

Average irradiance and polarization properties of a radially or azimuthally polarized beam in a turbulent atmosphere

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Abstract: Analytical formulas are derived for the average irradiance and the degree of polarization of a radially or azimuthally polarized doughnut beam (PDB) propagating in a turbulent atmosphere by adopting a beam coherence-polarization matrix. It is found that the radial or azimuthal polarization structure of a radially or azimuthally PDB will be destroyed (i.e., a radially or azimuthally PDB is depolarized and becomes a partially polarized beam) and the doughnut beam spot becomes a circularly Gaussian beam spot during propagation in a turbulent atmosphere. The propagation properties are closely related to the parameters of the beam and the structure constant of the atmospheric turbulence.

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1. Introduction

Recently, radially and azimuthally polarized beams attract more and more attention due to their unique focusing properties and important applications in optical data storage, particle trapping and acceleration, high-resolution microscopy, laser cutting, and determination of single fluorescent molecule orientation, etc [1-8]. Radially polarized beams can be generated either inside a laser resonator, e.g. by using a conical mirror or a conical Brewster element or outside a laser cavity, e.g. by using a space-invariant dielectric subwavelength gratings, a dual conical prism or an interferometric technique [6-9]. The focusing properties, paraxial and nonparaxial propagation properties through paraxial optical system or free space have been widely studied [1-3, 6, 10,11]. To the best of our knowledge, the propagation properties of a radially or azimuthally polarized beam in a turbulent atmosphere have not been studied so far.

Over the past several decades, many works have been carried out concerning the propagation of coherent and partially coherent laser beams through a turbulent atmosphere due to their wide applications in e.g. free-space optical communication, atmospheric imaging systems and remote sensing, and it has been found that the behavior of a laser beam in a turbulent atmosphere is closely related to its initial beam profile, coherence and polarization properties [12-26]. In this paper, we investigate the propagation properties (i.e., irradiance and polarization) of a radially or azimuthally polarized doughnut beam (PDB) in a turbulent atmosphere based on the extended Huygens-Fresnel method by adopting a beam coherence-polarization matrix. Some analytical formulae are derived and numerical examples are illustrated.

2. Formulation

Within the framework of the paraxial approximation, the vectorial electric field of a radially PDB is expressed as the coherent superposition of a TEM₀₁ with a polarization direction parallel to the x-axis and a TEM₁₀ with a polarization direction parallel to the y-axis [1, 7]

$$\mathbf{E}_r(x, y) = E_1 \mathbf{e}_x + E_2 \mathbf{e}_y = E_0 \left[\frac{x}{w_0} \exp\left(-\frac{r^2}{w_0^2}\right) \mathbf{e}_x + \frac{y}{w_0} \exp\left(-\frac{r^2}{w_0^2}\right) \mathbf{e}_y \right], \quad (1)$$

where $r^2 = x^2 + y^2$, w_0 denotes the beam waist size of a Gaussian beam, E_0 is a constant. In a similar way, the vectorial electric field of an azimuthally PDB is expressed as follows

$$\mathbf{E}_\theta(x, y) = E_0 \left[-\frac{y}{w_0} \exp\left(-\frac{r^2}{w_0^2}\right) \mathbf{e}_x + \frac{x}{w_0} \exp\left(-\frac{r^2}{w_0^2}\right) \mathbf{e}_y \right]. \quad (2)$$

Here we do not consider the longitudinal electric field component, since it is known to be negligible under the paraxial condition [11, 27]. The beam coherence-polarization (BCP) matrix provides the information of polarization and spatial correlation, and the BCP matrix for a vectorial electric field across a typical plane $z=\text{constant}$ (here z is the propagation axis) is defined as follows [28]

$$\hat{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, z) = \begin{pmatrix} \Gamma_{11}(\mathbf{r}_1, \mathbf{r}_2, z) & \Gamma_{12}(\mathbf{r}_1, \mathbf{r}_2, z) \\ \Gamma_{21}(\mathbf{r}_1, \mathbf{r}_2, z) & \Gamma_{22}(\mathbf{r}_1, \mathbf{r}_2, z) \end{pmatrix}, \quad (3)$$

where

$$\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z) = \langle E_\alpha(\mathbf{r}_1, \mathbf{r}_2, z) E_\beta^*(\mathbf{r}_1, \mathbf{r}_2, z) \rangle, \quad (\alpha, \beta = 1, 2) \quad (4)$$

E_1 and E_2 are the components of the vectorial electric field in the x and y directions, respectively, and the angle brackets denote an ensemble average over the medium statistics. The equivalent irradiance distribution of a polarized beam is given by [28]

$$I(\mathbf{r}, z) = \Gamma_{11}(\mathbf{r}, \mathbf{r}, z) + \Gamma_{22}(\mathbf{r}, \mathbf{r}, z), \quad (5)$$

and the degree of polarization is expressed as

$$P(\mathbf{r}, z) = \sqrt{1 - \frac{4 \det[\hat{\Gamma}(\mathbf{r}, \mathbf{r}, z)]}{\{\text{Tr}[\hat{\Gamma}(\mathbf{r}, \mathbf{r}, z)]\}^2}}, \quad (6)$$

where \det and Tr stand for determinant and trace of the BCP matrix, respectively.

By applying Eqs. (1)-(4), the BCP matrices for a radially PDB and an azimuthally PDB at source plane ($z=0$) are expressed as follows

$$\hat{\Gamma}_r(\mathbf{r}_1, \mathbf{r}_2, 0) = \frac{E_0^2}{w_0^2} \exp\left(-\frac{\mathbf{r}_1^2 + \mathbf{r}_2^2}{w_0^2}\right) \begin{pmatrix} x_1 x_2 & x_1 y_2 \\ y_1 x_2 & y_1 y_2 \end{pmatrix}, \quad (7)$$

$$\hat{\Gamma}_\theta(\mathbf{r}_1, \mathbf{r}_2, 0) = \frac{E_0^2}{w_0^2} \exp\left(-\frac{\mathbf{r}_1^2 + \mathbf{r}_2^2}{w_0^2}\right) \begin{pmatrix} y_1 y_2 & -x_2 y_1 \\ -x_1 y_2 & x_1 x_2 \end{pmatrix}. \quad (8)$$

One finds from Eqs. (5)-(8) that a radially PDB and an azimuthally PDB have the same irradiance distribution and degree of polarization $P_r(\mathbf{r}, 0) = P_\theta(\mathbf{r}, 0) = 1$ at $z=0$.

Figure 1 shows the propagation geometry of a radially or azimuthally PDB in a turbulent atmosphere. The paraxial propagation of a laser beam in a turbulent atmosphere can be treated with the well-known extended Huygens-Fresnel integral formula, and the elements of BCP matrix $\Gamma_{\alpha\beta}(\mathbf{r}, \mathbf{r}, z)$ at the output plane are given as follows [14-26]

$$\Gamma_{\alpha\beta}(\mathbf{r}, \mathbf{r}, z) = \frac{k^2}{4\pi^2 z^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma_{\alpha\beta}(\mathbf{r}'_1, \mathbf{r}'_2, 0) \exp\left[-\frac{ik}{2z}(\mathbf{r}'_1 - \mathbf{r})^2 + \frac{ik}{2z}(\mathbf{r}'_2 - \mathbf{r})^2\right] \\ \times \exp\left[-\frac{1}{\rho_0^2}(\mathbf{r}'_1 - \mathbf{r}'_2)^2\right] d\mathbf{r}'_1 d\mathbf{r}'_2, \quad (9)$$

$\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, 0)$ is given by Eq. (4) and $d\mathbf{r}_1 d\mathbf{r}_2 = dx_1 dy_1 dx_2 dy_2 \cdot \rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the coherence length (induced by the atmospheric turbulence) of a spherical wave propagating in the turbulent medium with C_n^2 being the structure constant [12-26], $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength of the light. In the derivation of Eq. (9), we have employed Kolmogorov spectrum and a quadratic approximation for Rytov's phase structure function [12-26]. The extended Huygens-Fresnel integral formula Eq. (9) has been approved to be reliable in e.g. Refs. [14]-[16], and has been used widely (see e.g. Refs. [17]-[26]).

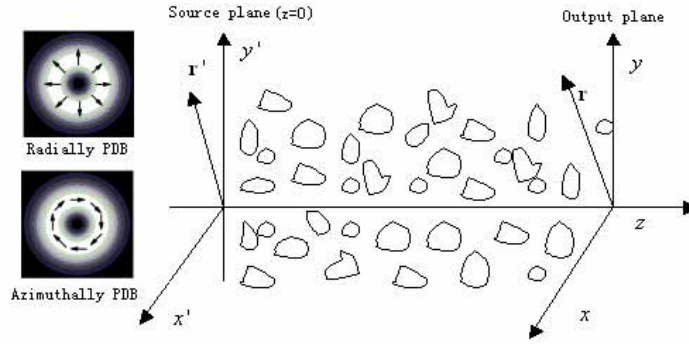


Fig. 1. Propagation geometry of a radially or azimuthally PDB in a turbulent atmosphere

Substituting $\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, 0)$ in Eq. (7) as $\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, 0)$ into Eq. (9), after some tedious integration, we obtain the following expressions for the elements of BCP matrix of a radially PDB in a turbulent atmosphere

$$\Gamma_{r11}(\mathbf{r}, \mathbf{r}, z) = E_0^2 \frac{2k^2 \rho_0^4 w_0^6 z^2}{A_1^2} \left[\frac{1}{\rho_0^2} + \frac{k^2 \rho_0^2 (k^2 w_0^4 + 4z^2)}{2z^2 A_1} x^2 \right] \exp \left[-\frac{2k^2 \rho_0^2 w_0^2}{A_1} \mathbf{r}^2 \right], \quad (10)$$

$$\Gamma_{r12}(\mathbf{r}, \mathbf{r}, z) = \Gamma_{r21}(\mathbf{r}, \mathbf{r}, z) = E_0^2 \frac{k^4 \rho_0^6 w_0^6 (k^2 w_0^4 + 4z^2) xy}{A_1^3} \exp \left[-\frac{2k^2 \rho_0^2 w_0^2}{A_1} \mathbf{r}^2 \right], \quad (11)$$

$$\Gamma_{r22}(\mathbf{r}, \mathbf{r}, z) = E_0^2 \frac{2k^2 \rho_0^4 w_0^6 z^2}{A_1^2} \left[\frac{1}{\rho_0^2} + \frac{k^2 \rho_0^2 (k^2 w_0^4 + 4z^2)}{2z^2 A_1} y^2 \right] \exp \left[-\frac{2k^2 \rho_0^2 w_0^2}{A_1} \mathbf{r}^2 \right], \quad (12)$$

with $A_1 = k^2 \rho_0^2 w_0^4 + 4z^2 (\rho_0^2 + 2w_0^2)$. In a similar way, we obtain the following expressions for the elements of BCP matrix of an azimuthally PDB

$$\Gamma_{\theta 11}(\mathbf{r}, z) = \Gamma_{r22}(\mathbf{r}, z), \Gamma_{\theta 22}(\mathbf{r}, z) = \Gamma_{r11}(\mathbf{r}, z), \Gamma_{\theta 12}(\mathbf{r}, z) = \Gamma_{\theta 21}(\mathbf{r}, z) = -\Gamma_{r12}(\mathbf{r}, z). \quad (13)$$

In the absence of turbulence ($\rho_0 \rightarrow \infty$, i.e., $C_n^2 = 0$), Eqs. (10)-(13) reduce to the expressions for a radially or azimuthally PDB in free space under the paraxial condition [6, 11]. One finds from Eqs. (5), (6) and (10)-(13) that a radially PDB and an azimuthally PDB have the same irradiance distribution and degree of polarization in a turbulent atmosphere. To distinguish a radially PDB and an azimuthally PDB in a turbulent atmosphere, it is necessary to introduce

some linear polarizer. Suppose that a linear polarizer is located at a position z , whose transmission axis forms an angle ϕ with the x -axis. Then the irradiances of a radially PDB and an azimuthally PDB at z becomes

$$I_r(\mathbf{r}, z) = \Gamma_{r11}(\mathbf{r}, z) \cos^2 \phi + \Gamma_{r22}(\mathbf{r}, z) \sin^2 \phi + \Gamma_{r12}(\mathbf{r}, z) \sin 2\phi, \quad (14)$$

$$I_\theta(\mathbf{r}, z) = \Gamma_{\theta11}(\mathbf{r}, z) \cos^2 \phi + \Gamma_{\theta22}(\mathbf{r}, z) \sin^2 \phi + \Gamma_{\theta12}(\mathbf{r}, z) \sin 2\phi. \quad (15)$$

One finds from Eqs. (10)-(15) that we can distinguish a radially PDB and an azimuthally PDB by measuring their irradiances behind the linear polarizer with proper ϕ .

3. Numerical examples

Now we give some numerical results. By applying Eqs. (5), (10) and (12), we calculate in Fig. 2 the cross line ($y=0$) of the normalized irradiance distribution $I(x,0,z)/I(x,0,0)_{\max}$ of a radially PDB at several propagation distances in a turbulent atmosphere for two different values of structure constant C_n^2 with $\lambda = 632.8nm$ and $w_0 = 2cm$. For comparison, the cross line of the far field ($z=15km$) normalized irradiance distribution $I(x,0,z)/I(x,0,0)_{\max}$ of a radially PDB in free space is also shown in Fig. 2. To learn about the vector structure of the intensity distribution of a radially PDB, we also calculate the normalized 3D-irradiance distribution $I(x,y,z)/I(x,0,0)_{\max}$ of a radially PDB and the corresponding contour graph at several propagation distances in a turbulent atmosphere with $\lambda = 632.8nm$, $w_0 = 2cm$ and $C_n^2 = 3 \times 10^{-15} m^{-2/3}$. One sees from Fig. 2 and 3 that the irradiance properties of a radially PDB in a turbulent atmosphere are quite different from those in free space. In free space, the irradiance distribution of a radially PDB keeps doughnut beam profile and beam spot spreads during propagation as shown in Refs. [6], [11] and Fig. 2, while we note the doughnut beam profile of a scalar doughnut beam (i.e., dark hollow beam) in free space disappears and converses into a quasi-Gaussian beam profile with a small bright ring around the brightest center as shown in Ref. [29]. In a turbulent atmosphere, the doughnut beam profile disappears gradually and the on-axis irradiance increases during propagation within certain propagation distance, the beam spot with flat-topped profile can be formed at certain propagation distance, and the doughnut beam spot becomes a circularly solid beam spot in the far field. The evolution properties of the irradiance of a radially PDB in a turbulent atmosphere are similar to those of a partially coherent radially PDB in a free space [30]. This phenomena can be explained by the fact that the atmospheric turbulence degrades the coherence of a radially PDB, and the stronger the atmospheric turbulence (i.e., larger structure constant C_n^2), the larger the degradation of the coherence, thus leading to the interesting irradiance distribution during propagation. Similar effects of turbulence on source coherence degradation were earlier noted in Refs. [13] and [15] for coherent Gaussian beams. One also finds from Fig. 3 that the azimuthal symmetry of the transverse intensity pattern is not affected by the turbulence although the doughnut beam profile disappears due to the isotropic influence of the atmospheric turbulence. Figure 4 shows the normalized on-axis irradiance distribution $I(0,0,z)/I(x,0,0)_{\max}$ of a radially PDB along z in a turbulent atmosphere for different values of w_0 and C_n^2 . One finds from Fig. 4 that the on-axis irradiance of a radially PDB increases gradually during propagation, its value becomes maximal at certain propagation distance, then its value decreases due to the spreading the beam spot as z increases further. The increase of the on-axis irradiance within certain propagation distance means that the light energy leaking into the central part of the beam from the outer ring during propagation, and the on-axis energy becomes maximal at certain propagation distance. The decrease of the on-axis energy after certain propagation distance is due to the spreading of the beam spot. One also finds from Figs. 2 and 4 that the conversion from a doughnut beam spot to a circularly solid beam

spot becomes quicker and the beam spot spreads more rapidly for a smaller w_0 and a larger C_n^2 . From the above discussions, one comes to the conclusion that the propagation properties of a radially PDB in a turbulent atmosphere are closely related to the parameters the beam and the structure constant of the turbulent media.

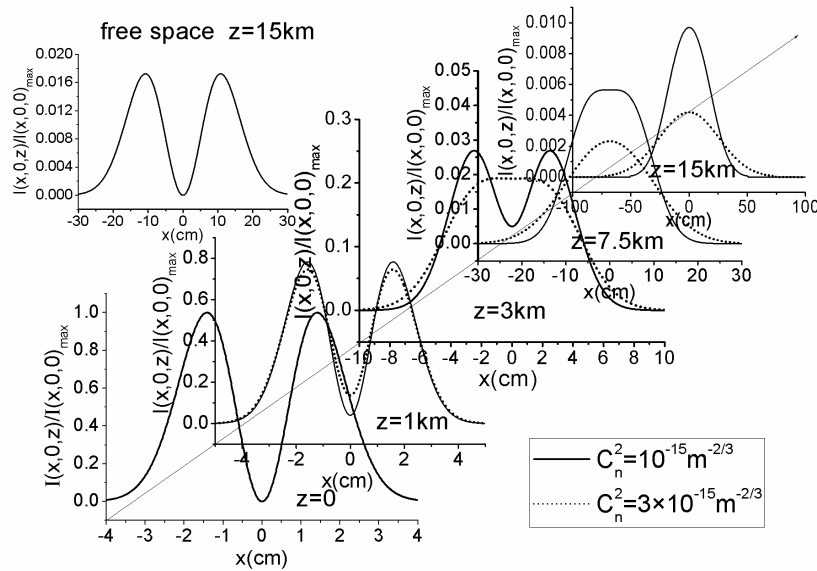


Fig. 2. Cross line ($y=0$) of the normalized irradiance distribution $I(x, 0, z)/I(x, 0, 0)_{\max}$ of a radially PDB at several propagation distances in a turbulent atmosphere for two different values of structure constant C_n^2 .

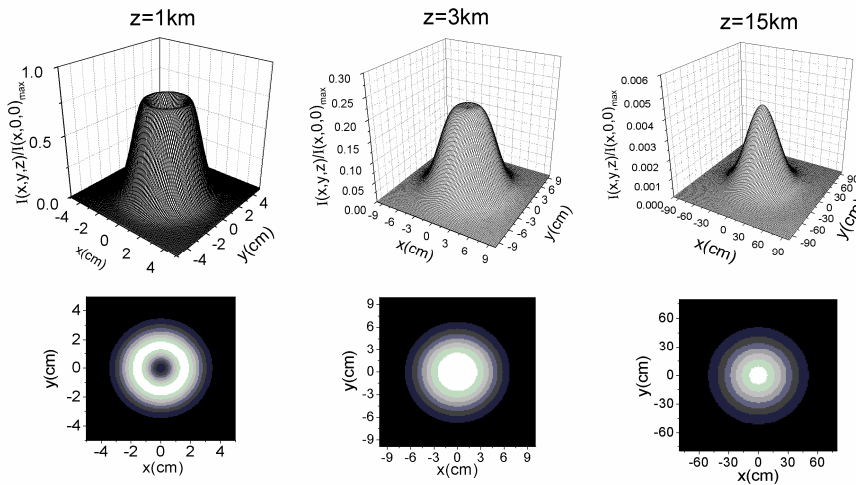


Fig. 3. Normalized 3D-irradiance distribution $I(x, y, z)/I(x, 0, 0)_{\max}$ of a radially PDB and the corresponding contour graph at several propagation distances in a turbulent atmosphere with $w_0 = 2\text{ cm}$ and $C_n^2 = 3 \times 10^{-15} \text{ m}^{-2/3}$.

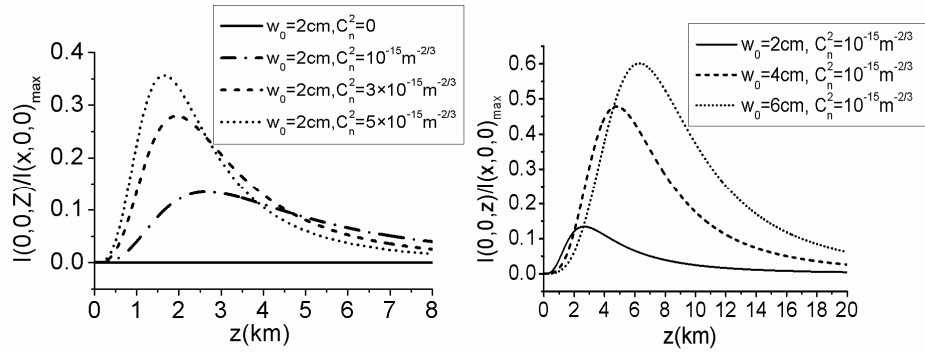


Fig. 4. Normalized on-axis irradiance distribution $I(0,0,z)/I(x,0,0)_{\max}$ of a radially PDB along z in a turbulent atmosphere for different values of w_0 and C_n^2

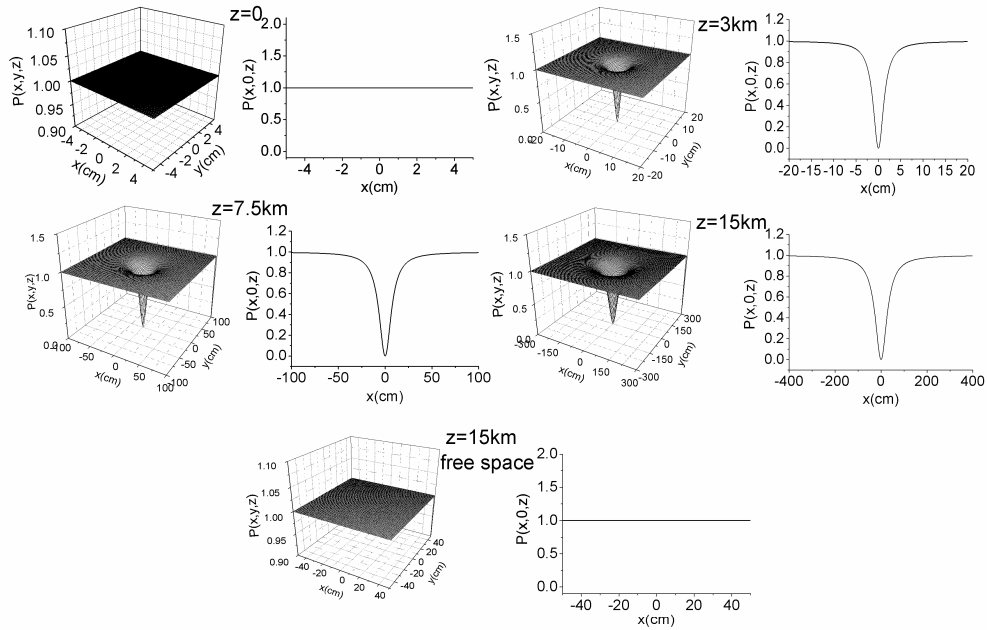


Fig. 5. Degree of polarization $P(x, y, z)$ of a radially PDB and the corresponding cross line ($y=0$) at several propagation distance with $w_0 = 2\text{ cm}$ and $C_n^2 = 10^{-15} \text{ m}^{-2/3}$

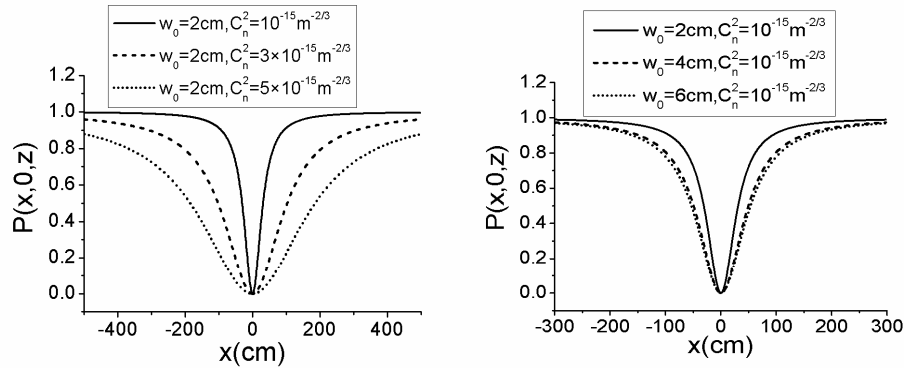


Fig. 6. Cross line ($y=0$) the degree of polarization $P(x, 0, z)$ of a radially PDB at $z=15\text{km}$ in a turbulent atmosphere for different values of w_0 and C_n^2

To learn about the influence of the atmospheric turbulence on the polarization properties of a radially PDB in a turbulent atmosphere, we calculate in Fig. 5 the degree of polarization $P(x, y, z)$ of a radially PDB and the corresponding cross line ($y=0$) at several propagation distance with $\lambda = 632.8\text{nm}$, $w_0 = 2\text{cm}$ and $C_n^2 = 10^{-15}\text{m}^{-2/3}$ by applying Eqs. (6) and (10)-(12). For comparison, the far-field ($z=15\text{km}$) degree of polarization of a radially PDB and the corresponding cross line ($y=0$) are also shown in Fig. 5. What's more, we also calculate in Fig. 6 the cross line ($y=0$) of the degree of polarization $P(x, 0, z)$ of a radially PDB at $z=15\text{km}$ in a turbulent atmosphere for different values of w_0 and C_n^2 with $\lambda = 632.8\text{nm}$. From the other tests conducted and from the illustration in Fig. 5, one finds that the degree of polarization of a radially PDB in free space equals 1 for all the points across the entire transverse plane and remains invariant during propagation, which means that radial polarization structure of a radially PDB in free space will not be destroyed during propagation in free space. While in a turbulent atmosphere, one finds from Fig. 5 that the a dip appears in the distribution of the degree of polarization, in other words, the degree of polarization of the on-axis point becomes zero after propagation and the degree of polarization of the off-axis point rises gradually towards the edges of the off-axis regions. One also finds that the width of the dip increases during propagation. Thus, one comes to the conclusion that the radial polarization structure of a radially PDB is destroyed during propagation in a turbulent atmosphere (i.e., a radially PDB is depolarized during propagation), and the radially PDB becomes a partially polarized beam. What's more, one finds from Fig. 6 that the depolarization of an off-axis point becomes larger for a larger w_0 or a larger C_n^2 .

4. Conclusion

In conclusion, we have derived the analytical propagation formulae for the BCP matrix, the average irradiance and the degree of polarization of a radially or azimuthally PDB propagating in a turbulent atmosphere based on the extended Huygens-Fresnel integral formula. We have investigated the irradiance and polarization properties of a radially or azimuthally PDB in a turbulent atmosphere. Our results show that the radial or azimuthal polarization structure of a radially or azimuthally PDB is destroyed and the doughnut beam spot becomes a circularly Gaussian beam spot during propagation in a turbulent atmosphere, which are quite different from those in free space. The propagation properties a radially or

azimuthally PDB are closely related to the parameters of the beam and the structure constant of the atmospheric turbulence. Our formulae provide a convenient and reliable way for studying the propagation of a radially or azimuthally PDB in a turbulent atmosphere.

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