Nonlinear Dyn (2023) 111:3701–3711
https://doi.org/10.1007/s11071-022-08015-z

ORIGINAL PAPER

Solitons in magnetized plasma with electron inertia under weakly relativistic effect

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Received: 30 August 2022 / Accepted: 15 October 2022 / Published online: 28 October 2022

Abstract In this relativistic consideration, the energy integral unlike others has been derived in a weakly relativistic plasma in terms of Sagdeev potential. Both compressive and rarefactive subsonic solitary waves are found to exist, depending on wave speeds in various directions of propagation. It is found that compressive relativistic solitons have potential depths that are higher than non-relativistic solitons in all directions of propagation, allowing for the presence of denser plasma particles in the potential well. Furthermore, it shows how compressive soliton amplitude grows as the propagation direction gets closer to the magnetic field's direction.

Keywords Sagdeev potential · Solitary waves · Mach number · Magnetic field · Energy integral

1 Introduction

Several authors have theoretically [1–23] and experimentally [24, 25] studied the existence of solitary waves under various physical phenomena in magnetized or unmagnetized plasma models using the standard reductive perturbation method. The works of Korteweg–De Vries (KdV) [26] and Washimi and Taniuti [27] have had a significant influence on the research of solitary waves. It is of paramount importance to include the presence of a magnetic field affecting a plasma medium that gives rise to different physical situations. The ratio between the collision and cyclotron frequencies determines how the magnetic

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field behaves. The behavior of the plasma is much more complicated with respect to the ion cyclotron frequency. Ion-acoustic solitons with finite amplitude and density hump that propagate obliquely to the external magnetic field have been shown to exist by Shukla and Yu [28]. The whole set of equations of motion for plane ion acoustic waves propagating at an angle to the magnetic field has been solved by Yu et al. [29] as stationary soliton solutions. The drifting influences of electrons on fully nonlinear ion-acoustic waves in a magnetoplasma have been studied by Kalita et al. [7]. They have observed that the drift motion of electrons along the magnetic field direction affects the ion-acoustic solitons. Yinhua and Yu [30] have investigated fully nonlinear ion-acoustic solitary waves propagating obliquely to the external magnetic field. Using Sagdeev's pseudo-potential method and the Thomas–Fermi density distribution for electrons, Chatterjee et al. [31] have investigated ion-acoustic solitary waves and double layers in a two-component dense magnetoplasma. Furthermore, many researchers have studied solitary waves in plasma in several techniques, which are widely available elsewhere.

Relativistic factors play a significant role in the formation of solitary waves when particle speeds are comparable to those of light. In the solar atmosphere and interplanetary space, for example, highly fast ions are commonly observed. Many researchers like Chain and Clemmow [32], Shukla et al. [33], and Arons [34] have investigated the nonlinear relativistic plasma waves in laser-plasma interactions and astronomical models. In a simple model of ion–electron plasma based on initial streaming, Das and Paul [35] have studied relativistic solitons. Chatterjee and Roychoudhury [36] have used Sagdeev's pseudo-potential method with electron inertia to study the influence of ion temperature in a relativistic plasma. They have demonstrated that the ion temperature limits the values of V, the soliton velocity. Using Sagdeev's pseudo-potential method in relativistic plasma, Roychoudhury et al. [37] have investigated the effect of ion and electron drifts on the presence of solitary waves. In a collisionless plasma made up of heated ions, Esfandyari et al. [38] have explored the influence of ion temperature and relativistic electron beam density on ion-acoustic solitons. Small amplitude relativistic solitons for electron inertia and mild relativistic effect were observed by Singh et al. [39]. In their investigation, electron inertia is not taken into account, and it is demonstrated that under constant plasma pressure, the soliton existence range between electron and ion speeds contracts. Das and Chatterjee [40] have studied large-amplitude solitary waves in relativistic plasmas with finite ion temperatures and electron inertia. In their investigation, they have noticed that there is a critical value u_0 of u at which $(u')^2 = 0$, beyond which the existence of solitary waves ceases to exist. Subject to a reasonable mathematical condition, Kalita and Das [41] have looked into higher and smaller-order relativistic effects in the formation of compressive solitons with large amplitudes in a defined range $u_0 - v_0$ and rarefactive solitons with small amplitudes in the small upper range of $|u_0 - v_0|$. It is said to be a reasonable justification for considering electron inertia in plasma that is susceptible to higher-order relativistic effects. In a warm magnetoplasma containing positive–negative ions and non-thermal electrons, El-Labany et al. [42] have studied the characteristics of small amplitude nonlinear ion-acoustic solitary waves by deriving Zakharov–Kuznetsov equation. It is shown that the mass and density ratios of the positive and negative ions as well as the non-thermal electron parameter have a significant impact on both compressive and rarefactive ion-acoustic solitary waves. Also, the variable-coefficient Zakharov–Kuznetsov equation that governs the two-dimensional ion-acoustic waves that are obliquely propagating in an inhomogeneous magnetized two-ion-temperature dusty plasma was explored by Qu et al. [43] using symbolic computation. Kalita et al. [22] have looked into the relativistic compressive solitons of fast acoustic mode in a magnetized ion-beam plasma. In a magnetized ion-beam plasma containing stationary warm ions, positive beam ions, and the usual electrons, Das [23] has investigated the formation of ion-acoustic solitary waves. In the weakly relativistic and magnetized plasma model, compressive solitons of low and high amplitudes have been studied by Kalita and Deka [44]. Their study reported that the increase in the initial flow velocity of electrons was found to be less effective in increasing the amplitude of compressed solitons due to mode one than the one corresponding to mode two. Rehmann et al. [45] have studied the ion sound waves via the nonlinear Zakharov–Kuznetsov equation in a magnetized plasma with two ionic components by using the perturbation method. They found that in such plasmas, two modes of ion sound waves can propagate at linear boundaries at fast and slow velocities. A variable-coefficient derivative nonlinear Schrödinger (vc-DNLS) equation explaining nonlinear Alfvén waves in inhomogeneous plasmas was studied by Wang et al. [46] using symbolic computation. They were able to find multi-solitonic solutions to the vc-DNLS equation in terms of the double Wronskian. Graphs are used to evaluate two- and three-solitonic interactions. The amplitudes and velocities of the solitonic waves are, respectively, controlled by plasma streaming and an inhomogeneous magnetic field. Sultana [47] has explored how non-Maxwellian κ-distributed electrons in a magnetized non-thermal collisional dusty plasma propagate ion-acoustic solitary waves in an oblique way.Kamalam and Ghosh [48] have used the Sagdeev pseudo-potential technique to investigate the ionic acoustic single wave of a three-component magnetized plasma consisting of warm fluid ions and two electrons at different temperatures in the Boltzmann distribution. El-Monier and Atteya [49] have investigated the nonlinear propagation of ionic noise in a collision-free dissipative ion-to-plasma magnetization system consisting of cold negatively charged non-relativistic ions and superthermal ions. Ullah et al. [50] have studied the propagation of ion single sound waves in an electron–positron plasma by using Tsalli distributed electrons and Maxwell positrons. They have found that changes in several numbers of important plasma characteristics, such as the non-extensive parameter, temperature ratio, direction cosine, positron concentration, and magnetic field intensity, have a considerable impact on the ion-acoustic solitary waves' distinctive qualities. In a homogeneous magnetized plasma for the bidirectional propagation close to the magneto-acoustic speed, Lan and Guo[51] have studied the coupled generalized nonlinear Schrödinger–Boussinesq system. The Hirota method is used to derive the expressions for the multi-soliton solutions. On the soliton, the effects of the group velocity, group dispersion coefficient for the upper hybrid, and magnetic field characteristics are discussed. Recently, Hassan and Sultana [52] have looked into the dust-ion-acoustic solitary waves in magnetized plasma, consisting of inertial ion species, non-inertial electron species following non-thermal κ-distribution, and immobile dust particles. In their investigation, they have reported that the basic characteristics of dissipative dust-ion-acoustic solitary waves and their modes of propagation are observed to significantly affect the variation of plasma configuration parameters as well as the variation of the superthermal index κ in the plasma system under consideration. Very recently, Nooralishahi and Salem [53] have studied the fully relativistic two-fluid hydrodynamic model used to examine the nonlinear propagation of stationary large-amplitude electromagnetic solitary waves in a magnetized electron–positron plasma. For more studies on different wave structures of nonlinear evolution equations, interested readers are referred to see [54–62]. The present paper analyzes the planar weakly relativistic ion-acoustic solitary waves in a magnetized cold plasma with electron inertia. In this relativistic consideration, the energy integral unlike others has been derived in a weakly relativistic plasma in terms of Sagdeev potential. The paper is structured as follows: The fundamental equations for our plasma model and the Sagdeev potential's derivation are presented in Sect. 2. In Sect. 3, the conditions for the existence of solitary waves are demonstrated. In Sect. 4, computational results for the presence of nonlinear structures are explored for various parametric ranges.

2 Governing equations

A magnetized plasma involving unidirectional weakly relativistic ions and highly magnetized electrons of constant temperature T_e is considered. Such a plasma model in the zx-plane is governed by the following fundamental equations:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_{ix})+\frac{\partial}{\partial z}(n_i u_{iz})=0, \tag{1}$$

$$\left(\frac{\partial}{\partial t}+u_{ix}\frac{\partial}{\partial x}+u_{iz}\frac{\partial}{\partial z}\right)\gamma_{i_x}u_{ix}=-\frac{\partial\varphi}{\partial x}+u_{iy}, \tag{2}$$

$$\left(\frac{\partial}{\partial t}+u_{ix}\frac{\partial}{\partial x}+u_{iz}\frac{\partial}{\partial z}\right)u_{iy}=-u_{ix}, \tag{3}$$

$$\left(\frac{\partial}{\partial t}+u_{ix}\frac{\partial}{\partial x}+u_{iz}\frac{\partial}{\partial z}\right)u_{iz}=-\frac{\partial\varphi}{\partial z}, \tag{4}$$

for the ions and

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial z}(n_e u_{ez})=0, \tag{5}$$

$$\left(\frac{\partial}{\partial t}+u_{ez}\frac{\partial}{\partial z}\right)u_{ez}=\frac{1}{Q}\left(\frac{\partial\varphi}{\partial z}-\frac{1}{n_e}\frac{\partial n_e}{\partial z}\right), \tag{6}$$

for the electrons, where

$$\gamma_{ix}=\frac{1}{\sqrt{1-V_x^2}}=1+\frac{V_x^2}{2}, V_x=\frac{u_{ix}}{c}, Q=\frac{m_e}{m_i},$$

is the electron-to-ion mass ratio and c is the speed of light. The authors have normalized the densities by the equilibrium plasma density n_0, time by the inverse of the ion gyro-frequency Ω_i, space by the ion gyro-radius $\rho_s=C_s/\Omega_i$, speed by $C_s[=(T_e/m_i)^{1/2}]$, and the potential by T_e/e in order to derive the set of Eqs. 1–6. Magnetized plasmas are anisotropic, i.e., the properties parallel to the magnetic field are quite different from those in directions perpendicular to it. Further, in the perpendicular direction of motion, relativistically length (so the space) is not contracted (otherwise doesn't change) and the proper time of the rest frame can be practically identical to the time t' of the moving frame. Choosing u_{iy}/c and u_{iz}/c to be very small inherent early beginning of submission to the propagation of solitary waves in one direction with constant speed will be rather more feasible along which, length contraction is mathematically justified. So, the components of velocity namely u_{iy}/c and u_{iz}/c in γ can be ignored.

We take a frame moving along with the wave given by

$$\xi=k_x x+k_z z-Mt, \tag{7}$$

for a stationary solution where

$$M=\text{Mach number}\left(=\frac{V}{C_s}=\frac{\text{pulse speed}}{\text{ion sound speed}}\right),$$

k_x and k_z are the direction cosines such that $k_x^2+k_z^2=1$. We can use the moving co-ordinate ξ to write from (7)

$$\frac{\partial}{\partial x}=k_x\frac{\partial}{\partial\xi}, \frac{\partial}{\partial z}=k_z\frac{\partial}{\partial\xi}, \frac{\partial}{\partial t}=-M\frac{\partial}{\partial\xi}.$$

Introducing the new co-ordinate ξ defined in (7), and using $u_{ix}=u_{iz}=0$, at $n_i=1$ as $|\xi|\to\infty$ after integration, Eq. 1 reduces to

$$k_x u_{ix}+k_z u_{iz}=M\left(1-\frac{1}{n_i}\right). \tag{8}$$

Using (7) and (8), Eqs. 2–4 can be simplified as

$$\frac{M}{n_i}\frac{\partial}{\partial\xi}(\gamma_{ix}u_{ix})=k_x\frac{\partial\varphi}{\partial\xi}-u_{iy}, \tag{9}$$

$$\frac{M}{n_i}\frac{\partial u_{iy}}{\partial\xi}=u_{ix}, \tag{10}$$

$$\frac{M}{n_i}\frac{\partial u_{iz}}{\partial\xi}=k_z\frac{\partial\varphi}{\partial\xi}. \tag{11}$$

Again making use of (7) in (5) and (6) and integrating once, we get

$$u_{ez}=\frac{M}{k_z}\left(1-\frac{1}{n_e}\right). \tag{12}$$

In acquiring Eq. 12, $u_{ez}=0$, at $n_e=1$ as $|\xi|\to\infty$ has been used.

Employing the co-ordinate ξ and using (12), Eq. 6 can be once integrated to give

$$n_e=e^{\varphi+A\left(1-\frac{1}{n_e^2}\right)}, A=\frac{QM^2}{2k_z^2}, \tag{13}$$

under the boundary conditions $\varphi=0$, at $n_e=1$ as $|\xi|\to\infty$.

Making use of (13) and $n_i=n_e=n$, Eq. 11 can be integrated once to yield

$$u_{iz}=\frac{k_z}{M}(n-1)\left(1-\frac{2A}{n}\right). \tag{14}$$

With the use of (14), from (8) we can get

$$u_{ix}=\frac{M}{k_z}(n-1)\left\{\frac{Q}{2A}\left(1-\frac{2A}{n}\right)\right\}. \tag{15}$$

Putting the value of u_{ix} from (15) into (9) u_{iy} can be determined as

$$u_{iy}=f(n)\frac{1}{n}\frac{dn}{d\xi}, \tag{16}$$

where $f(n)=\frac{A_1}{n^4}+\frac{A_2}{n^3}+\frac{A_3}{n^2}+A_4+A_5 n+A_6 n^2$ with

$$A_1 = -\frac{3M^4L^3H^3}{2c^2},$$

$$A_2 = -2A_1 + B,$$

$$A_3 = -(D - A_1 + 6B + E + G),$$

$$A_4 = H + 2B + 3E + A_6,$$

$$A_5 = -2(B + A_6),$$

$$A_6 = -\frac{3k_z^6H^3}{2c^2M^2},$$

where $B = \frac{3M^2k_z^2L^2H^2}{c^2}$, $D = M^2HL$, $E = \frac{3k_z^4LH^3}{2c^2}$, $G = \frac{2A}{H}$, $H = \frac{1}{k_x}$, and $L = 1 + Q$.

In deriving (16), we have used Eq. 13. With the values of u_{ix} and u_{iy} from (15) and (16) respectively, one can obtain from (10) the following expression

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left\{f(n)\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}\xi}\right\} = H(n-1)\left(L - \frac{Q}{2A}n\right). \tag{17}$$

Multiplying both sides of (17) by the term in the parenthesis, it can be integrated to recover the following energy integral for the classical particles with the Sagdeev potential ψ

$$\frac{1}{2}\left(\frac{\mathrm{d}n}{\mathrm{d}\xi}\right)^2 + \psi(n, M, k_z) = 0, \tag{18}$$

where

$$\psi(n, M, k_z) = g(n)h(n), \tag{19}$$

with

$$g(n) = \frac{n^2}{\{f(n)\}^2}, \tag{20}$$

and

$$\begin{aligned} h(n) = -H\Bigg[&\frac{A_1L}{4}\left(\frac{1}{n^4} - 1\right) - \frac{1}{3}\left\{\left(L + \frac{Q}{2A}\right)A_1 - A_2L\right\}\left(\frac{1}{n^3} - 1\right) \\ &- \frac{1}{2}\left\{\left(L + \frac{Q}{2A}\right)A_2 - A_3L - \frac{A_1Q}{2A}\right\}\left(\frac{1}{n^2} - 1\right) \\ &- \left\{\left(L + \frac{Q}{2A}\right)A_3 - \frac{A_2Q}{2A}\right\}\left(\frac{1}{n} - 1\right) - \left\{A_4L + \frac{A_3Q}{2A}\right\}\log n \\ &+ \left\{\left(L + \frac{Q}{2A}\right)A_4 - A_5L\right\}(n-1) + \frac{1}{2}\left\{\left(L + \frac{Q}{2A}\right)A_5 - A_6L - \frac{A_4Q}{2A}\right\}(n^2 - 1) \\ &+ \frac{1}{3}\left\{\left(L + \frac{Q}{2A}\right)A_6 - \frac{A_5Q}{2A}\right\}(n^3 - 1) - \frac{A_6Q}{8A}(n^4 - 1)\Bigg], \end{aligned} \tag{21}$$

and the boundary condition $\frac{\mathrm{d}n}{\mathrm{d}\xi} = 0$ at $n = 1$ has been used.

3 Conditions for the existence of solitary waves

By exploring the behavior of $\psi(n)$ near $n = 1$ and $n = N$, where N is the maximum value of n, i.e., the amplitude of the solitary wave pulse, the necessary conditions for the existence of localized solitary waves can be retrieved. We need to set $\psi(N) = 0$ give the amplitude ''N'' of the solitary wave pulse for the nonlinear dispersion relation such that

$$\begin{aligned} &\frac{A_1L}{4}\left(\frac{1}{N^4} - 1\right) - \frac{1}{3}\left\{\left(L + \frac{Q}{2A}\right)A_1 - A_2L\right\}\left(\frac{1}{N^3} - 1\right) - \frac{1}{2}\left\{\left(L + \frac{Q}{2A}\right)A_2 - A_3L - \frac{A_1Q}{2A}\right\}\left(\frac{1}{N^2} - 1\right) \\ &- \left\{\left(L + \frac{Q}{2A}\right)A_3 - \frac{A_2Q}{2A}\right\}\left(\frac{1}{N} - 1\right) - \left\{A_4L + \frac{A_3Q}{2A}\right\}\log N \\ &+ \left\{\left(L + \frac{Q}{2A}\right)A_4 - A_5L\right\}(N-1) + \frac{1}{2}\left\{\left(L + \frac{Q}{2A}\right)A_5 - A_6L - \frac{A_4Q}{2A}\right\}(N^2 - 1) \\ &+ \frac{1}{3}\left\{\left(L + \frac{Q}{2A}\right)A_6 - \frac{A_5Q}{2A}\right\}(N^3 - 1) - \frac{A_6Q}{8A}(N^4 - 1) = 0. \end{aligned} \tag{22}$$

Furthermore, the conditions for the existence of solitary waves are

$$\psi(1) = \psi(N) = \psi'(1) = 0, \tag{23}$$

and

$$\psi(n) < 0, \tag{24}$$

between $n = 1$ and $n = N$.

Now, to arrive at the mathematical conditions, we consider

$$\begin{aligned} h'(n) = -H\Bigg[&-\frac{A_1L}{n^5} + \left\{\left(L + \frac{Q}{2A}\right)A_1 - A_2L\right\}\frac{1}{n^4} + \left\{\left(L + \frac{Q}{2A}\right)A_2 - A_3L - \frac{A_1Q}{2A}\right\}\frac{1}{n^3} \\ &+ \left\{\left(L + \frac{Q}{2A}\right)A_3 - \frac{A_2Q}{2A}\right\}\frac{1}{n^2} - \left\{LA_4 + \frac{A_3Q}{2A}\right\}\frac{1}{n} + \left\{\left(L + \frac{Q}{2A}\right)A_4 - A_5L\right\} \\ &+ \left\{\left(L + \frac{Q}{2A}\right)A_5 - A_6L - \frac{A_4Q}{2A}\right\}n + \left\{\left(L + \frac{Q}{2A}\right)A_6 - \frac{A_5Q}{2A}\right\}n^2 - \frac{A_6Q}{2A}n^3\Bigg], \end{aligned} \tag{25}$$

and

$$\begin{aligned} h''(n) = -H\Bigg[&\frac{5A_1L}{n^6} - 4\left\{\left(L + \frac{Q}{2A}\right)A_1 - A_2L\right\}\frac{1}{n^5} \\ &- 3\left\{\left(L + \frac{Q}{2A}\right)A_2 - A_3L - \frac{A_1Q}{2A}\right\}\frac{1}{n^4} - \left\{\left(L + \frac{Q}{2A}\right)A_3 - \frac{A_2Q}{2A}\right\}\frac{1}{n^3} \\ &+ \left\{LA_4 + \frac{A_3Q}{2A}\right\}\frac{1}{n^2} + \left\{\left(L + \frac{Q}{2A}\right)A_5 - A_6L - \frac{A_4Q}{2A}\right\} \\ &+ 2\left\{\left(L + \frac{Q}{2A}\right)A_6 - \frac{A_5Q}{2A}\right\}n - \frac{3A_6Q}{2A}n^2\Bigg], \end{aligned} \tag{26}$$

where prime denotes the differentiation with respect to n.

It is observed from Eqs. 21, 25, and 26 that at $n = 1$

$$h(1) = 0,\ h'(1) = 0,$$

and

$$h''(1) = 2\left(L - \frac{Q}{2A}\right)\left(J + \frac{Q}{2A}\right)\frac{AH^2}{Q}, \quad (27)$$

with

$$J = Q + k_z^2.$$

With the above values and Eq. 20 at $n = 1$, we get

$$\psi(1) = 0,\ \psi'(1) = 0,$$

and

$$\psi''(1) = \frac{Q(2AL - Q)}{2A(2AJ - Q)}. \quad (28)$$

The nonlinear dispersion relation (22) is deduced by setting $\psi(N) = g(N)h(N) = 0$ for which $h(N) = 0$, since $g(N) \neq 0$ so that

$$h'(N) = -(N-1)H\left(\frac{L}{N} - \frac{Q}{2A}\right)\left[\frac{3H^3(N-1)^2}{2c^2}\frac{1}{N^4}\left(\frac{Q}{2A}N^2 - L\right)(Nk_z^2 - LM^2)^2 + \frac{H\{(N^2 - M^2) - 2A\}}{N^2}\right],$$

and

$$\psi'(N) = \frac{c^2QN^5(N-1)(NQ - 2AL)}{AH^2\left[3k_z^2(N-1)^2(QN^2 - 2AL)\left(\frac{QN}{2A} - L\right) + 2c^2k_x^2N^2\{(N^2 - M^2)Q - 2QA\}\right]}. \quad (29)$$

The set of conditions (23) is satisfied because of (28) and (22). The second set of conditions (24), $\psi(n)$ is expanded in Taylor's series near $n \approx 1$ and $n \approx N$ to give

$$\psi(n \approx 1) = \psi(1) + (n-1)\psi'(1) + \frac{(n-1)^2}{2!}\psi''(1) + \cdots,$$

and

$$\psi(n \approx N) = \psi(N) + (n-N)\psi'(1) + \frac{(n-N)^2}{2!}\psi''(N) + \cdots.$$

With the help of (28), (22), and (29), these can be expressed as

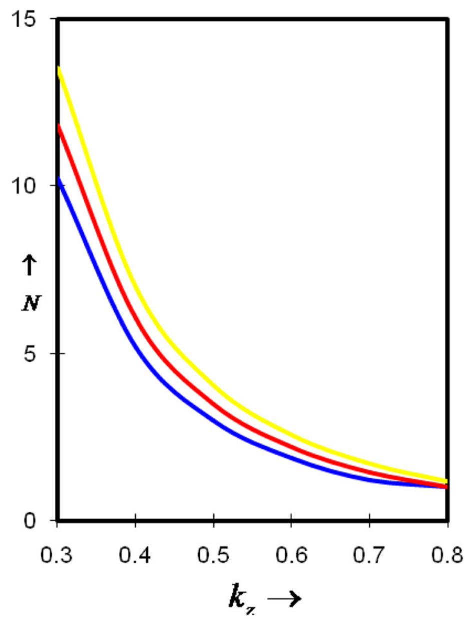

Fig. 1 Variation in the amplitude with k_z of the subsonic compressive soliton at various wave speeds $M =$ 0.75 (Blue), 0.80 (Red), 0.85 (Yellow) when $c = 300$

$$\psi(n \approx 1) = \frac{(n-1)^2Q(2AL - Q)}{4AM^2(2AJ - Q)}, \quad (30)$$

which is reducible from the works of Kalita et al. [7]exactly for $v_e' = 0$ in non-relativistic cases. As there is no initial streaming in this consideration, therefore, at the equilibrium stage where $n = 1$, there is no relativistic effect and so the above condition is justified and

$$\psi(n \approx N) = \frac{c^2Q(n-N)N^5(N-1)(NQ - 2AL)}{AH^2\left[3k_z^2(N-1)^2(QN^2 - 2AL)\left(\frac{QN}{2A} - L\right) + 2c^2k_x^2N^2\{(N^2 - M^2)Q - 2QA\}\right]}, \quad (31)$$

From (30) and (31), the following conditions finally are acquired for $\psi(n) < 0$ between $n = 1$ and $n = N$ to represent solitary waves:

$$\text{near } n = 1, Q + k_z^2 < \frac{k_z^2}{M^2} < 1 + Q, \quad (32)$$

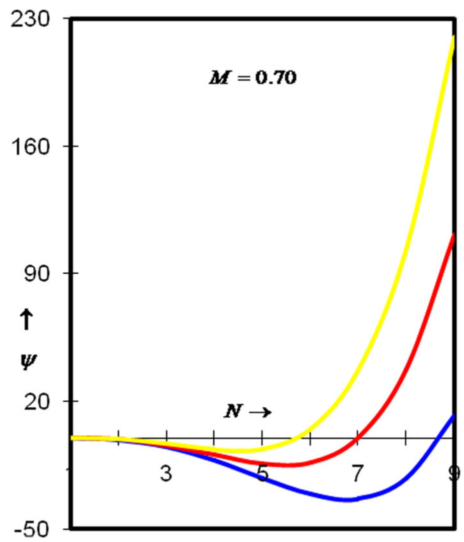

Fig. 2 The reflection of the potential wells characterized by $\psi(n)$ of the energy integral shows the amplitudes and corresponding depths of the relativistic compressive solitons for $k_z = 0.30$ (Blue), 0.33 (Red), 0.36 (Yellow) when $c = 300$ and $M = 0.70$

near $n = N,\ 1 > M \geq N,\ M \geq k_z,$ when $N < 1,$ (33)

$$\text{and}\, N > \frac{M^2}{k_z},\ M \geq k_z \text{ when } N > 1. \tag{34}$$

4 Results and discussion

In the current model of magnetized weakly relativistic plasma under consideration, both compressive $(N > 1)$ and rarefactive $(N < 1)$ subsonic $(M < 1)$ solitary waves are found to exist, depending on wave speeds in various directions of propagation. The amplitude of the compressive soliton is found to diminish rapidly (Fig. 1) with k_z for all soliton speeds $M = 0.75$ (Blue), 0.80 (Red) and 0.85 (Yellow). It is worthwhile to mention that, in the vicinity where the values of the direction of wave propagation k_z tend to attain the upper limit of subsonic soliton speeds $(k_z \to M)$, the relativistic compressive solitons appear to move almost with constant amplitudes. On the other hand, they attain various high amplitudes for small $k_z < M$ (Fig. 1). It is essential to report that the potential depths of compressive relativistic solitons (Fig. 2) increases with the decrease in k_z. Further, it demonstrates the increase in compressive soliton amplitude as the propagation direction approaches the direction of the magnetic field for $k_z = 0.30$ (Blue), 0.33 (Red) and 0.36 (Yellow) and $M = 0.70$. Unlike compressive solitons, the depths of potential wells of rarefactive relativistic solitons (Fig. 3a) for $M = 0.40$ and (Fig. 3b) for $M = 0.60$ are found to decrease with mach number M as it increases for $k_z = 0.10$ (Blue), 0.15 (Pink) and 0.20

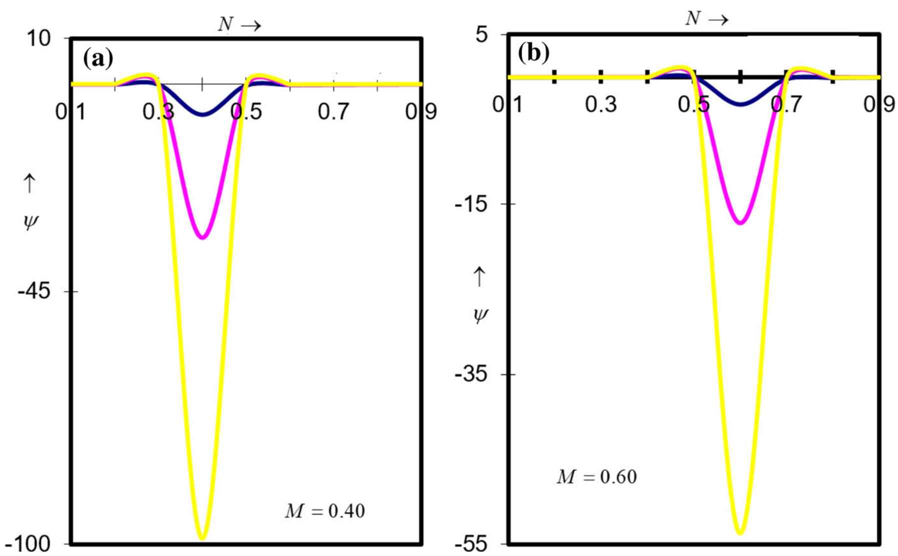

Fig. 3 The reflection of the potential wells characterized by $\psi(n)$ of the energy integral shows the amplitudes and corresponding depths of the relativistic compressive solitons for $k_z = 0.10$ (Blue), 0.15 (Pink), and 0.20 (Yellow) when $c = 300$ and $M = 0.40$ (**a**), $M = 0.60$ (**b**)

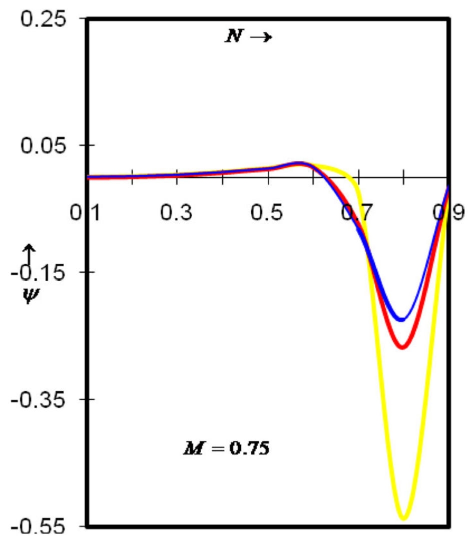

Fig. 4 The reflection of the potential wells characterized by $\psi(n)$ of the energy integral shows the amplitudes and corresponding depths of the relativistic compressive solitons for $k_z = 0.10$ (Yellow), 0.20 (Red), 0 .30 (Blue) when $c = 300$ and $M = 0.75$

(Yellow). This reveals that the plasma particles (lighter) moving with relativistic speeds are not constrained to be in deeper potential wells in case of relativistic but rarefactive solitons ascertaining submission to relativistic effects. The rarefactive soliton widths being dependent on potential depths are observed to decrease with the mach number M (Fig. 3a, b) as it increases slowly. Additionally, the potential depths of relativistic rarefactive solitons are seen to decrease with k_z for fixed higher values of $M = 0.75$(Fig. 4). Otherwise, as the direction of propagation deviates from that of the magnetic field, the potential depths of rarefactive solitons tend to be smaller. Interestingly, the amplitudes of the compressive solitons reflect a uniform increase (Fig. 5a) with subsonic wave speed M for all $k_z = 0.20\,(\text{Yellow})$, $0.30\,(\text{Red})$ and $0.40\,(\text{Blue})$. Besides, amplitudes of the rarefactive solitons which appear to exist far away from the direction of the magnetic field (Fig. 5b), i.e., for all (smaller) $k_z = 0.05\,(\text{Yellow})$, $0.10\,(\text{Red})$ and $0.15\,(\text{Blue})$ are observed to decrease almost linearly with M. But, the amplitudes of compressive solitons grow nonlinearly with M and at the increasing difference at the step-up increase in k_z (Fig. 5a). On the contrary, those of rarefactive solitons grow almost linearly with M maintaining an almost regular difference at the step-up increase in k_z except for smaller k_z. The higher and nearly constant amplitudes of the rarefactive solitons are found to decrease slowly initially (Fig. 6) for small k_z for all $M = 0.70\,(\text{Blue})$, $0.80\,(\text{Pink})$ and $0.90\,(\text{Yellow})$ which gradually increases with k_z. Besides, the higher is the wave speed, the higher is the corresponding rarefactive soliton amplitude.

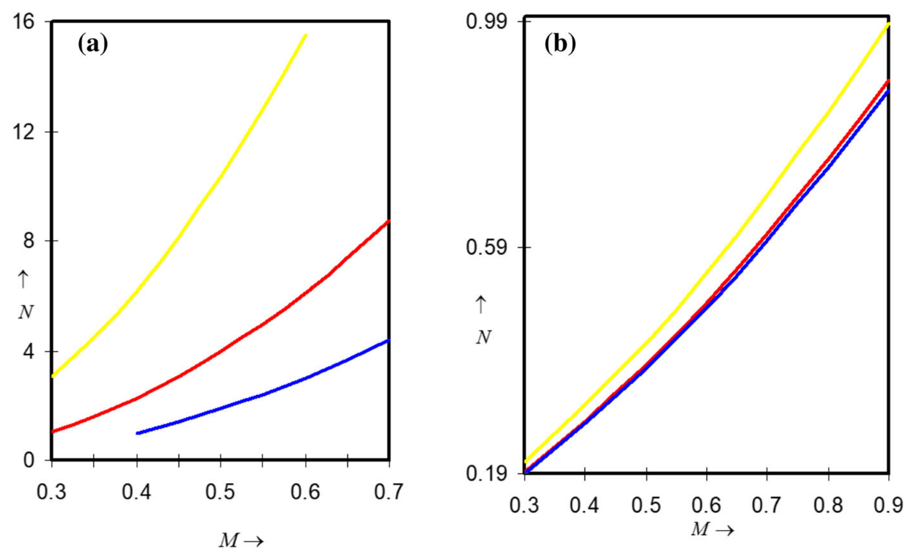

Fig. 5 Variation of subsonic compressive soliton amplitude (**a**) and rarefactive soliton amplitude (**b**) with M for different $k_z = 0.20$ (Yellow), 0.30 (Red), 0 .40 (Blue)(**a**) and $k_z = 0.05$(Yellow),0.10(Red),0.15(Blue)(**b**) when $c = 300$

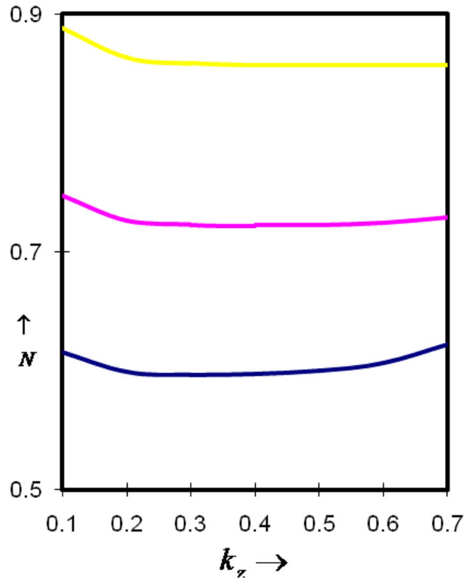

Fig. 6 Variation in the amplitude with k_z of the subsonic rarefactive soliton at various wave speeds $M = 0.70\,(\text{Blue}),\ 0.80\,(\text{Pink}),\ 0.90\,(\text{Yellow})$ when $c = 300$

The paper can be extended for future work by taking into account the relativistic effects on electrons but ions are non-relativistic.

Acknowledgements The authors are grateful to the anonymous referees for the suggestions made for the improvement of the paper from its earlier form.

Funding The authors have not disclosed any funding.

Data Availability Inquiries about data availability should be directed to the authors.

Declarations

Conflict of Interest The authors declare no conflict of interest.

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