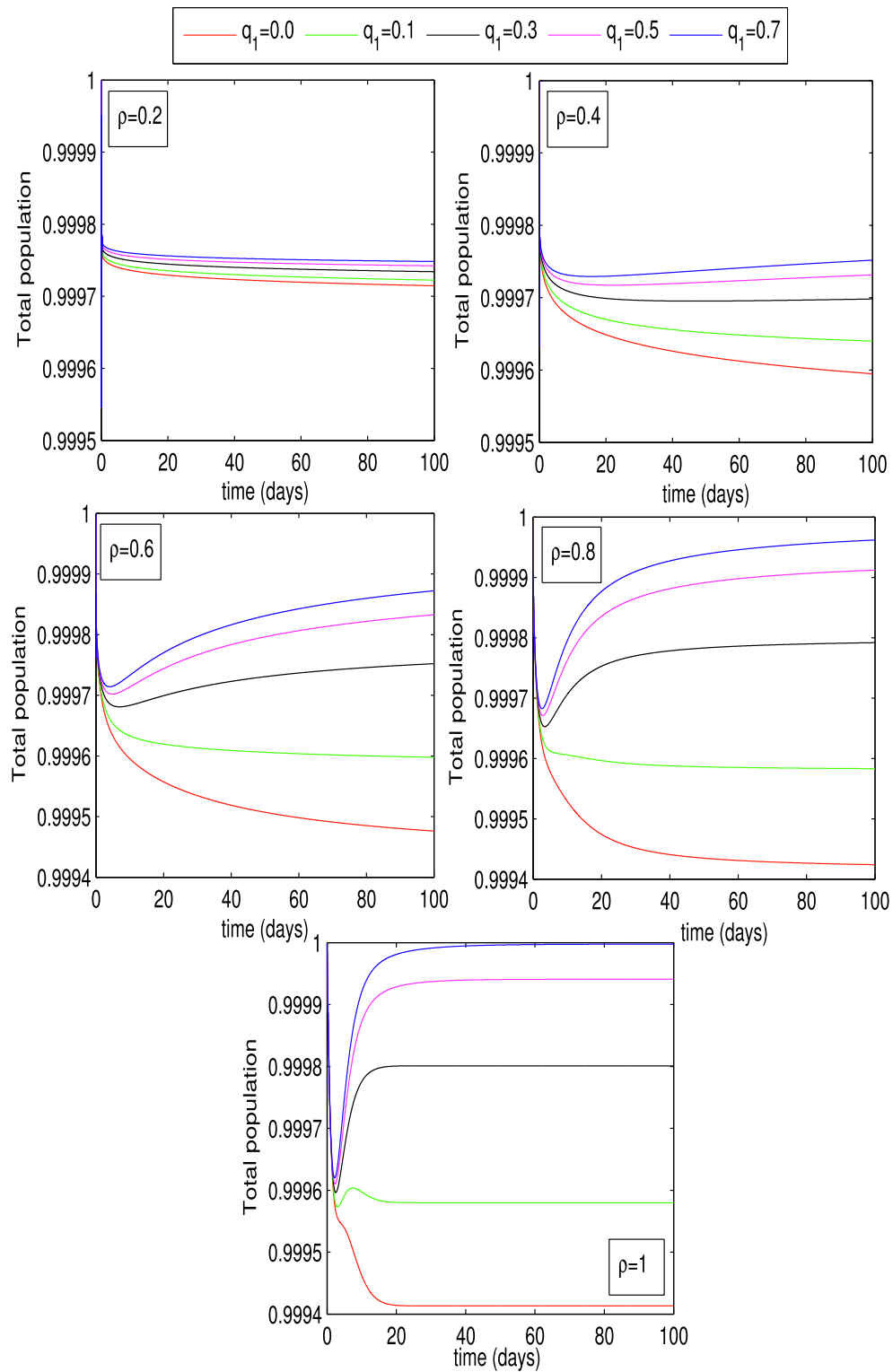


Fig. 8 Quarantine effect on the dynamics of total population $N(t)$ for four different values of ρ at endemic point F_1^* . Profiles for $N(t)$ are obtained using Toufik-Atangana numerical scheme with step size $h = 0.01$.

Table 2 Summary of quarantine effect on all classes for $\rho = 1$.

Levels of quarantine (%)	0.0	0.1	0.3	0.5	0.7
Population proportion of susceptible humans	0.4917	0.5752	0.7420	0.9089	1.0000
Population proportion of exposed humans	0.4312	0.3081	0.1450	0.0418	0.0000
Population proportion of infected humans	0.06104	0.04362	0.02053	0.005921	0.0000
Population proportion of recovered humans	0.01548	0.02333	0.02253	0.00983	0.0000

Table 3 Joint variability of populations.

Populations	ρ^*	Relationship
(Q, S)	0.5418	Direct
(Q, E)	-0.7624	Inverse
(Q, I)	-0.6542	Inverse
(Q, R)	-0.7560	Inverse

10. Conclusions

In this study, we assessed the dynamics of a novel Corona virus disease by developing a fractional order SEIQR model. We have used Atangana-Baleanu derivative operator with generalized Mittag-Leffler function to formulate the proposed fractional model. The Atangana-Baleanu derivative operator was preferred due to its non-local and non-singular kernel. The purpose of our study was to focus on a SEIQR model to depict the prevalent characteristics of Corona virus. To confirm the existence and uniqueness of solution of the proposed fractional model, the fixed point theorem is applied. Threshold parameter is computed theoretically and numerically to handle the dynamical behavior of the model. Disease free and endemic equilibrium points were obtained analytically. It is proved that both the equilibria are locally and globally stable in the feasible region. Global stability of equilibrium points is analyzed by the theory of Lyapunov functions. The solution of the model is then investigated by implementing a recently proposed numerical scheme from the existing literature. All the results obtained are simulated by figures and seen to be compatible with the model. We demonstrate the long term dynamical behavior of COVID-19 with the help of numerical simulations. The purpose was to investigate the impact of different values of fractional order ρ . Through the fractional analysis, it is observed that number of susceptible humans increases as the value of fractional order ρ decreases. However, as ρ decreases, the number of humans in exposed, infected, quarantined and the recovered class also decreases. Hence, Corona virus disease decreases slowly and can be controlled by reducing the value of fractional order ρ from 1. These fractional results show that fractional models will be more effective than integer-order ones in truly estimating the optimal quarantine level. As a second approach, it was suggested that the number of Corona infective cases can be reduced by increasing the value of fractional order ρ and levels of quarantine q_1 . Consequently, it is deduce that SEIQR fractional model is more efficient in fitting real data than integer-order SEIQR model. Finally, the joint variability of the quarantined class and the remaining classes is executed statistically. The obtained relationship along with the consequences among all pairs were also observed in all numerical simulations, one can validate our study. Our numer-

ical investigations prove that an earliest proper quarantine measure helps in controlling the Corona pandemic. To make it possible, the government especially in developing countries will have to take serious steps like providing less expensive suitable quarantine strategy to control the Corona virus disease. Mathematical analysis of the fractional model given in this paper makes an attempt to understand the dynamical behavior of a disease. To combat COVID-19 in the human population, we assure that the investigations in the current research work will be beneficial for the decision making and health authorities. In future, we will present a comprehensible depiction of Corona virus disease using different intervention strategies through SEIQHR fractional model with ABC operator. A general fractional order optimal control problem will be analyzed to find the best controls for the employed quarantine and hospitalization measures.

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Credit author statement

A. I. K. Butt: Supervision, Visualization, Investigation, Validation. **W. Ahmad:** Conceptualization, Methodology, Computations, Writing-original draft. **M. Rafiq:** Project administration, Resources, Data curation, Revision of original draft, Formal analysis. **D. Baleanu:** Writing-review and editing, Software.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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