Average intensity and spreading of an elegant Hermite–Gaussian beam in turbulent atmosphere

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Abstract: The propagation of an elegant Hermite-Gaussian beam (EHGB) in turbulent atmosphere is investigated. Analytical propagation formulae for the average intensity and effective beam size of an EHGB in turbulent atmosphere are derived based on the extended Huygens-Fresnel integral. The corresponding results of a standard Hermite-Gaussian beam (SHGB) in turbulent atmosphere are also derived for the convenience of comparison. The intensity and spreading properties of EHGBs and SHGBs in turbulent atmosphere are studied numerically and comparatively. It is found that the propagation properties of EHGBs and SHGBs are much different from their properties in free space, and the EHGB and SHGB with higher orders are less affected by the turbulence. What's more, the SHGB spreads more rapidly than the EHGB in turbulent atmosphere under the same conditions. Our results will be useful in long-distance free-space optical communications.

OCIS codes: (010.1300) Atmospheric propagation; (010.3310) Laser beam transmission.

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1. Introduction

Standard Hermite-Gaussian beams (SHGBs) are commonly encountered higher-order modes in practice. In the past decades, SHGBs have been extensively stuided both in theoretical and experimental aspects, and have found wide applications [1–10]. Carter first introduced the definition of effective spot size for a SHGB and stuided its divergence [2]. The diffraction properties of SHGBs by various gratings have been investigated [3,4]. Luk et al. produced the SHG modes in experiment by mulipoles with complex source points [5], and Chen produced the SHG modes in fiber-coupled laser-diode end-pumped lasers [6]. Laabs studied the propagation of SHGB beyond the paraxial approximation [7]. Cai and Lin introduced the elliptical Hermite-Gaussian beam to describe an astigmatic higher-order beam and studied its propagation properties [8,9]. More recently, SHGB was extended to the partially coherent

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case, and the propagation properties of partially coherent SHGB in free space and through paraxial ABCD optical system were studied [10,11].

It is well known that SHG modes are eigenmodes of the paraxial wave equation. The Gaussian part of the SHG mode has a complex argument, but its Hermite part is purely real. Siegman first introduced new Hermite-Gaussian solutions named elegant Hermite-Gaussian (EHG) modes that satisfy the paraxial wave equation but has a more symmetrical form [12]. The argument of the Hermite part of the EHG mode is complex. The EHG modes are not orthogonal in the usual sense, but they are biorthogonal. Up to now, various methods have been developed to obtain EHGBs [13,14]. Laabs et al. has demonstrated that the three-dimensional SHG modes can be converted to twisted Hermite-Gaussian modes with complex argument [15]. The propagation properties of EHGB and SHGB in free space and through paraxial ABCD optical system have been studied extensively [16–18].

The propagation of various laser beams in a turbulent atmosphere has been widely studied in the past due to their important applications in free-space optical communications and remote sensing, and various methods have been proposed to overcome or reduce the turbulence-induced degradation of laser beams [19–38]. It has been found that the use of a higher-mode source beam can reduce the turbulence-induced degradation [20,25,26]. Up to now, although the propagation of various standard higher-order beams in turbulent atmosphere has been investigated extensively, to our knowledge, the propagation of elegant higher-order beams in turbulent atmosphere hasn't been reported. In this paper, our aim is to investigate the propagation of an EHGB in turbulent atmosphere, and to explore the advantage of the EHGB for application in free-space optical communications. Analytical propagation formulae are derived and some numerical examples are given.

2. Formulation

The electric field of an EHGB at z = 0 is expressed as follows [12–18]

$$E(x_1, y_1, 0) = H_m(\frac{x_1}{w_0})H_n(\frac{y_1}{w_0})\exp(-\frac{x_1^2}{w_0^2}-\frac{y_1^2}{w_0^2}), \tag{1}$$

where H_m and H_n are the Hermite polynomials of order m or n in x and y directions, respectively, w_0 is the beam waist width of the fundamental Gausian mode. Figure 1 shows the normalized intensity distribution $I(x_1,y_1,0)/I(x_1,y_1,0)_{max}$ of an EHGB for different values of m with $m = n$ and $w_0 = 0.02m$ at z = 0. The intensity in Fig. 1 and following figures is normalized with respect to its maximum value.

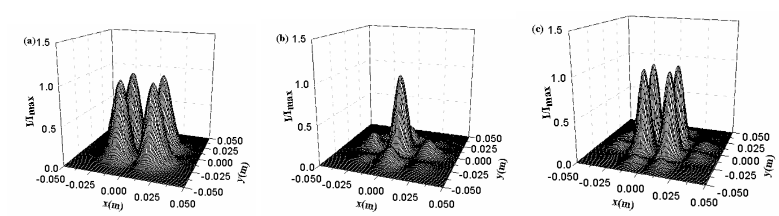

Fig. 1. Normalized intensity distribution of an EHGB for different values of m with $m = n$ at z = 0 (a) $m = 1$, (b) $m = 2$, (c) $m = 3$.

The electric field of a SHGB at z = 0 is expressed as follows [1,2]

$$E(x_1, y_1, 0) = H_m(\frac{\sqrt{2}x_1}{w_0})H_n(\frac{\sqrt{2}y_1}{w_0})\exp(-\frac{x_1^2}{w_0^2}-\frac{y_1^2}{w_0^2}). \tag{2}$$

One finds from Eqs. (1) and (2) that the main difference between EHGB and SHGB is $\sqrt{2}$ in Hermite polynomials. For $m = n = 0$, both EHGB and SHGB reduce to Gaussian beams. Figure 2 shows the normalized intensity distribution of a SHGB for different values of

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m with $m = n$ and $w_0 = 0.02m$ at z = 0. It is clear from Fig. 1 and Fig. 2 that the intensity distributions of SHGB and EHGB with the same orders are completely different for $m > 1$.

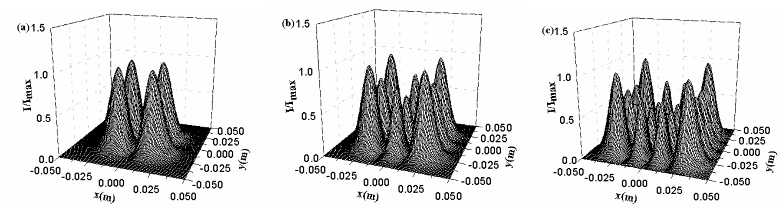

Fig. 2. Normalized intensity distribution of a SHGB for different values of m with $m = n$ and $w_0 = 0.02m$ at z = 0 (a) $m = 1$, (b) $m = 2$, (c) $m = 3$.

The paraxial propagation of a laser beam in a turbulent atmosphere can be treated with the well-known extended Huygens-Fresnel integral formula, and the average intensity of a laser beam at the output plane is given as follows [20–35]

$$\begin{aligned}\left\langle I\left(p_x, p_y, z\right)\right\rangle = &\frac{k^2}{4\pi^2 z^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left(x_1, y_1, 0\right) E^*\left(x_2, y_2, 0\right)\exp\left[-\frac{ik}{2z}\left(x_1 - p_x\right)^2 - \frac{ik}{2z}\left(y_1 - p_y\right)^2\right] \\ &\times\exp\left[\frac{ik}{2z}\left(x_2 - p_x\right)^2 + \frac{ik}{2z}\left(y_2 - p_y\right)^2 - \frac{1}{\rho_0^2}\left(x_1 - x_2\right)^2 - \frac{1}{\rho_0^2}\left(y_1 - y_2\right)^2\right] dx_1 dx_2 dy_1 dy_2,\end{aligned} \tag{3}$$

where $\rho_0 = \left(0.545 C_n^2 k^2 z\right)^{-3/5}$ is the coherence length (induced by the atmospheric turbulence) of a spherical wave propagating in the turbulent medium with C_n^2 being the structure constant [20–35], $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength of the light, z is the propagation axis. In the derivation of Eq. (3), we have employed Kolmogorov spectrum and quadratic approximation for Rytov's phase structure function [20–35]. The extended Huygens-Fresnel integral formula Eq. (3) has been approved to be reliable and has been used widely (see e.g [20]- [35].).

Substituting Eq. (1) into Eq. (3), after tedious integration, we obtain

$$\begin{aligned}< I(p_x, p_y, z) >= &\frac{k^2}{4\pi^2 z^2} w_0^2 \sqrt{\frac{\pi}{e_1}}(1 - \frac{1}{e_1})^{m/2} \exp(-\frac{k^2 w_0^2 p_x^2}{4 e_1 z^2}) \sum_{j_1=0}^{[\frac{m}{2}]} \frac{(-1)^{j_1} m!}{j_1!(m-2j_1)!} \sum_{l_1=0}^{[\frac{m}{2}]} \frac{(-1)^{l_1} m!}{l_1!(m-2l_1)!} \sum_{t_1=0}^{m-2l_1} \binom{m-2l_1}{t_1} \\ &\times (2)^{2m-2j_1-2l_1} (g_1)^{t_1} (-h_{1x})^{m-2l_1-t_1} (m+t_1-2j_1)! \exp(\frac{f_{1x}^2}{e_2})(\frac{f_{1x}}{e_2})^{m+t_1-2j_1} \sqrt{\frac{\pi}{e_2}} \sum_{q_1=0}^{[\frac{m+t_1-2j_1}{2}]} \frac{1}{q_1!\left(m+t_1-2j_1-2q_1\right)!} \\ &\times \left(\frac{e_2}{4 f_{1x}^2}\right)^{q_1} w_0^2 \sqrt{\frac{\pi}{e_1}}(1-\frac{1}{e_1})^{n/2} \exp(-\frac{k^2 w_0^2 p_y^2}{4 e_1 z^2}) \sum_{j'_1=0}^{[\frac{n}{2}]} \frac{(-1)^{j'_1} n!}{j'_1!(n-2j'_1)!} \sum_{l'_1=0}^{[\frac{n}{2}]} \frac{(-1)^{l'_1} n!}{l'_1!(n-2l'_1)!} \sum_{t'_1=0}^{n-2l'_1} \binom{n-2l'_1}{t'_1} (2)^{2n-2j'_1-2l'_1} \\ &\times (g_{1y})^{t'_1} (-h_{1y})^{n-2l'_1-t'_1} (n+t'_1-2j'_1)! \exp(\frac{f_{1y}^2}{e_2})(\frac{f_{1y}}{e_2})^{n+t'_1-2j'_1} \sqrt{\frac{\pi}{e_2}} \sum_{q'_1=0}^{[\frac{n+t'_1-2j'_1}{2}]} \frac{1}{q'_1!\left(n+t'_1-2j'_1-2q'_1\right)!} \left(\frac{e_2}{4 f_{1y}^2}\right)^{q'_1},\end{aligned} \tag{4}$$

where $[x]$ gives the greatest integer less than or equal to x, and

$$\begin{aligned}&e_1 = 1 - \frac{ikw_0^2}{2z} + \frac{w_0^2}{\rho_0^2}, \quad g_1 = (\frac{1}{e_1^2 - e_1})^{1/2} \frac{w_0^2}{\rho_0^2}, \; h_{1x} = \frac{ikw_0 p_x}{2z}(\frac{1}{e_1^2 - e_1})^{1/2}, \; h_{1y} = \frac{ikw_0 p_y}{2z}(\frac{1}{e_1^2 - e_1})^{1/2} \\ &e_2 = 1 + \frac{ik}{2z} w_0^2 + \frac{w_0^2}{\rho_0^2} - \frac{w_0^4}{e_1 \rho_0^4}, \quad f_{1x} = \frac{ikw_0 p_x}{2z} - \frac{w_0^3 ikp_x}{2ze_1\rho_0^2}, \quad f_{1y} = \frac{ikw_0 p_y}{2z} - \frac{w_0^3 ikp_y}{2ze_1\rho_0^2}.\end{aligned} \tag{5}$$

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In the above derivations, we have used the following formulae [39]

$$H_n(l)=\sum_{k=0}^{[\frac{n}{2}]}\frac{(-1)^k n!}{k!(n-2k)!}(2l)^{n-2k}, \tag{6}$$

$$\int_{-\infty}^{\infty}\exp[-\frac{(x-y)^2}{2u}]H_n(x)dx=\sqrt{2\pi u}(1-2u)^{n/2}H_n[y(1-2u)^{-1/2}], \tag{7}$$

$$\int_{-\infty}^{+\infty}x^n\exp(-px^2+2qx)dx=n!\exp(\frac{q^2}{p})(\frac{q}{p})^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{[\frac{n}{2}]}\frac{1}{k!(n-2k)!}\left(\frac{p}{4q^2}\right)^k. \tag{8}$$

Equation (4) is the analytical propagation formula for the average intensity of an EHGB in turbulent atmosphere, and it provides a convenient way for studying the propagation properties of an EHGB in turbulent atmosphere. In the absence of turbulence $\left(\rho_0\to\infty,\text{i.e., } C_n^2=0\right)$, Eq. (4) reduces to the propagation formula for an EHGB in free space.

According to [2], by use of twice the variance of *x* or *y*, the effective beam size of an EHGB in *x* direction at plane z is defined as

$$W_{xz}(z)=\sqrt{\frac{2\int x^2<I(x,y,z)>dxdy}{\int<I(x,y,z)>dxdy}}. \tag{9}$$

Substituting Eq. (4) into Eq. (9), after tedious integration, we obtain the expression for the effective beam size $W_{xz}(z)$ as follows

$$W_{xz}(z)=\sqrt{\frac{A_1}{A_2}}, \tag{10}$$

where

$$\begin{aligned}A_1=&\frac{k^2}{4\pi^2z^2}w_0^2\sqrt{\frac{\pi}{e_1}}(1-\frac{1}{e_1})^{m/2}\sum_{j_1=0}^{[\frac{m}{2}]}\frac{(-1)^{j_1}m!}{j_1!(m-2j_1)!}\sum_{l_1=0}^{[\frac{m}{2}]}\frac{(-1)^{l_1}m!}{l_1!(m-2l_1)!}\sum_{t_1=0}^{m-2l_1}\binom{m-2l_1}{t_1}\\&\times(2)^{2m-2j_1-2l_1}(g_1)^{t_1}(m+t_1-2j_1)!\sqrt{\frac{\pi}{e_2}}\sum_{q_1=0}^{[\frac{m+t_1-2j_1}{2}]}\frac{1}{q_1!(m+t_1-2j_1-2q_1)!}(\frac{1}{e_2})^{m+t_1-2j_1}\\&\times\left(\frac{e_2}{4}\right)^{q_1}(f_2)^{m+t_1-2j_1-2q_1}(-h_2)^{m-2l_1-t_1}\times\frac{\Gamma(\frac{2m-2j_1-2q_1-2l_1+3}{2})}{\sqrt{(\frac{k^2w_0^2}{4e_1z^2}-\frac{f_2^2}{e_2})^{2m-2j_1-2q_1-2l_1+3}}},\end{aligned} \tag{11}$$

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$$A_2 = \frac{k^2}{4\pi^2 z^2} w_0^2 \sqrt{\frac{\pi}{e_1}} (1-\frac{1}{e_1})^{m/2} \sum_{j_1=0}^{[\frac{m}{2}]} \frac{(-1)^{j_1} m!}{j_1!(m-2j_1)!} \sum_{l_1=0}^{[\frac{m}{2}]} \frac{(-1)^{l_1} m!}{l_1!(m-2l_1)!} \sum_{t_1=0}^{m-2l_1} \binom{m-2l_1}{t_1}$$
$$\times (2)^{2m-2j_1-2l_1} (g_1)^{t_1} (m+t_1-2j_1)! \sqrt{\frac{\pi}{e_2}} \sum_{q_1=0}^{[\frac{m+t_1-2j_1}{2}]} \frac{1}{q_1!(m+t_1-2j_1-2q_1)!} (\frac{1}{e_2})^{m+t_1-2j_1} \quad (12)$$
$$\times \left(\frac{e_2}{4}\right)^{q_1} (f_2)^{m+t_1-2j_1-2q_1} (-h_2)^{m-2l_1-t_1} \times \frac{\Gamma(\frac{2m-2j_1-2q_1-2l_1+1}{2})}{2\sqrt{(\frac{k^2 w_0^2}{4e_1 z^2} - \frac{f_2^2}{e_2})^{2m-2j_1-2q_1-2l_1+1}}},$$

with

$$f_2 = \frac{ikw_0}{2z} - \frac{w_0^3 ik}{2ze_1\rho_0^2}, h_2 = \frac{ikw_0}{2z}(\frac{1}{e_1^2 - e_1})^{1/2}. \quad (13)$$

Using Eq. (10), the spreading properties of an EHGB in turbulent atmosphere can be analysed. Equations (4) and (10) are the main analytical results of present paper.

For the convenience of comparison, we also derived the analytical propagation formula for the average intensity of a SHGB in turbulent atmosphere as follows

$$< I(p_x, p_y, z) >= \frac{k^2}{4\pi^2 z^2} \frac{w_0^2}{2} \sqrt{\frac{\pi}{e_{s1}}} (1-\frac{1}{e_{s1}})^{m/2} \exp(-\frac{k^2 w_0^2 p_x^2}{8e_{s1} z^2}) \sum_{j_1=0}^{[\frac{m}{2}]} \frac{(-1)^{j_1} m!}{j_1!(m-2j_1)!} \sum_{l_1=0}^{[\frac{m}{2}]} \frac{(-1)^{l_1} m!}{l_1!(m-2l_1)!} \sum_{t_1=0}^{m-2l_1} \binom{m-2l_1}{t_1}$$
$$\times (2)^{2m-2j_1-2l_1} (g_{s1})^{t_1} (-h_{s1x})^{m-2l_1-t_1} (m+t_1-2j_1)! \exp(\frac{f_{3x}^2}{e_{s2}}) (\frac{f_{3x}}{e_{s2}})^{m+t_1-2j_1}$$
$$\times \sqrt{\frac{\pi}{e_{s2}}} \sum_{q_1=0}^{[\frac{m+t_1-2j_1}{2}]} \frac{1}{q_1!(m+t_1-2j_1-2q1)!} \left(\frac{e_{s2}}{4f_{3x}^2}\right)^{q_1} \frac{w_0^2}{2} \sqrt{\frac{\pi}{e_{s1}}} (1-\frac{1}{e_{s1}})^{n/2}$$
$$\times \exp(-\frac{k^2 w_0^2 p_y^2}{8e_{s1} z^2}) \sum_{j'_1=0}^{[\frac{n}{2}]} \frac{(-1)^{j'_1} n!}{j'_1!(n-2j'_1)!} \sum_{l'_1=0}^{[\frac{n}{2}]} \frac{(-1)^{l'_1} n!}{l'_1!(n-2l'_1)!} \sum_{t'_1=0}^{n-2l'_1} \binom{n-2l'_1}{t'_1} (2)^{2n-2j'_1-2l'_1} (g_{s1})^{t'_1} \quad (14)$$
$$\times (-h_{s1y})^{n-2l'_1-t'_1} (n+t'_1-2j'_1)! \exp(\frac{f_{3y}^2}{e_{s2}}) (\frac{f_{3y}}{e_{s2}})^{n+t'_1-2j'_1} \sqrt{\frac{\pi}{e_{s2}}}$$
$$\times \sum_{q'_1=0}^{[\frac{n+t'_1-2j'_1}{2}]} \frac{1}{q'_1!(n+t'_1-2j'_1-2q'_1)!} \left(\frac{e_{s2}}{4f_{3y}^2}\right)^{q'_1}$$

where

$$e_{s1} = \frac{1}{2} - \frac{ikw_0^2}{4z} + \frac{w_0^2}{2\rho_0^2},\ g_{s1} = (\frac{1}{e_{s1}^2 - e_{s1}})^{1/2} \frac{w_0^2}{2\rho_0^2},\ h_{s1x} = \frac{ikw_0 p_x}{2\sqrt{2}z}(\frac{1}{e_{s1}^2 - e_{s1}})^{1/2},\ h_{s1y} = \frac{ikw_0 p_y}{2\sqrt{2}z}(\frac{1}{e_{s1}^2 - e_{s1}})^{1/2}$$
$$e_{s2} = \frac{1}{2} + \frac{ikw_0^2}{4z} + \frac{w_0^2}{2\rho_0^2} - \frac{w_0^4}{4e_{s1}\rho_0^4},\ f_{3x} = \frac{ikw_0 p_x}{2\sqrt{2}z} - \frac{w_0^3 ikp_x}{4\sqrt{2}ze_{s1}\rho_0^2},\ f_{3y} = \frac{ikw_0 p_y}{2\sqrt{2}z} - \frac{w_0^3 ikp_y}{4\sqrt{2}ze_{s1}\rho_0^2}. \quad (15)$$

In a similar way, we obtain the following expression for the effective beam size $W_{xz}(z)$ of a SHGB in turbulent atmosphere

$$W_{sxz}(z)=\sqrt{\frac{A_{s1}}{A_{s2}}}, \tag{16}$$

where

$$\begin{aligned}A_{s1}=&\frac{k^2}{4\pi^2 z^2}\frac{w_0^2}{2}\sqrt{\frac{\pi}{e_{s1}}}(1-\frac{1}{e_{s1}})^{m/2}\sum_{j_1=0}^{[\frac{m}{2}]}\frac{(-1)^{j_1}m!}{j_1!(m-2j_1)!}\sum_{l_1=0}^{[\frac{m}{2}]}\frac{(-1)^{l_1}m!}{l_1!(m-2l_1)!}\sum_{t_1=0}^{m-2l_1}\binom{m-2l_1}{t_1}\\&\times(2)^{2m-2j_1-2l_1}(g_{s1})^{t_1}(m+t_1-2j_1)!\sqrt{\frac{\pi}{e_{s2}}}\sum_{q_1=0}^{[\frac{m+t_1-2j_1}{2}]}\frac{1}{q_1!\left(m+t_1-2j_1-2q_1\right)!}\\&\times(\frac{1}{e_{s2}})^{m+t_1-2j_1}\left(\frac{e_{s2}}{4}\right)^{q_1}\left(f_4\right)^{m+t_1-2j_1-2q_1}(-h_{s2})^{m-2l_1-t_1}\times\frac{\Gamma(\frac{2m-2j_1-2q_1-2l_1+3}{2})}{\sqrt{(\frac{k^2w_0^2}{8e_{s1}z^2}-\frac{f_4^2}{e_{s2}})^{2m-2j_1-2q_1-2l_1+3}}},\end{aligned} \tag{17}$$

$$\begin{aligned}A_{s2}=&\frac{k^2}{4\pi^2 z^2}\frac{w_0^2}{2}\sqrt{\frac{\pi}{e_{s1}}}(1-\frac{1}{e_{s1}})^{m/2}\sum_{j_1=0}^{[\frac{m}{2}]}\frac{(-1)^{j_1}m!}{j_1!(m-2j_1)!}\sum_{l_1=0}^{[\frac{m}{2}]}\frac{(-1)^{l_1}m!}{l_1!(m-2l_1)!}\sum_{t_1=0}^{m-2l_1}\binom{m-2l_1}{t_1}\\&\times(2)^{2m-2j_1-2l_1}(g_{s1})^{t_1}(m+t_1-2j_1)!\sqrt{\frac{\pi}{e_{s2}}}\sum_{q_1=0}^{[\frac{m+t_1-2j_1}{2}]}\frac{1}{q_1!\left(m+t_1-2j_1-2q_1\right)!}\\&\times(\frac{1}{e_{s2}})^{m+t_1-2j_1}\left(\frac{e_{s2}}{4}\right)^{q_1}\left(f_4\right)^{m+t_1-2j_1-2q_1}(-h_{s2})^{m-2l_1-t_1}\times\frac{\Gamma(\frac{2m-2j_1-2q_1-2l_1+1}{2})}{2\sqrt{(\frac{k^2w_0^2}{8e_{s1}z^2}-\frac{f_4^2}{e_{s2}})^{2m-2j_1-2q_1-2l_1+1}}},\end{aligned} \tag{18}$$

with

$$f_4=\frac{ikw_0}{2\sqrt{2}z}-\frac{w_0^3ik}{4\sqrt{2}ze_{s1}\rho_0^2},h_{s2}=\frac{ikw_0}{2\sqrt{2}z}(\frac{1}{e_{s1}^2-e_{s1}})^{1/2}. \tag{19}$$

3. Numerical examples

Now we study the average intensity and spreading of EHGB in turbulent atmosphere numerically by use of the formulae derived in Section 2.

Figure 3 shows the normalized intensity distribution of an EHGB at several different propagation distances in free space with $m = n = 2$, $w_0 = 0.02m$, $\lambda = 632.8nm$ and $C_n^2 = 0$. One finds from Fig. 3 that the transverse pattern of the EHGB becomes a pattern with four-beamlets in the far field of free space. Our numerical results also show that any EHGB with m>1 and n>1 will converse into a laser beam with four-beamlets, and the distances between these beamlets increase by increasing the orders *m* and *n*. For a SHGB however, its beam profile remains invariant on propagation in free space, although its beam spot spreads on propagation. Our results agree well with those reported in [1,2,17,18].

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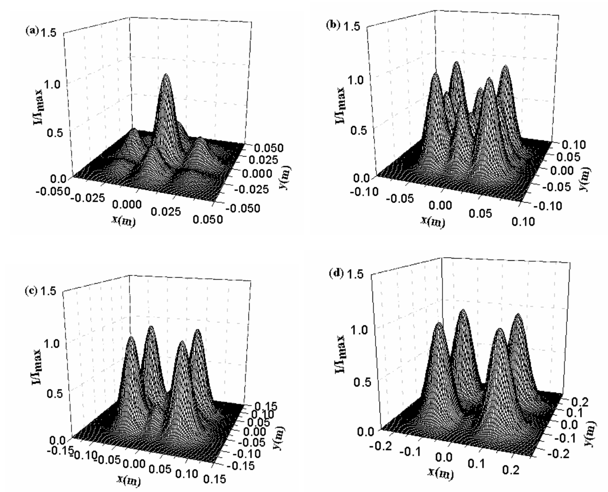

Fig. 3. Normalized intensity distribution of an EHGB with $m = n = 2$ at several different propagation distances in free space (a) $z = 1km$, (b) $z = 3km$, (c) $z = 5km$, (d) $z = 10km$.

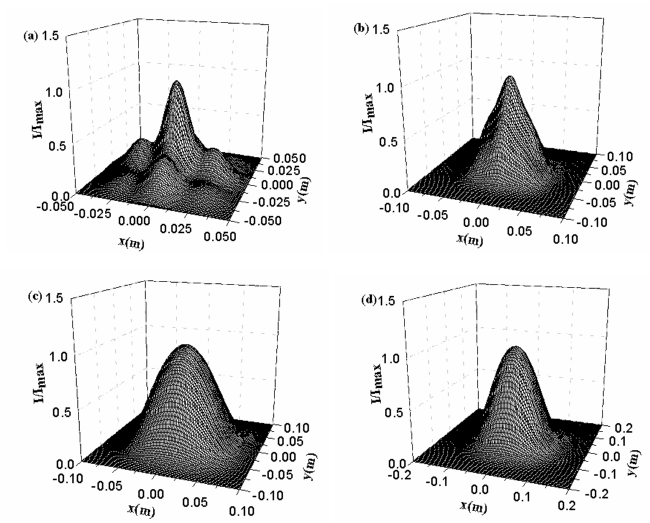

Fig. 4. Normalized intensity distribution of an EHGB with $m = n = 2$ at several different propagation distances in turbulent atmosphere with $C_n^2 = 10^{-14} m^{-2/3}$ (a) $z = 1km$, (b) $z = 1.5km$, (c) $z = 2km$, (d) $z = 3km$.

Figure 4 shows the normalized intensity distribution of an EHGB at several different propagation distances in turbulent atmosphere with $m = n = 2$, $w_0 = 0.02m$, $\lambda = 632.8nm$ and $C_n^2 = 10^{-14} m^{-2/3}$. Comparing Figs. 3 and 4, it is clear that the propagation properties of the EHGB in turbulent atmosphere are much different from its propagation properties in free space. In turbulent atmosphere, the EHGB finally becomes a circular Gaussian beam spot in the far field under the isotropic influence of the atmosphere turbulence. Figure 5 shows the normalized intensity distribution of an EHGB for different values of m with $m = n$ at $z = 3km$ in turbulent atmosphere. One finds from Fig. 5 that the intensity distribution of the EHGB with larger m and n in turbulent atmosphere is more similar to its far field intensity distribution in free space, which means the EHGB with larger m and n is less affected by the turbulence. Our numerical results (not shown here to save space) also show that the EHGB

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with larger values of w_0 and λ are less affected by turbulence. What's more, our numerical results (again not shown here to save space) also show that the SHGB also becomes a circular Gaussian beam in the far field of turbulent atmosphere, and the SHGB with larger values of m, n, w_0 and λ are less affected by the turbulence, which agree well with those reported in [20].

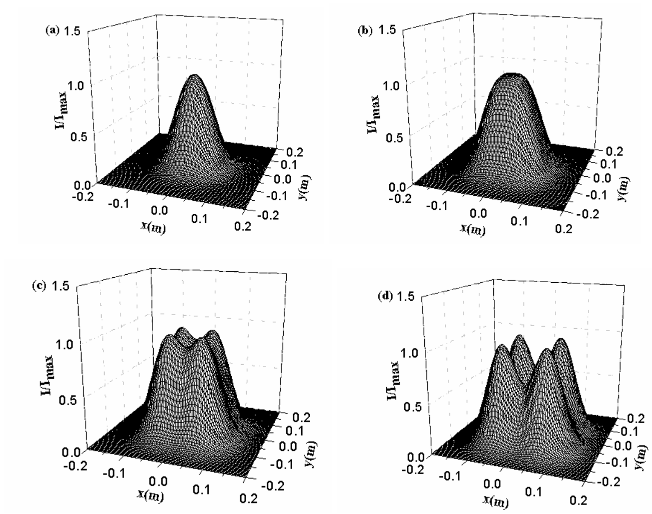

Fig. 5. Normalized intensity distribution of an EHGB for different values of m with $m = n$ at z = 3km in turbulent atmosphere with $C_n^2 = 10^{-14} m^{-2/3}$ (a) m = 1, (b) m = 3, (c) m = 5, (d) m = 8.

To learn about the spreading properties of the EHGB in turbulent atmosphere, we calculate in Figs. 6 (a)-6(c) the effective beam sizes of an EHGB (solid lines) and an SHGB (dotted line) with $m = n = 2$ and $\lambda = 632.8nm$ versus the propagation distance z in turbulent atmosphere for different values of the structure constant. The corresponding results in free space are also plotted in Fig. 6 (d). Note that in Fig. 6 we have set $w_0 = 0.02928m$ for the EHGB and $w_0 = 0.02m$ for the SHGB, so the EHGB and SHGB have the same effective beam size at the source plane (z = 0). One finds from Fig. 6 that the SHGB spreads more rapidly than the EHGB both in free space and in turbulent atmosphere. As the structure constant becomes larger (i.e., atmospheric turbulence becomes stronger), the advantage of the EHGB over the SHGB becomes smaller. So the EHGB has advantage over the SHGB for application in free-space optical communication in weakly turbulent atmosphere. What's more, our numerical results (not shown here to save space) also show that the EHGB spreads more rapidly in turbulent atmosphere for a larger structure constant, a shorter wavelength, larger beam orders, and a smaller waist size of the initial beam.

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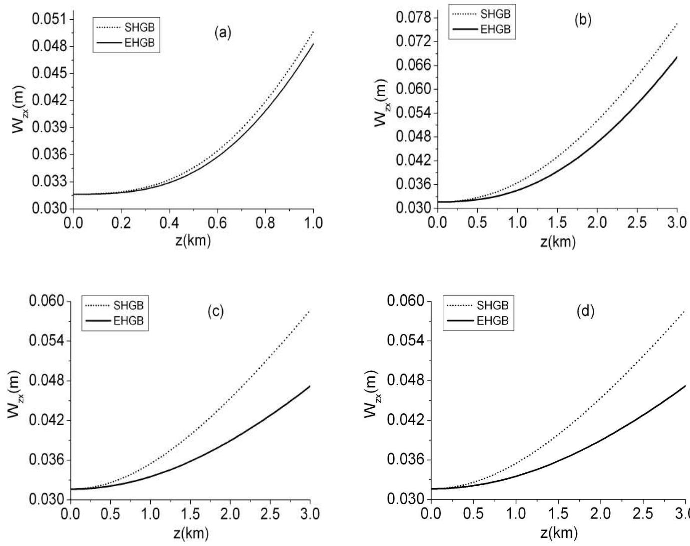

Fig. 6. Effective beam size of an EHGB (solid lines) and an SHGB (dotted line) with $m = n = 2$ and $\lambda = 632.8nm$ versus the propagation distance z in turbulent atmosphere for different values of the structure constant (a) $C_n^2 = 10^{-13} m^{-2/3}$ (b) $C_n^2 = 10^{-14} m^{-2/3}$ (c) $C_n^2 = 10^{-15} m^{-2/3}$ (d) $C_n^2 = 0$ (free space).

4. Conclusion

We have derived the analytical propagation formulae for the average intensity and effective beam size of an EHGB in turbulent atmosphere based on the extended Huygens-Fresnel integral. The propagation properties of the EHGB and SHGB have been studied numerically and comparatively. Our results show that the propagation properties of the EHGB and SHGB are much different from their properties in free space, and the EHGB and SHGB with higher orders are less affected by turbulence. What's more, the SHGB spreads more rapidly than the EHGB in turbulent atmosphere under the same conditions (i.e., the same initial effective beam size, orders and wavelength). The propagation properties of an EHGB are closely related to the beam parameters and the structure constant of the atmospheric turbulence. Our results will be useful in long-distance free-space optical communications.

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