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Research article

Unsteady Casson fluid flow over a vertical surface with fractional bioconvection

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Abstract: This paper deals with unsteady flow of fractional Casson fluid in the existence of bioconvection. The governing equations are modeled with fractional derivative which is transformed into dimensionless form by using dimensionless variables. The analytical solution is attained by applying Laplace transform technique. Some graphs are made for involved parameters. As a result, it is found that temperature, bioconvection are maximum away from the plate for large time and vice versa and showing dual behavior in their boundary layers respectively. Further recent literature is recovered from the present results and obtained good agreement.

Keywords: Casson fluid; bioconvection; fracional modeling; heat transfer; vertical plate
Mathematics Subject Classification: 35A22, 76A05

1. Introduction

Fractional calculus is the section of mathematics which deals with the analysis and applications of non-local differentiation and integration [1]. In many sections of engineering and science, fractional calculus and the applications of fractional calculus are now regarded as an important platform for accomplished calculations to create new strengths for complex non-local systems [2–6]. Butt et al. [7] accomplished solutions of semi analytical for the mass and heat transfer of a fractional MHD Jeffery fluid beyond an incalculable oscillating vertical plate by the Caputo Fabrizio fractional operator. Aman et al. [8] utilized fractional derivatives to seek the transfer of mass and heat of nanofluids (sodium alginate (SA) carrier fluid with graphene nanoparticles). Ali et al. [9] calculated the electrically conducted blended convection stream of summed up Jeffrey nanoliquid in a pivoted liquid cross an endless wavering plate soaked in a permeable medium through Atangana-Baleanu fractional approach. Khan et al. [10] used Caputo-Fabrizio fractional operator for the problem of generalized Maxwell fluid heat transfer above an incalculable perpendicular plate. Imran et al. [11, 12] used Caputo-fractional derivative to analyze the different problems related to fluid flow. Saad et al. [13] put forward approximations for model of a fractional cubic isothermal auto-catalytic chemical system by applying Caputo (C), Caputo-Fabrizio (CF) and Atangana-Baleanu fractional derivatives. In (2017), Shiekh et al. [14] evaluated the solutions of Jeffrey fluid with Caputo-Fabrizio and Atangana-Baleanu fractional derivative with the benefit of Laplace transform method. Ali et al. [15] analyzed the heat transfer of generalized Jeffery nanofluid in a rotating frame. Ahmad et al. [16] calculated the outcomes of Caputo and Caputo-Fabrizio for the mass and heat transmission of fractional Jeffrey fluid flow on incalculable perpendicular plate in motion aggressively at different temperature. For further study on fractional operators these references can be consulted [17–29].

In 2020, Baleanu et al. [30] suggested new fractional operator Constant Proportional Caputo (CPC) which is the mixture of Caputo derivative and Riemann-Liouville integral. Inspired by it, to contrast the new fractional approach with Caputo-Fabrizio (CF) and Caputo (C) fractional operators is of massive interest. In the existing literature there is no such type of comparison with new Constant Proportional Caputo (CPC). By introducing dimensionless variables, the dominant equations are turned into the position of non-dimensional equations. The classic model of Casson fluid is turned into a fractional model of form α. The current problem is determined by Constant Proportional Caputo (CPC) fractional operator. The exact solutions for concentrations, temperatures and velocity are attained with the technique of Laplace transform. The equivalent ratios of skin, heat friction and mass are also evaluated. We have drawn a contrast approach using MathCad software among the solutions of fractional models of (C) and (CPC) fractional derivatives graphically.

The event of bioconvection is one more extraordinary field which comprises of various natural applications. The convective motion of a matter because of gradient of density at tiny level is named as bioconvection. This flimsiness in thickness inclination happened because of aggregate swimming of microorganisms. This wonder generally happens at the uppermost level of fluid because of which the fluid in that particular district evolves into denser. Flimsiness in stream framework additionally happens because of the isolation in thickness of the lower and upper level of fluid. There are various clinical and organic cycles that require this actual marvel, for example, bio-energizes, chemicals, miniature framework, natural tissues, microbes and bio-innovation etc. The bioconvection measure is ordered into various classifications, for example, gyrotactic microorganism, chemotaxis and geotactic microorganisms. This order depends on the directional developments of different microorganisms. Kuznetsov [31, 32] explored the bioconvection by utilizing different kinds of nano particles. Mallikarjuna et al. [33] examined the consistent bio-convective flow for a nanofluid with gyrotactic microorganism over an upward chamber and changed the displayed issue into dimensionless structure by utilizing dimensionless factors and afterward have tackled resultant conditions in mathematical structure by limited contrast strategy. Uddin et al. [34] examined mathematically the numerical model to analysis the effects of speed slip of second request past a flat penetrable plate. Chebyshev strategy utilized in this examination for rough arrangement of issue. The viewer can additionally concentrate about the bioconvection liquid stream with various calculations and flow conditions in references [35–39].

Therefore, bioconvection related studies are carried out with classical models for different geometries, like plate sheet, cylinder, regular and irregular surfaces as discussed in the above literature. There is a gap in the existing literature with fractional approach of bioconvection. In 2021, Imran et al. [40] published the fractional bioconvection effect for viscous fluid over a vertical geometry (see Figure 1) and discussed the effect of fractional parameter and bioconvection number on the fluid flow with integral transform approach. In their work, the Caputo fractional model was obtained using Fourier and Ficks's Laws for energy and diffusion equation and got good response. In the current work, we extended [40] for non-Newtonian fluid with fractional modeling through generalized laws of heat and mass transfer by new fractional operator which is a linear combination of two fractional operators. Analytical solutions are obtained via Laplace transform method and some graphical results for different flow parameters drawn to see physical behavior. Furthermore, it is shown that newly applied fractional operator is stronger in decaying nature than the Caputo one.

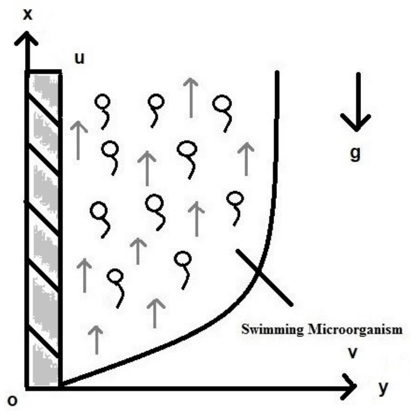

Figure 1. Geometry of the problem.

2. Mathematical formulation

Consider a turbulent heat flood of a thick liquid by a level area in a xy-coordinate framework arranged at $y = 0$. First and foremost at $t = 0$, both dish and fluid are at rest with mention concentration of microorganisms N_∞ and reference surface temperature T_∞. Ultimately after a few time $t = 0^+$, dish starts motion at consistent speed and concentration of microorganisms of dish N_w expand at surface temperature T_w. As length is incalculable, so y and t are the two functions of every physical quantity. First of all we make constitutive fractional model for bioconvection equations and energy balance equations then address them by Laplace transform technique, trailed with velocity field. The momentum equation to a thick liquid having bioconvection expression as [41–44].

$$\rho u_t(y,t) = \mu(1 + \frac{1}{\lambda})u_{yy}(y,t) + g[\rho\beta_T(T - T_\infty) - \gamma(\rho_m - \rho)(N - N_\infty)]. \tag{2.1}$$

The thermal balance equation given by [41–44]

$$(\rho C_p)T_t(y,t) = -q_y(y,t). \tag{2.2}$$

The fractional thermal flux equation of heat conduction by applying Fourier's principle became

$$q(y,t) = -k\,{}^{CPC}D_t^{\beta} T_y(y,t). \tag{2.3}$$

The diffusion balance equation given by [41–44]

$$N_t(y,t) = -J_y(y,t). \tag{2.4}$$

The fractional bioconvection concentration equation given by [41–44]

$$J(y,t) = -D_n\,{}^{CPC}D_t^{\gamma} N_y(y,t). \tag{2.5}$$

Based on the conditions,

$$u(y,0) = 0, \quad u(0,t) = u_0 H(t), \quad u(y,t) \to 0 \ \ as \ \ y \to \infty, \quad t > 0 \quad y \geq 0, \tag{2.6}$$

$$T(y,0) = T_\infty, \quad T(0,t) = T_w, \quad T(y,t) \to T_\infty \ \ as \ \ y \to \infty, \quad t > 0, \quad y \geq 0, \tag{2.7}$$

$$N(y,0) = N_\infty, \quad N(0,t) = N_w, \quad N(y,t) \to N_\infty \ \ as \ \ y \to \infty. \quad t > 0 \quad y \geq 0. \tag{2.8}$$

Introducing the following dimensionless variables into Eqs (2.1)–(2.5)

$$y^* = \frac{u_0 y}{\nu}, \qquad u^* = \frac{u}{u_0}, \qquad t^* = \frac{t u_0^2}{\nu}, \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$N^* = \frac{N - N_\infty}{N_w - N_\infty}, \qquad q^* = \frac{q}{q_0}, \qquad J^* = \frac{J}{J_0}. \tag{2.9}$$

We have the following dimensionless problem (ignoring *)

$$u_t(y,t) = A u_{yy}(y,t) + \mathrm{Gr}[\theta(y,t) - \mathrm{Ra} N(y,t)]. \tag{2.10}$$

The dimensionless thermal balance equation given by

$$\theta_t(y,t) = -B\, q_y(y,t). \tag{2.11}$$

The dimensionless thermal flux equation of heat conduction by applying Fourier's principle became

$$q(y,t) = -C\,{}^{CPC}D_t^{\beta}[\theta_y(y,t)]. \quad 0 < \beta \leq 1. \tag{2.12}$$

The dimensionless diffusion balance equation is

$$N_t(y,t) = -E J_y(y,t). \tag{2.13}$$

The dimensionless bioconvection concentration equation is

$$J(y,t) = -F\,{}^{CPC}D_t^{\gamma}[N_y(y,t)], \quad 0 < \gamma \leq 1, \tag{2.14}$$

with dimensionless conditions

$$u(y,0) = 0, \quad u(0,t) = H(t), \quad u(y,t) \to 0 \ \ as \ \ y \to \infty, \tag{2.15}$$

$$\theta(y,0) = 0, \quad \theta(0,t) = 1, \quad \theta(y,t) \to 0 \ \ as \ \ y \to \infty, \tag{2.16}$$

$$N(y,0)=0,\quad N(0,t)=1,\quad N(y,t)\to 0\ \ as\ \ y\to\infty. \tag{2.17}$$

where

$$\begin{aligned}
\mathrm{Lb} &= \frac{\nu}{D_n},\\
A &= 1+\frac{1}{\lambda},\\
B &= \frac{q_0\nu}{\mu C_p u_0(T_w-T_\infty)},\\
C &= \frac{k u_0(T_w-T_\infty)}{q_0\nu},\\
E &= \frac{J_0}{u_0(N_w-N_\infty)},\\
F &= \frac{D_n(N_w-N_\infty)u_0}{J_0\nu},\\
\mathrm{Pr} &= \frac{\mu C_p}{\mathrm{k}},\\
\mathrm{Gr} &= \frac{g\nu\beta_T(T_w-T_\infty)}{u_0^3},\\
\mathrm{Ra} &= \frac{\gamma(\rho_m-\rho)(N_w-N_\infty)}{\beta_T(T_w-T_\infty)\rho}.
\end{aligned} \tag{2.18}$$

Using Eq (2.12) in Eq (2.11) and Eq (2.14) in Eq (2.13), final forms of dimensionless thermal balance equation and bioconvection concentration equation become respectively,

$$\theta_t(y,t)=\frac{1}{\mathrm{Pr}}\,{}^{CPC}D_t^{\beta}\theta_{yy}(y,t). \tag{2.19}$$

$$N_t(y,t)=\frac{1}{\mathrm{Lb}}\,{}^{CPC}D_t^{\gamma}N_{yy}(y,t). \tag{2.20}$$

2.1. Heat equation

Taking Laplace transform on Eq (2.19), we have

$$s\bar{\theta}(y,s)=\frac{1}{\mathrm{Pr}}\left[\frac{k_1(\beta)}{s}+k_0(\beta)\right]s^{\beta}\bar{\theta}_{yy}(y,s). \tag{2.21}$$

Also apply Laplace on Eq (2.16)

$$\bar{\theta}(y,0)=0,\qquad \bar{\theta}(0,s)=\frac{1}{s},\qquad \bar{\theta}(y,s)\to 0\ \ as\ \ y\to\infty. \tag{2.22}$$

Using Eq (2.22) in Eq (2.21), the general solution of temperature profile becomes

$$\bar{\theta}(y,s)=\frac{1}{s}\exp\left(-y\sqrt{\frac{\mathrm{Pr}s^{1-\beta}}{\left[\frac{k_1(\beta)}{s}+k_0(\beta)\right]}}\right). \tag{2.23}$$

The Eq (2.23) is in exponential form is very complicated so it is hard to find the inverse formula with this form. Therefore, we used the series form of exponential function and expressed in more suitable form to obtain inverse analytically.

$$\bar{\theta}(y,s) = \frac{1}{s} + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty} \frac{(\mathrm{Pr})^{\frac{k}{2}}\,(-y)^k\,(-1)^l\,[k_1(\beta)]^l\,\Gamma(\frac{k}{2}+l)}{k!l!\,[k_0(\beta)]^{\frac{k}{2}+l}\ \ s^{1-(1-\beta)\frac{k}{2}+l}\Gamma(\frac{k}{2})}. \tag{2.24}$$

Taking Laplace inverse on the Eq (2.24), we get

$$\theta(y,t) = 1 + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty} \frac{(\mathrm{Pr})^{\frac{k}{2}}\,(-y)^k\,(-1)^l\,[k_1(\beta)]^l\,t^{-(1-\beta)\frac{k}{2}+l}\Gamma(\frac{k}{2}+l)}{k!l!\,[k_0(\beta)]^{\frac{k}{2}+l}\ \ \Gamma(1-(1-\beta)\frac{k}{2}+l)\Gamma(\frac{k}{2})}. \tag{2.25}$$

2.2. Bioconvection equation

Taking laplace transform on Eq (2.20), we get

$$s\bar{N}(y,s) = \frac{1}{\mathrm{Lb}}\left[\frac{k_1(\gamma)}{s} + k_0(\gamma)\right] s^{\gamma}\bar{N}_{yy}(y,s). \tag{2.26}$$

Also apply Laplace on Eq (2.17)

$$\bar{N}(y,0) = 0, \qquad \bar{N}(0,s) = \frac{1}{s}, \qquad \bar{N}(y,s) \to 0 \ \ as \ \ y \to \infty. \tag{2.27}$$

Using Eq (2.27) in Eq (2.26), the general solution of Bioconvection profile becomes

$$\bar{N}(y,s) = \frac{1}{s}\exp\left(-y\sqrt{\frac{\mathrm{Lb}\ s^{1-\gamma}}{[\frac{k_1(\gamma)}{s} + k_0(\gamma)]}}\right). \tag{2.28}$$

The Eq (2.28) can be compose in suitable form,

$$\bar{N}(y,s) = \frac{1}{s} + \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{(\mathrm{Lb})^{\frac{m}{2}}\,(-y)^m\,(-1)^n\,[k_1(\gamma)]^n\,\Gamma(\frac{m}{2}+n)}{m!n!\,[k_0(\gamma)]^{\frac{m}{2}+n}\ \ s^{1-(1-\gamma)\frac{m}{2}+n}\Gamma(\frac{m}{2})}. \tag{2.29}$$

Taking Laplace inverse on Eq (2.29), we get

$$N(y,t) = 1 + \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{(\mathrm{Lb})^{\frac{m}{2}}\,(-y)^m\,(-1)^n\,[k_1(\gamma)]^n\,t^{-(1-\gamma)\frac{m}{2}+n}\Gamma(\frac{m}{2}+n)}{m!n!\,[k_0(\gamma)]^{\frac{m}{2}+n}\ \ \Gamma(1-(1-\gamma)\frac{m}{2}+n)\Gamma(\frac{m}{2})}. \tag{2.30}$$

2.3. Momentum equation

Taking Laplace transform on Eq (2.10), we have

$$s\bar{u}(y,s) = A\bar{u}_{yy}(y,s) + \mathrm{Gr}[\bar{\theta}(y,s) - \mathrm{Ra}\bar{N}(y,s)]. \tag{2.31}$$

Also apply Laplace on Eq (2.15)

$$\bar{u}(y,0) = 0, \qquad \bar{u}(0,s) = \frac{1}{s}, \qquad \bar{u}(y,s) \to 0 \ \ as \ \ y \to \infty. \tag{2.32}$$

Using Eq (2.32) in Eq (2.31), the general solution of Momentum profile becomes

$$\bar{u}(y,s)=\left[\frac{1}{s}+\frac{\mathrm{Gr}}{As[\frac{\mathrm{Pr}\ s^{1-\beta}}{[\frac{k_1(\beta)}{s}+k_0(\beta)]}-s]}-\frac{\mathrm{GrRa}}{As[\frac{\mathrm{Lb}\ s^{1-\gamma}}{[\frac{k_1(\gamma)}{s}+k_0(\gamma)]}-s]}\right]e^{-y\sqrt{\frac{s}{A}}}$$
$$-\frac{\mathrm{Gr}e^{-y\sqrt{\mathrm{Pr}\ s^{1-\beta}[\frac{k_1(\beta)}{s}+k_0(\beta)]^{-1}}}}{As[\frac{\mathrm{Pr}\ s^{1-\beta}}{[\frac{k_1(\beta)}{s}+k_0(\beta)]}-s]}+\frac{\mathrm{GrRa}e^{-y\sqrt{\mathrm{Lb}\ s^{1-\gamma}[\frac{k_1(\gamma)}{s}+k_0(\gamma)]^{-1}}}}{As[\frac{\mathrm{Lb}\ s^{1-\gamma}}{[\frac{k_1(\gamma)}{s}+k_0(\gamma)]}-s]}. \tag{2.33}$$

The Eq (2.33) can be written in suitable form

$$\bar{u}(y,s)=\frac{1}{s}+\frac{1}{s}\sum_{k=1}^{\infty}\frac{[-y\sqrt{s}]^k}{(\sqrt{A})^k k!}$$
$$-\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\frac{\mathrm{Gr}\ [-y]^k\ [\mathrm{Pr}]^p\ [k_1(\beta)]^p\ \Gamma(p+1)}{k!\ p!(A)^{1+\frac{k}{2}}s^{p+2-\beta l-\frac{k}{2}}\ [k_0(\beta)]^l\ \Gamma(p+1-l)}$$
$$+\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\frac{\mathrm{GrRa}\ [-y]^k\ [\mathrm{Lb}]^p\ [k_1(\gamma)]^p\ \Gamma(p+1)}{k!\ p!(A)^{1+\frac{k}{2}}s^{p+2-\gamma l-\frac{k}{2}}\ [k_0(\gamma)]^l\ \Gamma(p+1-l)}$$
$$+\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\frac{\mathrm{Gr}\ [-y]^k\ [\mathrm{Pr}]^{m+\frac{k}{2}}\ [k_1(\beta)]^{l+p}}{Ak!s^{\beta k-\frac{k}{2}+\beta m+l+p+2}\ [k_0(\beta)]^{\frac{k}{2}m+l+p}}$$
$$-\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\frac{\mathrm{GrRa}\ [-y]^k\ [\mathrm{Lb}]^{m+\frac{k}{2}}[k_1(\gamma)]^{l+p}}{Ak!s^{\gamma k-\frac{k}{2}+\gamma m+l+p+2}\ [k_0(\gamma)]^{\frac{k}{2}m+l+p}}. \tag{2.34}$$

Taking Laplace inverse on Eq (2.34), we get final expression

$$u(y,t)=1+\sum_{k=1}^{\infty}\frac{[-y\sqrt{t}]^k}{(A)^{\frac{k}{2}}k!}$$
$$-\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\frac{\mathrm{Gr}\ [-y]^k\ [\mathrm{Pr}]^p\ t^{p+1-\beta l\frac{k}{2}}\ [k_1(\beta)]^p\ \Gamma(p+1)}{k!p!(A)^{1+\frac{k}{2}}[k_0(\beta)]^l\ \Gamma(p+1-l)\ \Gamma(p+2-\beta l-\frac{k}{2})}$$
$$+\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\frac{\mathrm{GrRa}\ [-y]^k\ [\mathrm{Lb}]^p t^{p+1-\gamma l\frac{k}{2}}\ [k_1(\gamma)]^p\ \Gamma(p+1)}{k!p!(A)^{1+\frac{k}{2}}[k_0(\gamma)]^l\ \Gamma(p+1-l)\ \Gamma(p+2-\gamma l-\frac{k}{2})}$$
$$+\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\frac{\mathrm{Gr}\ [-y]^k\ [\mathrm{Pr}]^{m+\frac{k}{2}}\ t^{\beta k-\frac{k}{2}+\beta m+l+p+1}\ [k_1(\beta)]^{l+p}}{Ak![k_0(\beta)]^{\frac{k}{2}m+l+p}\ \Gamma(\beta k-\frac{k}{2}+pm+l+p+2)}$$
$$-\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\frac{\mathrm{GrRa}\ [-y]^k\ [\mathrm{Lb}]^{m+\frac{k}{2}}\ t^{\gamma k-\frac{k}{2}+\gamma m+l+p+1}\ [k_1(\gamma)]^{l+p}}{Ak!\ [k_0(\gamma)]^{\frac{k}{2}m+l+p}\ \Gamma(\gamma k-\frac{k}{2}+pm+l+p+2)}. \tag{2.35}$$

3. Graphical results and discussion

Figures 2 and 3 are sketched to see the effect of fractional parameter γ for both large and small value of time in bioconvection field. Since the power law fractional operator has been used. As the fractional operator exhibits the memory of the function at the present time. It is found in Figures 2 and 3 that by increasing the values of fractional parameter and taking time t=2.5, the profile of bioconvection reduces. The concentration near the plate is maximum while decreasing in main stream region and finally asymptotically approaches to zero as $y \to 0$. This fact also justify the boundary condition as well.

Impact of fractional parameter β on temperature can be seen in Figures 4 and 5 and observed that for small and large values of time. For large time t of different values of fractional parameter, it is observed from Figure 4 that temperature is decreasing function near the plate for great values of β. This rapid decay in temperature is due to increase in thermal boundary layer for increasing β, while Figure 5 depicts for small time for different fractional parameter and demonstrates that temperature shows the opposite behavior for varying fractional parameter values. Temperature increases away from the plate in the main stream region and finally decay for greater values of y and then asymptotically approaches to zero as y goes to infinity. This is how fractional parameter shows dual behavior for temperature for smaller and larger values of time. The effect of fractional parameters on velocity field is presented in Figure 6. The fluid velocity rises as we raise the values of fractional parameters. This can be physically deliberated as when fractional parameters are raised, the momentum boundary layer became thickest as a result the velocity profiles increased. When $\lambda \to \infty$ casson fluid problem model becomes viscous fluid problem. It can be seen in Figure 7. It coincides with Imran et al. [40].

Figure 8 is designed to see the effect of Gr on the velocity field. It is realized that with high values of Gr, the velocity faster. Since Gr is related to buoyancy forces which rise the natural convection, so the velocity rises in speed. Figure 9 is planned to see the impact of Pr on velocity field. Since Pr is the dimensionless number that tests the comparative width of a boundary layer of thermal conductivity and momentum. So by increasing values of Pr, thermal conductivity is decreased, the viscosity of the fluid is enriched and, lastly, a decay in velocity is detected. It is also noted that the boundary layer width reduces. Figure 10 displays the result of the bioconvection Rayleigh number Ra on the velocity field. From figure, It is clear that the fluid velocity declines while rising the values of bioconvection Rayleigh number Ra. This is because of that, for higher values of Ra, the buoyancy impact from the movement of microorganisms falls.

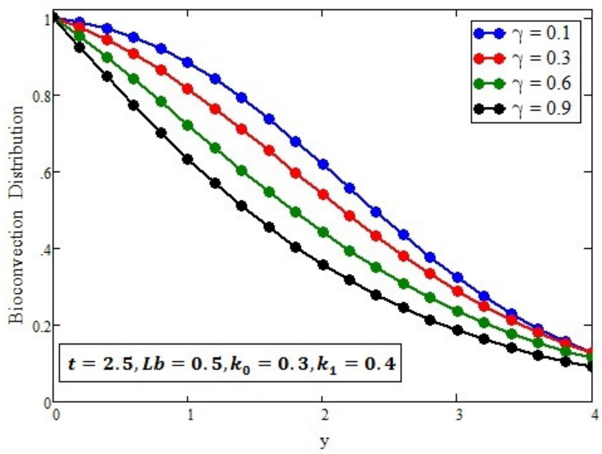

Figure 2. The result of fractional parameter γ on Bioconvection Distribution for large time.

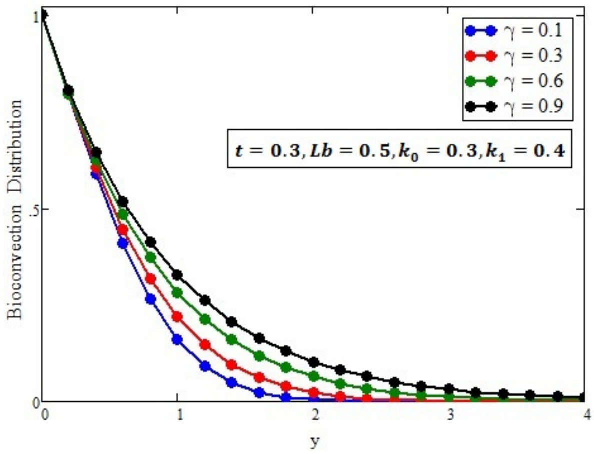

Figure 3. The result of fractional parameter γ on Bioconvection Distribution for small time.

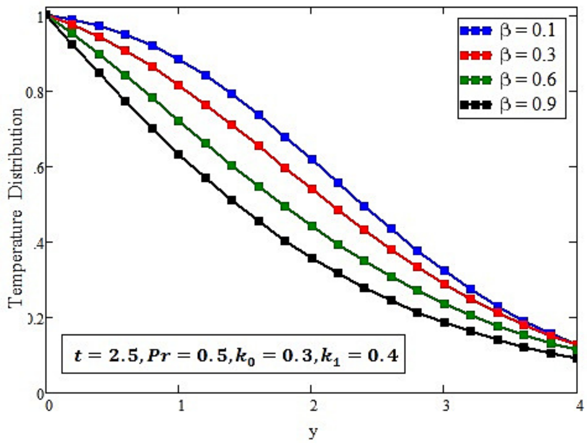

Figure 4. The result of fractional parameter β on Temperature Distribution for large time.

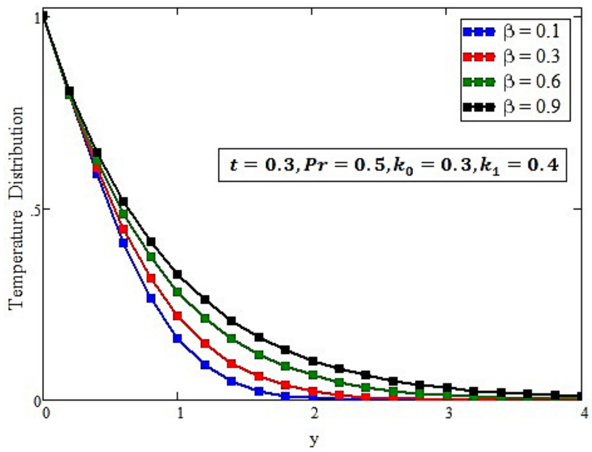

Figure 5. The result of fractional parameter β on Temperature Distribution for small time.

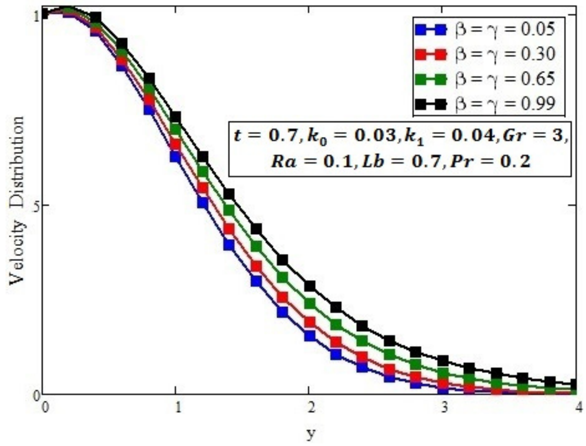

Figure 6. The effect of fractional parameters β and γ on Velocity Distribution.

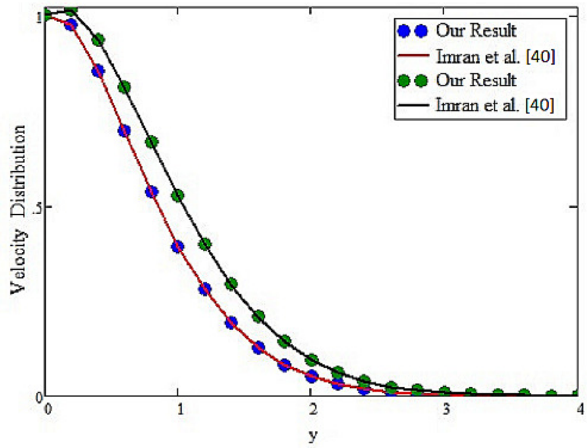

Figure 7. Comparison between our results when ($\lambda \to \infty$) and Imran et al. [40].

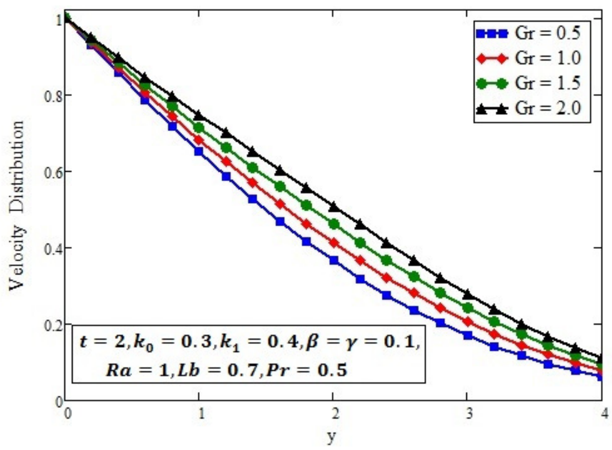

Figure 8. The effect of Gr on Velocity Distribution.

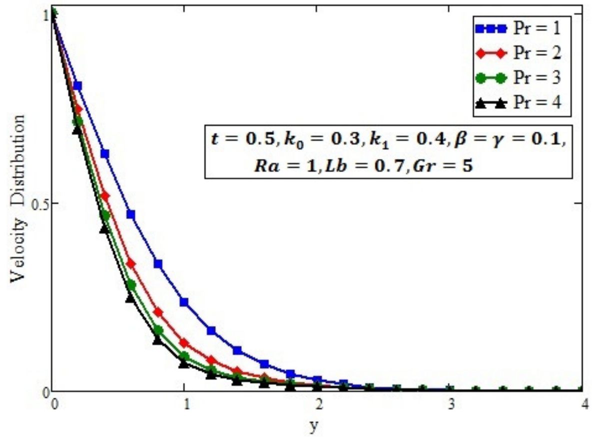

Figure 9. The effect of Pr on Velocity Distribution.

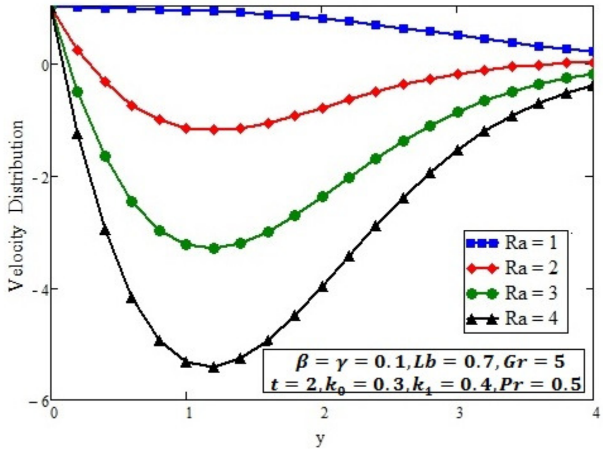

Figure 10. The effect of Ra on Velocity Distribution.

4. Conclusions

In this work, the unsteady fractional bioconvection for Casson fluid flow for vertical surface has been studied. The governing equations are modeled with fractional derivative and then transformed into dimensionless form by using dimensionless variables. The analytical solution is attained by applying Laplace transform technique. The physical effects of different parameters are explained graphically. Some key points are obtained as follow:

(1) Thermal, diffusion and momentum boundary layers can be controlled with variable values of fractional parameter and time respectively.
(2) Enhancement in temperature, concentration and velocity can be achieved with small values of time.
(3) The presence of bioconvection number Ra is responsible for decline of momentum boundary layer.
(4) A good agreement of the present results and the recent literature is obtained for the validation.

Nomenclature

Symbol	Name	Symbol	Name
(C)	Caputo	(CF)	Caputo-Fabrizio
(ABC)	Atangana-Baleanu	(CPC)	Constant proportional Caputo
ρ	density of fluid (kgm^{-3})	μ	dynamic viscosity ($kgm^{-1}s^{-1}$)
k	thermal conductivity ($Wm^{-2}K^{-1}$)	g	gravitational acceleration
β_T	thermal expansion coefficient	u	velocity field (ms^{-1})
C_p	specific heat ($jkg^{-1}K^{-1}$)	D_n	thermal diffusivity
T	temperature (K)	T_∞	ambient temperature (K)
N	bioconvection	N_∞	constant bioconvection
T_w	temperature at wall (K)	N_w	concentration of microorganisms of dish
t	Time (s)	β , γ	Fractional parameters
Pr	Prandtl number (Dimensionless)	Lb	Lewis number (Dimensionless)
Gr	Thermal Grashof number (Dimensionless)	Ra	Rayleigh number (Dimensionless)
λ	Casson fluid parameter		

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Conflict of interest

This work does not have any conflict of interest.

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