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Original Article

The dynamical behavior for a famous class of evolution equations with double exponential nonlinearities

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A R T I C L E I N F O

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A B S T R A C T

An analytical investigation for a famous class of evolution equations with double exponential nonlinearities that has vast applications in many nonlinear sciences is presented. These equations include the Tzitzéica Equation (TE), Dodd-Bullough-Mikhailov Equation (DBME), Tzitzéica-Dodd-Bullough-Mikhailov equation (TDBME) and the Peyrard Bishop DNA Equation (PB-DNA-E). Furthermore, the Kudryashov method for constructing exponential function solutions has been employed to reveal various sets of traveling wave solutions with different geometrical structures to the identified models. We also give the graphical illustrations of certain solutions to further analyze the results.

1. Introduction

Many natural phenomena in hydrodynamics, nonlinear optics, plasma physics, Bose-Einstein condensates, chemistry, engineering and fluid mechanics among others can be modeled via Partial Differential Equations (PDEs) [1–16] The studying of different categories of solutions for various models contribute immensely in the explanation of the physical meanings of such models [17–25]. Many important PDEs in applications as the celebrated Tzitzéica-based evolution equations are completely integrable, a generalization of the notion of integrable Ordinary Differential Equations (ODEs) to the case of PDEs which can be seen in this context as infinite dimensional ODE systems. In practice this means that powerful solution generating methods exist for these equations which allow the construction of interesting solutions as solitons, stable wave packets behaving in collision as particles. Consider the following generalized nonlinear evolution equation with double exponential nonlinearities

$$v_{xt} - \alpha e^{mv} - \beta e^{nv} = 0, \tag{1}$$

where $\alpha \neq 0, \beta \neq 0$ are real constant and m, n are integers. Eq. (1) encompasses Tzitzéica Equation (TE), Dodd-Bullough-Mikhailov Equation (DBME) and Tzitzéica-Dodd-Bullough-Mikhailov equation (TDBME). The equation is such regarded as the generalized Tzitzéica Dodd-Bullough-Mikhailov Equation. If we take $\alpha = 1$, $\beta = -1$, $m = 1$, $n = -2$ or $\alpha = -1$, $\beta = 1$, $m = -2$, $n = 1$, particularly Eq. (1) turns out to be the classical TE [26–28] as follows:

$$v_{xt} - e^{v} + e^{-2v} = 0, \tag{2}$$

which was originated in 1907 by G. Tzitzéica in the field of geometry and has many applications in different fields of science. Eq. (2) usually called the Dodd-Bullough equation, which was initiated by Bullough and Dodd [29] and Ziber and Sabat [30]. As a matter of fact, Eq. (2) has another form

$$v_{tt} - v_{xx} - e^{v} + e^{-2v} = 0, \tag{3}$$

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see [31,32] and the references therein. Taking $\alpha=1, \quad \beta=-1, \quad m=1, \quad n=-2$, Eq. (1) turns out to be the DBME given by

$$v_{xt}-e^{v}-e^{-2v}=0. \tag{4}$$

More, when $\alpha=1, \quad \beta=1, \quad m=1, \quad n=-2$, Eq. (1) turns out to be Tzitzéica Dodd-Bullough equation

$$v_{xt}+e^{v}+e^{-2v}=0. \tag{5}$$

The DBME and Tzitzéica-Dodd-Bullough equation appeared in many problems varying from fluid flow to quantum field theory [32]. Furthermore, when $m=1, \quad n=-1, \alpha=1/2, \beta=1/2$, or $m=-1, \quad n=1, \alpha=-1/2, \beta=1/2$, Eq. (1) turns out to be the sinh-Gordon equation

$$v_{xt}-\sinh v=0, \tag{6}$$

which was also shown in [33] and the references therein. Also, when $m=1, \quad n=-1, \alpha=1/2, \beta=1/2$, or $m=-1, \quad n=1, \alpha=1/2, \beta=1/2$, Eq. (1) turns out to be the cosh-Gordon equation

$$v_{xt}-\cosh v=0, \tag{7}$$

see [34] for details. Finally, when $m=1, \quad n=0$, particularly Eq. (1) becomes the Liouville equation

$$v_{xt}-\alpha e^{v}=0. \tag{8}$$

The above mentioned equations play extremely important roles in many scientific applications. It worth to be mention that several methods have been presented to find analytical solutions of these nonlinear PDEs. Such as in [35], where Wazwaz introduced solitary and periodic wave solutions for the Dodd-Bullough-Mikhailov and Dodd-Bullough equations, by using the tanh method. Andreev presented the Backlund transformation for Bullough-Dodd-Jiber-Shabat equation [36]. Cherdantzev and Sharipov obtained finite-gap solutions of the Bullough-Dodd-Jiber-Shabat equation [37]. In [38], Cherdantzev and Sharipov explored solitons on the finite-gap background in the Bullough-Dodd-Jiber-Shabat model. In addition, the Darboux transformation, self-dual Einstein spaces and consistency and general solution of TE were studied in [39–43].

This study use the Kudryashov method to find and discuss abundant analytical solutions for three types of models that emanated from Eq. (1) and also the Peyrard Bishop DNA Equation (PB-DNA-E) [44], having double exponential nonlinearities. Indeed, Kudryashov method has successfully been combined with the computer method and some transformations [45–47] for investigating exact traveling wave solutions of some nonlinear PDEs, see also [48–60] and the references therewith for more on the analytical methods for tackling a variety of nonlinear evolution equations. Therefore, using this method, we will obtain some new results which are different from those in the above references. The rest of this paper is organized as follows: in Section 2, we present an analysis of the model under study. In Sections 3 and 4, by using the Kudryashov method, we will obtain some new traveling wave solutions of the aiming governing equations; while in Sections 5 and 6 we give some results discussion and concluding remarks, respectively.

2. Governing equations

In this manuscript, we consider a famous class of the Tzitzéica-based evolution equations that occur with some exponential nonlinearities. These equations which need to be transformed to standard partial differential equations include the following

1. Tzitzéica Equation (TE) [26–28]

$$v_{tt}-v_{xx}-e^{v}+e^{-2v}=0. \tag{9}$$

2. Dodd-Bullough-Mikhailov Equation (DBME) [26–28]

$$v_{xt}+e^{v}+e^{-2v}=0. \tag{10}$$

3. Tzitzéica-Dodd-Bullough-Mikhailov Equation (TDBME) [26–28]

$$v_{xt}-e^{-v}-e^{-2v}=0. \tag{11}$$

4. Peyrard Bishop DNA Equation (PB-DNA-E) [44]

$$v_{tt}+\alpha_1 v_{xx}+\alpha_2 v_x^2 v_{xx}+\alpha_3 e^{-av}+\alpha_4 e^{-2av}=0, \tag{12}$$

where $a, \ \alpha_j, \ (j=1,2,3,4)$ are real constants.

2.1. Transformed equations

However, we transform the aforementioned partial differential equations in the presence of these exponential nonlinearities to standard nonlinear differential equations using the Painlevé transformation [28]. In fact, this transformation will allow us to tackle the equations squarely using the method to be given in the next session.

1. Transformed TE

The TE is transformed to standard nonlinear differential equation using $v=\ln(u)$ and yields

$$uu_{tt}-uu_{xx}-u_t^2+u_x^2-u^3+1=0. \tag{13}$$

2. Transformed DBME

The DBME is transformed to standard nonlinear differential equation using $v=\ln(u)$ and reveals

$$uu_{xt}-u_x u_t+u^3+1=0. \tag{14}$$

3. Transformed TDBME

The TDBME is transformed to standard nonlinear differential equation using $v=-\ln(u)$ and gives

$$uu_{xt}-u_x u_t+u^3+u^4=0. \tag{15}$$

4. Transformed PB-DNA-E

The PB-DNA-E is transformed to standard nonlinear differential equation using $v=-\frac{1}{a}\ln(u)$ and get

$$\begin{aligned}&-a^2u^3(u_{tt}+\alpha_1 u_{xx})+a^2u^2\left(u_t^2+\alpha_1 u_x^2\right)\\&-\alpha_2 u_x^2(uu_{xx}-u_x^2)+a^3u^5(\alpha_3+\alpha_4 u)=0,\end{aligned} \tag{16}$$

where $\alpha_j, \ j=1,2,3,4$ are constants.

We therefore tackle these transformed equations in Section 4.

3. Methodology

We consider the famous Kudryashov method while solving the above mentioned equations. Thus, considering the following generalized partial differential equation

$$Q(u,u_t,u_x,u_{xt},u_{tt},u_{xx},\cdots)=0. \tag{17}$$

Now, on using the traveling wave transformation of the form

$$u(x,t)=w(\eta), \qquad \eta=kx-ct, \tag{18}$$

with c and k are non-zero constants; then (17) reduces to the following ordinary differential equation

$$P(w,-cw',kw',-ckw'',c^2w'',k^2w'',\ldots)=0. \tag{19}$$

Therefore, the method offers a finite series of the following form

$$w(\eta)=a_0+\sum_{j=1}^{M}a_j\Psi^j(\eta), \tag{20}$$

where, $a_0, a_j, \ (j=1,2,\ldots,N)$ are constants that are not all equal to zero. More, M is a whole number to be determined by balancing the highest-order nonlinear and derivatives terms emerging in (19). Additionally, $\Psi(\xi)$ satisfies the following equation

$$\Psi'(\eta) = \Psi(\eta)(\Psi(\eta) - 1), \tag{21}$$

where the function $\Psi(\eta)$ is considered to be

$$\Psi(\xi) = \frac{1}{1 + fe^{\eta}}, \tag{22}$$

where f is an arbitrary constant. Putting (20) into Eq. (19) gives a polynomial in terms of $\Psi^j(\xi)$, $(j = 0, 1, 2, \ldots, M)$ which will then be set to zero for each j to obtain a system of algebraic equations. These equations are further then solved for $a_0, a_j, (j = 1, 2, \cdots, M)$ algebraically with the aid of mathematical software. We adopted *Mathematica* software in this study.

4. Application

In this section, we sought for the Kudryashov method as described above to demonstrate its application on the transformed TE, DBME, TDBME, and PB-DNA-E, respectively.

4.1. TE

Considering the transformed TE given in (13) with the application of the travelling wave transformation given in (18), (13) reduces to the following ordinary nonlinear differential equation

$$\left(c^2 - k^2\right)w(\eta)w''(\eta) + \left(k^2 - c^2\right)w'(\eta)^2 - w(\eta)^3 + 1 = 0. \tag{23}$$

Balancing ww'' and w^3 gives $M = 2$. Thus, (20) in this regards offers the following solution form

$$w(\eta) = a_0 + a_1\Psi(\eta) + a_2\Psi^2(\eta). \tag{24}$$

Substituting the above equation into (23) and do as explained in the procedure, we obtain the following system of equations

$$1 - a_0^3 = 0, \quad a_0a_1c^2 - a_0a_1k^2 - 3a_0^2a_1 = 0,$$

$$-3a_0a_1c^2 + 4a_0a_2c^2 + 3a_0a_1k^2 - 4a_0a_2k^2 - 3a_0a_1^2 - 3a_0^2a_2 = 0,$$

$$a_1^2c^2 + 6a_0a_2c^2 - 5a_1a_2c^2 - a_1^2k^2 - 6a_0a_2k^2 + 5a_1a_2k^2 - 3a_0a_2^2 - 3a_1^2a_2 = 0,$$

$$-2a_2^2c^2 + 4a_1a_2c^2 + 2a_2^2k^2 - 4a_1a_2k^2 - 3a_1a_2^2 = 0,$$
$$2a_2^2c^2 - 2a_2^2k^2 - a_2^3 = 0,$$

$$-a_1^2c^2 + 2a_0a_1c^2 + a_2a_1c^2 - 10a_0a_2c^2 + a_1^2k^2 - 2a_0a_1k^2$$
$$-a_2a_1k^2 + 10a_0a_2k^2 - a_1^3 - 6a_0a_2a_1 = 0.$$

Therefore, solving the above system reveals

Set-I

$$a_0 = 1, \quad a_1 = -6, \quad a_2 = 6, \quad k = \mp\sqrt{c^2 - 3}. \tag{25}$$

This solution set gives the following solutions

$$v_{1,2}(x,t) = \ln\left(1 - \frac{6}{1 + f\exp(kx - ct)} + \frac{6}{(1 + f\exp(kx - ct))^2}\right). \tag{26}$$

Set-II

$$\begin{cases} a_0 = \frac{1}{2}\left(-1 - i\sqrt{3}\right), \; a_1 = 3\left(1 + i\sqrt{3}\right), \\ a_2 = 3\left(-1 - i\sqrt{3}\right), \quad k = \pm\frac{\sqrt{2c^2 - 3i\sqrt{3} + 3}}{\sqrt{2}}. \end{cases} \tag{27}$$

This solution set gives the following solutions

$$v_{3,4}(x,t) = \ln\left(-\frac{1}{2}\left(1 + i\sqrt{3}\right) + \frac{3\left(1 + i\sqrt{3}\right)}{1 + f\exp(kx - ct)} - \frac{3\left(1 + i\sqrt{3}\right)}{(1 + f\exp(kx - ct))^2}\right). \tag{28}$$

Set-III

$$\begin{cases} a_0 = \frac{1}{2}\left(-1 + i\sqrt{3}\right), \quad a_1 = 3\left(1 - i\sqrt{3}\right), \\ a_2 = 3\left(-1 + i\sqrt{3}\right), \quad k = \pm\frac{\sqrt{2c^2 + 3i\sqrt{3} + 3}}{\sqrt{2}}. \end{cases} \tag{29}$$

This solution set gives the following solutions

$$v_{5,6}(x,t) = \ln\left(-\frac{1}{2}\left(1 - i\sqrt{3}\right) + \frac{3\left(1 - i\sqrt{3}\right)}{1 + f\exp(kx - ct)} - \frac{3\left(1 - i\sqrt{3}\right)}{(1 + f\exp(kx - ct))^2}\right). \tag{30}$$

4.2. DBME

Considering the transformed DBME given in (14) with the application of the travelling wave transformation given in Eq. (18), (14) reduces to the following ordinary nonlinear differential equation

$$-ck(w(\eta)w''(\eta) - w'(\eta)^2) + w(\eta)^3 + 1 = 0. \tag{31}$$

Balancing ww'' and w^3 gives $M = 2$. Thus, (20) in offers the following solution form

$$w(\eta) = a_0 + a_1\Psi(\eta) + a_2\Psi^2(\eta). \tag{32}$$

Substituting the above equation into (31) and do as explained in the procedure, we obtain the following system of equations

$$a_0^3 + 1 = 0, 3a_0^2a_1 - a_0a_1ck = 0,$$

$$3a_1a_0ck - 4a_2a_0ck + 3a_2a_0^2 + 3a_1^2a_0 = 0,$$

$$a_1^2ck - 2a_0a_1ck - a_2a_1ck + 10a_0a_2ck + a_1^3 + 6a_0a_2a_1 = 0,$$

$$a_1^2(-c)k + 5a_2a_1ck - 6a_0a_2ck + 3a_2a_1^2 + 3a_0a_2^2 = 0,$$

$$2a_2^2ck - 4a_1a_2ck + 3a_1a_2^2 = 0, a_2^3 - 2a_2^2ck = 0.$$

Therefore, solving the above system reveals

Set-I

$$a_0 = -1, \quad a_1 = 6, \quad a_2 = -6, \quad k = -\frac{3}{c}. \tag{33}$$

This solution set gives the following solution

$$v_1(x,t) = \ln\left(-1 + \frac{6}{1 + f\exp(-\frac{3}{c}x - ct)} - \frac{6}{(1 + f\exp(-\frac{3}{c}x - ct))^2}\right). \tag{34}$$

Set-II

$$\begin{cases} a_0 = \frac{1}{2}\left(1 \pm i\sqrt{3}\right), \quad a_1 = 3\left(-1 \mp i\sqrt{3}\right), \\ a_2 = 3\left(1 \pm i\sqrt{3}\right), \quad k = \frac{3\left(1 \pm i\sqrt{3}\right)}{2c}. \end{cases} \tag{35}$$

This solution set gives the following solutions

$$v_{2,3}(x,t) = \ln\left(\frac{1}{2}\left(1 \pm i\sqrt{3}\right) + \frac{3\left(-1 \mp i\sqrt{3}\right)}{1 + f\exp(kx - ct)} + \frac{3\left(1 \pm i\sqrt{3}\right)}{(1 + f\exp(kx - ct))^2}\right). \tag{36}$$

4.3. TDBME

Considering the transformed TDBME given in (15) with the application of the travelling wave transformation given in (18), (15) reduces to the following ordinary nonlinear differential equation

$$-ck(w(\eta)w''(\eta)-w'(\eta)^2)+w(\eta)^3+w(\eta)^4=0. \tag{37}$$

Balancing ww'' and w^4 gives $M=1$. Thus, (20) in this regards offers the following solution form

$$w(\eta)=a_0+a_1\Psi(\eta). \tag{38}$$

Substituting the above equation into (37) and do as explained in the procedure, we obtain the following system of equations

$$3a_0a_1ck+6a_0^2a_1^2+3a_0a_1^2=0,$$

$$a_0^4+a_0^3=0,\quad -a_1a_0ck+4a_1a_0^3+3a_1a_0^2=0,$$

$$a_1^4-a_1^2ck=0,\quad a_1^2ck-2a_0a_1ck+4a_0a_1^3+a_1^3=0.$$

Therefore, solving the above system reveals

Set-I

$$a_0=0,\quad a_1=-1,\quad k=\frac{1}{c}. \tag{39}$$

This solution set gives the following solution

$$v_1(x,t)=-\ln\left(-\frac{1}{1+f\exp(\frac{1}{c}x-ct)}\right). \tag{40}$$

Set-II

$$a_0=-1,\quad a_1=1,\quad k=\frac{1}{c}. \tag{41}$$

This solution set gives the following solution

$$v_2(x,t)=-\ln\left(-1+\frac{1}{1+f\exp(\frac{1}{c}x-ct)}\right). \tag{42}$$

4.4. PB-DNA-E

Considering the transformed DBME given in (16) with the application of the traveling wave transformation given in (18), (16) reduces to the following ordinary nonlinear differential equation

$$\left(w(\eta)w''(\eta)-w'(\eta)^2\right)\left(a^2w(\eta)^2\left(c^2+\alpha_1k^2\right)+\alpha_2k^4w'(\eta)^2\right)-a^3(\alpha_3+\alpha_4w(\eta))w(\eta)^5=0. \tag{43}$$

Balancing w'^2ww'' and w^6 gives $M=2$. Thus, (20) in offers the following solution form

$$w(\eta)=a_0+a_1\Psi(\eta)+a_2\Psi^2(\eta). \tag{44}$$

Substituting the above equation into (43) and do as explained in the procedure, we obtain the following system of equations

$$-6a^3a_1\alpha_4a_2^5-24\alpha_2a_2^4k^4+24a_1\alpha_2a_2^3k^4=0,\ 8a_2^4\alpha_2k^4-a^3a_2^6\alpha_4=0,$$

$$-a^3\alpha_4a_0^6-a^3\alpha_3a_0^5=0,\ -6a^3a_1\alpha_4a_0^5-5a^3a_1\alpha_3a_0^4+a^2a_1a_0^3c^2+a^2a_1\alpha_1a_0^3k^2=0,$$

$$-a^3\alpha_3a_2^5-6a^3a_0\alpha_4a_2^5-15a^3a_1^2\alpha_4a_2^4+2a^2a_2^4c^2+2a^2\alpha_1a_2^4k^2+24\alpha_2a_2^4k^4$$

$$+24a_0\alpha_2a_2^3k^4-76a_1\alpha_2a_2^3k^4+22a_1^2\alpha_2a_2^2k^4=0,$$

$$\vdots$$

Therefore, solving the above system reveals

Set-I

$$\begin{cases}a_0=0,\ a_2=-a_1,\ k=\mp\frac{a^{3/4}\sqrt{a_1}\sqrt[4]{\alpha_4}}{2^{3/4}\sqrt[4]{\alpha_2}},\\ \alpha_1=-\frac{\sqrt{\alpha_2}\left(4aa_1\alpha_3+aa_1^2\alpha_4+8c^2\right)}{2\sqrt{2}a^{3/2}a_1\sqrt{\alpha_4}}.\end{cases} \tag{45}$$

This solution set gives the following solutions

$$v_{1,2}(x,t)=-\frac{1}{a}\ln\left(\frac{a_1}{(1+f\exp(kx-ct))}-\frac{a_1}{(1+f\exp(kx-ct))^2}\right). \tag{46}$$

Set-II

$$\begin{cases}a_0=0,\ a_1=-\frac{\alpha_3}{\alpha_4},\ a_2=\frac{\alpha_3}{\alpha_4},\ k=\mp\frac{a^{3/4}\sqrt{\alpha_3}}{2^{3/4}\sqrt[4]{\alpha_2}\sqrt[4]{\alpha_4}},\\ \alpha_1=\frac{3\sqrt{2}a\sqrt{\alpha_2}\alpha_3^2-8\sqrt{2}\sqrt{\alpha_2}\alpha_4c^2}{4a^{3/2}\alpha_3\sqrt{\alpha_4}}.\end{cases} \tag{47}$$

This solution set gives the following solutions

$$v_{3,4}(x,t)=-\frac{1}{a}\ln\left(-\frac{\alpha_3}{\alpha_4(1+f\exp(kx-ct))}+\frac{\alpha_3}{\alpha_4(1+f\exp(kx-ct))^2}\right). \tag{48}$$

Set-III

$$\begin{cases}a_0=0,\ a_1=-\frac{\alpha_3}{\alpha_4},\ a_2=\frac{\alpha_3}{\alpha_4},\ k=\mp\frac{a^{3/4}\sqrt{\alpha_3}}{2^{3/4}\sqrt[4]{\alpha_2}\sqrt[4]{\alpha_4}},\\ c=-\frac{\sqrt{a}\alpha_3}{\sqrt{\alpha_4}},\ \alpha_1=-\frac{5\sqrt{\alpha_2}\alpha_3}{2\sqrt{2}\sqrt{a}\sqrt{\alpha_4}}.\end{cases} \tag{49}$$

This solution set gives the following solutions

$$v_{5,6}(x,t)=-\frac{1}{a}\ln\left(-\frac{\alpha_3}{\alpha_4(1+f\exp(kx-ct))}+\frac{\alpha_3}{\alpha_4(1+f\exp(kx-ct))^2}\right). \tag{50}$$

Set-IV

$$\begin{cases}a_0=0,\ a_1=-\frac{\alpha_3}{\alpha_4},\ a_2=\frac{\alpha_3}{\alpha_4},\ k=\mp\frac{a^{3/4}\sqrt{\alpha_3}}{2^{3/4}\sqrt[4]{\alpha_2}\sqrt[4]{\alpha_4}},\\ c=\frac{\sqrt{a}\alpha_3}{\sqrt{\alpha_4}},\ \alpha_1=-\frac{5\sqrt{\alpha_2}\alpha_3}{2\sqrt{2}\sqrt{a}\sqrt{\alpha_4}}.\end{cases} \tag{51}$$

This solution set gives the following solutions

$$v_{7,8}(x,t)=-\frac{1}{a}\ln\left(-\frac{\alpha_3}{\alpha_4(1+f\exp(kx-ct))}+\frac{\alpha_3}{\alpha_4(1+f\exp(kx-ct))^2}\right). \tag{52}$$

5. Results discussion

This section gives some graphical two-dimensional (2D) and three-dimensional (3D) illustrations to certain solutions of our models. Several exponential function solutions have been constructed via the application of the powerful Kudryashov method for the TE, DBME, TDBME and PB-DNA-E, respectively. Fig. 1 gives the graphical illustration of the TE $|v_1(x,t)|$ solution given in (26). Fig. 2 shows the graphical illustration of the DBME $|v_1(x,t)|$ solution given in (34). Figs. 3 and 4 depict the graphical illustration of the TDBME $|v_{1,2}(x,t)|$ solution given in (40) and (42), respectively. Figs. 5 and 6 plot the graphical illustration of the PB-DNA-E $|v_{1,3}(x,t)|$ solution given in (46) and (48), respectively. In fact, these solutions are different from those outlined in [6] for the Tzitzéica-based equations and also different with those solutions constructed in [22] for the DNA model. Thus, we can say that the employed method reveals unique set of solutions to each of the models under consideration.

Fig. 1 represents a periodic soliton solution given by Eq. (26) for $f=0.1$ and $c=0.1$ exhibited in 3D by limiting the coordinates of x and t as $-10\le x\le 10$ and $0\le t\le 2$. While, the 2D propagation is drawn for the values of $t=0,1,$ and 2, respectively.

Fig. 2 depicts a dark-bright soliton solution given by Eq. (34) for $f=3$ and $c=2$ sketched in 3D through the coordinates of x and t as $-10\le x\le 10$ and $0\le t\le 2$. While, the 2D

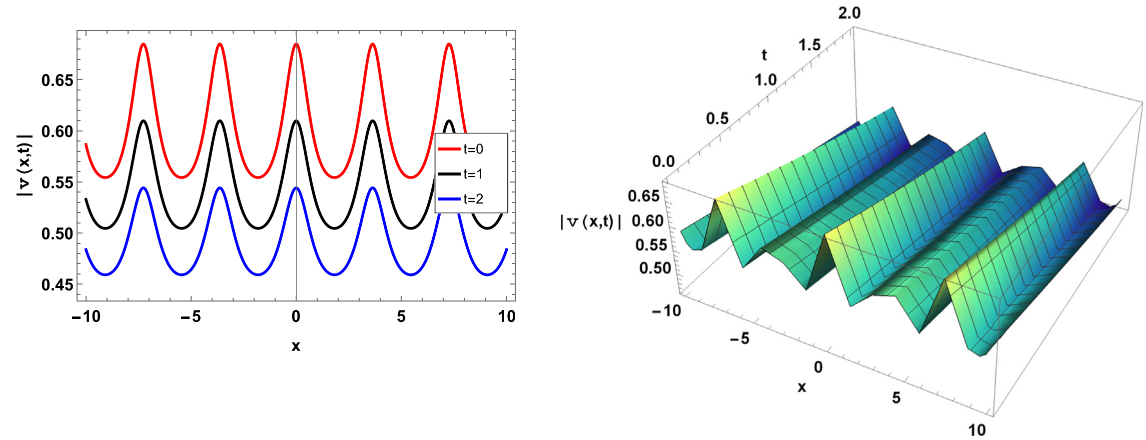

Fig. 1. 2D and 3D graphical illustrations of the TE solution $|v_1(x,t)|$ for (26) at $f = 0.1,\ c = 0.1$.

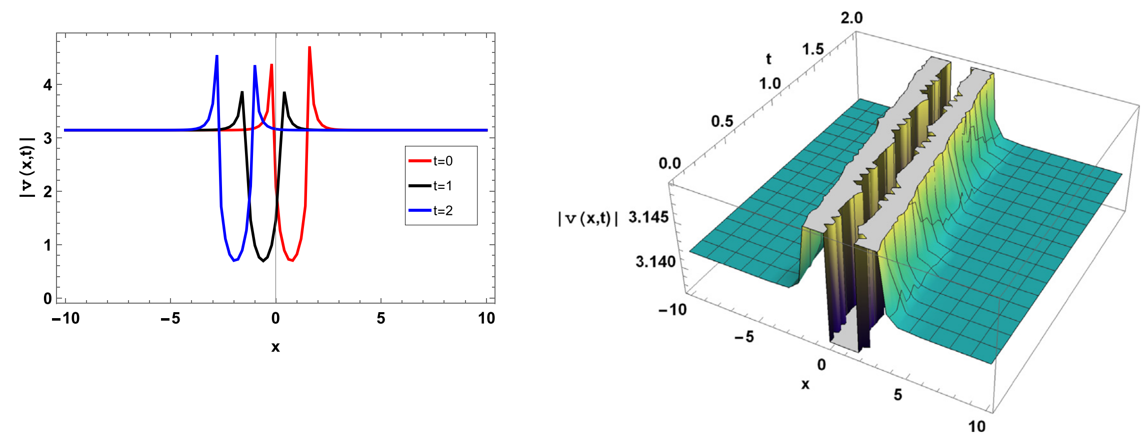

Fig. 2. 2D and 3D graphical illustrations of the DBME solution $|v_1(x,t)|$ for (34) at $f = 3,\ c = 2$.

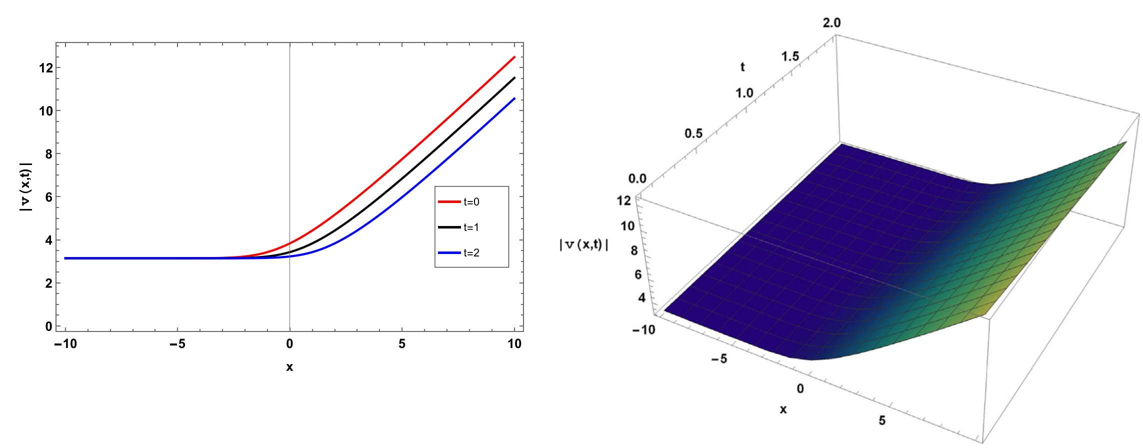

Fig. 3. 2D and 3D graphical illustrations of the TDBME solution $|v_1(x,t)|$ for (40) at $f = 8,\ c = 1$.

propagation is drawn for the values of $t = 0, 1$, and 2, respectively. It is clear that the solution is weakly dispersed (or disappeared) along the x-axis when $x \to \pm\infty$.

Fig. 3 depicts a kink soliton solution given by Eq. (40) for $f = 8$ and $c = 1$ sketched in 3D through the coordinates of x and t as $-10 \leq x \leq 10$ and $0 \leq t \leq 2$. While, the 2D propagation is drawn for the values of $t = 0, 1$, and 2, respectively. It is clear that the wave solution is propagated from left to right.

Fig. 4 depicts a kink soliton solution given by Eq. (42) for $f = 1$ and $c = 1$ sketched in 3D through the coordinates of x and t as $-10 \leq x \leq 10$ and $0 \leq t \leq 2$. While, the 2D propagation is drawn for the values of $t = 0, 1$, and 2, respectively. It is clear that the wave solution is propagated from right to left.

Fig. 5 depicts a dark soliton solution given by Eq. (46) for $f = 1.1,\ c = 0.2,\ \alpha_2 = 0.01,\ \alpha_3 = 0.3,\ \alpha_4 = 0.1,\ a = 0.8$, and $a_1 = 0.1$ sketched in 3D through the coordinates of x and t as $-10 \leq x \leq 10$ and $0 \leq t \leq 2$. While, the 2D propagation is drawn for the values of $t = 0, 1$, and 2, respectively. It is clear that the wave solution is symmetric about the origin.

Fig. 6 depicts a dark soliton solution given by Eq. (48) for $f = 1.1,\ c = 0.6,\ \alpha_2 = 0.01,\ \alpha_3 = 0.3,\ \alpha_4 = 0.1$, and $a = 0.8$ sketched in 3D through the coordinates of x and t as $-10 \leq x \leq 10$ and $0 \leq t \leq$

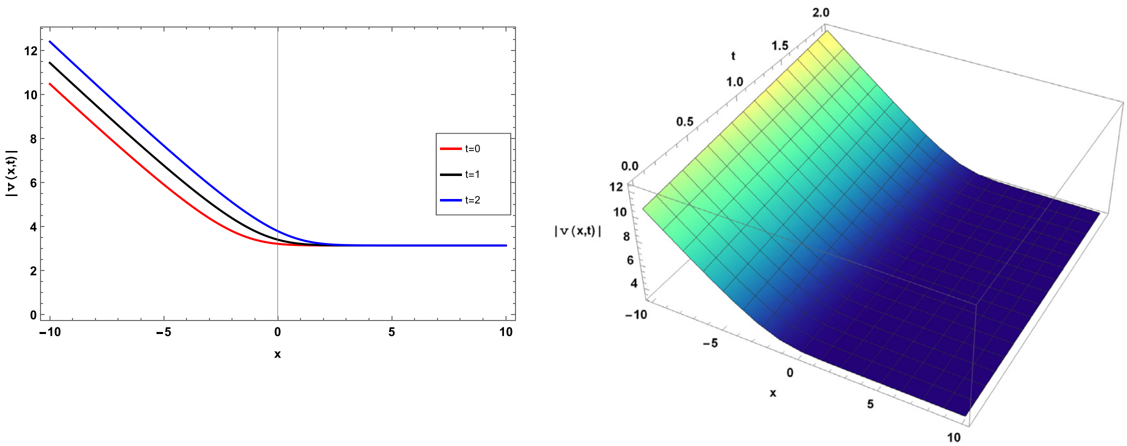

Fig. 4. 2D and 3D graphical illustrations of the TDBME solution $|v_2(x,t)|$ for (42) at $f=1,\ \ c=1$.

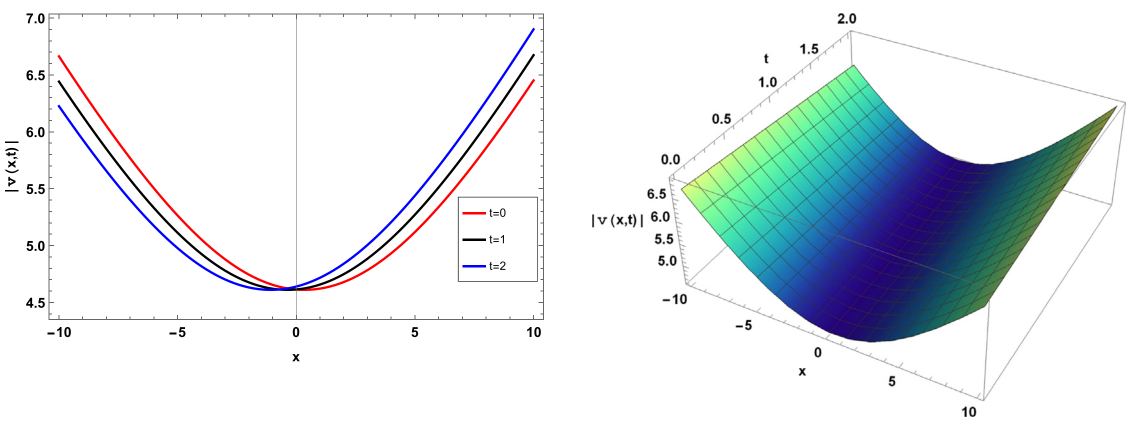

Fig. 5. 2D and 3D graphical illustrations of the TDBME solution $|v_1(x,t)|$ for (46) at $f=1.1,\ c=0.2,\ \alpha_2=0.01,\ \alpha_3=0.3,\ \alpha_4=0.1,\ a=0.8,\ a_1=0.1$.

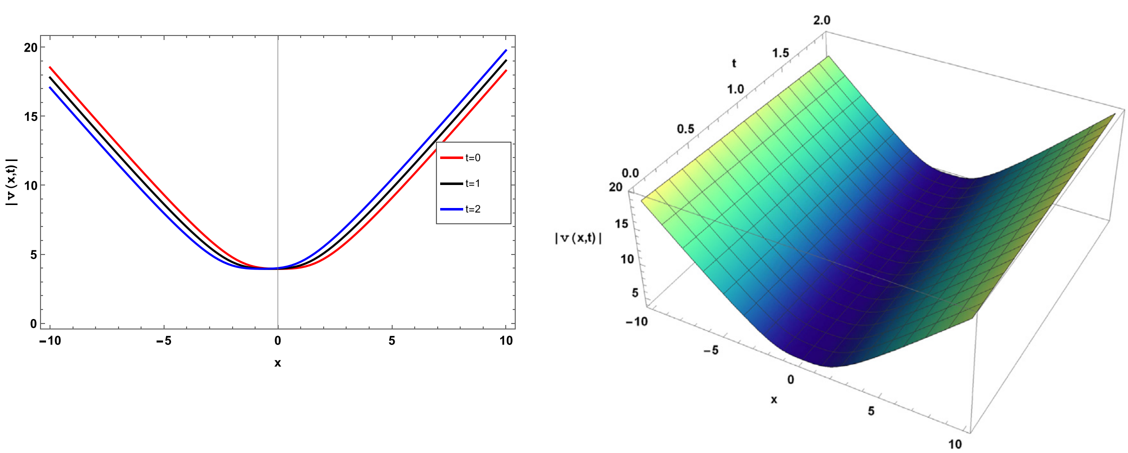

Fig. 6. 2D and 3D graphical illustrations of the TDBME solution $|v_3(x,t)|$ for (48) at $f=1.1,\ c=0.6,\ \alpha_2=0.01,\ \alpha_3=0.3,\ \alpha_4=0.1,\ a=0.8$.

2. While, the 2D propagation is drawn for the values of $t=0,1$, and 2, respectively. It is clear that the wave solution is symmetric about the origin.

6. Conclusion

In conclusion, we have employed the powerful Kudryashov method to construct various exponential traveling wave solutions to a class of evolution equations with double exponential nonlinearities. More precisely, the models include the TE, DBME, TDBME and PB-DNA-E, respectively. By these solutions, it is believed that a deep understanding of the evolution and dynamicity of these models will be brought to the light. As a result, new solitonic wave patterns attain, like as periodic, dark-bright, kink, and dark solitonic structures. The graphical 2D and 3D visualization of the obtained results is presented to express the pulse propagation behaviors by assuming the appropriate values of the involved parameters. Finally, we recommend the use of the used solution method in tackling different nonlinear evolution equation with various forms of nonlinearities; of course, after utilizing a required transformation (recall that we made use of the Painlevé transformation to recast the given PDEs to standard equations).

Credit author statement

All authors have contributed equally and must be given equal credit.

Data Availability

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Declaration of Competing Interest

Author has no conflict of interest.

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